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FULL LENGTH ARTICLE

Visual contact for two satellites orbits under J_2-gravity

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Abstract In this paper, general analytical and computational technique for satellite-to-satellite visibility will be established firstly under the keplerian force, secondary under the effect of earth's gravitational field (oblateness). The development is generally in the sense that the visibility conditions can be used whatever the types of the satellite orbits may be. Many data are taken to illustrate our technique.

1. Introduction

A satellite under the influence of an inverse square gravitational law has truly constant orbital elements, that is, the set $[a, e, M, i, \Omega, \omega]$ is composed of constants devoid of explicit time dependency. For many practical problems, the approximation of two-body motion is sufficient, especially if two closely points on a trajectory are under investigation. There are situations in which the cumulative effect of the gradual shift or variation of elements from true epoch values according to perturbative forces cannot be ignored (Brouwer and Clemence, 1961 and Brouwer, 1959).

Mutually visible satellites are defined as two satellites that can maintain direct line of sight between each other for a certain length of time. We primarily concerned with the rise and set time of a given satellite with respect to another, that is, the time of loss or gain of direct line of sight (Noton, 1998 and Maini and Agrawal, 2007).

2. Rise-set function

2.1. Relative rise-set geometry

Consider the geometry defined in Fig. 1. As illustrated, satellites (1) and (2) are in a state of relative rise or set. Indeed, if the vector S, which emanates from the dynamical center of the Earth, had magnitude equal to or less than the radius of the Earth and if it were perpendicular to C, the chord length vector between the satellites, it is evident that the satellites would not have direct line-of-sight communication. Owing to atmospheric interference, however, a realistic analysis would let the magnitude of S be slightly larger than a_e, the radius of the Earth. Letting Δ be the thickness of the atmosphere or suitable bias factor, it follows that

$$S^2 = \vec{S}\cdot\vec{S} = (a_e + \Delta)^2. \quad (1.1)$$

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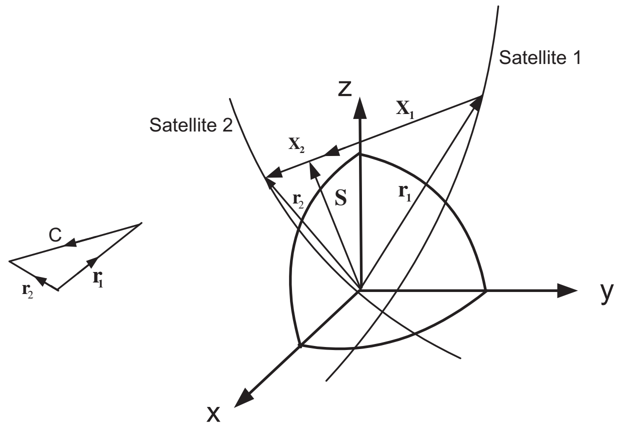

Fig. 1 Relative rise-set geometry.

2.2. Analytical expression of the relative rise-set function

Examination of Fig. 1 allows the two fundamental vector closure equations to be written as

$$\vec{r}_2 = \vec{S} + \vec{\chi}_2, \tag{2.2}$$

$$\vec{r}_1 = \vec{S} + \vec{\chi}_1, \tag{2.3}$$

where $\vec{r}_i; i = 1,2$ are the position vectors of the satellites and $\vec{\chi}_i; i = 1,2$ are two unknown vectors. At relative rise and set of satellite (1) with respect to satellite (2), we have

$$\vec{S}\cdot\vec{\chi}_1 = \vec{S}\cdot\vec{\chi}_2 = 0. \tag{2.4}$$

Then, from the figure

$$C = \sqrt{(\vec{r}_2 - \vec{r}_1)\cdot(\vec{r}_2 - \vec{r}_1)} = \sqrt{r_2^2 + r_1^2 - 2\alpha}, \tag{2.5}$$

where $\alpha = \vec{r}_1 \cdot \vec{r}_2$.

