NRIAG JOURNAL OF ASTRONOMY AND GEOPHYSICS
2020, VOL. 9, NO. 1, 238–244
https://doi.org/10.1080/20909977.2020.1738106

OPEN ACCESS Check for updates

Predicting the accurate period of true dawn using a third-degree polynomial model

Tono Saksono[a] and Mohamad Ali Fulazzaky[b,c]

[a]Islamic Science Research Network, University of Muhammadiyah Prof Dr Hamka, Jakarta, Indonesia; [b]Department for Management of Science and Technology Development, Ton Duc Thang University, Ho Chi Minh City, Vietnam; [c]Faculty of Environment and Labour Safety, Ton Duc Thang University, Ho Chi Minh City, Vietnam

ABSTRACT

Sky brightness data have been widely used to document the light pollution and to localise the direction of the sun during both dawn and dusk twilight. The use of the most common data analysis techniques of graphical interpretation can give less accurate final results. This study used a Sky Quality Metre to record the brightness of night sky in mag arcsec^{-2} at Depok city of Indonesia during 26 days for the months of June and July 2015. Analytical technique of third-degree polynomial was established to predict dawn astronomical twilight and provides a good fit to the experimental data. The modelling results have been experimentally verified that astronomical twilight in the morning at Depok city with its environment begins when the sun is at the position of −14° ± 0.6 below the eastern horizon and ends at sunrise. In conclusion, the Muslim communities of Indonesia begin their dawn prayer 6° too early and have been found to calculate the true dawn incorrectly when the centre of the sun is at a depression angle of −20° below the horizon.

ARTICLE HISTORY
Received 31 July 2019
Revised 19 February 2020
Accepted 26 February 2020

KEYWORDS
Dawn prayer; depression angle; morning twilight; sky brightness data; third degree polynomial model

1. Introduction

The presence of twilight in a region has always been of paramount importance for every Muslim community as it astronomically marks the beginning of the right time for the morning (fajr) prayer or evening (isha) prayer designed primarily for two among five obligatory (fard) daily prayers (Miftahi 2005; Bahammam 2011; Niri et al. 2012; Rennie 2014; Suzuki et al. 2016). Three daily obligatory prayers of early afternoon (zuhr), late afternoon (asr) and sunset (maghrib) can be determined from the observer's location and the time of day based on the position of the sun in the sky and are dependent on the latitude and longitude coordinates of the locality. In the meantime, two daily prayers of fajr and isha are determined based on daily presence of sunlight below the eastern horizon and daily disappearance of sunlight below the western horizon, respectively. The fact that the determination of the prayer times has been organised into many different schools of thought and must be linked with the analysis of twilight observations (Cook et al. 2013), with differing views of description of Earth's rotation on how the occurrence of astronomical dawn and dusk, especially in their latitudinal and local time dependencies (Kaye 1989; Brumberg 1995; Li et al. 2011; Sunda and Vyas 2013; Jin et al. 2017). Daily prayer times should be determined which precisely pinpoint the beginning of each prayer time usually starting with the call to prayer (adhan) from the minaret of the mosque (Aldrin 2010; Arab 2015), and it will be interesting to see if the traditional exemptions from daily prayer times as such from Ramadan fasting have also a modern scientific basis (Leiper et al. 2003). As an example, the accuracy of Muslim prayed facing towards the Kaaba can be examined in a scientific way by using three approaches of modelling the Earth (Saksono et al. 2018).

Even though the digital sky brightness data have been widely used by the environmentalists to determine the levels of light pollution (Pun and So 2012; Kyba et al. 2015; Falchi et al. 2016; Spitschan et al. 2016; de Miguel et al. 2017; Power et al. 2017) and by the astronomers to verify the presence of twilight during both dawn and dusk (Hunten 1967; Zhang et al. 2013; Pun 2014; Puschnig 2014a, 2014b), most astronomers analyse a vast number of sky brightness observations from graphical plots of the reliable data (Nor and Zainuddin 2012; Shariff et al. 2012; Herdiwijaya 2016) but still less accurate in predicting the presence of twilight. Different schools of thought of the Muslim communities of Indonesia may use a different approach for determining and measuring the time elapsed to indicate the occurrence of twilight at a region (Mateshvili et al. 2005). In this work, "dep" is defined as the sun vertical depression and can be measured from the local horizon to a line of sight from an observer to celestial object (Beutler et al. 2005; Lušić 2013; Boddice et al. 2017). The

CONTACT Mohamad Ali Fulazzaky mohamad.ali.fulazzaky@tdtu.edu.vn Ton Duc Thang University, Ho Chi Minh City, Vietnam

use of "dep" of the horizon has been found to be different in different schools of thought (Nor and Zainuddin 2012), which have not yet been confirmed scientifically through the valid data, and commonly ranged from −20° to −14°; however, when the "dep" of −20° is an indicator used to determine the beginning of false dawn.

