

## Full length article

## Twilight observation by the naked eye of the dawn sincere at Hail and other areas in Saudi Arabia

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## ABSTRACT

Naked eye observations of morning twilight phenomenon at Hail and other areas of Saudi Arabia are recorded. During the interval from 2014 to 2015, about 80 morning twilight observations were carried out in Hail at longitude  $\lambda = 41^{\circ}42'E$  and latitude  $\varphi = 27^{\circ}31'N$  for a desert background. The phenomena was followed over azimuth angles ranged from  $0^{\circ}$  to  $20^{\circ}$  of solar vertical direction and from  $0^{\circ}$  to  $10^{\circ}$  along the altitudinal range. By selecting 32 days with a very good visibility, it was found that Sun depression,  $D_o$ , lies in a range between  $13.48^{\circ}$  and  $14.69^{\circ}$  with an average of  $14.014^{\circ} \pm 0.317$ . The difference between our obtained value and that one which is currently applicable in Saudi Arabia is about  $4^{\circ}$ . The results indicate that dawn (white thread browser) occurs at a sun vertical depression angle  $D_o = 14.66^{\circ}$  (mean + 2SD) according to the normal eye estimations. The results at different areas in the deep desert in KSA showed that the beginning of morning twilight and true dawn is at sun vertical depression  $D_o = 14.88^{\circ}$  (mean + 1SD). The current study shows significant results, which are comparable with both naked eye observations and photoelectric measurements of true dawn in both Egypt and Libya for desert background.

## 1. Introduction

In some Arab countries, areas which are populated by some Muslims in U.S.A. and Europe, Pakistan and its surrounding areas like Bangladesh, Afghanistan and India, no religious signs for beginning and end of twilight are considered and the currently applicable value of beginning and end of twilight is fixed at Sun depression of  $18^{\circ}$  below the horizon which corresponds to that of astronomical twilight. For sun depression of  $18^{\circ}$  below the horizon, the eye receives the least possible non-perceptible light in all wavelengths which isn't enough to enable the normal eye to distinguish any horizon. So, people in the sea depend totally on stars of the sky to find their directions. North America, Canada, parts of U.S.A. and U.K. took a value of  $15^{\circ}$ . However, Saudi Arabia and all Gulf countries considered Um Al-Qura calendar which is adopting a value between  $18.5^{\circ}$  and  $19^{\circ}$  for beginning and  $22.5^{\circ}$  for the end except in Ramadan that value is increased to be  $30^{\circ}$ . Egypt, some African countries, Syria, Iraq and Lebanon follow the published values of the Egyptian General Authority of Survey  $19.5^{\circ}$  for beginning and  $17.5^{\circ}$  for end of twilight Hassan et al. (2009). The current degree of dawn prayer in Saudi Arabia is  $18.5^{\circ}$  (<http://www.prayer-now.com/calculations.html>).

The main aim of the present work is to determine the beginning of twilight (True dawn) in Hail city Peninsula desert and different locations (in the deep desert) in Saudi Arabia using the naked eye observations.

## 2. Methodology and sit observations

The observations were taken by the authors themselves. The authors based on naked eye morning twilight observations over an area of persistent light very light from neighboring areas for the desert background in the time interval (2014–2015) at Hail (Lat. =  $27^{\circ}31'N$ , Long. =  $41^{\circ}42'E$ , Elev. 1015 m). An overall of 80 observations were carried out with few of them carried out in an area of about 80 km in the way between Hail to al Medinah al Monawra. In addition to the observations of several different sites deep in the desert (see Table 3). To convert the local time of true dawn ( $B.T_N$ ) to Sun vertical depressions,  $D_o$ , the equations of Hassan et al. (2009, 2016), were used and the results were checked by Moon calculator 6.0 (Monzur Ahmed, <http://www.starlight.demon.co.uk/mooncalc>). The depth of the desert means that these places are far from civil activity with their airborne pollutants, light pollutants or any human activity.