It is then possible to obtain an analytical expression of the rise and set function, from the planer triangle, (Escobal, 1965), as

$$R = (\vec{r}_1 \cdot \vec{r}_2)^2 - r_2^2 r_1^2 + \left(r_2^2 + r_1^2\right)S^2 - 2S^2(\vec{r}_1 \cdot \vec{r}_2), \tag{2.6}$$

where S is obtained from Eqs. (2.2) and (2.3).

The rise-set function defined by Eq. (2.6) can be used to predict explicitly whether or not satellites are visible to one another. The sign of R associated with visibility can be obtained by constructing a case in which direct line-of-sight visibility is impossibility as shown in Fig. 2; consequently we can get the rule that

1. Negative value of R→ direct line-of-sight communication.
2. Positive value of R→ non-visibility.

3. Reduction of rise-set function to a two-parameter function

In terms of the orbital eccentricity e, semi-parameter p and true anomaly f, the equation of each orbit can be expressed by the relation

$$r_i = \frac{p_i}{1 + e_i\cos(f_i)}, \quad i = 1,2.$$

Also we have

$$\vec{r}_i = \xi_i\vec{P}_i + \eta_i\vec{Q}_i, \quad i = 1,2.$$

where

$$\xi_i = r_i\cos(f_i),$$
$$\eta_i = r_i\sin(f_i).$$

The standard orientation vectors $\vec{P}$ and $\vec{Q}$, where $\vec{P}$ is a unit vector from the dynamical center which points at perigee of the orbit and $\vec{Q}$ is advanced to $\vec{P}$ by a right angle in the plane and direction of motion that is

$$P_{xi} = \cos(\omega_i)\cos(\Omega_i) - \sin(\omega_i)\sin(\Omega_i)\cos(I_i), \tag{3.1.1}$$
$$P_{yi} = \cos(\omega_i)\sin(\Omega_i) + \sin(\omega_i)\cos(\Omega_i)\cos(I_i), \tag{3.1.2}$$
$$P_{zi} = \sin(\omega_i)\sin(I_i), \tag{3.1.3}$$
$$Q_{xi} = -\sin(\omega_i)\cos(\Omega_i) + \cos(\omega_i)\sin(\Omega_i)\cos(I_i), \tag{3.1.4}$$
$$Q_{yi} = -\sin(\omega_i)\sin(\Omega_i) + \cos(\omega_i)\cos(\Omega_i)\cos(I_i), \tag{3.1.5}$$
$$Q_{zi} = \cos(\omega_i)\sin(I_i), \tag{3.1.6}$$

where ω_i is the argument of perigee, Ω_i is longitude of the ascending node and I_i is the orbital inclination (Escobal, 1965 and Kozai, 1959).

Now

$$\begin{aligned}\vec{r}_1 \cdot \vec{r}_2 &= (\xi_1\vec{P}_1 + \eta_1\vec{Q}_1)\cdot(\xi_2\vec{P}_2 + \eta_2\vec{Q}_2)\\ &= A_1\xi_1\xi_2 + A_2\eta_1\xi_2 + A_3\eta_2\xi_1 + A_4\eta_1\eta_2,\end{aligned} \tag{3.2}$$

where

$$A_1 = \vec{P}_1\cdot\vec{P}_2, A_2 = \vec{Q}_1\cdot\vec{P}_2, \quad A_3 = \vec{P}_1\cdot\vec{Q}_2, \quad A_4 = \vec{Q}_1\cdot\vec{Q}_2. \tag{3.3}$$

Using Eqs. (3.1) and (3.3) into Eq. (3.2) we get

$$\vec{r}_1 \cdot \vec{r}_2 = \frac{p_1\,p_2\{D_1\cos(f_2)\cos(\gamma_1 - f_1) + D_2\sin(f_2)\cos(\Psi_1 - f_2)\}}{[1 + e_1\cos(f_1)][1 + e_2\cos(f_2)]}, \tag{3.4}$$

where

$$\sin(\gamma_1) = \frac{A_2}{\sqrt{A_1^2 + A_2^2}}, \quad \cos(\gamma_1) = \frac{A_1}{\sqrt{A_1^2 + A_2^2}};$$

