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On the role of cosmic mass in understanding the relationships among galactic dark matter, visible matter and flat rotation speeds

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ABSTRACT

With reference to our recently proposed Planck Scale White Hole Cosmology (PS-WHC) (Seshavatharam et al. 2021) or Flat Space Cosmology (PS-FSC) (Tatum et al. 2015), we make an attempt to quantify galactic dark matter and flat rotation speeds in terms of galactic visible matter and cosmic mass. Considering recently observed dwarf galaxies having very little dark matter and assuming a time-dependent reference mass unit of $M_X \cong (3\text{to}4) \times 10^{38}\text{kg}$, we suggest an empirical relation for galactic dark matter (M_d) via galactic visible mass (M_v) as, $M_d \cong M_v^{3/2} \big/ M_X^{1/2}$. This relation helps in fitting flat rotation speeds starting from 8 km/sec (for Segue-2) to 500 km/sec (for UGC12591). Following the Modified Newtonian Dynamics and understanding galactic flat rotation speed relation with Hubble mass $M_0 \cong (c^3/2GH_0)$ of the universe, ratio of galactic flat rotation speed to speed of light can be shown to be approximately $(V_G/c) \cong 0.5(M_v/M_0)^{1/4}$. Considering the sum of galactic dark matter and visible matter, ratio of galactic flat rotation speed to speed of light can be shown to be approximately $(V_G/c) \cong 0.25[(M_v + M_d)/M_0]^{1/4}$. With further study, dark matter's nature, effect and distribution can be understood in terms of visible matter's extended gravity and extended theories of gravity can be understood with "distance cosmic mass" rather than the empirical "minimum acceleration".

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1. Introduction

Dark matter, the "well-quantified" matter with completely "unknown" nature seems to be quite interesting and attracting the whole science community starting from "young scholars" to "experienced scientists" (Bertone and Hooper 2018). With lot of curiosity, particle physicists are spotlighting "n" number of efforts in detecting dark matter (Zyla et al. 2020). At present, many underground lab experiments, ground-based lab experiments and satellite experiments are being conducted to trace the "genes" of dark matter and all the results are tending towards "negative findings". Even then, most of the galactic observations are being understood with the "unidentified" dark matter and each time, "dark matter" concept is getting strengthened based on pure theoretical inferences (Doux et al. 2021). Here, the main issue is associated with the "identification and confirming the existence of dark matter". In this most complicated and ambiguous situation, we try to study dark matter and visible matter in a very simplified and unified approach. Clearly speaking, with our empirical formula (Tatum et al. 2015; Seshavatharam and Lakshminarayana 2020a, 2020b; Seshavatharam et al. 2021), by knowing the magnitude of visible matter, independent of galactic radii, dark matter can be quantified and by knowing the magnitude of dark matter, visible matter can be quantified. With future observations and further study, effects of dark matter can be understood with equivalent visible matter and an "effective model of dark gravity" can be developed.

2. Light speed expanding Planck scale cosmology models

It may be noted that, standard model of cosmology is based on general theory of relativity. Final unification point of view, it seems essential to merge general theory of relativity and quantum mechanics. In this context, modern cosmologists are having "different views" on "workable" quantum cosmology models (Ashtekar et al. 2020; Bojowald 2020). Based on quantum gravity, it seems inevitable to bring a change in currently believed "Lambda cosmology". It may also be noted that, "Spin" is basic property of quantum mechanics. In this context, we propose five different assumptions in line with Planck mass as the baby universe. First three assumptions are helpful in understanding the basic cosmic structure and 4th and 5th assumptions are helpful in understanding galactic structures.

Basic view: Planck mass as the baby universe, cosmic boundary is always moving at speed of light and growing like a ball with,

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(1) Continuous creation of matter.
(2) Decreasing Hubble parameter.
(3) Decreasing angular velocity.
(4) Decreasing temperature.
(5) No big bang, no inflation, no late acceleration and no dark energy.

Thus, our model closely follows a time reversed black hole (i.e. a white hole) (Seshavatharam et al. 2021). More clearly speaking, our model can be considered as a model of Planck scale growing and rotating black hole cosmology. We would like to stress that –

(1) Based on the assumption of "light speed expansion", it is very natural to relinquish the concepts "acceleration" and "dark energy".
(2) Big bang and inflation – both are observationally inferred ideal concepts and their origination mechanism is beyond the scope of General theory of relativity, Quantum mechanics and known physical logics.

Assumption-1: If $R_t \cong 2GM_t/c^2 \cong c/H_t$, $M_t \cong c^3/2GH_t$, and $tH_t \cong 1$. where R_t= Cosmic radius,M_t = Cosmic mass, H_t = Cosmic Hubble parameter and t= Cosmic age,

Assumption-2: With reference to Planck mass, cosmic temperature T_t follows a scaled form of Hawking's black hole temperature relation, $T_t \cong \frac{\hbar c^3}{8\pi G k_B\sqrt{M_t M_{pl}}}$ where $M_{pl} \cong$ Planckmass.

Assumption-3: Ratio of Hubble parameter to angular velocity ω_t can be expressed as, $\frac{H_t}{\omega_t} \cong \Upsilon_t \cong 1+\ln\left(\frac{H_{pl}}{H_t}\right)$ where H_{pl} is the Planck scale Hubble parameter.

Assumption-4: Galactic dark matter $(M_{Gd})_t$ and visible matter $(M_{Gv})_t$ are interrelated in such a way that, $M_{Gt} \cong (M_{Gd})_t + (M_{Gv})_t$ and $\frac{(M_{Gd})_t}{(M_{Gv})_t} \cong \sqrt{\frac{(M_{Gv})_t}{M_{Xt}}}$ where $M_{Xt} \cong \sqrt{\frac{3H_t^2c^2}{8\pi GaT_t^4}}\frac{(M_t^3 M_{pl})^{1/4}}{8\pi}$ = Time dependent dark-visible reference mass unit.

Assumption-5: Galactic flat rotation speed V_{Gt} can be expressed as, $\frac{V_{Gt}}{c} \cong \left(\frac{1}{2\Upsilon_t}\right)^{1/4}\left(\frac{M_{Gt}}{M_t}\right)^{1/4}$ where M_t = Cosmic total mass.

It may be noted that, based on assumptions (1) and (2), at any stage of cosmic evolution,

(1) Currently believed "critical energy density" and cosmic mass-energy density are equal in magnitude.
(2) Ratio of mass energy density to thermal energy density is always a constant, $\frac{3H_t^2c^2}{8\pi GaT_t^4} \cong 5760\pi$.
(3) Hubble parameter and temperature are always strongly interrelated via scaled Hawking's black hole temperature formula.
(4) Cosmological constant problem can be understood with,
$\left(\frac{3H_{pl}^2c^2}{8\pi G}\right)\Big/\left(\frac{3H_t^2c^2}{8\pi G}\right) \cong \frac{H_{pl}^2}{H_t^2} \cong \frac{T_{pl}^4}{T_t^4}$.

3. A review on galactic flat rotation speed formula pertaining to modified Newtonian dynamics

According to Modified Newtonian Dynamics (MOND), there exists no dark matter and galactic flat rotation speeds follow the following relation (Milgrom 1983; Brownstein & Moffat 2006) (Milgrom 1983; Brownstein and Moffat 2006).

$$V_G \cong \left[GM_{Gv}\left(\frac{cH_0}{6}\right)\right]^{1/4} \quad (1)$$

where, V_G=Galactic flat rotation speed,
M_{Gv}=Galactic visible mass, and
$\left(\frac{cH_0}{6}\right)$= MOND's empirical acceleration constant pertaining to a characteristic representation of "minimum acceleration due gravity".

It may be noted that,

(1) In MOND's approach, there is no clear physical reasoning for the proposed $\left(\frac{cH_0}{6}\right)$.
(2) As,(cH_0) is associated with cosmic expansion and based on the workability of MOND's approach, it seems possible to say that, directly and indirectly somehow, MOND's approach is connected with Mach's principle (Derek Raine 1975; Arto Annila 2012) (Annila 1973–1977; Raine 1975) of the effect of distance cosmic matter on individual galaxies.
(3) Other than data fitting, there seems no proper reasoning for the introduction of the numerical factor $\left(\frac{1}{6}\right)$ in $\left(\frac{cH_0}{6}\right)$.
(4) Based on assumption (1), current cosmic mass can be expressed as,

$$M_0 \cong \frac{c^3}{2GH_0} \quad (2)$$

Writing and simplifying equ. (1) with $\left(\frac{cH_0}{6}\right) \cong \left(\frac{c^4}{12GM_0}\right)$,

$$V_G \cong \left[GM_{Gv}\left(\frac{c^4}{12GM_0}\right)\right]^{1/4} \quad (3)$$

$$\frac{V_G}{c} \cong \left(\frac{1}{12}\right)^{1/4}\left[\frac{M_{Gv}}{M_0}\right]^{1/4} \cong 0.5373\left[\frac{M_{Gv}}{M_0}\right]^{1/4} \cong k_v\left[\frac{M_{Gv}}{M_0}\right]^{1/4} \cong 0.5\left[\frac{M_{Gv}}{M_0}\right]^{1/4} \quad (4)$$

where, k_v= Coefficient of proportionality pertaining to galactic visible mass $\approx$ 0.5

If one is willing to consider equ.(4) as a characteristic representation of galactic flat rotation speeds, then one can easily understand the significance of the ratio of galactic visible mass to cosmic mass in terms of Mach's principle associated with distance cosmic background. In addition to that, the numerical factor $k_v \cong 0.5373 \approx 0.5$ can be considered as an universal coefficient associated with galactic flat rotation speeds. Even though "actual physics" is still lagging, thinking in this way, retaining the concept of "galactic visible mass", "minimum acceleration" concept of MOND can be relinquished and a very simple model of galactic structures can be developed with respect to Mach's principle.

