



A Search for Evolutionary Changes in the Periods of Five Classical Cepheid Stars: R Cru, T Cru, XZ Car, X Lac and WZ Car

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ABSTRACT

In the present paper, we used the O-C diagrams for five classical Cepheid stars in our galactic plane to drive the rate of its period changes. The rate of period changes for individual Cepheid variables is being used to examine the parameters of stars as they crossing the instability strip (Effective temperature, Amplitude, Radius). So far, the studies have been very successful in mapping the pulsation amplitude of Cepheids across the instability strip. We found significant results and strong indications of the evolutionary period change in some Classical Cepheids understudy are not the results of the random cycle-to-cycle.

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1. Introduction

Cepheids are relatively yellow bright giants and super-giants stars with ages ranging from approximately 4×10^7 to 6×10^8 yr and have masses of roughly $3\text{--}9 M_{\odot}$. After leaving the main sequence, they evolve to the right in the colour-magnitude diagram (CMD), it is passing through the instability strip. As they evolve towards even cooler temperatures, they reach an effective temperature of 4000 K. Because such traits can be acquired by stars at different advanced stages of stellar evolution, massive stars become Cepheids several times during their late years, depending upon what fuel (and where) is being used to supply the radiation they emit. A simple diagnostic tool can be used to pinpoint the exact stage for individual Cepheids. All Cepheid variables undergo changes in the pulsation period as they evolve through the Cepheid instability strip, Turner et al. (2003).

Period changes are observed in several types of pulsating stars, and they are used, along with the theoretical model of the stars, to confirm (or not) our understanding of stellar evolution. Both observations and theoretical indicate whether the stars are expanding or contracting due to evolution, and at what rate, Neilson et al. (2016).

As evolved objects Cepheid populates the instability strip in the HR diagram according to the manner in which they generate energy, depend on strip crossing. The first dredge up phase may occur for Cepheid progenitors, but it is not at all clear that a second dredge up phase ever takes place. Cepheids represent a mixture of intermediate to high mass stars can become unstable as a result of the radial pulsation during shell hydrogen burning (first crossing), twice during core helium burning (second and third crossings), twice during shell helium burning (maybe fourth and fifth crossings?). In

all cases, the evolution of stars through the Cepheid instability strip should be associated with gradual changes in all dimensions, and hence periods of pulsation: increasing mean radius, and the period for evolution towards the cool edge of the instability strip, and decreasing mean radius and period for evolution towards the hot edge. Cepheids near the hot edge of the strip have larger masses and therefore evolved faster than Cepheids of identical period near the cool edge of the strip. The changing periods of pulsating variable stars can be used to detect and directly measure their evolution, Neilson et al. (2016).

Reinvestigation of Cepheid reddening, Turner et al. (2001a), has provided an empirical delineation of the Cepheid instability strip that offers important clues regarding the dependence of pulsation amplitude on location within the strip. As dictated from examination the amplitude map noticed that there is a correlation of pulsation amplitude with the rate of period change at a constant period that rebates directly to strip crossing mode, Berdnikov and Pastukhova (2019). In the present paper, we demonstrate the link in more detail. In section 2 we consider the data and technique of reduction. In section 3 we describe the results and discussions. Finally, section 4 contains the conclusions of this study.

2. Data and technique of reduction

Archival data are essential for such work since the rates of period change for most Cepheids are so small that they are difficult to establish from recent observations alone.

All observations for cepheids are from two primary data sets: (i) archival photographic data derived from

patrol plates in the Harvard College Observatory Photographic plate collections, and (ii) For a more detailed description of the plate series and some online search forms see the Harvard University Plates Stacks Digitisation Project “<http://tdc-www.harvard.edu/plates/>” and the Digital Access to a Sky Century and Harvard site “<http://dasch.rc.fas.harvard.edu/index.php>” & “<https://www.cfa.harvard.edu/hco/collect.html>”, and photoelectric data available from AAVSO website, “<https://www.aavso.org/category/tags/photoelectric-photometry>”. We used the method of Hertzsprung technique to study changes in the periods of Cepheids by high accuracy. This technique in current use has been described by Berdnikov (1992), Berdnikov and Turner (2001a), and Turner and Berdnikov (2003) Berdnikov and Pastukhova (2019). The phased data observed for a Cepheid over a restricted number of cycles are used to construct seasonal light curves that are matched to a standard light curve in order to detect phase shifts indication of period change. An alternative procedure used by Turner et al. (1999) relies on a set of high-quality observations as a standard curve, to which independent data sets are matched in magnitude and phase space by least-squares techniques. Both algorithms