The visualisation literacy of sky brightness data recorded at Teluk Kemang of Negeri Sembilan, Pantai Cahaya Bulan of Kelantan and Kuala Terengganu of Terengganu in Malaysia starting from half an hour before the beginning of dawn prayer until the sunrise to extract information from the data visualisation graphs has been concluded that the beginning of true dawn is at a depression angle of −20° below the east horizon (Nor and Zainuddin 2012). Even though a comprehensive approach to the numerical modelling of verifying the presence of dawn twilight has been proposed using the data of the naked-eye observations (Garstang 2000; Hassan et al. 2014; Hassan and Abdel-Hadi 2015), the use of digital data collected from monitoring the night sky brightness to determine the beginning of dawn twilight using third-degree polynomial model is still not fully understood. The aim of this study was to make a measurable improvement to be used in the verification of the presence of twilight zone in a region for specifically determining the exact "dep" position of beginning the obligatory dawn prayer.

2. Materials and methods

2.1. Equipment and location

This study used the Sky Quality Metre (SQM) of SQM-LU-DL-Unihedron for measuring the brightness of night sky in magnitudes per square arcsecond (mag arcsec^{-2}). The SQM was installed on the roof of a building at Sawangan of Depok city with latitude of 6° 26′ 53.73" S and longitude of 106° 48′ 8.01" E, the southern suburb of approximately 25 km from Jakarta, Indonesia. The measurements of the brightness of night sky were performed recorded using the SQM directed towards the zenith at such measurement point, where there was no light trespass from the neighbouring light sources and where part of the sky was not obscured by any objects of such as crowns of trees, walls, buildings, etc. Even though the SQM has the ability of reading sky brightness data of lower than −26 mag arcsec^{-2} for the brightness of a perfect sun spectrum at the full sunshine to greater than 23 mag arcsec^{-2} for a very limited visible light (total dark) in real-time to provide the digital data archiving, this study only recorded the sky brightness data that range from 6 to 19 mag arcsec^{-2}.

2.2. Data collection

The aim of the measurements of night sky brightness using the SQM that having its time resolution of 3 s was to observe the flow of sky brightness data at Depok city at least from midnight to sunrise during 26 days, i.e. 24 days of June and 2 days of July 2015. This study used the SQM as a pocket-size photometer designed to provide the measurements of night sky brightness in the astronomical unit of mag arcsec^{-2} (Zamorano et al. 2015), with a digital display for continuous measurement using the USB connection. In this work, we established the mathematical models of third-degree polynomial in accordance with principles of mathematical analysis for every daily data collection and then performed a numerical simulation of the night sky brightness data implemented with its proper model then to add a polynomial trendline to an Excel chart.

2.3. Numerical simulation

From direct observations, estimating human affective states through computational models have been proposed using an algorithm for the inverse of multivariate homogeneous polynomial of degree n (Grafarend et al. 1996) and a high-order multivariate polynomial regression for predicting affective valence and arousal (Wei et al. 2016). In this work, we used a third-degree polynomial with three roots with a positive leading coefficient to model the twilight period of dawn before sunrise during which the sky is partially lit by atmospheric scattering of sunlight. A general form of the polynomial of third-degree is given in the following equation:

$$y_i = p_0 + p_1 d_i + p_2 d_i^2 + p_3 d_i^3 \qquad (1)$$

where y_i is the calculated level of night sky brightness at "dep" angle d_i (in mag arcsec^{-2}), p_o, p_1, p_2, p_3 are the polynomial parameters (in mag arcsec^{-2}), and d_i is the "dep" angle recorded by the SQM as the astronomical data input (in °).

We can arrange Equation (1) to form a third-degree polynomial function of presenting the empirical data using a limited "dep" angle window of 5 degrees from −18° to −13° by plotting y_i versus d_i to yielding the coefficients p_0, p_1, p_2 and p_3. The selected "dep" angle interval could be appropriate for the detection of human perceptible twilight due to the ritualised behaviour of dawn prayer must be performed at that "dep" angle elapsed by the Muslim communities. In this work, we used the root-mean-square error (RMSE) in the unit of mag arcsec^{-2} as standard statistical metric to measure the difference between the values predicted by the third-degree polynomial model and the values actually observed during the twilight period (Ojo and Kangkolo 1997; Patat et al. 2006; Perez 2017). The RMSE value can be calculated by the following equation (Patat et al. 2006; Busch et al. 2014; Chai and Draxler 2014):

$$RMSE = \left(\sum_{i=1}^{n} \frac{(y_i - \bar{y}_i)^2}{n} \right)^{0.5} \tag{2}$$

where $\bar{y}_i$ is the modelled level of night sky brightness at "dep" angle d_i (in mag arcsec^{-2}) and n is the number of data recorded in magnitude per square arcsecond (dimensionless).