4. Dark matter dependent modified form of MOND's galactic flat rotation speed formula

If one is willing to believe in the "existence" of dark matter, then cosmic mass seems to be a representation of cosmic visible matter and cosmic dark matter. If so, in equ. (4), meaning of the ratio, $\left(\frac{M_{Gv}}{M_0}\right)$ seems to be incomplete because a ratio of galactic visible mass to cosmic (visible mass + dark) mass seems to be ambiguous.

Considering assumption (4) and giving some importance to "existence" of galactic dark matter, equ. (4) can be expressed in the following way. For the current case, let,

$$\frac{V_G}{c} \propto \left[\frac{M_{Gv}+M_{Gd}}{M_0}\right]^{1/4} \propto \left[\frac{M_G}{M_0}\right]^{1/4} \quad (4)$$

$$\frac{V_G}{c} \cong k_{vd}\left[\frac{M_G}{M_0}\right]^{1/4} \quad (5)$$

where, $M_G \cong M_{Gv} + M_{Gd}$= Galactic total mass
= Galactic visible mass + Galactic dark mass.

k_{vd}= Coefficient of proportionality with respect to galactic total mass. It can be estimated with our assumptions (3) and (5).

Considering, current cosmic microwave background radiation (CMBR) temperature (Planck Collaboration 2015) (Planck Collaboration: Planck 2015) as, $T_0 \cong 2.725\text{K}$, current cosmic physical parameters like, mass, radius, angular velocity, Hubble parameter and galactic dark-visible reference mass unit can be estimated with the following relations.

$$M_0 \cong \frac{1}{M_{pl}}\left(\frac{\hbar c^3}{8\pi G k_B T_0}\right)^2 \cong 9.31453 \times 10^{52}\text{kg} \quad (6)$$

$$H_0 \cong \frac{c^3}{2GM_0} \cong 2.1671 \times 10^{-18}\text{sec}^{-1} \cong 66.87\text{km.sce}^{-1}\text{Mpc}^{-1} \quad (7)$$

$$\Upsilon_0 \cong 1 + \ln\left(\frac{H_{pl}}{H_0}\right) \cong 140.61 \quad (8)$$

$$\omega_0 \cong \frac{H_0}{\Upsilon_0} \cong \frac{2.1671 \times 10^{-18}\text{sec}^{-1}}{140.61} \cong 1.5412 \times 10^{-20}\text{rad.sec}^{-1} \quad (9)$$

With reference to, current Hubble parameter and Planck scale Hubble parameter $H_{pl} \cong \frac{c^3}{2GM_{pl}} \cong \frac{1}{2}\sqrt{\frac{c^5}{G\hbar}} \cong 9.27445 \times 10^{42}\text{sec}^{-1}$, current value of $\Upsilon = \Upsilon_0 \cong 140.61$. Hence,

$$\left(\frac{1}{2\Upsilon_0}\right)^{1/4} \cong 0.244 \approx 0.25 \quad (10)$$

$$\frac{V_G}{c} \cong 0.25\left[\frac{M_G}{M_0}\right]^{1/4} \quad (11)$$

Equations (10) and (11) need a thorough verification (Stacy 2020; Pengfei et al. 2020),

(1) With reference to the standard model of estimated galactic visible masses, dark masses and flat rotation speeds.
(2) With reference to the standard model of estimated flat rotation speeds and dark matter estimated with assumption (4) where visible mass is an input.

5. Discussion on the proposed current dark-visible reference mass unit

The proposed dark-visible reference mass unit M_{Xt} can be considered as a reference line for the observable effects of dark matter. Clearly speaking, at any stage of cosmic evolution, M_{Xt} distinguishes the potential effects of visible matter and dark matter. It is a variable and its magnitude increases with increasing cosmic mass. With further study, it can be estimated in a better way, can be better understood and its potential role can be analysed at fundamental level. Since we are trying to connect galactic visible mass and dark mass, in this paper, we make an attempt to estimate the dark-visible reference mass unit via the nucleon mass m_n.

(1) As per assumption (4), current magnitude of dark-visible reference mass unit is $M_{X0} \cong M_X \cong 5.3524$ $\left(M_0^3 M_{pl}\right)^{\frac{1}{4}} \cong 3.47 \times 10^{38}\text{kg}$,
(2) In this paper, in a conceptual way, we proceed in the following way. At any stage of cosmic evolution,

$$\frac{M_{Xt}}{m_n} \propto (\text{Cosmicthermalradiationpressure})^{-1} \propto \left(\frac{1}{3}aT_t^4\right)^{-1} \quad (12)$$

$$\frac{M_{Xt}}{m_n} \propto \left(\frac{\text{Cosmicmassenergy}}{\text{Cosmicvolume}}\right) \cong \frac{3H_t^2c^2}{8\pi G} \quad (13)$$

$$\frac{M_{Xt}}{m_n} \propto \left(\frac{\text{Cosmicmass}}{\text{Planckmass}}\right) \cong \frac{M_t}{M_{pl}} \quad (14)$$

Thus,

$$\frac{M_{Xt}}{m_n} \cong 3\left(\frac{3H_t^2c^2}{8\pi GaT_t^4}\right)\left(\frac{M_t}{M_{pl}}\right) \quad (15)$$

$$\frac{M_{Xt}}{m_n} \cong 3(5760\pi)\left(\frac{M_t}{M_{pl}}\right) \cong 54286.72\left(\frac{M_t}{M_{pl}}\right)$$
$$\text{where,}\left(\frac{3H_t^2c^2}{8\pi GaT_t^4}\right) \cong 5760\pi \quad (16)$$

$$M_{Xt} \cong 3(5760\pi)\left(\frac{M_t}{M_{pl}}\right) m_n \cong 54286.72\left(\frac{M_t}{M_{pl}}\right) m_n \quad (17)$$

For the current case,

$$M_{X0} \cong M_X \cong 54286.72\left(\frac{M_0}{M_{pl}}\right) m_n \cong 3.89 \times 10^{38}\text{kg} \quad (18)$$

See Figure 1 for the estimated galactic dark matter. See Figure 2 for the estimated flat rotation speeds. Blue curve indicates flat rotation speeds estimated with MOND's relation, i.e. equation (4) where $k_v \approx 0.5$. Red curve indicates flat rotation speeds estimated with (Visible mass + Dark mass) of galaxy, i.e. equation (11) where $k_{vd} \approx 0.25$. Here, we have assumed the visible matter magnitude as $M_{Gv} \cong n^2 \times 10^5 M_\odot$ where $n = 1, 2, 3, \ldots$ up to $M_{Gv} \approx 10^{12} M_\odot$.

Regarding the estimation of "missing" visible mass, with advanced technology, very recently, by considering distance galaxies as "scintillating pins", cosmologists are seriously working on

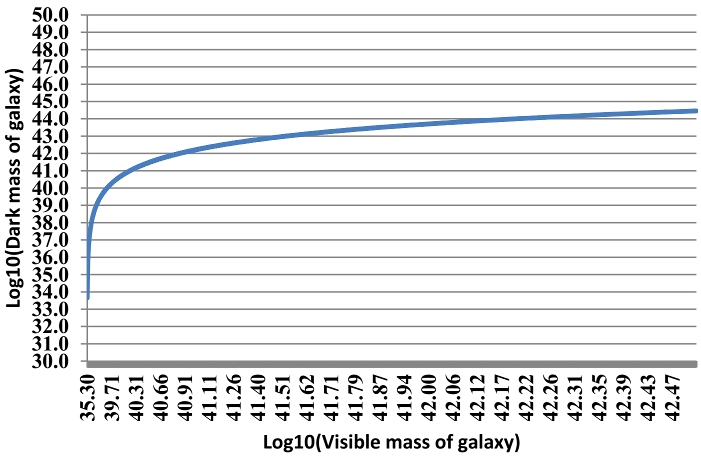

Figure 1. Galactic Visible mass Vs dark mass.