match photometric data for the variable stars to the reference light curve in the same fashion and produce virtually identical results are given of the usefulness of the Harvard college observatory photographic plate Collection for the study of period change in Cepheids Variable. When we used the photographic observations, the resulting photographic light curve for the stars was matched with existing photoelectric observations in the B-band. Very precise any photometry of stars under study by Pel, J.W., was used as a standard curve for matching data from all epochs in phase and magnitude to the reference set, and the result was tested with all available photometry for the star to validity over all epochs of observations.

Table 2 shows the five Cepheids examined (R Cru, T Cru, XZ Car, X Lac, and WZ Car), O-C during more than 100 years and the source of their observations. I concentrated on these stars because I have new observations for these stars from Harvard patrol plates don't use.

As illustrated in Figure 1, photographic observations for R Cru, T Cru, XZ Car, X Lac, and WZ Car are extracted from Harvard patrol plates covering the period 1893 to 1951 for R Cru, 1889 to 1954 for T Cru, 1899 to 1953 for XZ Car, and 1899 to 1953 for WZ Car interval.

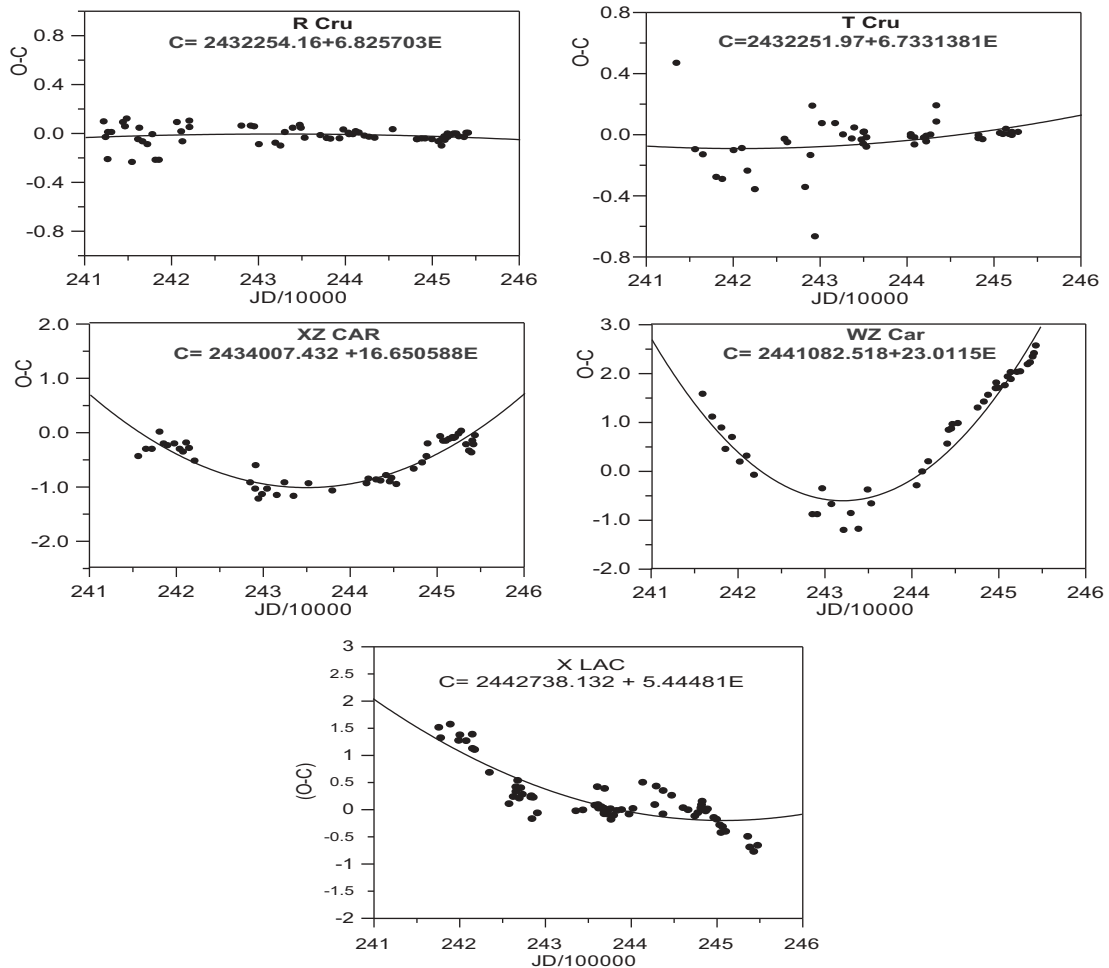


Figure 1. O-C data for four stars R Cru, T Cru, XZ Car, X Lac, and WZ Car plotted as a function of the observed Heliocentric Julian date of light maximum. “C” in all panels meaning the calculated time of max.

