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# The effect of thermal dispersion on unsteady MHD convective heat transfer through vertical porous

Ghada F. Mohamadien *

*National Research Institute of Astronomy and Geophysics, Helwan, Cairo, Egypt*

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**KEYWORDS**

Magneto-hydro-dynamics;
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**Abstract** The influence of thermal dispersion on unsteady two-dimensional laminar flow is presented. A viscous incompressible conducting fluid in the vicinity of a semi infinite vertical porous through a moving plate in the presence of a magnetic fluid is studied. A cod (FORTRAN) was constructed for numerical computations for the velocity and temperature for various values of the affected parameters were carried out.



## 1. Introduction

The study of flow and heat transfer for an electrically conducting fluid through a porous moving plate under the influence of a magnetic field has attracted the interest of many investigators. The applications have been presented in many scientific problems such as magneto-hydro-dynamic (MHD) studies, plasma studies, nuclear reactors, and the field of aerodynamics. Gribben (1965) has considered the MHD boundary layer flow over a semi-infinite plate with an aligned magnetic field in the presence of a pressure gradient. He has obtained solutions for large and small magnetic Prandtle numbers using the method of matched asymptotic expansion. Takhar and Ram (1991) have studied the MHD free porous convection heat transfer of water at 4 °C through a porous medium. Soudalgekar (1973) has obtained approximate solutions for two-dimensional flow of an incompressible, viscous fluid past with constant suction velocity normal to the plate. The difference between the temperature of the plate and the free steam is moderately large causing the free convection current. Raptis (1986) has studied mathematically the case of time-varying of two-dimensional natural convective heat transfer of an infinite vertical porous plate. The study of Darcian porous MHD is much complex. It is necessary to consider in detail the distribution of velocity and temperature in addition to the surface skin friction across the boundary layer. All these studies assume that thermal diffusivity is constant, however, under the conditions at which the internal effects are prevalent, the thermal dispersion effect become significant as observed in Plumb (1987), Hong and Tien (1987), Nield and Bejan (1992). Tomer et al., 2010 have studied the flow and heat transfer characteristics for natural convection along an inclined plate in a saturated porous medium with an applied magnetic field. Ramana Reddy et al. (2011) have analyzed the influence of first order homogenous chemical reaction and thermal radiation on hydro-magnetic free convection heat and mass transfer of a viscous fluid past a semi infinite vertical moving porous plate embedded in a porous medium.

In the present work, the investigation of the effects of a magnetic field, porous medium, thermal dispersion and the exponential index on unsteady MHD convective heat transfer

* Tel.: +20 02 25560046.
E-mail address: ghfm2006@yahoo.com

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through a semi-infinite vertical porous moving plate will be carried out. Numerical computations for the velocity and temperature for various values of the parameters will be obtained.

## 2. Mathematical analysis

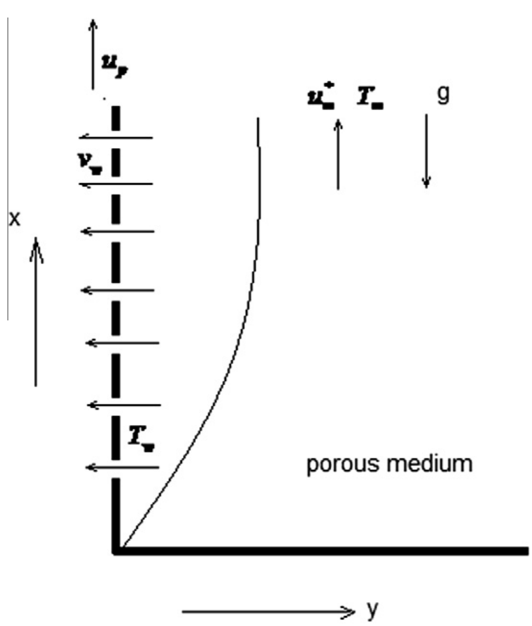
Suppose that two-dimensional unsteady flow of a laminar, incompressible fluid past a semi-infinite vertical porous moving plate embedded in a porous medium and subjected to a transverse magnetic field. Moreover, there is no applied voltage which implies the absence of an electric field. The induced magnetic field is negligible, viscous and Darcy's resistance terms are taken into account with constant permeability of the porous medium.

