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Studying the variation of eddy diffusivity on the behavior of advection-diffusion equation

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ABSTRACT

In this work, the advection-diffusion equation was solved in two dimensions to calculate the normalized crosswind integrated concentration by Laplace technique. Considering that the wind speed is constant and we have two models of the vertical eddy diffusivity, one depends on downwind distance and the other model depends on vertical distance. A comparison between our proposed two models, Gaussian, previous work and observed data measured at Copenhagen, Denmark, have been carried out. One finds that there is a good agreement between predicted (2) model and the observed concentrations than predicted (1), Gaussian and previous work.

From the statistical technique, one finds that all models are inside a factor of two with observed data. Regarding to Normalized mean square error (NMSE) and Fraction Bais (FB), proposed model (2) is performance well with observed data than the predicted (1), Gaussian and previous work in unstable condition.

1. Introduction

It is very important to be aware of how contaminants are dispersed through the atmosphere. Unfortunately, Air pollutants influence directly or indirectly on man and environment. Essa and El-Otaify (2008), Alharbi (2011) discussed the dispersion of pollutant mainly depends on meteorological and topographical conditions. In order to understand the dispersion of contaminants in the atmosphere we should study physics that describes the transport of these contaminants in the atmosphere in different boundary conditions. Logan (2001), Mazaher et al. (2013), Scott and Gerhard (2005), Essa et al. (2014) and Tirabassi et al. (2010) studied advection-diffusion equation which depends on Gaussian and non-Gaussian solutions.

Amruta and Pradhan (2013) solved advection-diffusion equation under various circumstances and using various methods.

In this work we solved the advection-diffusion equation in two dimensions to obtain normalized integrated crosswind concentration using Laplace technique. Two models of the vertical eddy diffusivity were developed, considering constant wind speed. One of them depends on downwind distance and the other depends on vertical distance. Comparisons between them, Gaussian, previous work (Sharan and Modani, 2006) and observed data measured at Copenhagen, Denmark were carried out (Gryning and Lyck, 1984, Gryning et al. 1987).

2. Mathematical models

Diffusion equation is the most important in studying of pollutants dispersion into the atmosphere by using the gradient transport theory, this diffusion equation of pollutants in air can be written as (Tiziano and Vilhena, 2012, Tirabassi et al., 2008, 2009):

$$u\frac{\partial c(x,y,z)}{\partial x} = \frac{\partial}{\partial y}\left(k_y\frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial c}{\partial z}\right) \quad (1)$$

where u is the wind speed (m/s), c(x,y,z) is the concentration of pollutant (g/m^3), K_y, k_z are the eddy diffusivities in lateral and vertical direction respectively.

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2.1. First model

In this model, one supposes that the vertical eddy diffusivity is a function of downwind distance i.e. $k_z = k(x)$. Integrating equation (1) with respect to y from $-\infty$to∞, then:

$$u\frac{\partial}{\partial x}\int_{-\infty}^{\infty} c(x,y,z)dy = k_y\frac{\partial c(x,y,z)}{\partial y}\Big|_{-\infty}^{\infty} + k(x)\frac{\partial^2}{\partial z^2}\int_{-\infty}^{\infty} c(x,y,z)dy \tag{2}$$

By supposing that:

$$\int_{-\infty}^{\infty} c(x,y,z)dy = c_y(x,z) \tag{3}$$

One gets that:

$$k_y\frac{\partial c(x,y,z)}{\partial y}\Big|_{-\infty}^{\infty} = 0 \tag{4}$$

By substituting from Eqs, (3) and (4) into Eq. (2), one gets:

$$u\frac{\partial c_y(x,z)}{\partial x} = \frac{\partial}{\partial z}\left[k(x)\frac{\partial c_y(x,z)}{\partial z}\right] \tag{5}$$

In first case the eddy diffusivity is supposed to be constant as a function of x.

$$u\frac{\partial c_{y(x,z)}}{\partial x} = k(x)\frac{\partial^2 c_y(x,z)}{\partial z^2} \tag{6}$$

Eq. (6) is solved under the boundary conditions:

(i) $c_y(0,z) = \frac{Q}{u}\delta(z-h_s)$, where$h_s$ is a stack height
(ii) $c_y(x,z) = 0$ at $x,z \to \infty$
(iii) $k_z\frac{\partial c_y}{\partial z} = 0$ at $z = 0,z_i$

where z_i is the mixing height.