3. Results and discussion

The results are divided into three parts: the first part is devoted to the fitting of the O-C curves and determining the period change. The second part is concerned with the variation of light amplitude with position in the instability strip. In the third part, the method of Eddington and Plakidis (1929) is used to examine the residuals in O-C.

3.1. Period change

Cepheids at the same pulsation period having the fastest rates of period change, so they must be located on the hot edge of the instability strip (since they correspond to highest mass stars), and that having the slowest rates of period change must be located on the cool edge of the instability strip (since they correspond to the lower mass stars) (Turner et al. 2006). About the comparison between LMC and Galactic Cepheid for period change, Michail et al. (2018) try to make it and he concluded that LMC Cepheids may experience slower rates of period change, compared to Milky Way Cepheids with similar periods. As seen from O-C figure, some stars have large scatters data, these scatter in the O-C diagram could arise if the star is a binary system or as a result of simple observational scatters, or they might be evidence of random cycle-to-cycle fluctuations in the periodicity of the light variation. O-C data are closely approximated by a parabolic trend which reflecting a regular period decrease or period increase. New parabolic elements of these Cepheids are given in Table 1 according to the relation:

$$HJD_{\max} = M_0 + PE + QE^2, \quad (1)$$

where M_0 is a new epoch, P is the new period and Q is a term that can be used to measure the values of period change (P(dot) or dP/dt) in seconds per year, and given by,

$$\frac{dp}{dt} = \left(\frac{2Q}{P}\right) 365.25 \times 24 \times 60 \times 60,$$

Her we have studied the O-C diagrams for five Cepheid stars and the least squares fit to the data results in an improved ephemeris for these Cepheids as represented in Table 1. The least square fit of

O-C results for four stars R Cru, T Cru, XZ Car and WZ Car can be given by the following quadratic equations:

$$\begin{aligned} \text{R Cru, } HJD_{\max} &= 2432254.152 + 5.825710E \\ &\quad - 0.0152591 \times 10^{-8} E^2, \end{aligned}$$

$$\begin{aligned} \text{T Cru, } HJD_{\max} &= 2432251.936 + 6.733163E \\ &\quad + 0.2603437 \times 10^{-8} E^2, \end{aligned}$$

$$\begin{aligned} \text{XZ Car, } HJD_{\max} &= 2434006.452 + 16.65110E \\ &\quad + 88.58850 \times 10^{-8} E^2, \end{aligned}$$

$$\begin{aligned} \text{X Lac, } HJD_{\max} &= 2442738.132 + 5.444990E \\ &\quad + 0.02322293 \times 10^{-8} E^2, \end{aligned}$$

$$\begin{aligned} \text{WZ Car, } HJD_{\max} &= 2444143.148 + 23.03600E \\ &\quad + 368.1399 \times 10^{-8} E^2, \end{aligned}$$

The final results of the least square fit for parabolic O-C data and values of period changes, are presented in Table 1 which represent the stars in negative and positive period change.

Table 1 includes the following columns: (1) – the name of the object, (2) – new epoch (Mo), (3) – the logarithmic Period, (4) – the Logarithmic of period change \pm Errors, (5) – Amplitude of the star, (6) to (8) the mathematical least square fits to the $\langle u_r(x) \rangle^2$ data for these stars, which can be represented in equation 4 and, (9) expected Crossing number.

From the figure, we found that the rate of period change dp/dt depends upon where a Cepheid is located in the instability strip. The observed period changes are deviated in a small but important way from what is expected according to predictions based upon specific stellar evolutionary models, see Turner (2001b), and Turner et al. (2012). Such scatter in our result makes it difficult to use the rate of period change for individual Cepheids to identify their exact location within the instability strip.