$$\sin(\Psi_1) = \frac{A_4}{\sqrt{A_3^2 + A_4^2}}, \quad \cos(\Psi_1) = \frac{A_4}{\sqrt{A_3^2 + A_4^2}};$$

and

$$D_1 = \sqrt{A_1^2 + A_2^2}, \quad D_2 = \sqrt{A_3^2 + A_4^2}. \tag{3.5}$$

Then the Eq. (2.6) become

$$\begin{aligned}R &= p_1^2\,p_2^2[D_1\cos(f_2)\cos(\gamma_1 - f_1) + D_2\sin(f_2)\cos(\Psi_1 - f_1)]^2\\ &\quad - p_1^2\,p_2^2 + S^2\left(p_1^2[1 + e_2\cos(f_2)]^2 + p_2^2[1 + e_1\cos(f_1)]^2\right)\\ &\quad - 2S^2 p_1\,p_2[D_1\cos(f_2)\cos(\gamma_1 - f_1) + D_2\sin(f_2)\cos(\Psi_1 - f_1)]\\ &\quad \times [1 + e_1\cos(f_1)][1 + e_2\cos(f_2)].\end{aligned} \tag{3.6}$$

If the two satellites in the same orbital plane we have

$$\vec{P}_1 = \vec{P}_2, \vec{Q}_1 = \vec{Q}_2 \Rightarrow A_1 = 1, A_2 = 0, A_3 = 0, A_4 = 1 \Rightarrow D_1 = 1, D_2 = 1, \gamma_1 = 0, \Psi_1 = 90^\circ.$$

4. Variation of the orbital's elements

The expansion of a set of elements $[a, e, M, i, \Omega, \omega]$ about some epoch time t_0 can be attempted by Taylor expansions as

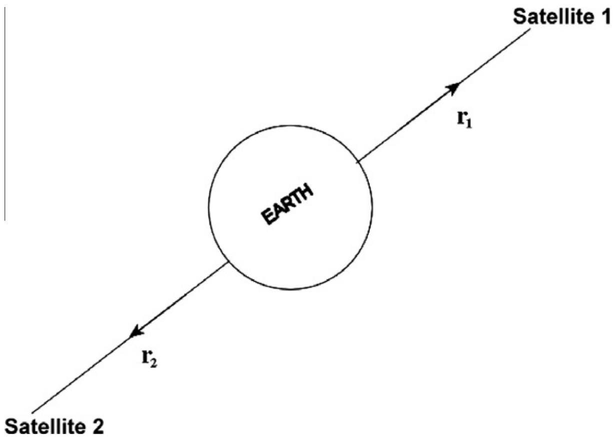

Fig. 2 Direct line-of-sight visibility.

$$a = a_0 + \dot{a}_0(t - t_0) + \frac{\ddot{a}_0(t - t_0)^2}{2!} + \cdots,$$
$$e = e_0 + \dot{e}_0(t - t_0) + \frac{\ddot{e}_0(t - t_0)^2}{2!} + \cdots$$
.
.
.
$$\omega = \omega_0 + \dot{\omega}_0(t - t_0) + \frac{\ddot{\omega}_0(t - t_0)^2}{2!} + \cdots$$

the previous series expansions are all encompassing in the sense of describing the physical situations. Analytical investigation of the oblateness effects of a central body on a satellite has shown that certain elements, such as M, Ω, ω, experience secular variations (increasing or decreasing) from the adopted epoch values and periodic variations about these epoch values. Other elements such as a,i and e are possessed of only periodic variations, a further distinction is made between short period variation and long period variations.

The oblateness variations are caused by the continuous variance of ω, owing the fact that the trigonometric functions of ω have secular variations with period 2π (Escobal, 1965).

Finally, secular variations are associated with a steady no-oscillatory. Continuous drift of an element from the adopted epoch value, short are associated with the trigonometric functions of linear combination of f and ω, and long period variations with trigonometric functions of ω (Escobal, 1965).

So that the total variance of the element q can be written as

$$q = q_0 + \dot{q}_0(t - t_0) + k_1\cos(2\omega) + k_2\cos(2\omega + 2f)$$

(Escobal, 1965).

