NRIAG Journal of Astronomy and Geophysics (2013) 2, 18–20

National Research Institute of Astronomy and Geophysics

NRIAG Journal of Astronomy and Geophysics

www.elsevier.com/locate/nrjag

Limb-effect of rapidly rotating stars

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Received 30 October 2012; accepted 30 January 2013
Available online 10 July 2013

Abstract Kerr metric is used to study the limb-effect phenomenon for axially rotating massive stars. The limb-effect phenomenon is concerned by the variation of the red-shift from the center to the limb of star. This phenomenon has been studied before for the sun. The solar gravitational field is assumed to be given by Schwarzschild and Lense-Thirring fields. In this trial, a study of the limb-effect for a massive axially symmetric rotating star is done. The line of site of inclination and the motion of the observer are taken into consideration to interpret a formula for this phenomenon using a general relativistic red-shift formula. A comparison between the obtained formula and previous formulae is given.

1. Introduction

One of the important observed phenomena is the solar limb-effect. This phenomenon indicates the variation in the gravitational red-shift value from a point to another along the Solar disk. Many authors tried to explain this phenomenon in the empirical frame work (cf. Adam, 1976, 1979; Peter, 1999). On the other hand, the theoretical studies using orthodox general relativity (GR) gave a constant value for gravitational red-shift which contradicts observations that show that this value increases as we move from the center to the limb of the Sun's disk. Many authors have attempted to find a satisfactory interpretation for this effect theoretically in the frame work of GR (cf. Mikhail et al., 2002; Wanas et al., 2008). In Mikhail et al. (2002), an attempt was made to find a more general formula for the gravitational red-shift in the context of GR. In that study, the gravitational field of the Sun is given by the Schwarzschild exterior solution, the observer, on the Earth's surface, moves in a circular orbit about the Sun. In Wanas et al. (2008), two changes have been considered. The first, is the Sun's gravitational field given by Lense-Thirring solution of GR field equations, in free space and the second is the observer trajectory which is an elliptic trajectory.

In the present work we are going to use the same general formula for the gravitational red-shift used in the above

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Peer review under responsibility of National Research Institute of Astronomy and Geophysics.

Production and hosting by Elsevier

2090-9977
http://dx.doi.org/10.1016/j.nrjag.2013.06.004

mentioned studies assuming that the star is a massive axially symmetric spinning object.

2. A general formula for the variation in the red-shift

Kermack et al. (1933), have constructed and proved two important theorems on null geodesics. A general expression for the red-shift, has been derived by Mikhail et al. (2002), using these theorems. Mikhail et al. (2002), assumed that, at the two points C_1 and C_2 on the equator of radiating celestial object, there are two identical atoms, having the wavelengths λ_1 and λ_2, respectively. The two points C_1 and C_2 lie on the same world line of the celestial object in space, as shown in Fig. 1. The wavelengths λ_1 and λ_2 are observed by an observer on the Earth's surface at point C_0, on the world line of the observer as $\underset{0}{\lambda_1}$ and $\underset{0}{\lambda_2}$, respectively. Due to KMW theorems these two wave lengths are given by:

$$\underset{0}{\lambda_1} = \frac{[\rho_\mu \eta^\mu]_{C_1}}{[\varpi_\mu \eta^\mu]_{C_0}} \lambda_1 \tag{1}$$

and

$$\underset{0}{\lambda_2} = \frac{[\rho_\mu \zeta^\mu]_{C_2}}{[\varpi_\mu \zeta^\mu]_{C_0}} \lambda_2 \tag{2}$$

where ρ^μ and ϖ^μ are the unit vectors tangent to the world lines of the celestial object and observer, respectively. η^μ and ζ^μ are the transport null-vectors along the radial null trajectory Λ_1 and the oblique null trajectory Λ_2, respectively.

The radial null trajectory Λ_1 passes through the points C_1 and C_0 while the oblique Λ_2 passes through the points C_2 and C_0.

The suffixes C_0, C_1 and C_2 indicate that the expressions between the brackets are evaluated at C_0, C_1 and C_2, respectively.

Mikhail et al. (2002), have used (1) and (2) to construct a general formula for the variation in red-shift. They have assumed that the wavelengths at C_1 and C_2 are equal; their general formula takes the following form

$$\Delta Z = \frac{[\rho_\mu \zeta^\mu] C_2}{[\varpi_\mu \zeta^\mu] C_0} - \frac{[\rho_\mu \eta^\mu] C_1}{[\varpi_\mu \eta^\mu] C_0} \tag{3}$$

Mikhail et al. (2002) and Wanas et al. (2008) have used this general formula to study the Solar limb-effect.