3.1.1. The behaviour of O-C for WZ Car AND X Lac

As known, all cepheids undergo changes in the pulsation period as they evolve through the instability strip.

Table 1. A least square fit to the data results in an improved ephemeris for Cepheids understudy and the least squares fit to the random test ($\langle u_r(x) \rangle^2$) data for the same stars and expected crossing number.

Star	M_0 2,400,000+	Log P	LogP(dot) s/y	Amp.	A (Error)	B (Error)	Sigma	Expected No. of Crossing
R Cru	32,254.152	0.765	-2.78178 (0.021)	1.200	0.0057 (0.1102)	0.000 (0.0002)	0.0169	2 nd
T Cru	32,251.936	0.828	-1.61255 (0.059)	0.740	0.0096 (0.1008)	0.000 (0.0002)	0.0934	3 rd
WZ Car	44,143.148	1.362	1.004 (0.451)	1.840	0.1008 (0.0694)	0.000 (0.0001)	0.1693	3 rd
X Lac	42,738.132	0.736	-0.56996 (0.1082)	0.600	0.6143 (0.0428)	0.000 (0.0000)	2.9578	3 rd or 5 th
XZ Car	34,006.452	1.221	0.526 (0.166)	1.560	0.2664 (0.0678)	0.000 (0.0001)	0.6905	3 rd

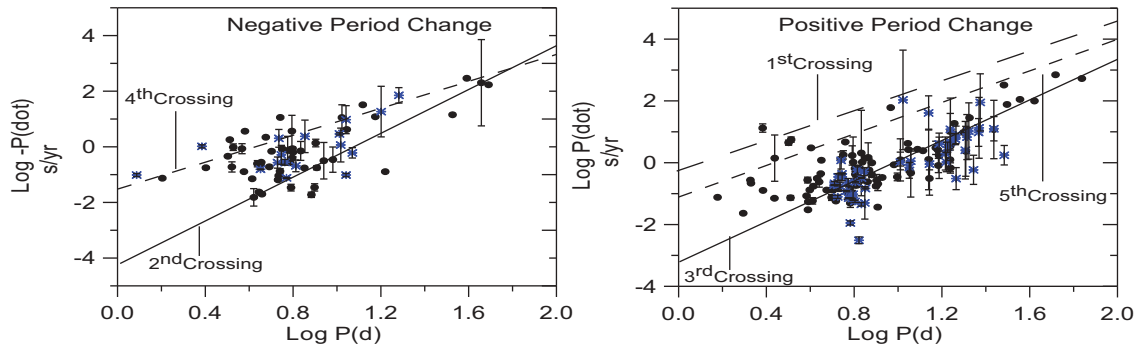


Figure 2. Rate of period change with uncertainties is compared with predictions from evolutionary models. From left to right the lines in the left plot (period decreasing) corresponding to fourth and second crossings, while the right plot (period increasing) corresponding to first, fifth, and third crossings. Cepheids investigated in the present work (asterisk) as well as Cepheids from Turner (points) (private communication).

Here we consider yet another interesting feature for Cepheids, WZ Car and X Lac, which have changed the direction of its evolution recently as seen from O-C light curve. Prior to about 1972 WZ Car behaved like a Cepheid undergoing a period increase at a rate of 15.3 ± 0.05 s/year. Since 1972 the trend has reversed, and WZ Car presently behaves like a Cepheid undergoing a period decrease at a very similar rate of -11.5 ± 1.4 s/year (reference here). So the rates of period change during intervals of increasing and decreasing period are nearly similar. The difference between the two rates is small than a change from red ward to blue ward evolution in the H-R diagram. The O-C data during the interval from 1920 to 1970 exhibit particularly large scatter that may be indicative of chaotic fluctuations in period for WZ Car star. Also from Figure 3, the behaviour of O-C data for X Lac prior to 1971 has a period increase at a rate of 1.881 ± 0.0233 s/year, since 1971 the trend of O-C is reversed decreasing by a similar rate of 1.534 ± 0.315 s/year (Figure 3). The recent photoelectric

data imply that the O-C diagram of X Lac can be properly interpreted in terms of phase jumps (Figure 3). The behaviour of period change from increasing to decreasing period depending on the helium burning. Iben (1967) suggested that the most massive cepheids should switch from redward to blueward evolution through the instability strip during the advanced stages of helium burning associated with third and fourth crossings. Also, Szabados (1991) mentioned, the phase jumps seen in the O-C diagram gives independent evidence for the duplicity of X Lac.