A tangent line occurs at the extreme points however when $\tan(\alpha)$ equals zero where α is the slope of the line, and this algebraically occurs when $\frac{\partial y}{\partial t} = 0$. Therefore, practice finding the first derivative of the third polynomial expression can be written as:

$$\frac{\partial y}{\partial t} = 3p_3 d_i^2 + 2p_2 d_i + p_1 = 0 \tag{3}$$

Using Equation (3) permits us to calculate d_i to verifying the accurate "dep" angle of true dawn begins because the level of night sky brightness continuously decreases after passing "dep" angle d_i at the horizontal line with its $\tan(\alpha)$ equals zero hence the astronomical twilight begins at the "dep" angle after which the sky is no longer completely dark after astronomical dawn. We noted that the highest point occurs on the graph of third-degree polynomial when $\frac{\partial^2 y}{\partial t^2} < 0$.

3. Results and discussion

3.1. Results

3.1.1. Recording of midnight to morning sky brightness evolution

A plot of the night sky brightness versus either "dep" angle observed from the midnight to the sun starting to rise (see Figure 1(a)) or timeline observed from 00:00:00 am to 06:21:32 am (see Figure 1(b)) for the night sky brightness data recorded every 3 s, then we can visually define a peak of curve occurred when the morning astronomical twilight begins but it is still less accurate than a calculation to measure the forecast accuracy. Therefore, we proposed the use of third-degree polynomial model to analyse the night sky brightness data that cover only a limited "dep" angle window of around the beginning of the dawn twilight to find out the most accurate "dep" angle of true dawn such as for the city of Depok in Indonesia.

3.1.2. Third-degree polynomial regression analysis

One of the most important applications of the polynomial regression analysis is fitting the night sky brightness data to the third-degree polynomial models. A plot of the night sky brightness versus either "dep" angle observed at least 5 degrees from −18° to −13° (see Figure 2(a)) or timeline observed for the period of time from 04:58:54 am as starting point to 05:15:58 am as endpoint of the curve (see Figure 2(b)) within which the beginning of the morning twilight occurs for a limited window of "dep" angle elapsed recorded every 3 s, thus we can determine the horizontal line with its $\tan(\alpha)$ equals zero occurred at a peak of curve that indicates the beginning of true dawn.

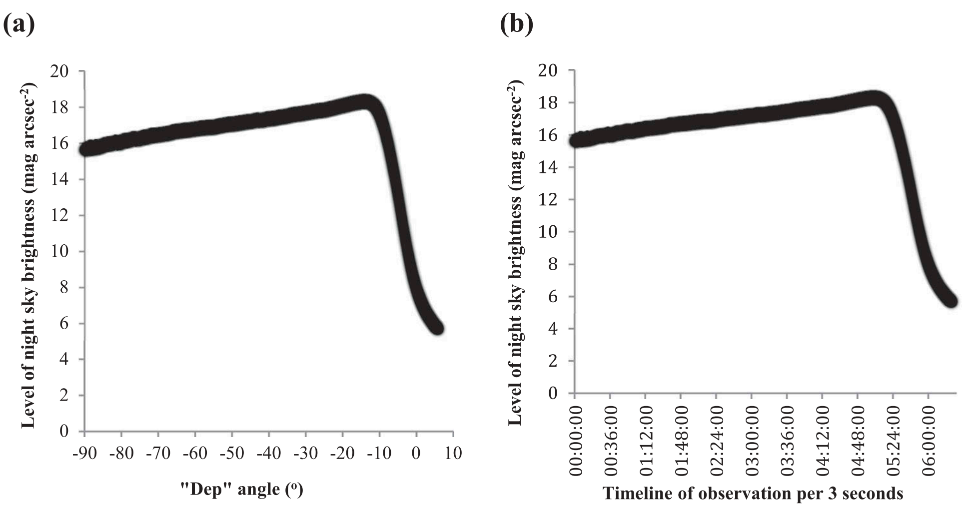

Figure 1. Level of the night sky brightness observed from midnight to sunrise on 1 July 2015; with (a) curve of plotting the level of night sky brightness versus "dep" angle of the horizon and (b) curve of plotting the level of night sky brightness versus timeline of observation per 3 s.