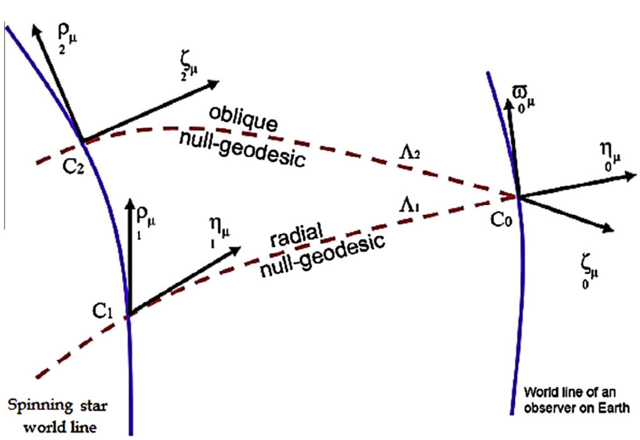

Fig. 1 After Wanas et al. (2008).

Here we are going to use this general formula, in other astrophysical applications, in order to describe a general formula for the red-shift in case of the relativistic massive, axially symmetric rapidly spinning star.

3. Evaluation of the vectors and null vectors

We are going to assume that the exterior field of the massive star is represented by Kerr solution of the field equations of general relativity (GR). This solution represents the field exterior to a rapidly rotating axially symmetric body and possesses interesting features in the region of very strong fields.

As it is well known, the metric of Kerr solution (cf. Boyer and Lindquist, 1967) has the form

$$\begin{aligned} dS^2 = {} & \left(1 - \frac{2m\ r}{r^2 + a^2 \cos^2 \theta}\right) c^2 dt^2 + \frac{(r^2 + a^2 \cos^2 \theta)}{r^2 - 2m\ r + a^2} dr^2 \\ & - (r^2 + a^2 \cos^2 \theta) d\theta^2 \\ & - \sin^2 \theta \left(r^2 + a^2 + \frac{2m\ r\ a^2 \sin^2 \theta}{r^2 + a^2 cos^2 \theta}\right) d\varphi^2 \\ & - \frac{4m\ a\ r \sin^2 \theta}{r^2 + a^2 \cos^2 \theta} c\ d\varphi dt \end{aligned} \tag{4}$$

In this case there is no matter or non-gravitational fields are present in the regions outside the star. In order to calculate the red-shift of the pulses coming out from a massive axially symmetric rotating star by using the general formula (3), we assume that:

1. The signals from the star are traveling along a null geodesic, where η^μ is the transport null vector along the radial null geodesic (see Fig. 1), ζ^μ is a transport null vector along the oblique null geodesic.
2. The star is spinning around its axis of symmetry, such that the atom on its equator moves in a circular orbit with radius (a) the radius of the star at an orbital velocity v_2.
3. The observer on the Earth is rotating at a velocity v_1 with respect to Earth's center.
4. We are going to evaluate the null vectors $\underset{1}{\eta^\mu}$, $\underset{2}{\eta^\mu}$, $\underset{1}{\zeta^\mu}$ and $\underset{2}{\zeta^\mu}$ at the star assuming that the gravitational field is axially symmetric, produced by the star, given by Kerr solution (4); while to evaluate the unit vector $\underset{0}{\varpi^\mu}$ and the null vectors, $\underset{0}{\eta^\mu}$ and $\underset{0}{\zeta^\mu}$ at the Earth, we assume that the field of the Earth is spherically symmetric (Schwarzschild field).

In this case the values of these vectors and null vectors are:

$$\rho^\mu = \left[\frac{A}{\gamma(a)}, 0, 0 \cdot \frac{v_1 A}{\gamma(a)}\right] \tag{5}$$

$$\varpi^\mu = \left[\frac{B}{\gamma(b)}, 0, 0 \cdot \frac{v_2 B}{\gamma(b)}\right] \tag{6}$$

$$\underset{1}{\eta^\mu} = \left[\frac{C}{\gamma(a)},\ \sqrt{C^2}, 0, 0\right] \tag{7}$$

$$\underset{0}{\eta^{\mu}} = \left[\frac{C}{\gamma(b)},\ \sqrt{C^{2,0,0}}\right] \tag{8}$$

$$\underset{2}{\zeta^{\mu}} = \left[\frac{D}{\gamma(a)},\ \sqrt{D^2 - \frac{\gamma(a)l^2}{a^2}},\ 0,\ \frac{l}{a^2}\right] \tag{9}$$

$$\underset{0}{\zeta^{\mu}} = \left[\frac{D}{\gamma(b)},\ \sqrt{D^2 - \frac{\gamma(b)l^2}{b^2}},\ 0,\ \frac{l}{b^2}\right] \tag{10}$$