3.2. Pulsation amplitude

The investigation by Kraft (1963) was the first study of Cepheid's amplitude as a function of position in the strip, and he found that; small amplitude Cepheids were found only on the hot edge of the strip. Yakimova et al. (1975) have examined more extensive but less accurate photographic data and concluded that irrespective of period

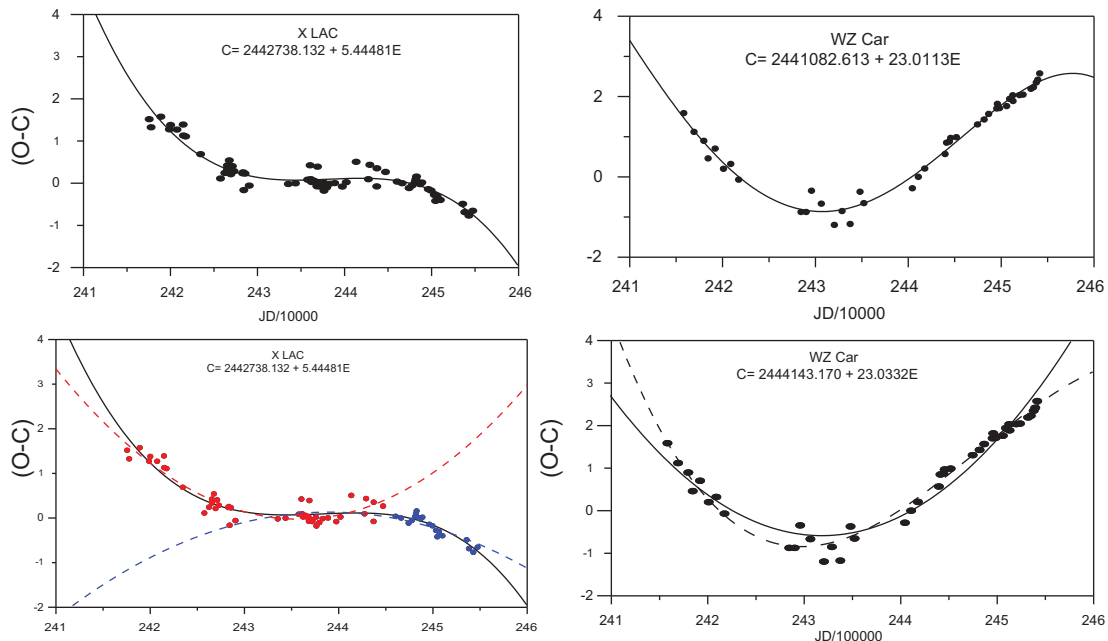


Figure 3. O-C data for the southern hemisphere Cepheid X Lac and WZ Carinae.

range, amplitudes are large on the red edge of the instability strip. On the other hand, Hofmeister (1967), Sandage and Tammann (1971), Pel and Lub (1978), Turner (2001b), and Sandage et al. (2004) found similar results, maximum amplitude on the hot edge of the strip and decline towards the cool edge. As an illustration of the usefulness of Cepheid period changes, Figure 4 represents the light curve amplitude ΔB (we applied the Fourier decomposition technique to the light curves in B-band). See Sabour et al. (2015) as a function of the rate of period change dp/dt for Cepheids under study. As indicated from the correlation of pulsation amplitude with the rate of period change that relates directly to strip crossing mode. The two stars (WZ Car, XZ Car) are far from our prediction of the relation between the rate of period change and pulsation amplitude, it may be attributed to its long period. The data for $p > 16^d$ Cepheids display a tendency for large-amplitude Cepheids which have a rate of period increase typical of stars lying near the centre of the instability strip, while Cepheids with smaller amplitude falling towards the hot and cool edges of the instability strip, Shashi et al. (2006).

The variation of light amplitude with a position in the instability strip, as established by the $P(\dot{P})$ parameter (reference here), is exactly what is found in the present analysis. We noticed that the large value of $P(\dot{P})$ should correspond to the hot side of the instability strip and small values to the cool side.

3.3. Residual of O-C

The final test of the pulsation mode of Cepheids is to consider the residuals of the O-C data from the best fit. The residual scatter arises primarily from observational uncertainties.