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**Table 1**

Monthly variation of the metrological conditions at Hail ([http://www.weatherreports.com/Saudi\\_Arabia/Tabuk/averages.html](http://www.weatherreports.com/Saudi_Arabia/Tabuk/averages.html)).

Month	T. High, °C	T. Low, °C	Prec., mm	R.H%	Pressure (hpa)
Jan	27	−10	20.8	40.9	1021
Feb	29	−7	19.1	33.4	1017
Mar	34	−4	21.4	37.6	1017
Apr	37	0	27.6	23.1	1016
May	42	10	15.5	15.9	1014
Jun	44	16	0	13.1	1012
Jul	44	17	0	11.1	1011
Aug	45	16	0	12.9	1013
Sep	42	14	5.4	14.2	1015
Oct	38	5	12.9	33.1	1018
Nov	31	5	50.6	58.4	1019
Dec	38	5	12.3	54.8	1022
Year	36	5.17	15.47	29.1	1016.3

High temperature (T. High °C), Low temperature (T. Low °C), Prec. (precipitation, mm), Relative humidity (R.H%).

There are two types of dawn, the false dawn (pseudo dawn) is a pyramid shape that shows with difficulty and the best time to see it on the second or third day after the beginning of the lunar month in the autumn season and first appears around the 18° under the horizon and then disappears and shows the true dawn as a white thread increases and expands horizontally more than vertically at a lower degree from 15 under the horizon. The observer here focuses and monitors the beginning of the true dawn (Hassan et al., 2009).

The monthly variation of the metrological conditions at Hail during the observation interval to give an impression of the monitoring zone is summarized in Table 1. It is important to note that the lower temperature (T. Low °C) to be pre dawn and this entails that the thickness of the atmosphere as a function of the temperature is less than the thickness of the atmosphere at the end of twilight heat.

### 3. Results and discussion

#### 3.1. Naked eye observation at Hail in Saudi Arabia

Distribution of  $D_o$  (degree) indicates that the range of normalization of data is around  $\pm 2$  as it illustrated in Fig. 1. Fig. 2 represents the third order of sort data fitting of  $D_o$  and visibility (observed by the naked eye, 0–8 Okta) as a function of air transparency, for the total data (80 days). Fig. 3 represents the frequency of  $D_o$  (degree) which consists of 13 bars of 0.1° width and cover by Gaussian distributions.

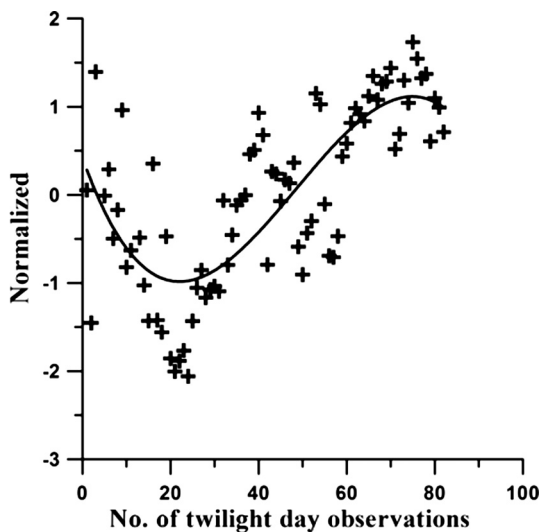


Fig. 1. Normalized distribution of  $D_o$  at Hail.

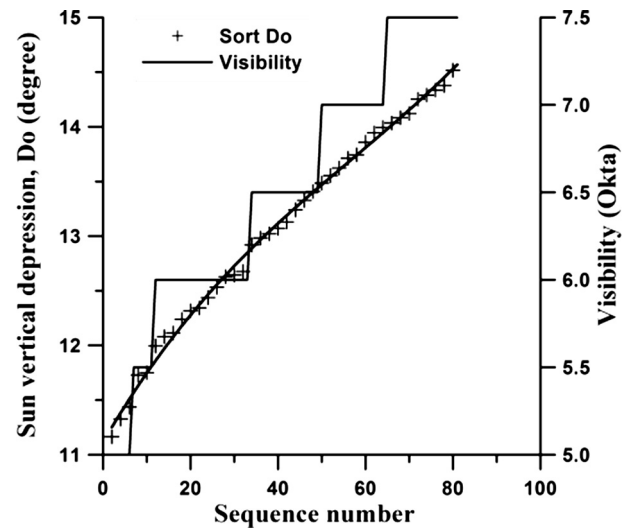


Fig. 2. Sort of  $D_o$  and visibility variation.