Substituting the values (5)–(10) in the general formula (4) one has:

$$\begin{aligned} z = \frac{1}{2}\Big[&\sqrt{-(-2a^2 + 4v_1^2a^2 - 12v_1^2ma + 8v_1^2m^2)a(a-m)}(b^2 - 2mb + a^2)(-b^3 + v_2^2b^3 - 4v_2^2mb^2 + v_2^2ba^2 + 4v_2^2m^2b - 2v_2^2ma^2) \\ &\times \gamma(b)l\big(2b^5v_1\gamma(a)a^2 - 6b^5v_1\gamma(a)ma + 4b^5v_1\gamma(a)m^2 - a^4v_2\gamma(b)b^3 + 4a^4v_2\gamma(b)b^2m - a^6v_2\gamma(b)b - 4a^4v_2\gamma(b)m^2b + 2a^6v_2\gamma(b)m\big)\Big] \\ \Big/ \Big[&a^3(a-m)(-a^2 + 2v_1^2a^2 - 6v_1^2ma + 4v_1^2m^2)\gamma(a)\sqrt{-(-b^3 + v_2^2b^3 - 4v_2^2mb^2 + v_2^2ba^2 + 4v_2^2m^2b - 2v_2^2ma^2)b(b^2 - 2mb + a^2)} \\ &\times \big(b^5(D) - v_2l\gamma(b)b^3 + 4v_2l\gamma(b)b^2m - v_2l\gamma(b)ba^2 - 4v_2l\gamma(b)m^2b + 2v_2l\gamma(b)ma^2\big)b^2\Big] \end{aligned} \tag{11}$$

where $\gamma(a) = 1 - (2m/a)$, $\gamma(b) = 1 - (2m/b)$. D and l are constants of integrations and a as mentioned before the radius of the star while b is the distance from star to Earth.

4. Discussion and concluding remarks

1. MKW theorems take into account many factors affecting the red-shift in addition to the gravitational effect.
2. These theorems are usually used in cosmological applications. However it has been used in astrophysics in an attempt to interpret Solar and stellar limb-effect.
3. It is clear from Eq. (11), that the variation in the red-shift has different parameters, one of them is related to the change in the inclination of line of sight. It also depends on some constant of integration.
4. The relation (11) is more general for finding the variation in red-shift for any celestial objects.
5. Eq. (11) for the red-shift variation seems very complicated but if one put $a = 0$, the formula reduces to the same case studied before by Mikhail et al. (2002).
6. In a future work, we are going to adapt this relation to be suitable to study the shift in the signals of x-ray which may through a light on twisting of light around rotating black holes [cf. Fabrizio et al., 2011].

References

Adam, M.G., 1976. The solar limb effect - Observations of line contours and line shifts. Mon. Not. Roy. Asrton. Soc. 177, 687.

Adam, M.G., 1979. A determination of solar rotation using sunspot spectra. Mon. Not. Roy. Asrton. Soc. 188, 819.

Boyer, R.H., Lindquist, R.W., 1967. Maximal Analytic Extension of the Kerr Metric. J. Math. Phys. 8, 265–281.

Fabrizio, Tamburini, Bo, Thidé, Gabriel, Molina-Terriza, Gabriele, Anzolin, 2011. Twisting of light around rotating black holes. Nat. Phys. 7, 195–197.

Kermack, W.O., Mc Crea, W.H., Wittaker, E.T., 1933. On Properties of Null Geodesics and the Application to the Theory of Radiation. Proc. Roy. Soc. Edin. 53, 31.

Mikhail, F.I., Wanas, M.I., Morcos, A.B., 2002. Application of theorems on null-geodesics on the solar limb effect. Astrophys. Space Sci. 223, 233.

Peter, H., 1999. Analysis of transition region emission line profiles from full disk scans of the Sun using the SUMER instrument on SOHO. Astrophys. J. 516, 490.

Wanas, M.I, Morcos, A.B., El Gamal, S.I., 2008. Lense-thirring field and the solar limb effect. Proc. MERIEM-IAU Regional Meeting, vol. 1. p. 173. Available from: <http://arxiv.org/abs/1008.1028>.