A statistical test was developed by Eddington and Plakidis (1929) to establish the regularity and irregularity of pulsation in pulsating stars. They divided the types of irregularity into five types and attributed the source of the irregularity to chaotic variations in the period of pulsation as well as to change arising

from errors in observations. The average value for all accumulated delays between light maxima separated by x -cycles or period, denoted by $\langle u_r(x) \rangle$, can be written as:

$$\langle u_r(x) \rangle = a_{r+x} - a_r, \quad (2)$$

where a_r is the average delay of the r^{th} maximum compared with the ephemeris, and the probable value of the sum of “ x ” fluctuations is timed the probable value of one fluctuation. It was correlated with **random fluctuations in period “e”** by;

$$\langle u_r(x) \rangle^2 = 2a^2 + xe^2, \quad (3)$$

where “ a ” represents the magnitude of random errors in the measured times of light maxima. The $\langle u_r(x) \rangle$ is drawn as a function of increasing cycle count difference “ x ” derived from the residuals in the O-C data from stars under study as shown in Figure 5. As comparable data for SV Vul in Figure 5, no evidence for chaotic behaviour in its pulsation and the latter is recognised for a small degree of random period changes, Turner and Berdnikov (2003). Chaotic fluctuations in period should be relatively uncommon in Cepheids, except possible for those of long period. The mathematical least square fits the $\langle u_r(x) \rangle^2$ data for these stars, can be represented as: $\langle u_r(x) \rangle^2 = A + Bx$, Table 1 presents negative and positive period change, the null value for the slope of the relation is consistent with no random cycle-to-cycle fluctuation in period for Cepheids.

As shown from Figure 6, the dependence of the parameter “ e ” as a function of the pulsation period for Cepheid is less straight forward than for Mira and RV Tauri stars. Our result is therefore in a good agreement with expectation based upon the quality of the observational sample to date. By contrast, the situation for the O-C data of the Cepheid SV Vul is clear evidence for random fluctuations in the period as demonstrated by Turner (1998). The available ($\langle u_r(x) \rangle^2$) data are illustrated in Figure 5. For SV Vul the slope of the ($\langle u_r(x) \rangle^2$) data for cycle

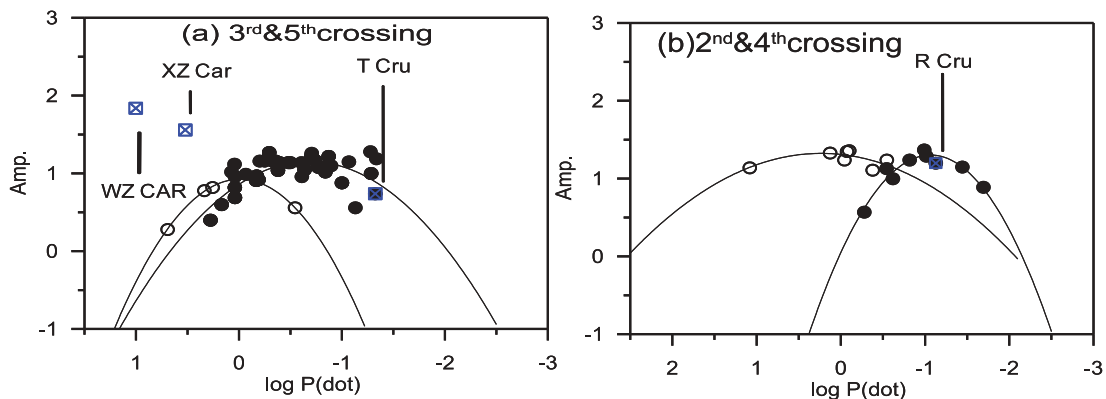


Figure 4. Light curve amplitude ΔB as a function of the rate of period change dp/dt for Cepheids with the number of Crossing mode, in addition between the four stars (XZ Car, WZ Car, T Cru, and R Cru).