The MHD term is derived from an order of magnitude analysis of the full Navier–Stokes equations. Under these conditions, the governing equations, which mean the continuity, momentum and energy conservation equations expressed in a Cartesian coordinates as:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \upsilon \frac{\partial^2 u^*}{\partial y^*} + g\beta(T - T_\infty) - \upsilon \frac{u^*}{k^*} - \frac{\sigma}{\rho}\beta_0^2 u^* \tag{2}$$

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{\partial}{\partial y^*}\left(\alpha_{y^*}\frac{\partial T}{\partial y^*}\right) \tag{3}$$




Eq. (1) is the continuity equation, Eq. (2) is the momentum equation, and Eq. (3) is the energy equation Youn (2000). Where $x^*$ and $y^*$ are the dimensional distances in the directions along and perpendicular to the plate plan, respectively $u^*$, $v^*$ are the components of dimensional velocities along $x^*$ and $y^*$ directions, respectively, $t^*$ is the dimensional time, $\rho$ is the density of the medium, $^*$superscript for dimensional properties, $g$ is the gravitational constant, $\infty$ subscript evaluated at the outer edge of the boundary, $p^*$ is the pressure, $\beta$ is the thermal expansion coefficient, $\beta_0$ is the magnetic flux density, $\upsilon$ is the kinematic viscosity, $k^*$ is permeability of the porous medium, and $T^*$ is dimensional temperature. $\alpha_{y^*}$ is the component of thermal diffusion in $y^*$ direction, it is a variable quantity which is the sum of molecular thermal diffusivity $\alpha$ and dispersion thermal diffusivity $\alpha_d$. The expansion for dispersion thermal diffusivity will be $\alpha_d = \gamma du$ Plumb (1987). Where $\gamma$ is mechanical dispersion coefficient whose value depends on the experiment method, and d is the pore diameter.

The third term at the RHS of the momentum Eq. (2) denotes buoyancy effects. The fourth term is the bulk matrix linear resistance i.e., Darcy term. The fifth is the MHD term. It is assumed that porous plate moves with constant velocity in the direction of fluid flow, and the free stream velocity follows the exponentially increasing small perturbation law. Moreover, the plate temperature and velocity are exponentially varying with time.

Under these assumptions, the appropriate boundary conditions for the velocity and temperature are

$$u^* = u_p^* \; T = T_w + \varepsilon(T_w + T_\infty)e^{n^* t^*} \quad \text{at} \quad y^* = 0 \tag{4}$$

$$u^* \to u_\infty^* = u_0(1 + \varepsilon\, e^{n^* t^*}), \quad T \to T_\infty \quad \text{as} \quad y^* \to \infty \tag{5}$$

where subscripts $w$ evaluated on the wall, and $p$ evaluated on the plate. $n$ exponential index. From the continuity Eq. (1), it is clear that the suction velocity normal to the plate is a function of time only which yields to

$$v^* = -v_0(1 + \varepsilon\, A e^{n^* t^*}) \tag{6}$$

where $A$ is a real positive constant, $\varepsilon$ is a small parameter, so that $\varepsilon\ A$ still smaller than unity, and $v_0$ is a scale of suction velocity which has non-zero positive constants.

The dimensionless variables will be introduced as

$$\begin{aligned} u &= \frac{u^*}{u_0}, \quad v = \frac{v^*}{v_0}, \quad y = \frac{v_0}{\upsilon} y^* \\ u_\infty &= \frac{u_\infty^*}{u_0}, \quad u_p = \frac{u_p^*}{u_0}, \quad t = \frac{t^* v_0^2}{\upsilon} \\ \theta &= \frac{T - T_\infty}{T_w - T_\infty}, \quad n = \frac{n^* \upsilon}{v_0^2}, \quad k = \frac{k^* v_0^2}{\upsilon^2} \end{aligned} \tag{7}$$

Then

$$\begin{aligned} P_r &= \frac{\upsilon\, \rho\, c_p}{k} = \frac{\upsilon}{\alpha}, \quad M = \frac{\sigma\, B_0^2\, \upsilon}{\rho\, v_0^2} \\ G &= \frac{\upsilon\, B\, g(T_w - T_\infty)}{u_0\, v_0^2} \end{aligned} \tag{8}$$

where $p_r$ Prandtl number, $G$ Grashof number, and $M$ magnetic field parameter. In view of Eqs. (6)–(8) the governing Eqs. (2) and (3) are reduced to the non-dimensional form

$$\frac{\partial u}{\partial t} - (1 + \varepsilon\, A e^{nt})\frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + \frac{\partial^2 u}{\partial y^2} + G\theta + N(U_\infty - u) \tag{9}$$