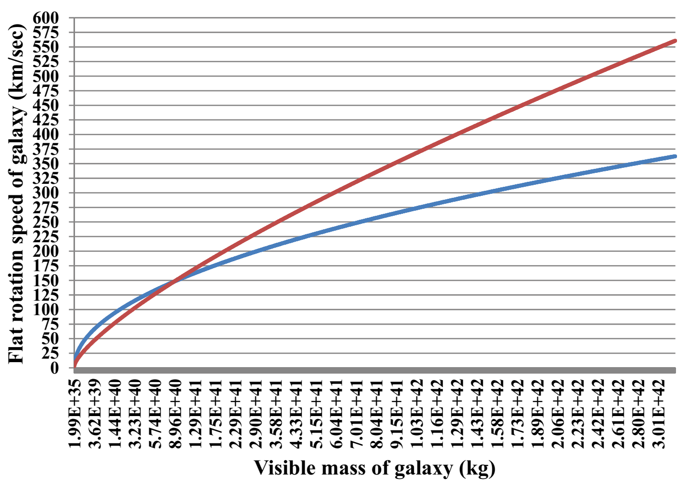

Figure 2. Galactic flat rotation speeds.

detecting cold hydrogen hidden in galaxies and intergalactic space (Yuanming Wang et al 2021) (Yuanming et al. 2021). This certainly helps in quantifying the missing galactic visible mass to a very good approximation. Some cosmologists are having difference of opinion on the "existence of dark matter" (Erik Verlinde 2017; Andre Maeder 2017; Kyu-Hyun Chae 2020; Tatum 2021; Deur 2021)(Tatum 2021; Verlinde 2017; Maeder 2017; Chae et al. 2020; Deur 2021). Meanwhile, qualitatively, our approach can be recommended for estimating galactic dark matter. Alternatively, by following our approach and by considering galactic flat rotation speeds, galactic total mass and hence dark and visible masses can be estimated independent of galactic radii. We sincerely appeal that, our approach is very flexible and based on observations, coefficients like 0.25 and 0.5 can be modified for a better fit.

6. Discussion on assumptions (3), (4) and (5)

Assumptions (3), (4) and (5) need a critical review for their scope and applicability for the past and future cosmic evolution.

With reference to MOND's relation for galactic flat rotation speeds, existence of dark matter and considering cosmic angular velocity as per assumption (3) and following equations (9), (10) and (11), galactic flat rotation speeds can be expressed as,

$$V_G \cong [(GM_G)(c\omega_0)]^{1/4} \tag{19}$$

where, $M_G \cong M_{Gv} + M_{Gd}$= Galactic total mass

= Galactic visible mass + Galactic dark mass.

ω_0= Current cosmic angular velocity.

$(c\omega_0)$= Current cosmic (maximum) angular acceleration.

It may be noted that, equ. (19) also directly and indirectly reflects the effect of distance cosmic "angular acceleration" on galaxies. If found logical and meaningful, by knowing the galactic total mass and flat rotation speeds, cosmic angular velocity (Vladimir Korotky et al; 2020; Shamir 2020) (Korotky and Eduard Masár Yuri 2020; Shamir 2020) can be estimated in a very simplified approach.

According to very recent study (Wang et al. 2021), filaments of galaxies that are cylindrical tendrils spreading millions of light years across the galaxies are found to be spinning. Most probably, with further study, galactic rotations and galactic filament spinning collectively may shed light on cosmic rotation.

With reference to MOND's relation,

$$\begin{aligned} V_G &\cong \left(\frac{1}{\Upsilon_0}\right)^{1/4}[(GM_G)(cH_0)]^{1/4} \\ &\cong 0.29[(GM_G)(cH_0)]^{1/4} \cong 0.454\left[(GM_G)\left(\frac{cH_0}{6}\right)\right]^{1/4} \end{aligned} \tag{20}$$

Using assumption (4) and by knowing the galactic visible mass, galactic dark matter can be quantified. Using assumption (5) and by knowing the galactic total mass, galactic flat rotation speeds can be fitted. Alternatively, considering both the assumptions and by knowing the galactic flat rotation speed, galactic total mass can be estimated and with a simple computer program, galactic visible mass and dark mass, both can be estimated.

But, based on assumption (4), relation between galactic dark mass and visible mass is not linear and magnitude of galactic dark mass is proportional to $\left[(M_{Gv})_0\right]^{\frac{3}{2}}$. Based on this idea,

(1) Observed low dark matter content of NGC1052-DF2 and NGC1052-DF4 can be understood (Pieter van Dokkum et al 2019; Shany Danieli et al 2019; Zili Shen et al 2021) (van Dokkum et al. 2019; Danieli et al. 2019; Shen et al. 2021).

(2) At present there seems to be around 20 ultra light dwarf galaxies having very little dark matter (Guo et al 2020) (Guo et al. 2020). By finding more number of such galaxies, proposed dark matter relation can be verified.

(3) By considering a low dark matter galaxy of visible mass of $\text{Log}_{10}\left(\frac{M_{Gv}}{M_\odot}\right) \approx 9$, flat rotation speed estimated with equ. (11) is around 38.6 km/sec and is matching with recent (Pavel E. Mancera Piña et al 2019,2020) (Pavel 2019; Pavel and Piña et al. 2020) baryon dominated galactic circular speeds falling in the range of $(20\text{to}37)^{+5}_{-5}$km/sec. Its corresponding MOND value is 57.4 km/sec.

(4) Very recent studies conducted on super spiral galaxies with Southern African Large Telescope (SALT) suggest (Patrick M. Ogle 2019) (Patrick et al. 2019) high rotation speeds in the range of (240 to 570) km/sec indicating a dynamical mass range of $(0.6\text{to}4) \times 10^{12}M_\odot$kg. It may be noted that, with a visible galactic mass of $\sqrt{0.6 \times 4} \times 10^{12}M_\odot\text{kg} \cong 3.08 \times 10^{42}$kg, estimated flat rotation speed is 554 km/sec. Its corresponding MOND's value is close to 380 km/sec (Milgrom 2020) (Milgrom 2020).

(5) Sun's estimated dark mass is 1.42×10^{26}kg.

(6) Nucleon's estimated dark mass is 3.5×10^{-60}kg.

(7) Electron's estimated dark mass is 4.4×10^{-65}kg.

Considering the estimated dark masses of Sun and nucleons, it is reasonable to say that, starting from nuclear volume to solar volume, dark matter effects are negligible. It can be confirmed with current experiments and observations. Interesting point to be noted is that, for understanding the nature of dark matter at atomic and nuclear scales, experimental setup should have an access to find ultra light masses of the order of $(10^{-65}\text{to}10^{-60})$kg.

7. Milky Way – its flat rotation speed, total mass, dark mass and visible mass

By considering Milky Way's flat rotation speed of 214^{+17}_{-15} km/sec (Steven Phelps et al 2013; Przemek Mrózl et al 2019; Laura L. Watkins et al 2019; Yoshiaki Sofue 2020; Marius Cautun et al 2020) (Phelps et al. 2013; Przemek et al. 2019; Laura et al. 2019; Sofue 2020; Cautun et al. 2020), its total mass, dark mass and visible mass can be estimated. As per our proposal, total mass of galaxy is proportional to 4$^{\text{th}}$ power of its flat rotation speed and a small increment in flat rotation speed greatly increases the total mass. For calculation purpose, we consider, $V_{MW} \cong 214$ km/sec. Obtained total mass of Milky Way is $M_{MW} \cong (V_{MW}/0.25c)^4(c^3/2GH_0) \cong 3.113 \times 10^{12}M_\odot$ and its earlier estimation range is, $M_{MW} \cong (1.5\text{to}4.5) \times 10^{12}M_\odot$. Its recent estimation range is (Lorenzo Posti and Amina Helmi 2019; Robert Grand et al 2019)(Posti and Helmi 2019; J J Grandet Al 2019) $M_{MW} \cong 1.54^{+0.75}_{-0.44} \times 10^{12}M_\odot \cong (1.1\text{to}2.3) \times 10^{12}M_\odot$. Considering $V_{MW} \cong 214$ km/sec as a characteristic representation of Milky Way, its total radius can be expressed as, $R_{MW} \cong GM_{MW}/V_{MW}^2 \cong 16G\sqrt{M_0 M_G}/c^2 \cong 292.4$kpc and is comparable with the recent estimation of (292 ± 61) kpc (Deason et al 2020) (Deason et al. 2020). With a simple computer program, estimated dark and visible masses of Milky Way are, $M_{MWd} \cong 2.99 \times 10^{12}M_\odot$ and $M_{MWv} \cong 1.21 \times 10^{11}M_\odot$ respectively. As per 2016 reference paper (Nicastro et al 2016) (Nicastro et al. 2016), total baryonic mass of the Milky Way is, $M_{MWv} \cong (0.8\text{to}4.0) \times 10^{11}M_\odot$. As per 2006 reference paper (Brownstein and Moffat 2006) (Brownstein and Moffat 2006), $M_{MWv} \cong (9.12 \pm 0.28) \times 10^{10}M_\odot$ and $M_{MWv} \cong (10.6 \pm 0.37) \times 10^{10}M_\odot$.

Considering SPARC data base [http://astroweb.cwru.edu/SPARC/] and with reference to the available data for 175 galaxies, for 51 galaxies, data pertaining to flat rotation speed is not available. Based on NFW method of total estimated mass of galaxies, for the remaining 175–51 = 124 galaxies, considering an error bar of $\pm$ 30km/sec in our approach of estimated flat rotation speeds, for 94 galaxies, average error is -4km/sec. For 30 galaxies out of 124, error in estimated flat rotation speed is $(-114\text{to} + 145)$km/sec. Out of 30 odd galaxies, for 17 galaxies, error in estimated flat rotation speed is in the range of (± 60)km/sec. It may be due to, estimated total mass is on higher side or lower side and error in recommended flat rotation speed is on higher side or lower side. For example, recommended total NFW mass of ESO563-G021 seems to be on higher side compared to its recommended flat rotation speed. For UGC02487, recommended total NFW mass seems to be on lower side compared to its recommended flat rotation speed. For UGC11914, recommended total NFW mass seems to be on higher side and error bar in recommended flat speed is also on higher side. For other galaxies, error bar in recommended flat speed is on higher side. As so many assumptions are involved in total mass estimation of galaxies, to validate our approach, it seems reasonable to fix the error bar in flat rotation speed around $\pm$ 30km/sec. We are working in all other possible ways.

