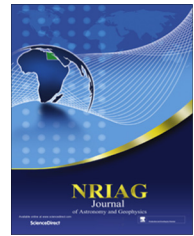




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## The solar activity by wavelet-based multifractal analysis



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**Abstract** The interest in the relation between the solar activity and climate change is increasing. As for the solar activity, a fractal property of the sunspot series was studied by many works. In general, a fractal property was observed in the time series of dynamics of complex systems. The purposes of this study were to investigate the relationship between the sunspot number, solar radio flux at 10.7 cm (F10.7 cm) and total ozone from a view of multifractality. To detect the changes of multifractality, we examined the multifractal analysis on the time series of the solar activity and total ozone indices. The changes of fractality of the sunspot number and F10.7 cm are very similar. When the sunspot number becomes maximum, the fractality of the F10.7 cm changes from multifractality to monofractality. The changes of fractality of the F10.7 cm and the total ozone are very similar. When the sunspot number becomes maximum, the fractality of the total ozone changes from multifractality to monofractality. A change of fractality of the F10.7 cm and total ozone was observed when the solar activity became maximum. The influence of the solar activity on the total ozone was shown by the wavelet coherence, phase and the similarity of the change of fractality. These findings will contribute to the research of the relationship between the solar activity and climate.

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### 1. Introduction

The influence of solar activity on climate has been a matter of debate for a long time.

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Recent measurements from space indicate that the total solar irradiance changes associated with the 11-year solar sunspots cycle are negligibly small (0.1%), although larger (4–8%) variations are found in the ultraviolet (UV) range of 200–250 nm (Lean et al., 1997). Even if we do not expect direct solar cycle impacts at Earth's surface, a significant influence should be detected in the stratopause region (Kodera and Kuroda, 2002).

A decadal variation of tropical lower stratospheric ozone and temperature has previously been identified that correlates positively with the 11 year solar activity cycle. However, the El Niño-Southern Oscillation (ENSO) also influences lower stratospheric ozone and temperature (Hood et al., 2010).

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Various objects in nature show the so-called self-similarity or fractal property. Monofractal shows an approximately similar pattern at different scales and is characterized by a fractal dimension. Multifractal is a non-uniform, more complex fractal and is decomposed into many subsets characterized by different fractal dimensions. Fractal property can be observed in the time series representing dynamics of complex systems as well. A change of fractality accompanies a phase transition and changes of state. The multifractal properties of daily rainfall were investigated in two contrasting climates: an East Asian monsoon climate with extreme rainfall variability and a temperate climate with moderate rainfall variability (Svensson et al., 1996). In both the climates, the frontal rainfall shows monofractality and the convective-type rainfall shows multifractality.

Hence, climate change can be interpreted from the perspective of fractals. A change of fractality may be observed when the climate changes. We attempt to explain changes in climate, referred to as regime shifts, by analyzing fractality. We use the wavelet transform to analyze the multifractal behavior of the climate index. Wavelet methods are useful for the analysis of complex non-stationary time series. The wavelet transform allows reliable multifractal analysis to be performed (Muzy et al., 1991).

In our previous paper (Maruyama et al., 2015), in terms of the multifractal analysis, we conclude that a climatic regime shift corresponds to a change from multifractality to monofractality of the Pacific Decadal Oscillation (PDO) index.

We examined the relationships between the solar activity and total ozone in this study. To detect the changes of multifractality, we examine the multifractal analysis on the sunspot number (SSN), solar radio flux at 10.7 cm (F10.7 cm), and total ozone using the wavelet transform. Moreover we examined the wavelet coherence and phase of those indices.

## 2. Data and method of analysis

The sunspot number (SSN) provided by Solar Influences Data Analysis Center (sidc.oma.be), the solar radio flux at 10.7 cm provided by NOAA's space weather prediction center ([www.swpc.noaa.gov](http://www.swpc.noaa.gov)), and the total ozone provided by NASA ([nasasearch.nasa.gov](http://nasasearch.nasa.gov)) were used. The F10.7 cm is an excellent indicator of the solar activity.