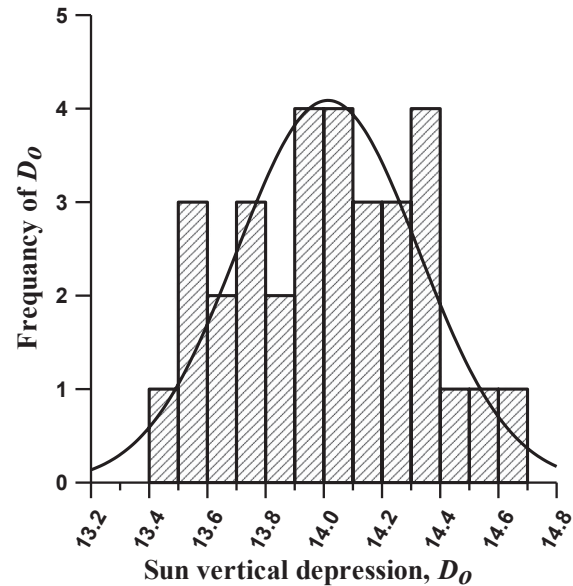


Fig. 3. Frequency of  $D_o$  at Hail.

**Table 2**

Statistical analysis of the naked eye twilight observations of the Sun vertical depression ( $D_o$ ) and visibility (Vis.) for the total cloudless morning (80 days) and selected of high visibility days (32 days).

Statistical Results	80 days		32 days	
	$D_o$	Vis.	$D_o$	Vis.
Minimum	11.1651	5	13.4851	7
Maximum	14.692	7.5	14.692	7.5
Range	3.52696	2.5	1.20691	0.5
Mean	13.0824	6.51	14.0146	7.27
Median	13.1039	6.5	14.0186	7.5
SD	0.93067	0.733	0.31714	0.254
$D_o$ (mean + 2 SD)			14.664	

The statistical analysis of naked eye twilight observations of the Sun vertical depression ( $D_o$ ) and visibility (Vis.) for the total cloudless morning are summarized in Table 2. For the overall 80 days, the ranges of the data are 2.5, 3.5° and 0.93° for visibility,  $D_o$  and SD (Standard deviation) respectively. For the selected 32 days of high visibility, the ranges of data are 0.5, 1.2° and 0.32° for visibility,  $D_o$  and SD

**Table 3**

Naked eye twilight observations at different locations in Saudi Arabia (in the deep desert).

Point	North	East	Elev. (m)	Date dd-mm-yyyy	L.M.T hh:mm	$D_o$
1	25°:29'	39°:22'	752	28-5-2016	4:23	14.6°
2	25°:29'	39°:22'	752	29-5-2016	4:21	14.91°
3	25°:45'	39°:18'	791	30-5-2016	4:21	14.74°
4	25°:31'	41°:00'	1092	4-6-2016	4:21	14.46°
5	19°:33'	43°:38'	1110	12-12-2016	5:30	13.97°
6	19°:40'	43°:31'	1049	13-12-2016	5:30	14.23°
7	24°:17'	39°:40'	770	19-12-2016	5:54	14.62°
8	24°:17'	39°:40'	770	20-12-2016	5:56	14.30°
9	24°:19'	39°:39'	731	21-12-2016	5:56	14.44°
10	24°:19'	39°:39'	731	7-1-2017	6:00	14.95°
11	24°:19'	39°:39'	731	8-1-2017	6:00	14.99°

**Table 4**

Statistical analysis of the naked eye twilight observations at different locations in Saudi Arabia (in the deep desert).