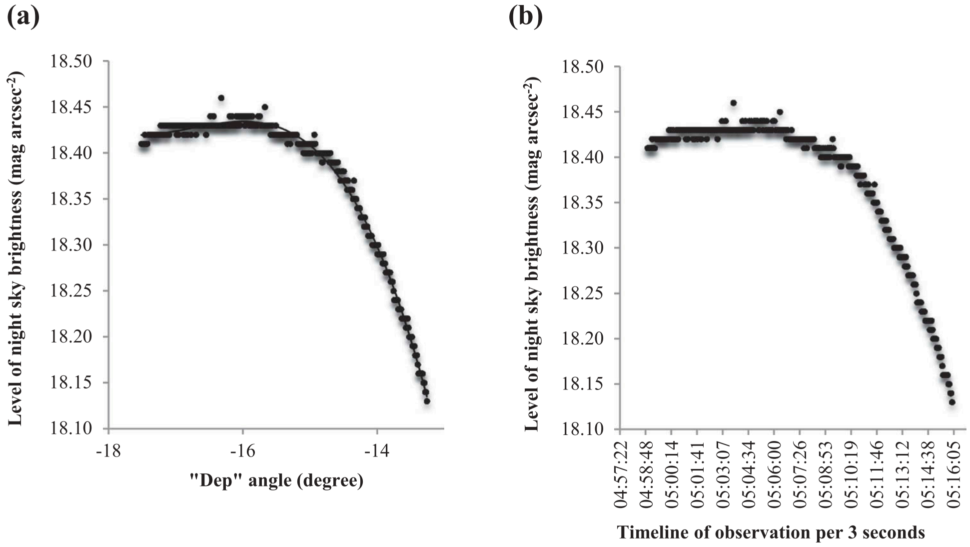

Figure 2. Model of third-degree polynomial of 5-degree "dep" angle window of the night sky brightness data observed on 29 June 2015; with (a) curve of plotting the level of night sky brightness versus "dep" angle of the horizon and (b) curve of plotting the level of night sky brightness versus timeline of observation per 3 s.

Table 1. Results of "dep" of the horizon and RMSE for noise-free data set of 18 days.

	The sun's position to the horizon in "dep" and RMSE value	
Date	"dep" angle of the horizon (°)	RMSE (mag arcsec^{-2})
12 June 2015	−13.96	0.12
13 June 2015	−14.04	0.01
14 June 2015	−13.42	0.18
15 June 2015	−14.23	0.13
16 June 2015	−14.50	0.10
17 June 2015	−13.38	0.03
18 June 2015	−14.78	0.22
19 June 2015	−14.98	0.26
21 June 2015	−13.14	0.15
22 June 2015	−13.98	0.03
23 June 2015	−13.43	0.09
25 June 2015	−13.75	0.15
26 June 2015	−13.29	0.21
27 June 2015	−13.38	0.19
28 June 2015	−14.36	0.03
29 June 2015	−14.52	0.07
1 July 2015	−14.29	0.01
7 July 2015	−14.44	0.02
Average	−14.00	0.11
Standard deviation	± 0.60	

Table 2. Values of the polynomial parameters p_0, p_1, p_2 and p_3 in the equations.

	Polynomial parameters (mag arcsec^{-2})				
Date	p_0	p_1	p_2	p_3	R^2
12 June 2015	5.922	−3.306	−0.288	−0.0083	0.99921
13 June 2015	7.222	−2.834	−0.232	−0.0063	0.99662
14 June 2015	12.652	−1.306	−0.090	−0.0020	0.97273
15 June 2015	15.646	−0.590	−0.038	−0.0008	0.94976
16 June 2015	7.052	−2.883	−0.235	−0.0063	0.99690
17 June 2015	11.390	−1.772	−0.138	−0.0036	0.99656
18 June 2015	−0.457	−4.344	−0.323	−0.0080	0.99847
19 June 2015	4.959	−3.470	−0.284	−0.0077	0.99894
21 June 2015	−2.655	−4.676	−0.362	−0.0094	0.99703
22 June 2015	9.220	−2.266	−0.183	−0.0049	0.99017
23 June 2015	9.454	−2.072	−0.155	−0.0038	0.98857
25 June 2015	8.586	−2.406	−0.185	−0.0047	0.99460
26 June 2015	5.848	−3.111	−0.245	−0.0064	0.99721
27 June 2015	9.439	−2.144	−0.158	−0.0038	0.99308
28 June 2015	8.565	−2.425	−0.198	−0.0054	0.99287
29 June 2015	−18.695	−6.670	−0.399	−0.0079	0.99436
1 July 2015	4.562	−3.497	−0.291	−0.0080	0.99804
7 July 2015	11.177	−1.711	−0.130	−0.0032	0.98429

3.1.3. Depression angle for determination of the morning twilight

Even though the curves in Figures 1 and 2 may present the examples of curves that address the goal of regression analysis to fit the experimental data to a third-degree polynomial equation, we analysed the night sky brightness data monitored for 26 days and then only keeping 18 days of the reliable data to determine the sun's position to the horizon. The results (Table 1) show that the average "dep" angle of the horizon equals −14.0° with a standard deviation of ± 0.6°. All values of the "dep" angle of horizon interpolated from a third-degree polynomial have a very good fit to those recorded using the SQM, which has already known from the RMSE value of 0.11 mag arcsec^{-2}. Meaning that the use of such as the equation: $y_i = -18.695 - 6.6670\, d_i - 0.399\, d_i^2 - 0.0079\, d_i^3$ (see Figure 2(a)) as one form of graphing Equation (1) could be useful to determine the beginning of morning twilight and to verify the accurate "dep" angle for dawn prayer begins at Depok city of Indonesia. All values of the polynomial parameters p_0, p_1, p_2 and p_3 completed with the R^2 value for the reliable data of keeping 18-day observations can be seen in Table 2. It is interesting because of the naked eye observations for the morning twilight have been reported to have an average value of −14.7° for the "dep" angle of the horizon at different sites in Egypt (Hassan et al. 2014) and that of −14.7° for that at Tubruq of Libya through 4 years observations (Hassan and Abdel-Hadi 2015).