Since the perturbative function

$$\widetilde{R} = \Phi - V,$$

where

$\widetilde{R}$: the perturbative function, Φ: the potential of the Earth, V: the potential of purely spherical Earth.

The potential of Earth is given (Escobal, 1965), by

$$\Phi = \frac{k^2 m}{r}\left[1 + \frac{J_2}{2r^2}(1 - 3\sin^2\delta) + \frac{J_3}{2r^3}(3 - 5\sin^2\delta)\sin\delta - \frac{J_4}{8r^4}(3 - 30\sin^2\delta - 35\sin^4\delta) - \frac{J_5}{8r^5}(15 - 70\sin^2\delta + 63\sin^4\delta)\sin\delta + \cdots\right]$$

where

m: the mass of the Earth, k: the gravitational constant, J_i: coefficient of the ith harmonic.

Since the equation of a conic is

$$r = \frac{a(1 - e^2)}{1 + e\cos(f)},$$

and we have the relation

$$\sin(\delta) = \sin(i)\sin(f + \omega),$$

then perturbative function will be in the following form to the order of J_4 (Escobal, 1965)

$$\widetilde{R} = k^2 m\left[\frac{3}{2}\frac{J_2}{a^3}\left(\frac{a}{r}\right)^3\left\{\frac{1}{3} - \frac{1}{2}\sin^2 i + \frac{1}{2}\sin^2 i\cos 2(f + \omega)\right\} - \frac{J_4}{a^4}\left(\frac{a}{r}\right)^4\left\{\left(\frac{15}{8}\sin^2 i - \frac{3}{2}\right)\sin(f + \omega) - \frac{5}{8}\sin^2 i\sin 3(f + \omega)\right\} \sin i - \frac{35}{8}\frac{J_4}{a^5}\left(\frac{a}{r}\right)^5\left\{\frac{3}{35} - \frac{3}{7}\sin^2 i + \frac{3}{8}\sin^4 i + \sin^2 i\left(\frac{3}{7} - \frac{1}{2}\sin^2 i\right)\cos 2(f + \omega) + \frac{1}{8}\sin^4 i\cos 4(f + \omega)\right\}\right]$$

where

a : is the semi-major axis of the orbit, e : is the orbital eccentricity, f: is the true anomaly, i: is the orbit inclination, ω: is the argument of perigee.

We interest with the secular variation, so we omitted the short and long period terms from the perturbative function, which will be written as

$$\widetilde{R} = k^2 m\left[\frac{3}{2}\frac{J_2}{a^3}\left(\frac{a}{r}\right)^3\left\{\frac{1}{3} - \frac{1}{2}\sin^2 i\right\} - \frac{35}{8}\frac{J_4}{a^5}\left(\frac{a}{r}\right)^5\left\{\frac{3}{35} - \frac{3}{7}\sin^2 i + \frac{3}{8}\sin^4 i\right\}\right].$$

Or for only J_2–gravity

$$\widetilde{R} \approx k^2 m\left[\frac{3}{2}\frac{J_2}{a^3}\left(\frac{a}{r}\right)^3\left\{\frac{1}{3} - \frac{1}{2}\sin^2 i\right\}\right].$$

Since we are not interested with the periodic variation of the elements, the previous equation may be averaged over the given revolution as

$$\overline{\left(\frac{a}{r}\right)^3} = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{a}{r}\right)^3 dM = \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{a}{r}\right)^3\left(\frac{r}{a}\right)^2\frac{1}{\sqrt{1 - e^2}}df = (1 - e^2)^{3/2}.$$

Then

$$\widetilde{R} = k^2 m\left[\frac{3}{2}\frac{J_2}{a^3}(1 - e^2)^{3/2}\left\{\frac{1}{3} - \frac{1}{2}\sin^2 i\right\}\right]. \tag{4.1}$$