Statistical results	$D_o$
Number of values	11
Minimum	13.97
Maximum	14.99
Range	1.02
Mean	14.565
Median	14.6
SD, Standard deviation	0.3234

respectively. The high confidence of the true dawn is  $D_o = 14.66^\circ$  (mean + 2 SD) which is less than maximum value 14.69°.

### 3.2. Naked eye observation at different location in Saudi Arabia

Parallel observations in another location of the true dawn in the deep desert at King Saudi Arabia (KSA) are done. Table 3 represents the naked eye twilight observations at different locations in Saudi Arabia (in the deep desert), while Table 4 represents the statistical analysis for these observations. It is noticed that there is a short range between the maximum and the minimum data (about 01.02°) and  $D_o = 14.56^\circ \pm 0.323$ , which means that the very high value of these observations. Then the high confidence for the true dawn at the different locations from Table 3 is 14.88° (mean + 1SD).

## 4. Comparison between the current results and the preceding results

Comparing of the current results with the preceding ones is considered to be a good method to assess the current work. Many authors were interested in studying the beginning and end of twilight all over the world.

**Table 5**

Summarization of the published work of observing twilight using naked eye (N.E) and photoelectric (P.E.) instruments in Egypt.

Location	Lat. N	Long. E	Elev. (m)	N. L.	Method	$D_o$	Sources
Bahria, (Egypt)	28° 42.9'	29° 59.82'	150	Desert	P. E.	$15.5^\circ \pm 0.5$	Issa and Hassan (2008b)
Bahria, (Egypt)	28° 42.9'	29° 59.82'	150	Desert	P. E.	$14^\circ \leq D_o \leq 15.5^\circ$	Issa and Hassan (2008c)
Matrouh (Egypt)	31° 0.2'	27° 51'	75	Sea- Desert	P. E.	14.5°	Hassan et al. (2009)
Kottamia (Egypt)	29° 55.9'	31° 49.5'	470	Desert	P. E.	14.5°	Issa and Hassan (2010)
Matrouh (Egypt)	31° 0.2'	27° 51'	75	Sea- Desert	P. E.	$14^\circ \leq D_o \leq 16^\circ$	Hassan et al. (2013)
Bahria, (Egypt)	28° 42.9'	29° 59.82'	150	Desert	P. E.	$14^\circ \leq D_o \leq 15.5^\circ$	Hassan et al. (2014a, 2014b)
Bahria	28° 42.9'	29° 59.82'	150	Desert	N.E	$12.6^\circ \leq D_o \leq 15^\circ$	Hassan et al. (2014a, 2014b)
Matrouh	31° 0.2'	27° 51'	75	Sea- Desert	N.E	$12.3^\circ \leq D_o \leq 14.5^\circ$	Hassan et al. (2014a, 2014b)
Kottamia	29° 55.9'	31° 49.5'	470	Desert	N.E	$14.46^\circ \leq D_o \leq 14.86^\circ$	Hassan et al. (2014a, 2014b)
Aswan	23° 48.22'	32° 29.5'	250	Desert	N.E	$12.46^\circ \leq D_o \leq 13.96^\circ$	Hassan et al. (2014a, 2014b)
Sinai	31° 04' N	32° 52' E	10	Desert	N.E	14.61°	Hassan et al. (2016)
Assiut	27° 10' N	31° 10' E	74	Agricultural land	N.E	13.48°	Hassan et al. (2016)

### 4.1. Morning twilight observation by the photoelectric measurement and naked eye observations in Egypt

Based on the photoelectric measurements at different sites in Egypt, the beginning of twilight is observed. It is found to be at Sun depression,  $D_o$ , ranged from 14° to 15°. In addition to the naked eye observations which recorded on four locations in Egypt; Matrouh, Bahria, Kottamia and Aswan and all had the same results. In 2014, Hassan et al. (2014a, 2014b) concluded that there is a good agreement between the photoelectric measurements and the naked eye observations of the beginning of morning twilight (true dawn) in four locations in Egypt (Issa and Hassan, 2008a, 2008b, 2008c; Issa et al., 2009, 2011; Hassan et al., 2013, 2014a, 2014b; Hassan and Abdel Hadi, 2015). All the above mentioned results are summarized in Table 5.