3.2. Discussion

It is well known that the "dep" angle for dawn prayer lasts from the onset of true dawn determined by the presence of true morning twilight until the sun starts to rise (Kenworthy and Hinz 2003; Bahammam et al. 2012). The beginning of true dawn varies due to the variation of the angular velocity of Earth's rotation occurs over a period of 1 year in responding to astronomical forcing and terrestrial dynamics (Morrison and Stephenson 2001; Scottili et al. 2015; Levin et al. 2017). As can be seen from the graphs of Figure 1 that the level of night sky brightness observed from midnight to sunrise (see Figure 1(a)) and the presence of true dawn (see Figure 1(b)) at around 5 am (local time) with an abrupt change of slope is likely to occur. This study suggests that the determination of twilight angle can be used to calculate the exact "dep" angle of true dawn for Depok city of Indonesia and needs to go through the verification process of using a tangent line that touches a function at only one point. In any case, the slope of the tangent line to the level of night sky brightness versus "dep" angle curve beyond a limited "dep" angle window of performing the third-degree polynomial curve seems to continuously decrease towards the negative values. In spite of the visual observation of Figure 1(b) showed that an abrupt change of the slope seems to be at around 5 am, the relative effects of precision and accuracy on the exact "dep" angle of true dawn depend on the proper mathematical model to describe the behaviour of slope change to determine the "dep" angle of the beginning of astronomical twilight in the morning.

The darkness (or brightness) of the sky varies with "dep" angle after astronomical twilight (Benn and Ellison 1998) and is a topic of interest to the astronomers, environmentalists, and astrophotographers. This is mainly because of the natural and anthropogenic disruptions of the night sky darkness in urban and rural areas (Zamorano et al. 2015; Bará 2016; Falchi et al. 2016; Jechow et al. 2017) due to the presences of meteor, comet, lightning, thunderstorm, and other light sources (Jenniskens et al. 2011) in the darkness of night can yield a tropospheric turbulence that affects the photons received by the SQM. But the most important source of disturbances is due to the presence of literal moonlight is the major cause of light pollution in the observation of night sky brightness (Davies et al. 2013; Ugolnikov et al. 2016). Disturbances are not problems with the prayer itself but abnormal conditions that occur during both dawn and dusk can interfere the determination of prayer time. Like many other forms of light pollution, noise appears to disproportionately affect the twilight observations and disadvantaged determination of the true dawn and is also an environmental issue impacted religious beliefs (Hope and Jones 2014). If there was no major noise disturbance observed through a limited "dep" angle window of the polynomial curve around the beginning of morning twilight, we can then propose a polynomial regression to model the abrupt change of slope for fixing the recorded sky brightness data with a third-degree polynomial equation. For example, the values of p_0, p_1, p_2 and p_3 obtained from a third-degree polynomial regression are −18.695, −6.670, −0.399, and −0.0079 mag arcsec^{-2}, respectively, with R^2 = 0.99436 (see Figure 2(a); see also Table 2). The figure shows a near-perfect matching for all data of recorded using the SQM (see Figure 2(a)) and verifies the beginning of dawn twilight at around 05:01 am local time (see Figure 2(b)).

In this work, it has only been feasible to cover a noise-free data set of 18 days from the observation of 26 days in relation to a third-degree polynomial regression analysis (Bergé et al. 2012) because of the data recorded for another 8 days of night sky brightness observations cannot be used due to the light pollution caused by a light shining up and not down, and emphasis has been upon the use of 18 noise-free data to illustrate some of the relevant "dep" angle determinations rather than a naked eye determination of dawn such as from the graph of Figure 1. In spite of the third-degree polynomial model only covers a limited part of night sky brightness data of around 17 min from 04:59 am when "dep" angle is at its zero point to 05:16 am when "dep" angle is defined as its endpoint (see Figure 2(a,b)) to determine the time period of true dawn, the residual data recorded from 00:00 to 04:59 am (see Figure 1(a,b)) can be used to verify the presence of light pollution which probably causes dome of sky glow but represent valid data that would otherwise distort the modelling process. Table 1 shows that the average "dep" angle of the horizon could be as high as −14.0° and is valid because the third-degree polynomial model fitted usable data from monitoring the night sky brightness of 18 days. Assuming that if a "dep" angle of −20° is one of the oldest and widely used indicators to determine the beginning of true dawn that has been settled some decades in many parts of Indonesia, it is much too early for the Muslim communities to begin performing dawn prayer and has been accidentally praying fajr too early thinking it was the right time. Thus, a difference in the "dep" value of 6° (see (−14°)–(−20°)) refers to the false dawn. In addition, it will be interesting to perform more measurements of the brightness of night sky at different places mainly in five largest islands of Indonesia and at different times when sunlight shines on the northern and southern hemispheres, and further studies will help to make more comprehensive conclusions.