4.1. Rate of change of the elements

It is possible to verify that the perturbative function, the elements of the orbit and time are related by Lagrange's planetary equations

$$\frac{da}{dt} = \frac{2}{na}\frac{\partial \widetilde{R}}{\partial M},$$
$$\frac{de}{dt} = \frac{1-e^2}{na^2 e}\frac{\partial \widetilde{R}}{\partial M} - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial \widetilde{R}}{\partial \omega},$$
$$\frac{d\omega}{dt} = -\frac{\cos I}{na^2 \sin I\sqrt{1-e^2}}\frac{\partial \widetilde{R}}{\partial I} - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial \widetilde{R}}{\partial e}, \quad (4.2.1)$$
$$\frac{dI}{dt} = \frac{\cos I}{na^2 \sin I\sqrt{1-e^2}}\frac{\partial \widetilde{R}}{\partial \omega},$$
$$\frac{d\Omega}{dt} = \frac{1}{na^2 \sin I\sqrt{1-e^2}}\frac{\partial \widetilde{R}}{\partial I},$$
$$\frac{dM}{dt} = n - \frac{1-e^2}{na^2 e}\frac{\partial \widetilde{R}}{\partial e} - \frac{2}{na}\frac{\partial \widetilde{R}}{\partial a}.$$

From the Eq. (4.1) of $\widetilde{R}$ and by the previous equations we find that parameters a, e, i experience no secular variations (Escobal, 1965; Sterne, 1960 and Brouwer, 1959).

Mathematically, the mean anomaly M is defined as

$$M = n(t - T),$$

where

T: the time of perifocal passage, n: the unperturbed mean motion.

To calculate the variations in the parameters which experience the secular variations, we find that

$$\frac{dM}{dt} = n\left[1 + \frac{3}{2}J_2\frac{\sqrt{1-e^2}}{p^2}\left\{1 - \frac{3}{2}\sin^2 i\right\}\right] \equiv \bar{n},$$

and

$$\int d\Omega = \int \left(\frac{1}{na^2 \sin I\sqrt{1-e^2}}\right)\left(-\frac{3}{2}K^2 m\frac{J_2}{a^3}(1-e^2)^{3/2}\right)\sin I \times \cos I dt = -\frac{3}{2}\frac{J_2}{p^2}\cos I\int \bar{n}dt$$

Or

$$\Omega = \Omega_0 - \left(\frac{3}{2}\frac{J_2}{p^2}\cos i\right)\bar{n}(t - t_0).$$

By same way we find that

$$\omega = \omega_0 + \frac{3}{2}\frac{J_2}{p^2}\left(2 - \frac{5}{2}\sin^2 i\right)\bar{n}(t - t_0),$$

and

$$M = M_0 + \left(\frac{3}{2}J_2\frac{\sqrt{1-e^2}}{p^2}\left(2 - \frac{5}{2}\sin^2 i\right)\right)\bar{n}(t - t_0) + n_0(t - t_0).$$

5. Computational algorithm

In what follows computational algorithm of the mutual visibility between two Earth Satellites will be established whatever the types of their orbits may be.

Purpose: Mutual visibility between two Earth satellites.

Input:

a_i or $q_i, e_i, I_i, \omega_i, \Omega_i, T_i;\ i = 1, 2, S, \Delta, t, k, \mu.$

Computational sequence:

1. If $e_i > 1$ then $n_i = k\sqrt{\frac{\mu}{-a_i^3}}$ and $q_i = a_i(1 - e_i)$.
2. If e_i 1 then $n_i = k\sqrt{\frac{\mu}{2q_i^3}}$.
3. If $e_i < 1$ then $n_j = k\sqrt{\frac{\mu}{a_i^3}}$ and q_i a_i $(1 - e_i)$.
4. $M_i = n_i(t - T_i)$.
5. If $e_i > 1$ then solve F_i from Kepler equation of hyperbolic orbit using Newton method and then f_i as follows
   (a) Let $(F_i)_0 = 6M_i$,
   (b) $(F_i)_{n+1} = (F_i)_n + \frac{M_i - e_i \sin h(F_i)_n + (F_i)_n}{e_i \cos h(F_i)_n - 1}$,
   (c) If $|(F_i)_{n+1} - (F_i)_n| > 0.00000001$ go to b else $F_i = (F_i)_{n+1}$,
   (d) $f_i = \tan^{-1}\left(\frac{-\sin h(F_i)\sqrt{e_i^2 - 1}}{\cos h(F_i) - e_i}\right)$ and end.
6. If e_i 1 then solve f_i from Barkar's equation as follows
   (a) Let $A_i = \frac{3}{2}M_i$,
   (b) $B_i = \left(\sqrt{A_i^2 + 1} + A_i\right)^{1/3}$,
   (c) $C_i = B_i - \frac{1}{B_i}$,
   (d) $f_i = 2\tan^{-1}(C_i)$.
7. If $e_i < 1$ then solve for E_i from Kepler's equation using Newton method and then f_i as follows
   (a) Let $(E_i)_0 = M_i$,
   (b) $(E_i)n + 1 = (E_i)n + \frac{M_i + e_i \sin(E_i)_n - (E_i)_n}{1 - e_i \cos(E_i)_n}$,
   (c) If $|(E_i)_{n+1} - (E_i)_n| > 0.00000001$ go to b else $E_i = (E_i)_{n+1}$,
   (d) $f_i = \tan^{-1}\left(\frac{\sin(E_i)\sqrt{1 - e_i^2}}{\cos(E_i) - e_i}\right)$ and end.
8. $r_i = \frac{(1 + e_i)q_i}{1 + e_i \cos f_i}$.
9. $\vec{P}$ and $\vec{Q}$ from Eq. (3).
10. $\xi_i = r_i\cos(f_i)$ and $\eta_i = r_i\sin(f_i)$.
11. $\vec{r}_i = \xi_i P_i + \eta_i Q_i$.
12. Compute the mutual visibility function R from Eq. (2.6). Whenever this value is negative, the satellites can see each other at the given time t.
13. The algorithm is completed.

6. Results and conclusion

6.1. Test orbits

We will take as an example the following seven satellites. Satellite_1, Satellite_2, Satellite_3 and Satellite_4 are nearly circular, SatellitPe_5 is elliptical orbit, but Satellite_6 is parabolic orbit and finally Satellite_7 is hyperbolic orbit (http://celestrak.com). The three-line elements of seven satellites are:

Satellite_1:
EGYPTSAT 1

```
1 31117U 07012A   08142.74302347  .00000033  00000-0  13654-4 0  2585
2 31117  098.0526 218.7638 0007144 061.2019 298.9894 14.69887657 58828
```

Satellite_2:
TRMM

```
1 25063U  97074A  08141.84184490  .00002948  00000-0  41919-4  0  7792
2 25063  034.9668  053.5865  0001034  271.1427  088.9226  15.55875272598945
```

Satellite_3:
GOES 3

```
1 10953U  78062A  08140.64132336  -.00000110  00000-0  10000-3  0  1137
2 10953  014.2164  003.1968  0001795  336.4858  023.4617  01.00280027  62724
```

Satellite_4:
NOAA 3

```
1 06920U  73086A  08141.92603915 -.00000030+00000-0 +10000-3  0  00067
2 06920  101.7584  171.9430  0006223  187.3360  172.7614  12.40289355563642
```

Satellite_5:
NAVSTAR 46

```
1 25933U  99055A  08142.14123352  .00000019  00000-0  10000-3  0  00126
2 25933  051.0650  222.9439  0079044  032.8625  327.6958  02.00568102  63184
```

Satellite_6:
Parabola

```
1 00000U  00000A  08141.53396007  .00000000  00000-0  00000-0  0  00001
2 00000  035.3423  067.8765  001.000  253.7654  138.0987  02.65786544  63184
```

*Satellite_7:*Hyp_1

```
1 00000U  00000A  08141.89332000  .00000000  00000-0  00000-0  0  00001
2 00000  072.8721  105.6746  001.164  065.8757  221.4654  02.00568102  63184
```

6.2. Numerical results

We apply the above algorithm on some of the seven satellites to get the time and date of what satellite observes the other. Tables 1–7 give these results.

Table 1 EGYPTSAT_1 and TRMM are visible during the times.