### 4.2. Morning twilight observation at Tubruq in Libya

623 naked eye observations on Tubruq2 (2010–2013) in Libya emphasis that the beginning is at sun vertical depression,  $D_o = 14.7^\circ$  (desert background), while on Tubruq1 (2007–2009) as the 429 naked eye observations,  $D_o = 13.43^\circ$  (sea background) as it summarized in Table 6 (Hassan et al., 2009; Hassan and Abdel Hadi, 2015). This refers to the high statistical value through a number of observations that reached the 1052 balances.

### 4.3. Morning twilight observation at Riyadh in Saudi Arabia

Al Mostafa et al. (2005) studied the true dawn in the deep desert which is 170 km far from Riyadh city in Saudi Arabia at latitude = 25°45'41"N, longitude = 47°12'10"E and 540 m height over the sea level. Their observations were carried out during one year twice in every month by four groups each one consists of two observers, the best 12 monitors were selected to be handled statistically, as well as 11 best camera images. The team based on using both naked eye observations ( $B.T_N$ ) and camera (Nikon type) measurements ( $B.T_c$ ) parallel to each other. The result showed that the true dawn is at  $D_o = 14.6^\circ \pm 0.3$  for naked eye observations, while it's at  $D_o = 14.48^\circ \pm 0.62$  for Camera records as it clear in Table 7. However, the currently applied dawn now is at 19° which is corresponded to the pseudo-dawn (zodiacal light). The statistical analysis of the results showed that there is a difference of 1.1° between the maximum and minimum for naked eye and 2° for camera records as it clear in Table 8.

### 4.4. Twilight measurements in non-Arab Countries

Spitschan et al. (2016) studied the ambient illumination intensity changes. Intensity is systematically and most rapidly decreasing at dusk or increasing at dawn as a function of solar elevation ( $\theta_s$ ). As Sun set below horizon and no longer directly illuminates the Earth (i.e.  $\theta_s < 0^\circ$ ), the sky light is considered to be a result of refraction and

**Table 6**

Summarization of the published work of observing twilight using naked eye (N.E) In Tubruq, Libya.

Location	Lat. N	Long. E	Elev. (m)	N. L.	Method	$D_o$	Sources
Tubruq1 (Libya)	32° 05′	23° 59′	10	Sea	N.E	13.43°	Hassan et al. (2009)
Tubruq2 (Libya)	32° 04′	23° 59′	40	Desert	N.E	14.7°	Hassan and Abdel-Hadi (2015)

**Table 7**

Summarization of the results of observing twilight using naked eye (N.E) and camera measurements in Riyadh, Saudi Arabia.

Location	Lat. N	Long. E	Elev. (m)	N. L.	Method	$D_o$	Source
Riyadh (Saudi Arabia)	25° 46′	47° 12.16′	540	Desert	N.E and Camera	14.6° ± 0.3	Al Mostafa et al. (2005)

**Table 8**Statistical analysis of naked eye observations ( $B.T_N$ ) and Camera records ( $B.T_c$ ) for twilight in Riyadh, Saudi Arabia.

Statistical Results	$D_o = B.T_N$	$D_o = B.T_c$
Number of values	12	11
Minimum	14	13.5
Maximum	15.1	15.5
Range	1.1	2
Mean	14.6	14.48
Median	14.6	14.5
SD (Standard deviation)	0.3	0.621

scattering of Sun rays in the upper atmosphere. Twilight is classified into three distinct phases according to solar elevation,  $\theta_s$ , and the prevailing visibility conditions due to the illumination level: (a) Civil twilight ( $-6^\circ < \theta_s < 0^\circ$ ), when terrestrial objects can still be distinguished by human observers, (b) nautical twilight ( $-12^\circ < \theta_s < -6^\circ$ ), when only object outlines are visible, and (c) astronomical twilight ( $-18^\circ < \theta_s < -12^\circ$ ), when the illumination level is low enough such that stars and other astronomical objects are available for observation. All measurements were taken with two customized USB spectrometers (USB2000+, Ocean Optics, Inc.; Dunedin, FL), referred to as the 'A' and 'B' spectrometers. These spectrometers are coupled to a