8. Discussion on dark matter clumps and mass limits of dark matter candidates

Very recently it has been established that (Massimo Meneghetti et al 2020; Nierenberg et al 2020; Daniel Gilman et al 2020) (Massimo et al. 2020; Nierenberg et al. 2020; Daniel 2020), within galaxies and galactic clusters, like stars, dark matter exists in compact form (Dark matter clumps) with a mass roughly (10,000 to 100,000) times smaller than the massive Milky Way dark hallow. In view of galactic astrophysical observations, this is a very good signal for confirming the existence of dark matter. Even then, there is no single piece of information on the nature of dark matter. Again it is an ambiguous situation and needs a critical review.

Particle physics point of view, based on quantum gravity and considering the massive nature of dark matter, scientists are trying to apply limits on the mass of observable dark matter particles. Very recently, for scalar dark particles, proposed mass range (Xavier Calmet and Folkert Kuipers 2021) (Calmet and Kuipers 2021) is, $m_\phi c^2 \cong (10^{-3}\text{to}10^7)$eV. In this context, fundamental questions to be answered are,

(1) What is the basic characteristic of any dark particle?
(2) If dark particles are having a real existence, why are they escaping from detection?
(3) Why are the dark particles assembling together to form galaxies?
(4) Why is galactic dark matter content heavier than galactic visible mass?

By confirming the existence of pure dark galaxies (Weisberg, Michael et al 2018) (Weisberg et al. 2018) having no visible matter, it seems possible to confirm the existence of dark matter on galactic scales. Anyhow, from now onwards it seems inevitable to design experiments or observations in view of "quantity" of dark matter as well as "nature" of dark matter. Clearly speaking,

(1) Combined study of "quantity and nature" of dark matter may help in resolving the issue at fundamental level.
(2) New techniques and methods should be developed for probing the structure of dark matter (Sebastian Baum et al 2020; Margot M. Brouwer et al 2021) (Baum et al. 2020; Margot et al. 2021).

9. Toy model of NGC 3521 rotation curve with total matter and visible matter

Considering the estimated visible mass and total mass, galactic flat rotation curves can be understood with the following toy model relation and needs a critical review. Advantage of this relation is that, it constitutes galactic visible mass, total mass, galactic black hole radius and galactic core radius. Starting from a radial distance equal to the galactic core radius and up to 300 kpc, it can be expressed as,

$$V_{Gr} \cong \frac{1}{y}\left[1+\ln\left(\frac{r}{r_c}\right)+\ln(x)\right]\sqrt{\frac{GM_G}{\sqrt{R_{GB}r}}} \tag{21}$$

where, V_{Gr}= Orbiting speed of star

r= Radial distance of orbiting star.

$$r_c = \text{Galactic core radius} = \sqrt{\frac{M_0}{M_G}}\left(\frac{2GM_{Gv}}{c^2}\right) \text{where,} M_0 \cong \frac{c^3}{2GH_0}$$

R_{GB}= Hypothetical galactic black hole radius = $\left(\frac{2G(M_{Gv}+M_{Gd})}{c^2}\right) \cong \left(\frac{2GM_G}{c^2}\right)$

$x \cong \sqrt{\frac{M_G}{M_{Gv}}}$= Square root of ratio of galactic total matter to visible matter

y= Assumed to be a common factor for all galaxies = 320 and needs a

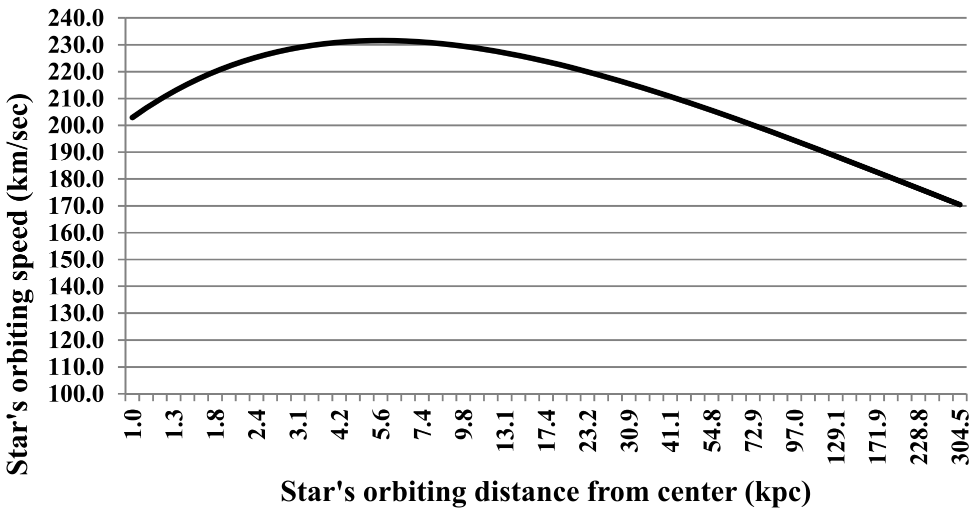

Figure 3. Toy model of NGC3520 flat rotation curve up to 300 kpc.

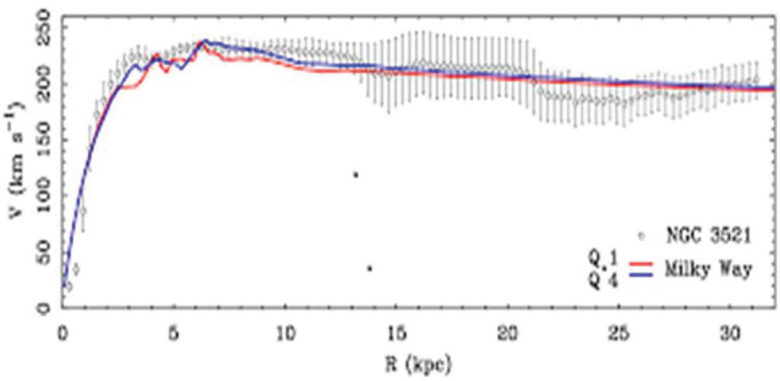

Figure 4. Open source NGC3520 flat rotation curve up to 30 kpc.

Table 1. Galactic flat rotation speeds, baryon & total masses taken from SPARC data base.