4. Conclusions

This study used the SQM towards the zenith as an astronomical instrument to measure the night sky brightness at Depok city for 26 days. We developed the third-degree polynomial equation and then validated it to having a very good fit to the "dep" data recorded using the SQM, and this becomes a powerful tool to predict the accurate "dep" degree of true dawn. The results indicated that using the third-degree polynomial model permits us to accurately estimate the start of true dawn for Indonesia when the sun is at the position of −14° ± 0.6 below the eastern horizon; this means that the dawn degree now applied in Indonesia is about 5.5° before the true dawn.

Acknowledgements

The authors gratefully acknowledge financial supports from the University of Muhammadiyah Prof Dr Hamka, Indonesia, for Research Grant No. 230/F.03.07/2017 and from Ton Duc Thang University, Vietnam, for Research Funding Contract No. 551/2019/TĐT-HĐLV-NCV.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

The authors gratefully acknowledge financial supports from the University of Muhammadiyah Prof Dr Hamka, Indonesia, for Research Grant No. 230/F.03.07/2017 and from Ton Duc Thang University, Vietnam, for Research Funding Contract No. 551/2019/TĐT-HĐLV-NCV.

ORCID

Mohamad Ali Fulazzaky http://orcid.org/0000-0002-4600-5742

References

Aldrin V. 2010. Prayer in peasant communities [Doctor of Theology Dissertation]. Sweden: University of Gothenburg.

Arab PT. 2015. "A minaret of light": transducing the islamic call to prayer? Mater Relig. 11:136–163. doi:10.1080/17432200.2015.1059126.

Bahammam AS. 2011. Sleep from an islamic perspective. Ann Thorac Med. 6:187–192. doi:10.4103/1817-1737.84771.

Bahammam AS, Sharif MM, Spence DW, Pandi-Perumal SR. 2012. Sleep architecture of consolidated and split sleep due to the dawn (Fajr) prayer among Muslims and its impact on daytime sleepiness. Ann Thorac Med. 7:36–41. doi:10.4103/1817-1737.91560.

Bará S. 2016. Anthropogenic disruption of the night sky darkness in urban and rural areas. R Soc Open Sci. 3:160541. doi:10.1098/rsos.160541.

Benn CR, Ellison SL. 1998. Brightness of the night sky over La Palma. New Astron Rev. 42:503–507. doi:10.1016/S1387-6473(98)00062-1.

Bergé J, Price S, Amara A, Rhodes J. 2012. On point spread function modelling: towards optimal interpolation. Mon Not R Astron Soc. 419:2356–2368. doi:10.1111/j.1365-2966.2011.19888.x.

Beutler G, Drewes H, Verdun A. 2005. The integrated global geodetic observing systems (IGGOS) viewed from the perspective of history. J Geodynamics. 40:414–431. doi:10.1016/j.jog.2005.06.005.

Boddice D, Metje N, Tuckwell G. 2017. Capability assessment and challenges for quantum technology gravity sensors for near surface terrestrial geophysical surveying. J Appl Geophys. 146:149–199. doi:10.1016/j.jappgeo.2017.09.018.

Brumberg VA. 1995. General relativistic description of Earth, s rotation in different reference systems. J Geodynamics. 20:181–197. doi:10.1016/0264-3707(94)00027-S.

Busch P, Lahti P, Werner RF. 2014. Quantum root-mean-square error and measurement uncertainty relations. Rev Mod Phys. 86:1261. doi:10.1103/RevModPhys.86.1261.

Chai T, Draxler RR. 2014. Root mean square error (RMSE) or mean absolute error (MAE)? - Arguments against avoiding RMSE in the literature. Geosci Model Dev. 7:1247–1250. doi:10.5194/gmd-7-1247-2014.

Cook JC, Stern SA, Feldman PD, Gladstone GR, Retherford KD, Tsang CCC. 2013. New upper limits on numerous atmospheric species in the native lunar atmosphere. Icarus. 225:681–687. doi:10.1016/j.icarus.2013.04.010.

Davies TW, Bennie J, Inger R, Gaston KJ. 2013. Artificial light alters natural regimes of night-time sky brightness. Sci Rep. 3:1722. doi:10.1038/srep01722.

de Miguel ASM, Zamorano AJ, Kocifaj M, Roby J, Tapia C. 2017. Sky quality meter measurements in a colour changing world. Mon Not R Astron Soc. 467:2966–2979. doi:10.1093/mnras/stx145.