$$\frac{\partial \theta}{\partial t} - \left(1 + \varepsilon\, A e^{n^* t^*}\right)\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} + \frac{\gamma d u_0}{\upsilon}\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y} \tag{10}$$

where $\theta N = \left(M + \frac{1}{k}\right)$

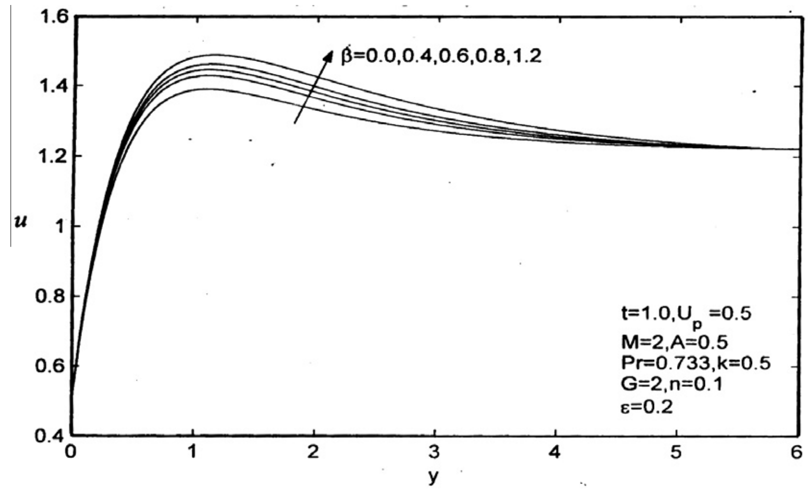



**Fig. 1** Velocity profiles against $y$ for different values of thermal dispersion $\beta$.

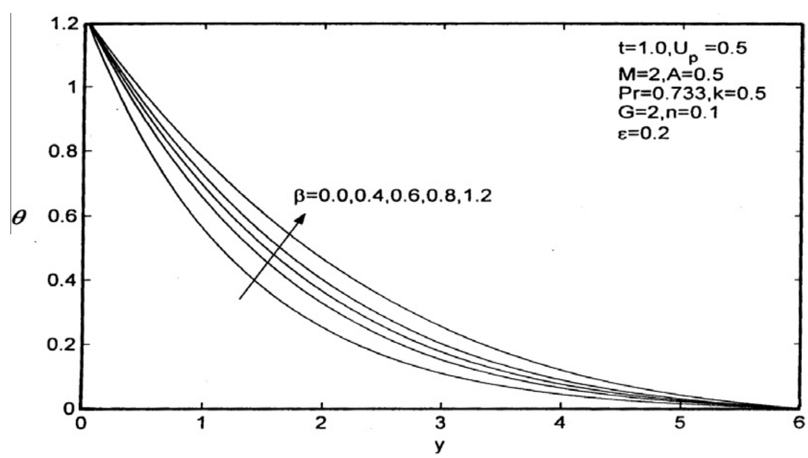



**Fig. 2** Temperature profiles against $y$ for different values of thermal dispersion $\beta$.

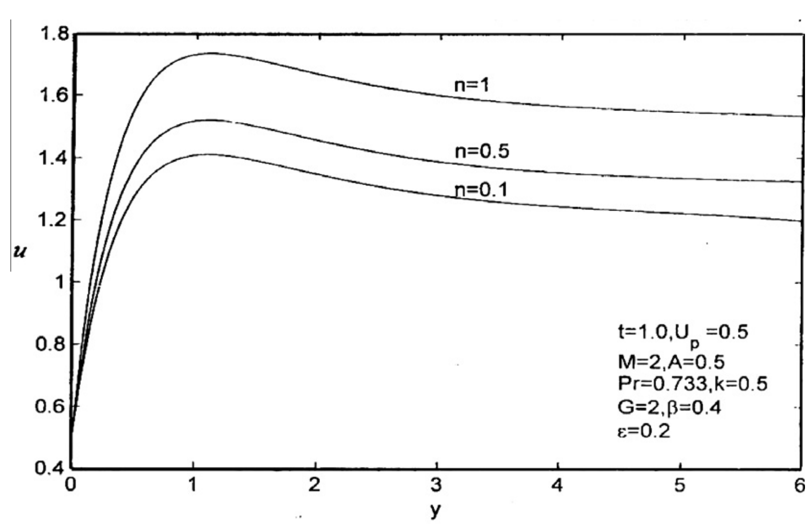



**Fig. 3** Velocity profiles against $y$ for different values of exponential index $n$.

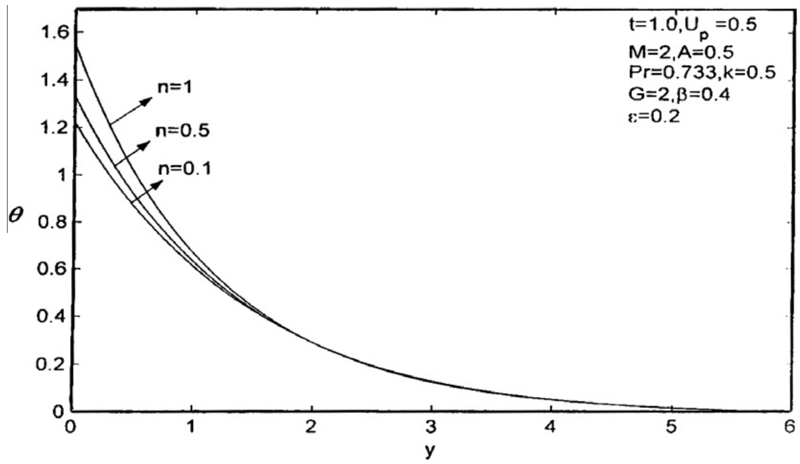



**Fig. 4** Temperature profiles against $y$ for different values of exponential index $n$.

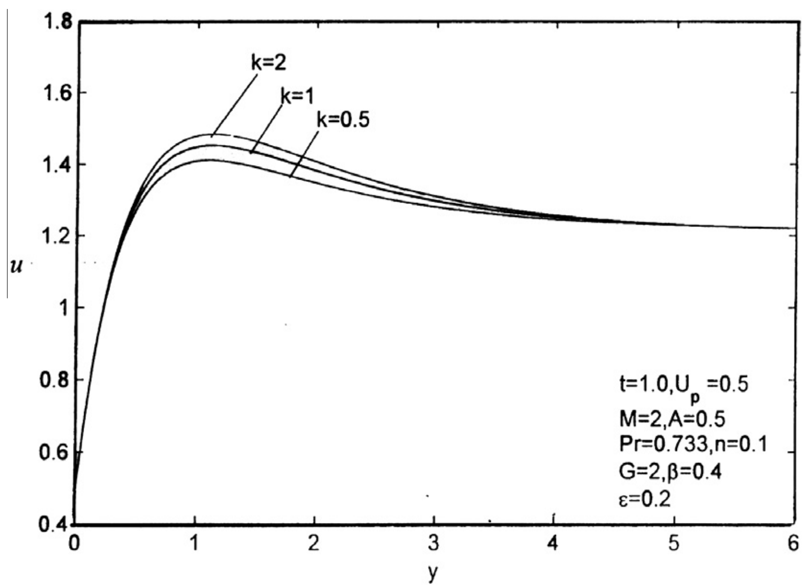



**Fig. 5** Velocity profiles against $y$ for different values of permeability $k$.

The boundary conditions (4) and (5) are then given in terms of dimensionless form