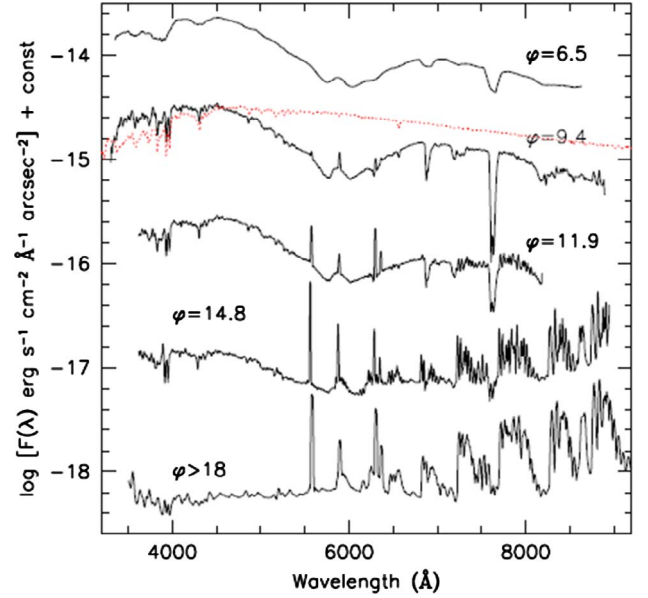


Fig. 5. Twilight spectra obtained at Paranal at different Sun depression angles (Fig. 9 in Patat et al., 2006).

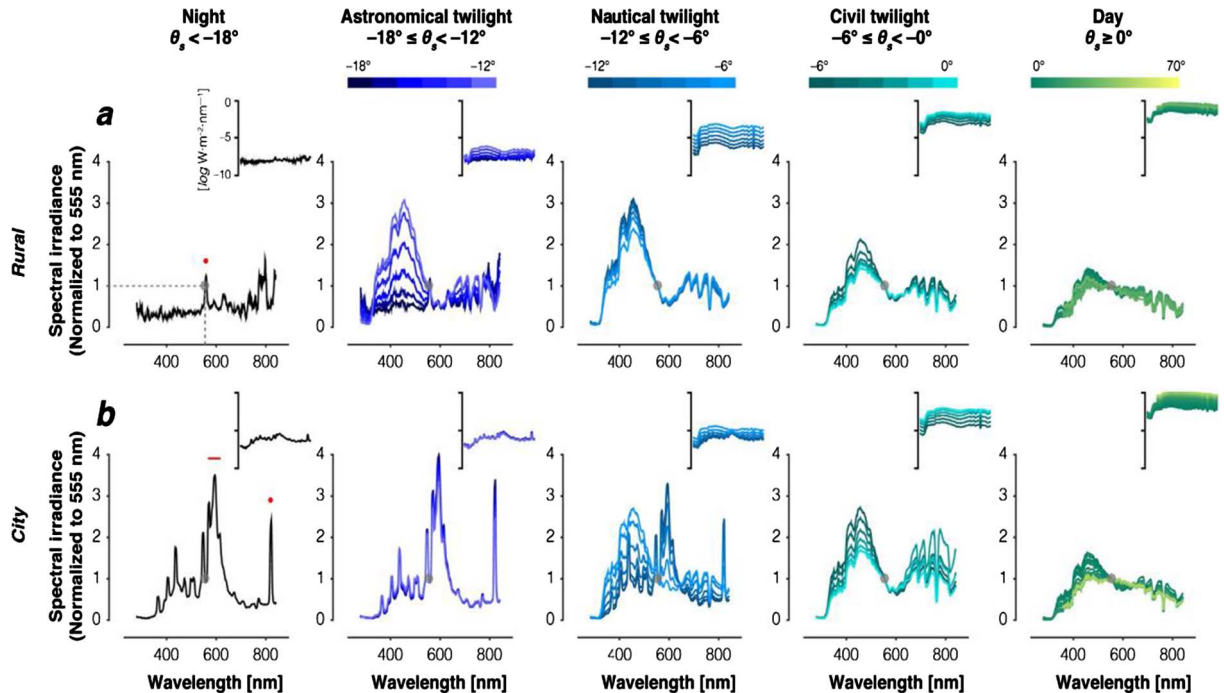


Fig. 4. Variation of outdoor illumination as a function of solar elevation and light pollution (Fig. 1 in Spitschan et al., 2016).