Galaxy	Flat rotation speed (km/sec) & Error		Baryon mass $\log_{10}\left(\frac{M_{Gbar}}{M_\odot}\right)$ & Error		NFW Model $\log_{10}\left(\frac{M_{Gtotal}}{M_\odot}\right)$ & Error		Einasto model $\log_{10}\left(\frac{M_{Gtotal}}{M_\odot}\right)$ & Error		DC14 Model $\log_{10}\left(\frac{M_{Gtotal}}{M_\odot}\right)$ & Error	
(1)	(2A)	(2B)	(3A)	(3B)	(4A)	(4B)	(5A)	(5B)	(6A)	(6B)
UGC09992	33.6	11.6	8.77	0.26	10.64	0.14	10.62	0.16	10.69	0.11
KK98–251	33.7	2.7	8.29	0.26	10.34	0.09	10.18	0.16	10.50	0.09
UGCA444	37.0	6.3	7.98	0.06	10.07	0.07	10.04	0.08	10.06	0.08
DDO064	46.1	6.4	8.56	0.26	10.53	0.10	10.45	0.13	10.69	0.09
DDO154	47.0	1.7	8.59	0.06	10.58	0.04	10.13	0.07	10.26	0.03
NGC3741	50.1	3.1	8.41	0.06	10.39	0.06	10.27	0.07	10.32	0.05
DDO168	53.7	3.7	8.81	0.06	10.67	0.07	10.55	0.08	10.59	0.06
UGC01281	55.2	4.2	8.75	0.06	10.74	0.07	10.67	0.08	10.67	0.07
UGCA442	56.4	4.2	8.62	0.06	10.64	0.06	10.49	0.08	10.45	0.05
UGC08550	56.9	2.3	8.72	0.26	10.65	0.09	10.57	0.10	10.86	0.09
UGC07690	57.4	7.3	8.98	0.27	10.89	0.09	10.75	0.12	10.99	0.09
D631–7	57.7	4.1	8.68	0.05	10.70	0.06	10.54	0.07	10.65	0.06
DDO170	60.0	3.7	9.1	0.26	10.65	0.09	10.52	0.14	10.68	0.10
UGC07603	61.6	3.3	8.73	0.26	10.77	0.08	10.62	0.11	10.86	0.09
UGC07125	65.2	2.8	9.88	0.26	10.72	0.08	10.64	0.12	10.76	0.13
NGC3109	66.2	3.9	8.86	0.06	11.06	0.06	10.58	0.08	10.75	0.05
DDO161	66.3	4.6	9.32	0.26	10.87	0.07	10.50	0.14	10.77	0.07
IC2574	66.4	6.0	9.28	0.06	11.29	0.06	10.90	0.07	10.99	0.06
UGC06818	71.2	6.4	9.35	0.13	10.90	0.08	10.85	0.09	10.99	0.08
UGC10310	71.4	12.3	9.39	0.27	10.96	0.12	10.92	0.14	11.06	0.10
UGC12632	71.7	4.9	9.47	0.26	10.83	0.09	10.73	0.13	10.85	0.08
UGC04499	72.8	4.4	9.35	0.26	10.91	0.09	10.78	0.13	10.99	0.10
UGC05716	73.1	9.4	9.24	0.22	10.77	0.10	10.72	0.12	10.87	0.09
UGC00731	73.3	4.0	9.41	0.26	10.72	0.06	10.58	0.12	10.63	0.06
UGC07151	73.5	3.7	9.29	0.08	11.07	0.08	11.03	0.08	11.14	0.09
UGC07261	74.7	23.3	9.43	0.26	11.00	0.13	10.96	0.14	11.07	0.11
UGC08490	78.6	5.2	9.17	0.11	10.74	0.08	10.82	0.08	10.87	0.07
UGC06923	79.6	5.1	9.4	0.14	11.07	0.08	11.02	0.09	11.13	0.09
UGC07524	79.6	5.7	9.55	0.06	11.07	0.07	10.99	0.08	10.93	0.06
UGC05721	79.7	7.8	9.01	0.26	10.79	0.08	10.72	0.10	10.80	0.07
UGC06446	82.2	7.0	9.37	0.26	10.89	0.08	10.85	0.11	10.94	0.09
UGC08286	82.4	2.5	9.17	0.06	10.87	0.06	10.91	0.09	10.70	0.05
NGC2915	83.5	8.0	9	0.06	10.82	0.06	10.74	0.07	10.78	0.05
F571–V1	83.6	26.0	9.41	0.1	10.98	0.09	10.99	0.09	10.98	0.09
UGC06667	83.8	3.9	9.25	0.13	11.11	0.07	10.94	0.09	11.00	0.06
UGC06399	85.0	5.2	9.31	0.14	11.10	0.08	11.00	0.09	11.12	0.08
NGC2976	85.4	9.6	9.28	0.11	11.13	0.08	11.11	0.08	11.18	0.08
NGC0055	85.6	5.4	9.64	0.08	11.27	0.07	11.05	0.08	11.11	0.05
F583–1	85.8	7.9	9.52	0.22	11.02	0.08	10.70	0.14	10.95	0.07
UGC02259	86.2	6.2	9.18	0.26	10.94	0.09	10.95	0.10	11.02	0.08
NGC0100	88.1	8.7	9.63	0.27	11.19	0.08	10.96	0.13	11.21	0.09
NGC5585	90.3	3.7	9.57	0.27	11.26	0.06	11.07	0.08	11.13	0.05
UGC04325	90.9	6.4	9.28	0.27	11.03	0.09	11.02	0.11	11.08	0.08
UGC04278	91.4	7.3	9.33	0.26	11.16	0.07	10.93	0.12	11.16	0.08
NGC0300	93.3	20.1	9.43	0.08	11.11	0.08	11.10	0.08	11.07	0.08
F574–1	97.8	9.3	9.9	0.1	11.27	0.09	11.21	0.10	11.22	0.08
UGC05005	98.9	22.3	9.79	0.18	11.10	0.12	11.06	0.14	11.14	0.11
UGC07399	103.0	6.4	9.2	0.27	11.12	0.08	10.94	0.11	11.11	0.08
NGC0247	104.9	9.1	9.78	0.08	11.37	0.06	11.24	0.08	11.26	0.06
NGC0024	106.3	8.5	9.45	0.09	11.30	0.10	11.20	0.09	11.30	0.10
UGC06930	107.2	16.4	9.94	0.13	11.30	0.11	11.28	0.11	11.30	0.10
UGC06917	108.7	6.2	9.79	0.14	11.32	0.08	11.22	0.10	11.33	0.08
UGC06983	109.0	7.8	9.82	0.13	11.23	0.07	11.17	0.09	11.27	0.07
ESO116-G012	109.1	4.0	9.55	0.27	11.49	0.08	11.07	0.12	11.34	0.07
NGC1003	109.8	5.8	10.05	0.26	11.59	0.08	11.43	0.10	11.55	0.06
NGC4183	110.6	6.6	10	0.14	11.22	0.06	11.28	0.08	11.30	0.06
UGC05986	113.0	4.9	9.77	0.27	11.71	0.10	11.29	0.11	11.49	0.12
NGC6503	116.3	3.3	9.94	0.09	11.24	0.03	11.28	0.05	11.27	0.02

(*Continued*)

Table 1. (Continued).

Galaxy	Flat rotation speed (km/sec) & Error		Baryon mass $\log_{10}\left(\frac{M_{Gbar}}{M_\odot}\right)$ & Error		NFW Model $\log_{10}\left(\frac{M_{Gtotal}}{M_\odot}\right)$ & Error		Einasto model $\log_{10}\left(\frac{M_{Gtotal}}{M_\odot}\right)$ & Error		DC14 Model $\log_{10}\left(\frac{M_{Gtotal}}{M_\odot}\right)$ & Error	
(1)	(2A)	(2B)	(3A)	(3B)	(4A)	(4B)	(5A)	(5B)	(6A)	(6B)
NGC3769	118.6	10.2	10.22	0.14	11.42	0.07	11.36	0.09	11.46	0.07
NGC4559	121.2	5.6	10.24	0.27	11.43	0.07	11.28	0.12	11.49	0.08
NGC4010	125.8	5.5	10.09	0.14	11.60	0.11	11.38	0.12	11.68	0.13
UGC03580	126.2	5.6	10.09	0.23	11.50	0.09	11.25	0.09	11.50	0.07
UGC00128	129.6	15.0	10.2	0.14	11.48	0.10	11.47	0.11	11.49	0.07
NGC2403	131.2	6.1	9.97	0.08	11.40	0.05	11.32	0.05	11.39	0.04
NGC4085	131.5	6.5	10.1	0.15	11.51	0.11	11.37	0.12	11.59	0.13
NGC3972	132.7	5.4	9.94	0.15	11.57	0.11	11.41	0.12	11.65	0.12
NGC3917	135.9	5.1	10.13	0.15	11.75	0.12	11.55	0.13	11.98	0.33
F571–8	139.7	5.8	9.87	0.19	12.05	0.15	10.91	0.11	11.68	0.06
NGC4138	147.3	18.0	10.38	0.16	11.83	0.17	11.73	0.13	11.72	0.31
NGC3198	150.1	4.7	10.53	0.11	11.64	0.03	11.49	0.05	11.65	0.04
UGC09037	152.3	14.9	10.78	0.11	12.09	0.18	11.60	0.17	11.96	0.11
NGC2683	154.0	11.4	10.62	0.11	11.81	0.11	11.81	0.09	11.82	0.08
NGC6015	154.1	8.1	10.38	0.27	11.46	0.05	11.73	0.09	11.75	0.05
NGC4051	157.0	10.9	10.71	0.16	11.92	0.22	11.79	0.16	11.84	0.83
NGC4100	158.2	8.2	10.53	0.15	11.67	0.09	11.76	0.10	11.83	0.06
NGC6946	158.9	14.8	10.61	0.28	11.80	0.12	11.80	0.14	11.97	0.11
NGC0289	163.0	17.8	10.86	0.22	11.82	0.10	11.88	0.12	11.89	0.11
NGC3949	163.0	16.1	10.37	0.15	11.71	0.17	11.62	0.16	11.69	0.22
NGC1090	164.4	5.7	10.68	0.23	11.71	0.07	11.53	0.14	11.91	0.09
NGC3726	168.0	13.2	10.64	0.15	11.93	0.15	11.80	0.20	12.53	0.35
NGC3877	168.4	6.2	10.58	0.16	11.68	0.10	11.60	0.15	11.79	0.07
NGC4088	171.7	8.4	10.81	0.15	11.94	0.18	11.82	0.18	12.33	0.36
NGC4013	172.9	7.5	10.64	0.16	12.27	0.15	12.27	0.19	12.44	0.12
NGC3893	174.0	14.6	10.57	0.15	11.93	0.17	11.80	0.17	11.97	0.12
ESO079-G014	175.0	4.8	10.48	0.24	12.39	0.14	11.62	0.24	12.23	0.14
NGC5055	179.0	14.2	10.96	0.1	11.72	0.05	11.70	0.05	11.80	0.05
NGC4217	181.3	9.1	10.66	0.16	12.15	0.18	11.65	0.22	12.15	0.13
UGC08699	182.4	11.3	10.48	0.24	11.99	0.15	12.00	0.13	12.03	0.12
UGC02916	182.7	17.3	10.97	0.15	11.95	0.10	11.54	0.08	11.72	0.10
NGC2903	184.6	7.5	10.65	0.28	11.64	0.06	11.42	0.07	11.65	0.07
NGC4157	184.7	9.3	10.8	0.15	12.12	0.18	12.02	0.19	12.42	0.26
NGC5033	194.2	4.7	10.85	0.27	11.88	0.04	11.59	0.07	11.95	0.04
UGC03546	196.9	15.3	10.73	0.24	11.89	0.09	11.80	0.17	12.03	0.09
UGC06614	199.8	51.4	10.96	0.12	12.64	0.22	12.67	0.27	12.54	0.15
NGC5371	209.5	7.4	11.27	0.24	11.75	0.05	12.00	0.09	11.85	0.05
NGC2998	209.9	9.5	11.03	0.15	12.00	0.09	12.16	0.08	12.09	0.05
NGC3521	213.7	16.9	10.68	0.28	12.41	0.22	12.12	0.32	12.50	0.23
UGC05253	213.7	21.4	11.03	0.23	12.08	0.11	11.72	0.10	12.05	0.10
NGC5907	215.0	3.8	11.06	0.1	11.88	0.02	12.17	0.06	12.03	0.03
NGC0891	216.1	6.2	10.88	0.11	11.93	0.06	11.70	0.14	12.03	0.05
NGC7814	218.9	7.2	10.59	0.11	12.05	0.12	12.09	0.15	12.11	0.07
UGC06786	219.4	9.8	10.64	0.24	12.08	0.06	12.02	0.12	12.18	0.05
UGC03205	219.6	10.9	10.84	0.2	12.28	0.13	12.13	0.09	12.08	0.06
NGC0801	220.1	7.0	11.27	0.13	12.32	0.14	12.33	0.27	12.16	0.06
NGC3953	220.8	8.0	10.87	0.16	12.23	0.25	12.10	0.13	12.08	0.93
UGC09133	226.8	18.4	11.27	0.19	12.15	0.07	12.32	0.09	12.21	0.07
UGC12506	234.0	17.5	11.07	0.11	12.08	0.04	12.11	0.13	12.24	0.04
NGC7331	239.0	6.2	11.15	0.13	12.42	0.08	12.50	0.11	12.42	0.05
NGC3992	241.0	9.3	11.13	0.13	12.12	0.10	12.28	0.12	12.29	0.06
NGC6674	241.3	19.1	11.18	0.19	12.80	0.17	13.17	0.23	13.49	0.25
IC4202	242.6	11.2	11.03	0.13	11.94	0.05	11.15	0.09	11.92	0.02
UGC06787	248.1	7.7	10.75	0.24	13.38	0.11	13.35	0.13	13.20	0.09
NGC6195	251.7	15.1	11.35	0.13	12.73	0.22	12.66	0.39	13.44	0.20
NGC5005	262.2	22.8	10.96	0.13	12.86	0.30	12.63	0.38	12.50	0.41
UGC02953	264.9	16.7	11.15	0.28	12.23	0.07	12.17	0.08	12.32	0.06
UGC11455	269.4	8.1	11.31	0.16	13.03	0.19	11.63	0.08	13.10	0.15