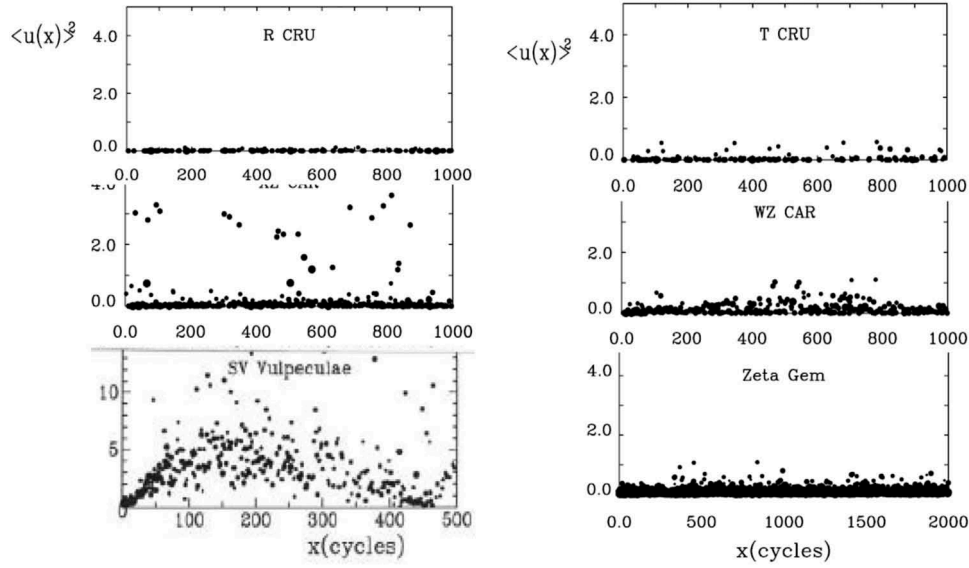


Figure 5. Randomness diagrams (plots of $\langle u_r(x) \rangle^2$ versus cycle count difference x) derived from residuals in O-C data for R Cru, T Cru, XZ Car, WZ Car, SV Vul, and Zeta Gem.

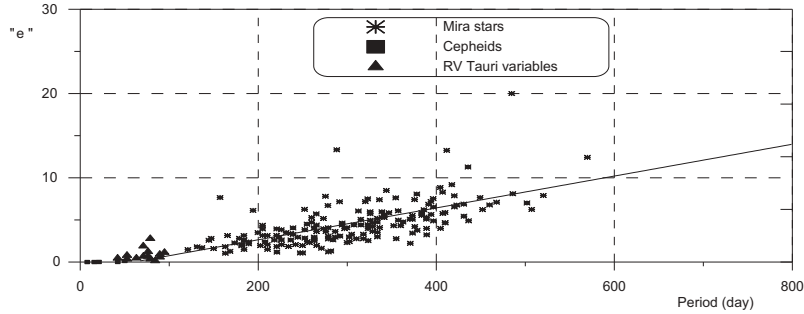


Figure 6. The dependence of the parameter “e” as a function of the pulsation period for Cepheids (square), RV Tauri Variable (triangle), and Mira (asterisk).

differences of 100 or less gives clear evidence for random cycle-to-cycle fluctuations in the star’s period of pulsation.

4. Conclusion

Our intent here is to demonstrate that the rate of period change and amplitudes for Cepheids are useful parameters that permits one to characterise the variable in terms of specific evolutionary state. About the information on $P(\dot{P})$ for our samples, in conjunction with its known period and light amplitudes, can be used to identify the strip crossing mode for the object as well as its location within the strip.

- Specific stellar evolutionary models presented in Figure 2 predict a smaller variation in $\log P(\dot{P})$ than what is observed. In that regard, observed rates of period change in Cepheids can play an important role as a check on how closely stellar evolutionary models match real stars.

Period changes can be compared with predictions from evolutionary models. In general, the agreement is good, but there are many cases of disagreement. These disagreement can potentially be used to identify important physical processes, such as rotation, magnetic fields, or mass loss, which need to be incorporated into the evolutionary models.

- We can interpret from the light curve of Figure 4 as a generic indicator of how pulsation amplitude varies during a cross the instability strip.

Also as shown in Figure 4, the stars XZ Car and WZ Car are far away from most stars because all stars period in the range <10 days but theses two stars the periods are greater than 16 days. Also, as we know increasing and decreasing pulsation periods as a function of normalised rate of period change.

The number of Cepheids inferred to be in the first, second and third crossing of the instability strip on basis of derived rates of period change is not fully consistent with time scales based upon stellar evolution cross the inferred width of the strip. There are

fewer Cepheids in fifth crossing than predicted on that basis and fewer at a long period may be because not all massive stars cross the instability strip during the period of shell helium burning. Recall that large value of $P(\dot{P})$ should correspond to the hot side of the instability strip, and small values to the cool side. Cepheids which have longer periods should have longer amplitudes and period changes also.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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