Falchi F, Cinzano P, Duriscoe D, Kyba CCM, Elvidge CD, Baugh K, Portnov BA, Rybnikova NA, Furgoni R. 2016. The new world atlas of artificial night sky brightness. Sci Adv. 2:e1600377. doi:10.1126/sciadv.1600377.

Garstang RH. 2000. Limiting visual magnitude and night sky brightness. J Italian Astron Soc. 71:83–92.

Grafarend EW, Krarup T, Syffus R. 1996. An algorithm for the inverse of a multivariate homogeneous polynomial of degreen. J Geod. 70:276–286.

Hassan AH, Abdel-Hadi YA. 2015. Naked eye determination of the dawn at Tubruq of Libya through four years observations. Middle-East J Sci Res. 23:2627–2632.

Hassan AH, Abdel-Hadi YA, Issa IA, Hassanin NY. 2014. Naked eye observations for morning twilight at different sites in Egypt. NRIAG J Astron Geophys. 3:23–26. doi:10.1016/j.nrjag.2014.02.002.

Herdiwijaya D. 2016. Sky brightness and twilight measurements at Yogyakarta city, Indonesia. J Phys Conference Ser. 771:012033. doi:10.1088/1742-6596/771/1/012033.

Hope ALB, Jones CR. 2014. The impact of religious faith on attitudes to environmental issues and Carbon Capture and Storage (CCS) technologies: A mixed methods study. Technol Soc. 38:48–59. doi:10.1016/j.techsoc.2014.02.003.

Hunten DM. 1967. Spectroscopic studies of the twilight airglow. Space Sci Rev. 6:493–573. doi:10.1007/BF00173704.

Jechow A, Kolláth Z, Ribas SJ, Spoelstra H, Hölke F, Kyba CCM. 2017. Imaging and mapping the impact of clouds on skyglow with all-sky photometry. Sci Rep. 7:6741. doi:10.1038/s41598-017-06998-z.

Jenniskens P, Gural PS, Dynneson L, Grigsby BJ, Newman KE, Borden M, Koop M, Holman D. 2011. CAMS: cameras for Allsky Meteor Surveillance to establish minor meteor showers. Icarus. 216:40–61. doi:10.1016/j.icarus.2011.08.012.

Jin S, Jin R, Kutoglu H. 2017. Positive and negative ionospheric response to the March 2015 geomagnetic storm from BDS observations. J Geod. 91:613–626. doi:10.1007/s00190-016-0988-4.

Kaye JA. 1989. Vertical, meridional, seasonal, and local time dependence of non-LTE effects in stratospheric NO and implications for infrared remote sensing. Planet Space Sci. 37:847–857. doi:10.1016/0032-0633(89)90135-9.

Kenworthy MA, Hinz PM. 2003. Spectrophotometry with a transmission grating for detecting faint occultations. Publ Astron Soc Pacific. 155:322. doi:10.1086/368099.

Kyba CCM, Tong KP, Bennie J, Birriel I, Birriel JJ, Cool A, Danielsen A, Davies TW, den Outer PN, Edwards W, et al. 2015. Worldwide variations in artificial skyglow. Sci Rep. 5:4809.

Leiper JB, Molla AM, Molla AM. 2003. Effects on health of fluid restriction during fasting in Ramadan. Eur J Clin Nutr. 57:S30–38. doi:10.1038/sj.ejcn.1601899.

Levin BW, Sasorova EV, Steblov GM, Domanski AV, Prytkov AS, Tsyba EN. 2017. Variations of the Earth's rotation rate and cyclic processes in geodynamics. Geod Geodynamics. 8:206–212. doi:10.1016/j.geog.2017.03.007.

Li Z, Hao T, Xu Y, Xu Y. 2011. An efficient and adaptive approach for modeling gravity effects in spherical coordinates. J Appl Geophys. 73:221–231. doi:10.1016/j.jappgeo.2011.01.004.

Lušić Z. 2013. The use of horizontal and vertical angles in terrestrial navigation. Trans Marit Sci. 1:5–14. doi:10.7225/toms.v02.n01.001.

Mateshvili N, Fussen D, Vanhellemont F, Bringen C, Kyrölä E, Mateshvili I, Mateshvili G. 2005. Twilight sky brightness measurements as a useful tool for stratospheric earosol investigations. J Geophys Res. 110:D09209. doi:10.1029/2004JD005512.

Miftahi YA. 2005. Fajar and isha. London (UK): Hizbul Ulama.

Morrison LV, Stephenson FR. 2001. Historical eclipses and the variability of the Earth's rotation. J Geodynamics. 32:247–265. doi:10.1016/S0264-3707(01)00024-2.

Niri MA, Zainuddin MZ, Man S, Nawawi MSAM, Wahab RA, Ismail K, Zaki NHA, Ghani AA, Lokman MAA. 2012. Astronomical determinations for the beginning prayer time of isha. Middle-East J Sci Res. 12:101–107.