(*Continued*)

Table 1. (Continued).

Galaxy	Flat rotation speed (km/sec) & Error		Baryon mass $\log_{10}\left(\frac{M_{Gbar}}{M_\odot}\right)$ & Error		NFW Model $\log_{10}\left(\frac{M_{Gtotal}}{M_\odot}\right)$ & Error		Einasto model $\log_{10}\left(\frac{M_{Gtotal}}{M_\odot}\right)$ & Error		DC14 Model $\log_{10}\left(\frac{M_{Gtotal}}{M_\odot}\right)$ & Error	
(1)	(2A)	(2B)	(3A)	(3B)	(4A)	(4B)	(5A)	(5B)	(6A)	(6B)
NGC2841	284.8	15.3	11.03	0.13	12.57	0.07	12.66	0.07	12.64	0.06
UGC11914	288.1	43.2	10.88	0.28	13.59	0.14	13.57	0.16	13.36	0.17
UGC02885	289.5	15.9	11.41	0.12	12.67	0.10	12.75	0.11	12.72	0.10
NGC5985	293.6	10.5	11.08	0.24	12.20	0.05	12.06	0.13	12.37	0.05
ESO563-G021	314.6	12.3	11.27	0.16	13.45	0.13	11.70	0.09	13.51	0.09
UGC02487	332.0	40.8	11.43	0.16	12.26	0.08	12.19	0.13	12.41	0.08

physical explanation. See Note-2 and equ. (23) for a better understanding.

Note-1: Our defined galactic core radii are roughly two times less than the currently adopted galactic core radii.

Note-2: $\frac{c}{320^2} \cong 2.9297 \approx 3\text{km/sec} \cong (\text{V}_\text{G})_{\text{min}}$ may be an indication of minimum galactic flat rotation speed. Its corresponding minimum total mass of a galaxy seems to be 2.17×10^{35}kg. Corresponding visible and dark masses are 2.12×10^{35}kg. and 4.95×10^{33}kg respectively. Corresponding core radius and acceleration due to gravity are, 2.063×10^{17}m and $3.324 \times 10^{-10}\text{m/sec}^{-2}$. To a great surprise, estimated acceleration due to gravity at core radius is $0.51 * (cH_0)$. As per the observations, if core radius is really two times of our estimation, acceleration due to gravity at observable core radius is roughly $0.127 * (cH_0) \cong (cH_0)/7.84$. This approach seems to be the basic form of Mondian approach. Clearly speaking, concepts of "minimum mass of galaxy" and "core radius of galaxy" help in understanding the concepts of "minimum flat rotation speed" and MOND's view of "minimum acceleration due to gravity". We are working in this direction.

Note-3: Considering the assumed "whole radius" of Milky Way, it is possible to show that, for any galaxy, $R_w \cong \frac{GM_G}{V_G^2} \cong \frac{16G\sqrt{M_0 M_G}}{c^2}, M_G \cong \left(\frac{cH_0}{128G}\right)R_w^2 \text{and} \frac{GM_G}{R_w^2} \cong \frac{cH_0}{128}$. Interesting points to be noted are,

(1) By knowing the radius of a galaxy, its total mass can be estimated by the simple relation, $M_G \cong 0.07605 R_w^2$. It needs observational study and support. According to very recent study (Igor D. Karachentsev et al 2021) (Igor et al. 2021), newly discovered dwarf galaxies, Do II, Do III and Do IV have effective radii of approximately 323, 495 and 596 light years, respectively. Based on this data, their estimated total masses seem to be 7.11×10^{35}kg, 1.67×10^{36}kg, and 2.42×10^{36}kg respectively.

(2) Considering the relation $\frac{GM_G}{R_w^2} \cong \frac{cH_0}{128}$, equ. (20), can be obtained as, $V_G \cong 0.29[(GM_G)(cH_0)]^{1/4}$.

On simplification,

$$V_{Gr} \cong \frac{1}{320}\left[1 + \ln\left(\frac{r}{r_c}\right) + \ln\left(\sqrt{\frac{M_G}{M_{Gv}}}\right)\right]\left(\frac{GM_G c^2}{2r}\right)^{1/4} \tag{22}$$

See Figures 3,4 pertaining to NGC3521 flat rotation curves. Figure 3 is our prepared toy model curve up to 300 kpc and Figure 4 is a open source curve up to 30 kpc. Equations. (21) and (22) seem to be applicable for distances above galactic core radii and needs a review for distances less than galactic core radii.

Considering the concept of "minimum galactic flat rotation speed", above equ. (22) can be simplified as follows.

$$V_{Gr} \cong \left[1 + \ln\left(\frac{r}{r_c}\right) + \ln\left(\sqrt{\frac{M_G}{M_{Gv}}}\right)\right]\left(\frac{GM_G(\text{V}_\text{G})^2_{\text{min}}}{2r}\right)^{1/4} \tag{23}$$

Advantage of this relation is that, it constitutes only two terms. First term seems to help in increasing star's rotation speed with increasing distance and second term seems to help in reducing star's orbiting speed with increasing distance. In the first term, ratio of total mass to visible mass and increasing ratio of star's orbiting distance to galactic core radius, seem to play an interesting role in maintaining a practically constant rotation speed of the galaxy. In the second term, total mass of galaxy, minimum galactic flat rotation speed of 3 km/sec and increasing star's distance play a crucial role.

It may be noted that, by considering $\ln\left(1 + \frac{r}{r_c}\right)$ in place of $\ln\left(\frac{r}{r_c}\right)$, for Sun-Earth system, estimated rotation speed is 7.9 km/sec and is comparable with actual speed of 30 km/sec. For Earth-Moon system, estimated rotation speed is 1.47 km/sec and is comparable with actual speed of 1.0 km/sec. Hence, qualitatively, it seems appropriate to write equ. (23) as,

Table 2. Estimated galactic masses and radii based on assumed flat rotation speeds.