We used the Daubechies wavelet as the analyzing wavelet because it is widely used in solving a broad range of problems, e.g., self-similarity properties of a signal or fractal problems and signal discontinuities. The data used were a discrete signal that fitted the Daubechies Mother wavelet with the capability of precise inverse transformation. Hence, precisely optimal value of  $\tau(q)$  could be calculated as explained below. We then estimated the scaling of the partition function  $Z_q(a)$ , which is defined as the sum of the  $q$ -th powers of the modulus of the wavelet transform coefficients at scale  $a$ . In our study, the wavelet-transform coefficients did not become zero, and therefore, for a precise calculation, the summation was considered for the entire set. Muzy et al. (1991) defined  $Z_q(a)$  as the sum of the  $q$ -th powers of the local maxima of the modulus to avoid division by zero. We obtained the partition function  $Z_q(a)$  as follows:

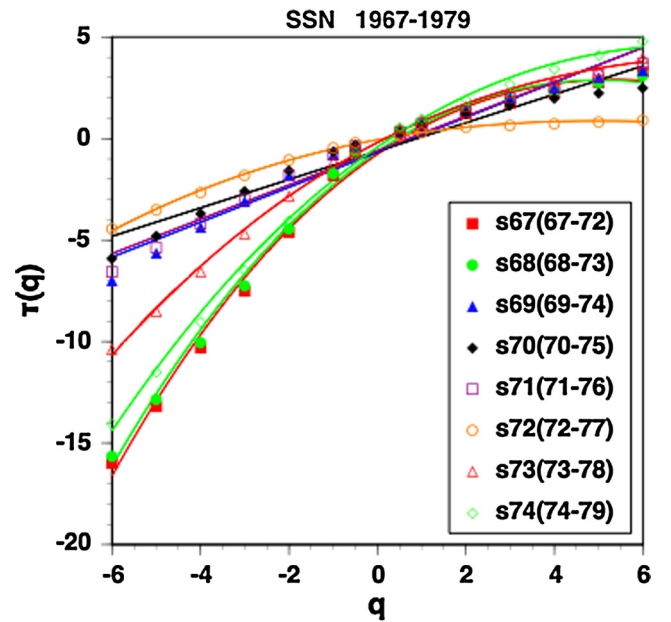
$$Z_q(a) = \sum |W_\varphi[f](a, b)|^q, \quad (1)$$

where  $W_\varphi[f](a, b)$  is the wavelet coefficient of the function  $f$ ,  $a$  is a scale parameter and  $b$  is a space parameter. The time window was set to six years for the following outlined reasons. We calculated the wavelets using a time window of various periods, 10, 6 and 4 years. For a time window of 10 years, a slow change of fractality was observed. The overlap of the first and subsequent data was 9 years, which is longer than the 3 years in the case of the 4-year calculation and thus the change of fractality was small. Thus, this case was inappropriate to find a rapid change of regime shift. For four years, a fast change of fractality was observed. The overlap of the first and subsequent data was 3 years, which is shorter than the 9 years in the case of the 10-year calculation and thus the change of fractality was large. For six years, a moderate change of fractality was observed and hence the time window was set to this period. For small scales, we expect

$$Z_q(a) \sim a^{\tau(q)}. \quad (2)$$

First, we investigate the changes of  $Z_q(a)$  in time series at a different scale  $a$  for each  $q$ . A plot of the logarithm of  $Z_q(a)$  against the logarithm of timescale  $a$  was created. Here  $\tau(q)$  is the slope of the linear fitted line on the log-log plot for each  $q$ . Next, we plot  $\tau(q)$  vs  $q$ . The time window was then shifted forward one year and the process repeated. We define monofractal and multifractal as follows: if  $\tau(q)$  is linear with respect to  $q$ , then the time series is said to be monofractal and if  $\tau(q)$  is convex upwards with respect to  $q$ , then the time series is classified as multifractal (Frish and Parisi, 1985). We define that the value of  $R^2$  is the coefficient of determination, for fitting straight line; if  $R^2 \geq 0.98$  the time series is monofractal and if  $0.98 > R^2$  that is multifractal.

We calculated the multifractal spectrum  $\tau(q)$  of different moments  $q$  for individual records for the SSN index. In Fig. 1, the multifractal spectrum  $\tau(q)$  for individual SSN between 1967 and 1979 is shown. The data were analyzed in 6 year sets, for example, the multifractal spectrum  $\tau(q)$  of s67