$$u = u_p, \quad \theta = 1 + \varepsilon\, e^{nt} \quad \text{at} \quad y = 0$$
$$u \to u_\infty, \quad \theta \to 0 \quad \text{as} \quad y \to 0 \tag{11}$$

### 3. Solution of the problem

In order to reduce the above system of partial differential Eqs. (10) and (11) in dimensionless form, the velocity and temperature may be represented as, the following formulae will be assumed:

$$u = f_0(y) + \varepsilon\, e^{nt} f_1(y) + o(\varepsilon^2) + \cdots \tag{12}$$

$$\theta = g_0(y) + \varepsilon e^{nt} g_1(y) + o(\varepsilon^2) + \cdots \tag{13}$$

Substituting from Eqs. (12) and (13) into Eqs. (9) and (10), then equating the corresponding terms neglecting the coefficients of order $\varepsilon^2$ we get

$$f_0' = N f_0 - N - G g_0 - f_0' \tag{14}$$

$$g_0' = \frac{1}{1 + B g_0}\left(B {g_0'}^2 + P_r g_0'\right) \tag{15}$$

$$f_1'' = (N + n)(f_1 - 1) - A f_0' - G g_1 - f_1' \tag{16}$$

$$G_1'' = \frac{1}{1 + g_0 B}\left[B g_1 - 2B g_0' g_1' - P_r g_1' + n P_r g_1\right] + B g_1 \times \frac{B {g_0'}^2 - P_r g_1'}{(1 + B g_0)^2} \tag{17}$$