Assumed Flat rotation speed (km/sec)	$\log_{10}\left(\frac{M_G}{M_\odot}\right)$	$\log_{10}\left(\frac{M_{Gd}}{M_\odot}\right)$	$\log_{10}\left(\frac{M_{Gv}}{M_\odot}\right)$	$\left(\frac{M_{Gd}}{M_{Gv}}\right)$	$\%M_{Gd}$	Core radius (kpc)	Whole radius (kpc)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
4.0	5.58	4.20	5.56	0.04	4.14	0.01	0.1
7.0	6.55	5.60	6.50	0.13	11.28	0.03	0.3
10.0	7.17	6.47	7.08	0.25	19.79	0.06	0.6
13.0	7.63	7.08	7.48	0.39	28.28	0.10	1.1
16.0	7.99	7.54	7.79	0.56	36.06	0.13	1.6
19.0	8.29	7.92	8.04	0.75	42.90	0.16	2.3
22.0	8.54	8.23	8.25	0.95	48.82	0.20	3.1
25.0	8.76	8.49	8.43	1.17	53.90	0.23	4.0
28.0	8.96	8.73	8.58	1.40	58.25	0.26	5.0
31.0	9.14	8.93	8.72	1.63	62.00	0.29	6.1
34.0	9.30	9.11	8.84	1.88	65.25	0.32	7.4
37.0	9.44	9.28	8.95	2.13	68.06	0.35	8.7
40.0	9.58	9.43	9.05	2.39	70.53	0.38	10.2
43.0	9.71	9.57	9.14	2.66	72.69	0.40	11.8
46.0	9.82	9.70	9.23	2.94	74.60	0.43	13.5
49.0	9.93	9.81	9.31	3.22	76.30	0.45	15.3
52.0	10.04	9.93	9.38	3.51	77.82	0.48	17.3
55.0	10.13	10.03	9.45	3.80	79.18	0.50	19.3
58.0	10.23	10.13	9.52	4.10	80.40	0.53	21.5
61.0	10.31	10.22	9.58	4.41	81.51	0.55	23.8
64.0	10.40	10.31	9.64	4.72	82.51	0.57	26.2
67.0	10.48	10.40	9.70	5.03	83.43	0.59	28.7
70.0	10.55	10.48	9.75	5.35	84.26	0.62	31.3
73.0	10.62	10.55	9.80	5.68	85.03	0.64	34.0
76.0	10.69	10.63	9.85	6.01	85.73	0.66	36.9
79.0	10.76	10.70	9.90	6.34	86.38	0.68	39.8
82.0	10.83	10.77	9.94	6.68	86.98	0.70	42.9
85.0	10.89	10.83	9.98	7.03	87.54	0.72	46.1
88.0	10.95	10.89	10.03	7.37	88.06	0.74	49.4
91.0	11.01	10.95	10.07	7.72	88.54	0.76	52.9
94.0	11.06	11.01	10.11	8.08	88.99	0.78	56.4
97.0	11.12	11.07	10.14	8.44	89.40	0.80	60.1
100.0	11.17	11.12	10.18	8.80	89.80	0.81	63.9
103.0	11.22	11.18	10.22	9.17	90.16	0.83	67.7
106.0	11.27	11.23	10.25	9.54	90.51	0.85	71.7
109.0	11.32	11.28	10.28	9.91	90.83	0.87	75.9
112.0	11.37	11.33	10.32	10.29	91.14	0.89	80.1
115.0	11.41	11.38	10.35	10.67	91.43	0.90	84.4
118.0	11.46	11.42	10.38	11.05	91.70	0.92	88.9
121.0	11.50	11.47	10.41	11.44	91.96	0.94	93.5
124.0	11.55	11.51	10.44	11.83	92.20	0.96	98.2
127.0	11.59	11.55	10.47	12.22	92.44	0.97	103.0
130.0	11.63	11.59	10.49	12.62	92.66	0.99	107.9
133.0	11.67	11.63	10.52	13.02	92.87	1.01	112.9
136.0	11.71	11.67	10.55	13.42	93.06	1.02	118.1
139.0	11.74	11.71	10.57	13.82	93.25	1.04	123.4
142.0	11.78	11.75	10.60	14.23	93.44	1.06	128.7
145.0	11.82	11.79	10.62	14.64	93.61	1.07	134.2
148.0	11.85	11.82	10.65	15.06	93.77	1.09	139.9
151.0	11.89	11.86	10.67	15.47	93.93	1.10	145.6
154.0	11.92	11.90	10.69	15.89	94.08	1.12	151.4
157.0	11.96	11.93	10.72	16.32	94.23	1.14	157.4
160.0	11.99	11.96	10.74	16.74	94.36	1.15	163.5
163.0	12.02	12.00	10.76	17.17	94.50	1.17	169.6
166.0	12.05	12.03	10.78	17.60	94.62	1.18	175.9
169.0	12.08	12.06	10.80	18.03	94.75	1.20	182.4
172.0	12.11	12.09	10.82	18.47	94.86	1.21	188.9
175.0	12.14	12.12	10.84	18.91	94.98	1.23	195.5

(*Continued*)

Table 2. (Continued).

Assumed Flat rotation speed (km/sec)	$\log_{10}\left(\frac{M_G}{M_\odot}\right)$	$\log_{10}\left(\frac{M_{Gd}}{M_\odot}\right)$	$\log_{10}\left(\frac{M_{Gv}}{M_\odot}\right)$	$\left(\frac{M_{Gd}}{M_{Gv}}\right)$	$\%M_{Gd}$	Core radius (kpc)	Whole radius (kpc)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
178.0	12.17	12.15	10.86	19.35	95.09	1.24	202.3
181.0	12.20	12.18	10.88	19.79	95.19	1.26	209.2
184.0	12.23	12.21	10.90	20.24	95.29	1.27	216.2
187.0	12.26	12.24	10.92	20.69	95.39	1.29	223.3
190.0	12.29	12.27	10.94	21.14	95.48	1.30	230.5
193.0	12.31	12.29	10.96	21.59	95.57	1.32	237.8
196.0	12.34	12.32	10.98	22.04	95.66	1.33	245.3
199.0	12.37	12.35	11.00	22.50	95.75	1.34	252.9
202.0	12.39	12.37	11.01	22.96	95.83	1.36	260.5
205.0	12.42	12.40	11.03	23.42	95.91	1.37	268.3
208.0	12.44	12.43	11.05	23.89	95.98	1.39	276.2
211.0	12.47	12.45	11.06	24.36	96.06	1.40	284.3
214.0	12.49	12.48	11.08	24.83	96.13	1.42	292.4
217.0	12.52	12.50	11.10	25.30	96.20	1.43	300.7
220.0	12.54	12.52	11.11	25.77	96.26	1.44	309.0
223.0	12.56	12.55	11.13	26.25	96.33	1.46	317.5
226.0	12.59	12.57	11.15	26.72	96.39	1.47	326.1
229.0	12.61	12.60	11.16	27.20	96.45	1.48	334.8
232.0	12.63	12.62	11.18	27.68	96.51	1.50	343.7
235.0	12.66	12.64	11.19	28.17	96.57	1.51	352.6
238.0	12.68	12.66	11.21	28.65	96.63	1.52	361.7
241.0	12.70	12.68	11.22	29.14	96.68	1.54	370.9
244.0	12.72	12.71	11.23	29.63	96.74	1.55	380.1
247.0	12.74	12.73	11.25	30.12	96.79	1.56	389.5
250.0	12.76	12.75	11.26	30.62	96.84	1.58	399.1
253.0	12.78	12.77	11.28	31.12	96.89	1.59	408.7
256.0	12.80	12.79	11.29	31.61	96.93	1.60	418.5
259.0	12.82	12.81	11.30	32.11	96.98	1.62	428.3
262.0	12.84	12.83	11.32	32.62	97.03	1.63	438.3
265.0	12.86	12.85	11.33	33.12	97.07	1.64	448.4
268.0	12.88	12.87	11.34	33.62	97.11	1.66	458.6
271.0	12.90	12.89	11.36	34.13	97.15	1.67	468.9
274.0	12.92	12.91	11.37	34.64	97.19	1.68	479.4
277.0	12.94	12.93	11.38	35.15	97.23	1.69	489.9
280.0	12.96	12.95	11.40	35.67	97.27	1.71	500.6
283.0	12.98	12.97	11.41	36.18	97.31	1.72	511.4
286.0	13.00	12.99	11.42	36.70	97.35	1.73	522.3
289.0	13.02	13.00	11.43	37.22	97.38	1.74	533.3
292.0	13.03	13.02	11.44	37.74	97.42	1.76	544.4
295.0	13.05	13.04	11.46	38.26	97.45	1.77	555.7
298.0	13.07	13.06	11.47	38.78	97.49	1.78	567.0
301.0	13.09	13.07	11.48	39.31	97.52	1.79	578.5
304.0	13.10	13.09	11.49	39.84	97.55	1.81	590.1
307.0	13.12	13.11	11.50	40.37	97.58	1.82	601.8
310.0	13.14	13.13	11.51	40.90	97.61	1.83	613.6
313.0	13.15	13.14	11.53	41.43	97.64	1.84	625.5
316.0	13.17	13.16	11.54	41.97	97.67	1.85	637.6
319.0	13.19	13.18	11.55	42.50	97.70	1.87	649.8
322.0	13.20	13.19	11.56	43.04	97.73	1.88	662.0
325.0	13.22	13.21	11.57	43.58	97.76	1.89	674.4
328.0	13.24	13.23	11.58	44.12	97.78	1.90	686.9
331.0	13.25	13.24	11.59	44.66	97.81	1.91	699.6
334.0	13.27	13.26	11.60	45.21	97.84	1.93	712.3
337.0	13.28	13.27	11.61	45.75	97.86	1.94	725.1
340.0	13.30	13.29	11.62	46.30	97.89	1.95	738.1
343.0	13.31	13.30	11.63	46.85	97.91	1.96	751.2
346.0	13.33	13.32	11.64	47.40	97.93	1.97	764.4
349.0	13.34	13.33	11.65	47.96	97.96	1.99	777.7

(*Continued*)

$$V_{Gr} \cong \left[1+\ln\left(1+\frac{r}{r_c}\right)+\ln\left[\left(\frac{M_G}{M_{Gv}}\right)^{1/4}\right]\right]\left(\frac{GM_G(V_G)^2_{min}}{2r}\right)^{1/4} \quad (24)$$

Table 2. (Continued).