**Figure 1** Multifractal spectrum  $\tau(q)$  for individual SSN between 1967 and 1979.

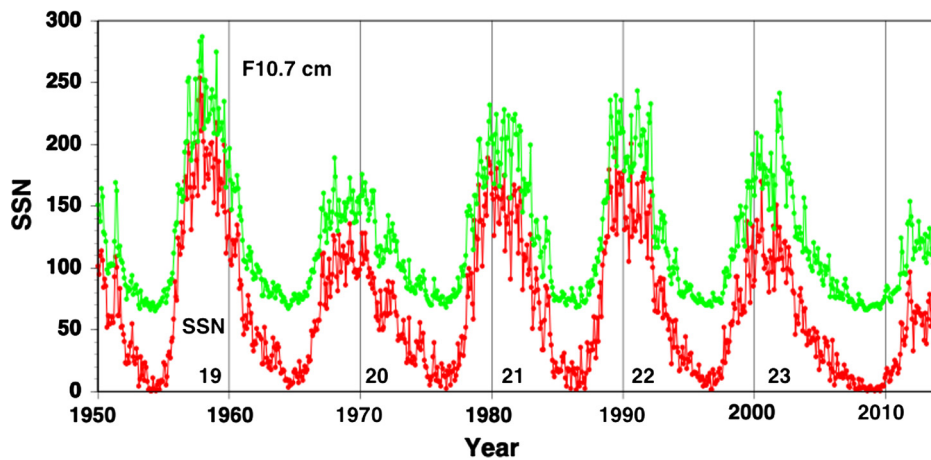
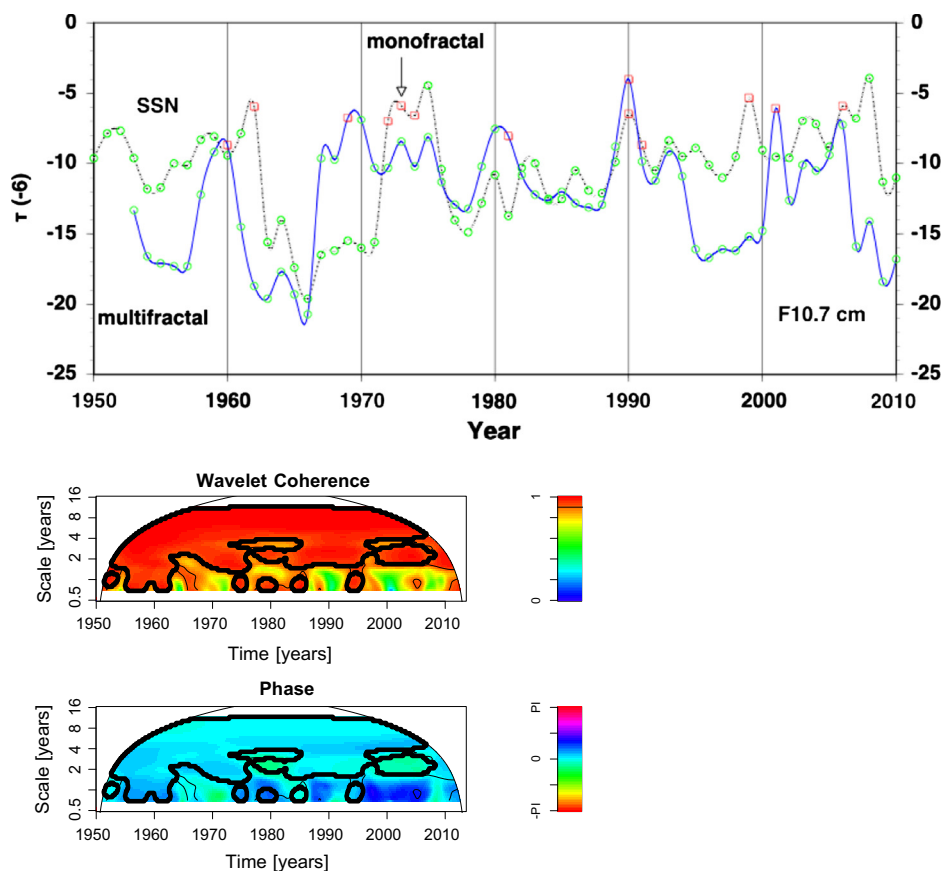


Figure 2 Time series of the SSN and F10.7 cm.