Nor SAM, Zainuddin MZ. 2012. Sky brightness for the determination of fajr and isha prayer by using sky quality meter. Int J Sci Eng Res. 3:1–3.

Ojo SB, Kangkolo R. 1997. Shortcomings in the determination of regional fields by polynomial fitting: A simple solution. J Appl Geophys. 36:205–212. doi:10.1016/S0926-9851(96)00043-2.

Patat F, Ugolnikov OS, Postylyakov OV. 2006. UBVRI twilight sky brightness at ESO-Paranal. Astron Astrophys. 455:385–393. doi:10.1051/0004-6361:20064992.

Perez RO. 2017. Ionospheric error contribution to GNSS single-frequency navigation at the 2014 solar maximum. J Geod. 91:397–407. doi:10.1007/s00190-016-0971-0.

Power M, Del Campo AG, Espey B. 2017. Light pollution: spatial analysis and potential ecological effects in rural Ireland. Irish Geogr. 50:1–18.

Pun CSJ. 2014. Contributions of artificial lighting sources on light pollution in Hong Kong measured through a night sky brightness monitoring network. J Quant Spectrosc Radiat Transfer. 139:90–108. doi:10.1016/j.jqsrt.2013.12.014.

Pun CSJ, So CW. 2012. Night-sky brightness monitoring in Hong Kong: a city-wide light pollution assessment. Environ Monit Assess. 184:2537–2557. doi:10.1007/s10661-011-2136-1.

Puschnig J, Posch T, Uttenthaler S. 2014a. Night sky photometry and spectroscopy performed at the Vienna University Observatory. J Quant Spectrosc Radiat Transfer. 139:64–75. doi:10.1016/j.jqsrt.2013.08.019.

Puschnig J, Schwope A, Posch T, Schwarz R. 2014b. The night sky brightness at Potsdam-Babelsberg including overcast and moonlit conditions. J Quant Spectrosc Radiat Transfer. 139:76–81. doi:10.1016/j.jqsrt.2013.12.011.

Rennie SJ. 2014. Common orientation and layering of migrating insects in southeastern Australia observed with a Doppler weather radar. Meteorol Appl. 21:218–229. doi:10.1002/met.1378.

Saksono T, Fulazzaky MA, Sari Z. 2018. Geodetic analysis of disputed accurate qibla direction. J Appl Geod. 12:129–138. doi:10.1515/jag-2017-0036.

Scottili G, Palladino DM, Cuffaro M, Doglioni C. 2015. Earth's rotation variability triggers explosive eruptions in subduction zones. Earth Planet Sp. 67:208. doi:10.1186/s40623-015-0375-z.

Shariff NNM, Muhammad A, Zainuddin MZ, Hamidi ZS. 2012. The application of Sky Quality Meter at twilight for islamic prayer time. Int J Appl Phys Math. 2:143–145. doi:10.7763/IJAPM.2012.V2.71.

Spitschan M, Aguirre GK, Brainard DH, Sweeney AM. 2016. Variation of outdoor illumination as a function of solar elevation and light pollution. Sci Rep. 6:26756. doi:10.1038/srep26756.

Sunda S, Vyas BM. 2013. Local time, seasonal, and solar cycle dependency of longitudinal variations of TEC along the crest of EIA over India. J Geophys Res Space Phys. 118:6777–6785. doi:10.1002/2013JA018918.

Suzuki H, Sakanoi K, Nishitani N, Ogawa T, Ejiri MK, Kubota M, Kinoshita T, Murayama Y, Fujiyoshi Y. 2016. First imaging and identification of a noctilucent cloud from multiple sites in Hokkaido (43.2-44.4°N), Japan. Earth Planets Space. 68:182. doi:10.1186/s40623-016-0562-6.

Ugolnikov OS, Maslov IA, Kozelov BV, Dlugach JM. 2016. Noctilucent cloud polarimetry: twilight measurements in a wide range of scattering angles. Planet Space Sci. 125:105–113. doi:10.1016/j.pss.2016.03.010.

Wei J, Chen T, Liu G, Yang J. 2016. Higher-order multivariable polynomial regression to estimate human affective states. Sci Rep. 6:23384. doi:10.1038/srep23384.

Zamorano J, de Miguel AS, Ocaña F, Pila-Díez B, Castaño JG, Pascual S, Tapia C, Gallego J, Fernández A, Nievas M. 2015. Testing sky brightness models against radial dependency: a dense two dimensional survey around the city of Madrid, Spain. J Quant Spectrosc Radiat Transfer. 181:52–66. doi:10.1016/j.jqsrt.2016.02.029.

Zhang -H-H, Liu X-W, Yuan H-B, Zhao H-B, Yao J-S, Zhang H-W, Xiang M-S. 2013. Atmospheric extinction coefficients and night sky brightness at the Xuyi Observational Station. Res Astron Astrophys. 13:490–500. doi:10.1088/1674-4527/13/4/010.