Date			Time			to	Date			Time		
Year	Month	Day	Hour	Minute	Seconds		Year	Month	Day	Hour	Minute	Seconds
2008	5	22	12	22	25.99	To	2008	5	22	12	29	50.99
			13	10	39.98					13	16	49.98
			13	58	56.96					14	3	41.96
			14	47	26.95					14	50	24.94
			22	43	18.79					22	46	30.79
			23	30	05.78					23	34	54.77
2008	5	23	0	16	56.77	to	2008	5	23	0	23	15.76
			1	3	58.74					1	11	25.74
			1	50	59.73					1	59	35.73
			2	38	07.71					2	47	37.71
			3	25	15.70					3	35	40.70
			4	12	30.68					4	23	36.68
			4	59	45.67					5	11	31.66
			5	47	06.65					5	59	20.65
			6	34	28.64					6	47	08.63
			7	21	57.62					7	34	50.62
			8	9	26.61					8	15	16.60
			8	16	39.60					8	22	30.60
			8	57	02.59					9	2	52.59
			9	4	16.59					9	10	4.59
			9	44	39.58					9	57	36.57
			10	32	23.56					10	45	03.56
			11	20	08.54					11	32	28.54

Table 2 EGYPTSAT_1 and GOES_3 are visible during the times.

Date			Time			to	Date			Time		
Year	Month	Day	Hour	Minute	Seconds		Year	Month	Day	Hour	Minute	Seconds
2008	05	22	12	14	01.00	To	2008	05	22	13	17	55.97
			13	47	46.96					14	09	51.96
			14	28	06.95					14	47	22.95
			15	23	50.93					15	39	18.93
			16	06	46.92					16	21	13.91
			17	00	01.90					17	13	25.90
			17	43	17.89					17	56	41.88
			18	35	44.87					18	49	30.87
			19	18	05.86					19	32	47.85
			20	10	12.84					20	27	18.83
			20	49	25.83					21	08	59.82
			21	41	22.81					22	44	05.79
			23	04	17.78		2008	05	23	00	08	51.76
2008	05	23	00	27	23.76	To	2008	05	23	01	30	38.74
			02	02	05.73					02	22	33.72
			02	43	22.71					03	01	10.71
			03	38	15.69					03	53	15.69
			04	21	27.68					04	35	24.68
			05	14	24.66					05	27	48.66
			05	57	42.65					06	11	02.64
			06	49	58.63					07	04	09.63
			07	32	07.62					07	47	11.61
			08	24	04.60					08	42	26.60
			09	02	25.59					09	23	18.58
			09	54	14.57					10	57	40.55
			11	15	36.55					12	00	00.00

Table 3 Hyp_1 and Parabola are visible during the times.

Date			Time				Date			Time		
Year	Month	Day	Hour	Minute	Seconds	to	Year	Month	Day	Hour	Minute	Seconds
2008	05	22	12	00	00.00	To	2008	05	23	12	00	00.00

Table 4 EGYPTSAT_1 and Hyp_1are visible during the times.

Date			Time			to	Date			Time		
Year	Month	Day	Hour	Minute	Seconds		Year	Month	Day	Hour	Minute	Seconds
2008	05	22	12	24	36.00	to	2008	05	22	12	46	47.98
			13	13	38.98					13	35	45.97
			14	02	32.96					14	24	43.95
			14	51	35.94					15	13	41.94
			15	40	28.93					16	02	39.92
			16	29	32.91					16	51	37.91
			17	18	25.90					17	40	36.89
			18	07	29.88					18	29	34.87
			18	56	22.86					19	18	32.86
			19	45	26.85					20	07	30.84
			20	34	19.83					20	56	28.83
			21	23	23.82					21	45	26.81
			22	12	17.80					22	34	25.79
			23	01	20.78					23	23	23.78
			23	50	14.77		2008	05	23	00	12	22.76
2008	05	23	00	39	17.75	to	2008	05	23	01	01	19.75
			01	28	11.74					01	50	18.73
			02	17	14.72					02	39	16.71
			03	06	08.71					03	28	15.70
			03	55	11.69					04	17	13.68
			04	44	05.67					05	06	12.67
			05	33	08.66					05	55	09.65
			06	22	02.64					06	44	08.63
			07	11	05.63					07	33	06.62
			08	00	00.61					08	22	05.60
			08	49	03.59					09	11	03.59
			09	37	57.58					10	00	02.57
			10	27	00.56					10	49	00.55
			11	15	54.55					11	37	59.54

Table 5 GOES_3 and Parabola are visible in the time.