**Figure 3** The  $\tau(-6)$  of the SSN and F10.7 cm (top). The red squares show monofractality and the green circles show multifractality for a 6-year period centered on the year shown. Wavelet coherence (middle) and phase (bottom) between the SSN and F10.7 cm. The thick black contour encloses regions of greater than 95% confidence. The thin black contour encloses regions of greater than 90% confidence. The cone of influence, which indicates the region affected by edge effects, is shown with a black line. In the wavelet phase, the positive value shown by the blue and pink shading means that the SSN leads the F10.7 cm and the negative value shown by the green, yellow and red shading means that the F10.7 cm leads the SSN.

was calculated between 1967 and 1972, and that of s68 was calculated between 1968 and 1973. To examine the change of fractality, the time window was then shifted forward one year and the multifractal spectrum  $\tau(q)$  was calculated from s67 to

s76. A monofractal signal would correspond to a straight line for  $\tau(q)$ , while for a multifractal signal  $\tau(q)$  is nonlinear. In Fig. 1, the constantly changing curvature of the  $\tau(q)$  curves

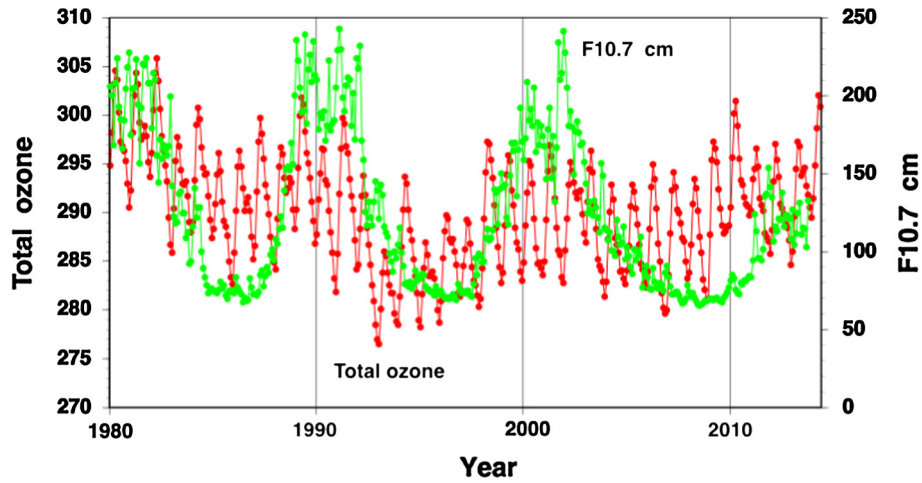


Figure 4 Time series of the F10.7 cm and total ozone.