Assumed Flat rotation speed (km/sec)	$\log_{10}\left(\frac{M_G}{M_\odot}\right)$	$\log_{10}\left(\frac{M_{Gd}}{M_\odot}\right)$	$\log_{10}\left(\frac{M_{Gv}}{M_\odot}\right)$	$\left(\frac{M_{Gd}}{M_{Gv}}\right)$	$\%M_{Gd}$	Core radius (kpc)	Whole radius (kpc)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
352.0	13.36	13.35	11.66	48.51	97.98	2.00	791.1
355.0	13.37	13.36	11.67	49.07	98.00	2.01	804.7
358.0	13.39	13.38	11.68	49.62	98.02	2.02	818.3
361.0	13.40	13.39	11.69	50.18	98.05	2.03	832.1
364.0	13.42	13.41	11.70	50.74	98.07	2.04	846.0
367.0	13.43	13.42	11.71	51.30	98.09	2.06	860.0
370.0	13.44	13.44	11.72	51.87	98.11	2.07	874.1
373.0	13.46	13.45	11.73	52.43	98.13	2.08	888.3
376.0	13.47	13.46	11.74	53.00	98.15	2.09	902.7
379.0	13.49	13.48	11.75	53.57	98.17	2.10	917.2
382.0	13.50	13.49	11.76	54.14	98.19	2.11	931.7
385.0	13.51	13.51	11.77	54.71	98.20	2.12	946.4
388.0	13.53	13.52	11.78	55.28	98.22	2.13	961.2
391.0	13.54	13.53	11.79	55.85	98.24	2.15	976.2
394.0	13.55	13.55	11.79	56.43	98.26	2.16	991.2
397.0	13.57	13.56	11.80	57.01	98.28	2.17	1006.3
400.0	13.58	13.57	11.81	57.59	98.29	2.18	1021.6
403.0	13.59	13.59	11.82	58.17	98.31	2.19	1037.0
406.0	13.61	13.60	11.83	58.75	98.33	2.20	1052.5
409.0	13.62	13.61	11.84	59.33	98.34	2.21	1068.1
412.0	13.63	13.62	11.85	59.91	98.36	2.22	1083.8
415.0	13.64	13.64	11.85	60.50	98.37	2.24	1099.7
418.0	13.66	13.65	11.86	61.09	98.39	2.25	1115.6
421.0	13.67	13.66	11.87	61.67	98.40	2.26	1131.7
424.0	13.68	13.67	11.88	62.26	98.42	2.27	1147.9
427.0	13.69	13.69	11.89	62.86	98.43	2.28	1164.2
430.0	13.71	13.70	11.90	63.45	98.45	2.29	1180.6
433.0	13.72	13.71	11.90	64.04	98.46	2.30	1197.1
436.0	13.73	13.72	11.91	64.64	98.48	2.31	1213.8
439.0	13.74	13.73	11.92	65.23	98.49	2.32	1230.5
442.0	13.75	13.75	11.93	65.83	98.50	2.33	1247.4
445.0	13.76	13.76	11.94	66.43	98.52	2.34	1264.4
448.0	13.78	13.77	11.94	67.03	98.53	2.35	1281.5
451.0	13.79	13.78	11.95	67.63	98.54	2.37	1298.7
454.0	13.80	13.79	11.96	68.24	98.56	2.38	1316.1
457.0	13.81	13.80	11.97	68.84	98.57	2.39	1333.5
460.0	13.82	13.82	11.97	69.45	98.58	2.40	1351.1
463.0	13.83	13.83	11.98	70.06	98.59	2.41	1368.8
466.0	13.85	13.84	11.99	70.66	98.60	2.42	1386.6
469.0	13.86	13.85	12.00	71.27	98.62	2.43	1404.5
472.0	13.87	13.86	12.00	71.89	98.63	2.44	1422.5
475.0	13.88	13.87	12.01	72.50	98.64	2.45	1440.6
478.0	13.89	13.88	12.02	73.11	98.65	2.46	1458.9
481.0	13.90	13.89	12.03	73.73	98.66	2.47	1477.3
484.0	13.91	13.91	12.03	74.34	98.67	2.48	1495.7
487.0	13.92	13.92	12.04	74.96	98.68	2.49	1514.3
490.0	13.93	13.93	12.05	75.58	98.69	2.50	1533.1
493.0	13.94	13.94	12.06	76.20	98.70	2.51	1551.9
496.0	13.95	13.95	12.06	76.82	98.72	2.52	1570.8
499.0	13.96	13.96	12.07	77.45	98.73	2.53	1589.9
502.0	13.97	13.97	12.08	78.07	98.74	2.54	1609.1

By understanding the actual meaning of "core radius" and considering $\left[\ln\left(1+\frac{r}{r_c}\right)\right]$ in place of $\ln\left(\frac{r}{r_c}\right)$ and considering $\ln\left[\left(\frac{M_G}{M_{Gv}}\right)^{1/4}\right]$ in place of $\ln\left(\sqrt{\frac{M_G}{M_{Gv}}}\right)$, it may be possible to develop an universal relation for flat rotation speeds starting from solar system to gigantic galaxies. It is for further study.

10. Estimation of galactic masses and radii

See Table 1 for the galactic baryonic and total masses and flat rotation speeds (in ascending order) taken from SPARC data base.

Assuming flat rotation speeds starting from 4 to 500 km/sec. we tried to estimate galactic total masses, visible masses and radii in Table 2.

Considering equ. (11), based on assumed galactic flat rotation speeds starting from 4.0 km/sec to 500 km/sec and by considering a simple C++ program, we have developed Table 2 pertaining to estimation of galactic masses and radii. In Table-2,

Column-1: Assumed flat rotation speed in km/sec.

Column-2: $\log_{10}$(total mass of galaxy/solar mass).

Column-3: $\log_{10}$(dark mass of galaxy/solar mass).

Column-4: $\log_{10}$(visible mass of galaxy/solar mass).

Column-5: Ratio of dark mass of galaxy to visible mass of galaxy.

Column-6: % of dark mass of galaxy.

Column-7: Galactic core radius in kpc = $\sqrt{\frac{M_0}{M_G}\left(\frac{2GM_{Gv}}{c^2}\right)}$where,$M_0 \cong \frac{c^3}{2GH_0}$.

Column-8: Galactic whole radius in kpc = $R_w \cong \frac{GM_G}{V_G^2} \cong \sqrt{\frac{M_0}{M_G}\left(\frac{16GM_G}{c^2}\right)} \cong \frac{16G\sqrt{M_0 M_G}}{c^2}$.

11. Cosmic visible matter density and dark matter density

With reference to our earlier published article (Seshavatharam & Lakshminarayana 2015) (Seshavatharam and Lakshminarayana 2015), recent paper (Seshavatharam & Lakshminarayana 2021) (Seshavatharam et al. 2021) and recent book on Flat Space Cosmology (Tatum and Seshavatharam 2021) (Tatum and Seshavatharam 2021), we are presenting the following semi-empirical relations pertaining to cosmic visible matter density $\left(\rho_v c^2\right)_t$ and dark matter density $\left(\rho_d c^2\right)_t$. We are working on understanding their physical significance in all possible ways. For the current case,

$$\begin{aligned}\left(\rho_v c^2\right)_0 &\cong [1+\ln(\Upsilon_0)]\sqrt{\left(\frac{3H_0^2c^2}{8\pi G}\right)\left(aT_0^4\right)} \\ &\cong \frac{[1+\ln(\Upsilon_0)]}{\sqrt{5760\pi}}\left(\frac{3H_0^2c^2}{8\pi G}\right) \cong 0.04425\left(\frac{3H_0^2c^2}{8\pi G}\right)\end{aligned} \quad (25)$$

$$\begin{aligned}\left(\rho_d c^2\right)_0 &\cong [1+\ln(\Upsilon_0)]^2\sqrt{\left(\frac{3H_0^2c^2}{8\pi G}\right)\left(aT_0^4\right)} \\ &\cong \frac{[1+\ln(\Upsilon_0)]^2}{\sqrt{5760\pi}}\left(\frac{3H_0^2c^2}{8\pi G}\right) \cong 0.2634\left(\frac{3H_0^2c^2}{8\pi G}\right)\end{aligned} \quad (26)$$

$$\left(\frac{\rho_d c^2}{\rho_v c^2}\right)_0 \cong [1+\ln(\Upsilon_0)] \cong 5.953 \quad (27)$$

Curiosity point of view, it is very interesting to note that, MOND's model of "minimum acceleration" can be expressed in the following way.

$$\begin{aligned}&\frac{cH_0}{6} \cong \left(\frac{\rho_v c^2}{\rho_d c^2}\right)_0 (cH_0) \\ &\text{where}\,\frac{1}{6} \cong \left(\frac{\rho_v c^2}{\rho_d c^2}\right)_0 \cong \frac{1}{5.953}\end{aligned} \quad (28)$$

12. Conclusion

In this paper, we have proposed a simple relation for estimating galactic dark masses based on the known galactic visible masses and tried to estimate the galactic flat rotation speeds. In doing so, we have introduced a cosmological increasing reference mass unit, by using which boundary line of dark effects can be understood. By increasing the number count of ultra light dwarf galaxies having very little dark matter and by finding super massive galaxies having dark matter in very large quantity, our approach can be reviewed and analysed for a better understanding and better formula. Proceeding further, MOND's concept of "minimum acceleration" can be replaced with Mach's concept of "distance cosmic mass". Finally, "effective models of dark gravity" can be developed in terms of galactic visible mass.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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