custom-built probe of down welling irradiance 28, 29. The 'A' spectrometer was manufacturer-optimized for high sensitivity measurements by Sony ILX511B linear silicon CCD array and reporting between 180 and 875 nm which was used for twilight and night measurements. The 'B' spectrometer was less sensitive with a wavelength range of 340–1025 nm which was used for daylight measurements.

Fig. 4 showed the relative and absolute down welling illuminations at night, twilight and day. Measurements were carried out in different locations as follows:

- In *rural* location where red dot in night time spectrum panel indicates 558 nm.
- In *City* location where red dot in night time spectrum panel indicates 819 nm; red line indicates 570–615 nm. Spectral irradiance measurements were binned by  $1^\circ$  steps of solar elevation and averaged (except for night, where all measurements for  $\theta_s < 18^\circ$  were averaged). The color bars above the individual plots indicate the solar elevation bins. All measurements were normalized to a value of 1 at 555 nm to emphasize changes in spectral composition (gray dots). Insets show absolute spectral irradiance distribution on a log scale.

For Rural measurements (a) of the astronomical twilight which appears at the range ( $-18 < \theta_s < -12$ ) where  $\theta_s$  in that article is equivalent to  $D_o$  here, Its notice on color bar that the color change from blue into the dark starting from about  $15^\circ$  and become more and more dark to reach black at  $18^\circ$  (i.e. during the interval from  $15^\circ$  to  $18^\circ$ , it is more dark that the normal eye can't sense any light).

Patat et al. (2006), studied the *UBVRI* twilight brightness at dome C (Kenyon and Storey, 2006) and found that, in all pass bands the night sky brightness levels is reached at around zenith angle  $Z = 105\text{--}106^\circ$ . As it noticed in Table 2 and Fig. 9 of Patat article, the line spectrum coup fallen between the lines and lines of the reflected spectrum at  $D_o = 14.8^\circ$  below the horizon happens as it clear in the following figure.

## 5. Conclusion

From this study we conclude that:

- Naked eye observations of morning twilight are carried out in Hail and other areas, KSA over a period of 80 cloudless days (at Hail) and different locations 11 cloudless days at deep desert (see Tables 3 and 4).
- The results at Hail showed that the beginning of morning twilight and true dawn is at sun vertical depression of  $D_o = 14.66^\circ$  (mean + 2SD) below the horizon.
- There is about  $4^\circ$  difference between the currently applicable of Sun vertical depression for false dawn ( $D_o = 18.5^\circ$ ) and our results for true dawn ( $D_o = 14.66^\circ$ ), which gives a time difference of 18 min at

the equinoxes and increased with increasing latitude and the declination of the Sun (<http://www.prayer-now.com/calculations.html>).

- The results at different areas in the deep desert at KSA (see Table 4) showed that, the true dawn is at sun vertical depression of  $D_o = 14.56^\circ \pm 0.32^\circ$  below the horizon and for the high confidences values is  $D_o = 14.88^\circ$  ( $14.56^\circ + 1\text{SD}$ ).
- By comparing the current result with the proceeding, it is found that:
  - The current result is well agreed with all the previous published work of the author and his team (see Tables 5 and 6).
  - The current result is agreed with those results of Spitschan et al. (2016) (see Fig. 4).
  - The current result is well agreed with those results of Al Mostafa et al. (2005), (see Tables 7 and 8).
  - The current result is well agreed with those results of Patat et al. (2006) (see Fig. 5).

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