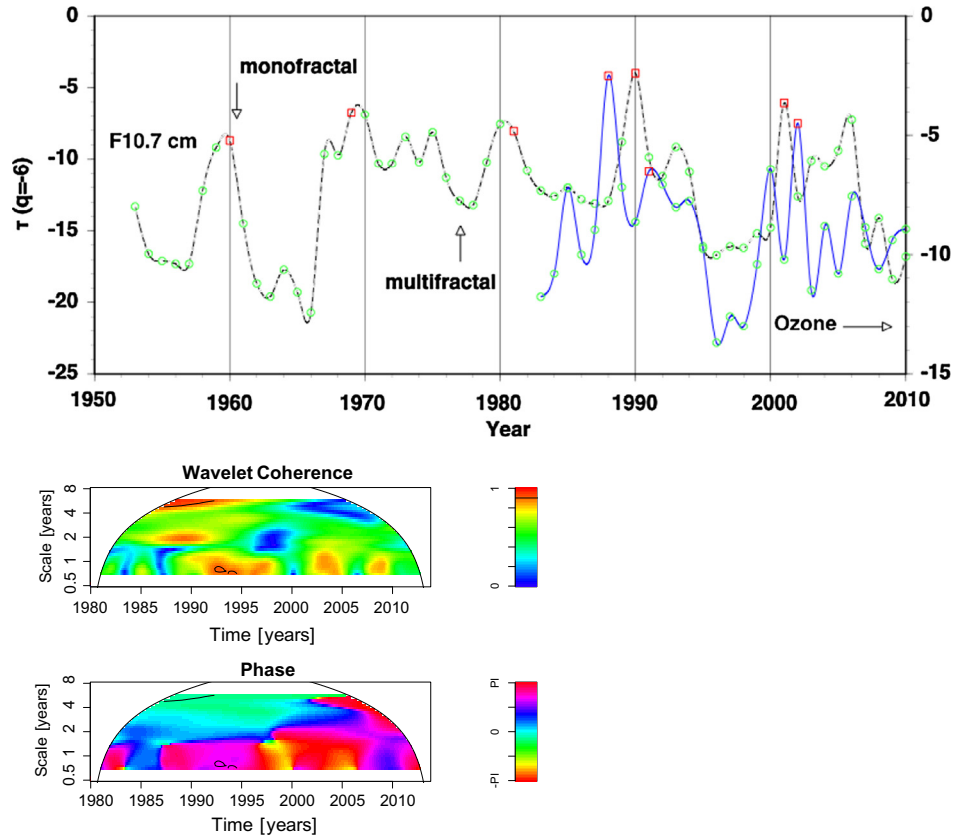


Figure 5 The  $\tau(-6)$  of the F10.7 cm and total ozone (top). Wavelet coherence (middle) and phase (bottom) between the F10.7 cm and total ozone. In the wavelet phase, the positive value means that the F10.7 cm leads the total ozone and the negative value means that the total ozone leads the F10.7 cm.

for s67, s68, and s72 – s74 suggests multifractality. In contrast,  $\tau(q)$  is linear for s69 – s71, which indicates monofractality.

We plot the value of the  $\tau(-6)$  in each index. The negative large values of the  $\tau(-6)$  show large multifractality. For the  $\tau(-6)$ ,  $q = -6$  is the appropriate number to show the change of  $\tau$ . The value of the  $\tau(-6)$  does not always correspond to the fractality obtained from the value of  $R^2$ .

### 3. Results and discussions

#### 3.1. The solar activity observed by the SSN and F10.7 cm

To investigate the solar activity, the time series of the SSN and F10.7 cm is shown in Fig. 2. The  $\tau(-6)$  of the SSN and F10.7 cm are shown in Fig. 3(top). The red square shows

monofractality and the green circle shows multifractality for the 6 years centered on the year plotted. For example, the green circle for 1980 in the SSN shows multifractality between 1977 and 1982. The data were excluded from Fig. 3(top) for cases where we could not distinguish between monofractality and multifractality. Time series of the SSN and F10.7 cm was very similar. When the SSN is maximum and minimum, the  $\tau(-6)$  of the SSN was maximum and the multifractality became weak. This tendency was similar to that of the F10.7 cm. We show the wavelet coherence and phase using the Morlet wavelet between the SSN and F10.7 cm in Fig. 3 middle and bottom, respectively. The coherence between the SSN and F10.7 cm was strong and the lead of the SSN was observed. When the sunspot number becomes maximum, the fractality of the F10.7 cm changes from multifractality to monofractality.

### 3.2. The influence of the solar flux on the total ozone

We investigated the relationship between the F10.7 cm and total ozone. The time series of the F10.7 cm and total ozone is shown in Fig. 4. The  $\tau(-6)$  of the F10.7 cm and total ozone are also shown in Fig. 5(top). We show the wavelet coherence and phase using the Morlet wavelet between the F10.7 cm and total ozone in Fig. 5 middle and bottom, respectively. The coherence between the F10.7 cm and total ozone in 4–8 year scale was strong for 1985–1995 as shown in Fig. 5(middle). The lead of the F10.7 cm was observed. The changes of fractality of the F10.7 cm and the total ozone are very similar. When the sunspot number becomes maximum, the fractality of the total ozone changes from multifractality to monofractality. The influence of the solar activity on the total ozone was shown by the wavelet coherence and phase and the similarity of the change of fractality.

## 4. Conclusions

We investigated the change of multifractal behavior of the climate index. We used the wavelet transform to analyze the multifractal behavior of the SSN, F10.7 cm, and total ozone indices. We showed the change of multifractality by plotting the  $\tau(-6)$ . In this study, we examined the relationship between the sunspot number, F10.7 cm, and total ozone. To detect the changes of multifractality, we examined the multifractal analysis using the wavelet transform on the SSN, and F10.7 cm

which is used as a solar index, and total ozone indices. Moreover we investigated the wavelet coherence and phase of these indices. And, the results of this study are summarized as follows:

- (1) The changes of fractality of the sunspot number and F10.7 cm are very similar. When the sunspot number becomes maximum, the fractality of the F10.7 cm changes from multifractality to monofractality.
- (2) The changes of fractality of the F10.7 cm and the total ozone are very similar.

The influences of the solar activity on the total ozone were shown. When the sunspot number becomes maximum, the fractality of the total ozone changes from multifractality to monofractality. The influence of the solar activity on the total ozone was shown by the wavelet coherence, phase and the similarity of the change of fractality.

These findings will contribute to the research of the relationship between the solar activity and climate.

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