Date			Time			to	Date			Time		
Year	Month	Day	Hour	Minute	Seconds		Year	Month	Day	Hour	Minute	Seconds
2008	05	22	12	00	00.00		2008	05	23	12	00	00.00

Table 6 EGYPTSAT_1 and Parabola are visible in times.

Date			Time			to	Date			Time		
Year	Month	Day	Hour	Minute	Seconds		Year	Month	Day	Hour	Minute	Seconds
2008	05	22	12	00	00.00	to	2008	05	22	12	01	50.00
			12	29	50.99					12	50	53.98
			13	18	51.97					13	39	47.97
			14	07	48.96					14	28	51.95
			14	56	49.94					15	17	45.94
			15	45	46.93					16	06	49.92
			16	34	47.91					16	55	43.90
			17	23	44.89					17	44	47.89
			18	12	45.88					18	33	41.87
			19	01	42.86					19	22	45.86
			19	50	44.85					20	11	39.84
			20	39	40.83					21	00	43.82
			21	28	42.81					21	49	37.81
			22	17	38.80					22	38	41.79
			23	06	40.78					23	27	35.78
			23	55	36.77		2008	05	23	00	16	39.76
2008	05	23	00	44	38.75	to	2008	05	23	01	05	33.74
			01	33	34.74					01	54	37.73
			02	22	36.72					02	43	31.71
			03	11	32.70					03	32	35.70
			04	00	34.69					04	21	29.68
			04	49	30.67					05	10	33.66
			05	38	32.66					05	59	27.65
			06	27	28.64					06	48	31.63
			07	16	30.62					07	37	25.62
			08	05	26.61					08	26	29.60
			08	54	28.59					09	15	23.58
			09	43	24.58					10	04	27.57
			10	32	26.56					10	53	21.55
			11	21	22.54					11	42	25.54

Table 7 TRMM and Hyp_1 are visible during the times.

Date			Time			to	Date			Time		
Year	Month	Day	Hour	Minute	Seconds		Year	Month	Day	Hour	Minute	Seconds
2008	05	22	12	10	49.00	to	2008	05	22	12	20	58.99
			12	57	3.98					13	7	14.98
			13	43	22.97					13	53	32.96
			14	29	37.95					14	39	48.95
			15	15	56.94					15	26	06.93
			16	02	11.92					16	12	22.92
			16	48	30.91					16	58	41.90
			17	34	45.89					17	44	56.89
			18	21	04.88					18	31	15.87
			19	07	19.86					19	17	30.86
			19	53	38.85					20	3	48.84
			20	39	53.83					20	50	04.83
			21	26	12.82					21	36	22.81
			22	12	27.80					22	22	38.80
			22	58	46.79					23	8	56.78
			23	45	01.77					23	55	12.77
2008	05	23	00	31	20.76	to	2008	05	23	00	41	30.75
			01	17	35.74					01	27	46.74
			02	03	54.73					02	14	04.72
			02	50	09.71					03	00	20.71
			03	36	28.70					03	46	38.69
			04	22	43.68					04	32	54.68
			05	09	01.67					05	19	12.66
			05	55	17.65					06	05	27.65
			06	41	35.63					06	51	46.63

Table 7 (*continued*) TRMM and Hyp_1 are visible during the times.

Date			Time			to	Date			Time		
Year	Month	Day	Hour	Minute	Seconds		Year	Month	Day	Hour	Minute	Seconds
			07	27	51.62					07	38	01.62
			08	14	09.60					08	24	19.60
			09	00	24.59					09	10	35.59
			09	46	43.57					09	56	53.57
			10	32	58.56					10	43	09.56
			11	19	17.54					11	29	27.54

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