The primes referred to differentiation with respect to y. The corresponding boundary conditions can be written as

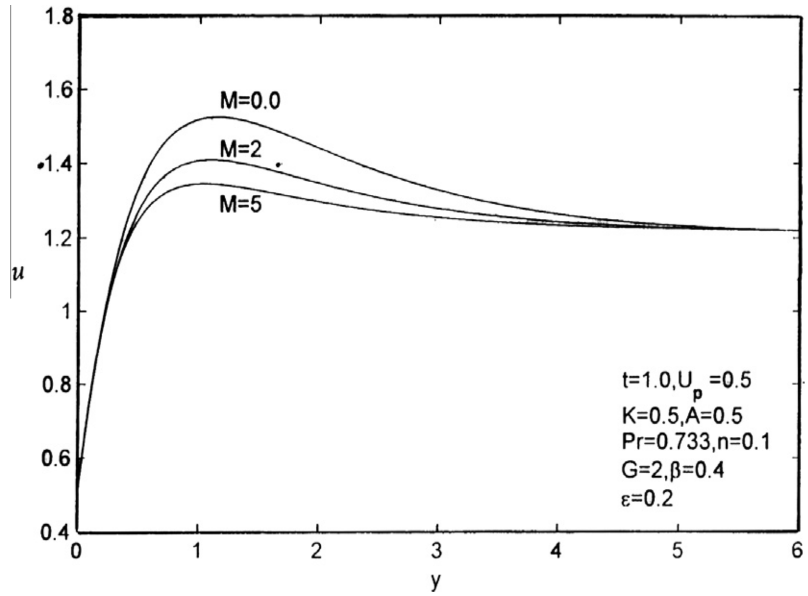



**Fig. 6** Velocity profiles against $y$ for different values of magnetic parameter $M$.

$$f_0 = u_p, \quad f_1 = 0, \quad g_0 = 1, \quad g_1 = 1 \quad \text{at} \quad y = 0 \tag{18.1}$$

$$f_0 = 1, \quad f_1 = 1, \quad g_0 \to 0, \quad g_1 \to 0 \quad \text{at} \quad y = \infty \tag{18.2}$$

## 4. Results and discussions

The study of unsteady boundary layer owes its importance to the fact that all boundary layers that occur in real life are, in a sense, unsteady. Consequently, the solution of many practical fluid mechanics problems hinges on the understanding of the behavior of the unsteady boundary layer. The investigation of the effects of a magnetic field, porous medium, thermal dispersion and the exponential index on unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate has been carried out. This enables the numerically computations for the velocity and temperature for various values of these parameters to be carried out. A cod (FORTRAN) was constructed to apply these assumptions numerically.

Numerical computations for the velocity and temperature for various values of the affected parameters such as, thermal dispersion, exponential index, permeability, and magnetic field, were carried out, which led to the following, results:

(i) The dimensionless velocity u is proportional to the thermal dispersion parameter β, the exponential index n, the dimensionless porous medium parameter k, and inversely to the magnetic field parameter M.
(ii) The dimensionless temperature $\theta$ is proportional to the thermal dispersion parameter β, and the exponential index n,
(iii) Figs. 1 and 2 show that the dimensionless velocity u and the dimensionless temperature $\theta$ are increasing as the thermal dispersion parameter β is increasing, respectively.
(iv) Figs. 3 and 4 show that the dimensionless velocity u and the dimensionless temperature $\theta$ are increasing as the exponential index n is increasing, respectively.
(v) Fig. 5 shows that the dimensionless velocity u is increasing with increasing the dimensionless porous medium parameter k. Physically, this result could be attributed to the fact that when the holes of the porous medium are very large the resistance of the medium could be neglected.
(vi) Fig. 6 represents the relation between the magnetic field parameter M and the velocity profiles from which it is clear that the increase of the magnetic field decreases the velocity.

## 5. Conclusion

- Both dimensionless velocity $u$ and dimensionless temperature $\theta$ are proportional to the thermal dispersion parameter $\beta$ and the exponential index $n$.
- The dimensionless velocity $u$ is proportional to dimensionless porous parameter $k$ and inversely to the magnetic field parameter $M$.

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