Taking $k(x) = \alpha\bar{u}x$, where α is the turbulence parameter such that: $\alpha = \left(\frac{\sigma_w}{u}\right)^2$, σ_w is the vertical velocity standard deviation (Moreira et al., 2014; Essa et al., 2007; Torbern, 2012, Pramod and Sharan, 2016).

$$\therefore k(x) = \frac{\sigma_w^2}{u}x$$

Taking Laplace transform on x as follows:

$$\tilde{c}(s,z) = \int_0^{\infty} c_y(x,z)e^{-sx}dx \tag{7}$$

Eq. (6) becomes:

$$\int_0^{\infty} u\frac{\partial c_y}{\partial x}e^{-sx}dx = \int_0^{\infty}\frac{\sigma_w^2 x}{u}\frac{\partial^2 c_y}{\partial z^2}e^{-sx}dx \tag{8}$$

Integrating Eq. (8), one gets:

$$-uc_y(0,z) + su\tilde{c}_y(s,z) = \frac{\sigma_w^2}{su}\frac{\partial^2\tilde{c}_y(s,z)}{\partial z^2} \tag{9}$$

Condition (i) is applied in Eq. (9) then.

$$\frac{\sigma_w^2}{su}\frac{\partial^2\tilde{c}_y(s,z)}{\partial z^2} - su\tilde{c}_y(s,z) = -Q\delta(z-h_s) \tag{10}$$

Now applying Laplace transform on z one gets:

$$\int_0^{\infty} e^{-pz}\frac{\sigma_w^2}{su}\frac{\partial^2\tilde{c}_y(s,z)}{\partial z^2}dz - \int_0^{\infty} sue^{-pz}\tilde{c}_y(s,z)dz = -\int_0^{\infty} e^{-pz}Q\delta(z-h_s)dz \tag{11}$$

$$\frac{\sigma_w^2}{su}\left[p^2\tilde{\tilde{c}}_y(s,p) - pc_y(s,0) - \frac{\partial\tilde{c}_y(s,0)}{\partial z}\right] - us\tilde{\tilde{c}}_y(s,p) = -Q\int_0^{\infty} e^{-pz}\delta(z-h_s)dz \tag{12}$$

After application the condition (iii), Eq. (12) becomes:

$$\frac{\sigma_w^2}{su}[p^2\tilde{\tilde{c}}_y(s,p) - pc_y(s,0)] - us\tilde{\tilde{c}}_y(s,p) = -Qe^{-ph_s} \tag{13}$$

$$\tilde{\tilde{c}}_y(s,p) = \frac{\frac{\sigma_w^2}{su}c_y(s,0)p}{\left(\frac{\sigma_w^2}{su}p^2 - us\right)} - \frac{Qe^{-ph_s}}{\left(\frac{\sigma_w^2}{su}p^2 - us\right)} \tag{14}$$

$$\tilde{\tilde{c}}_y(s,p) = \frac{c_y(s,0)p}{(p^2 - \frac{u^2s^2}{\sigma_w^2})} - \frac{Qe^{-ph_s}}{(p^2 - \frac{u^2s^2}{\sigma_w^2})} \tag{14}$$

$$\tilde{\tilde{c}}_y(s,p) = c_y(s,0)F(s,p) - Qe^{-ph_s}G(s,p) \tag{15}$$

where $F(s,p) = \frac{p}{(p^2 - \frac{u^2s^2}{\sigma_w^2})}$ and $G(s,p) = \frac{su/\sigma_w^2}{(p^2 - \frac{u^2s^2}{\sigma_w^2})}$

Take the inverse of Laplace transform on "z" i.e.$\mathscr{L}^{-1}\{(\tilde{\tilde{c}}_y(s,p),z\} = \tilde{c}_y(s,z)$

$$\tilde{c}_y(s,z) = \frac{c_y(s,0)}{2}\left[e^{\frac{su}{\sigma_w}z} + e^{-\frac{su}{\sigma_w}z}\right] - \frac{Q}{2}\frac{\sigma_w}{su}\left[e^{\frac{su}{\sigma_w}(z-h_s)} - e^{-\frac{su}{\sigma_w}(z-h_s)}\right]H(z-h_s) \tag{16}$$

where H is a Heaviside function.

Let $R_n = \frac{su}{\sigma_w}$

$$\tilde{c}_y(s,z) = \frac{c_y(s,0)}{2}[e^{R_nz} + e^{-R_nz}] - \frac{Q}{2R_n}[e^{R_n(z-h_s)} - e^{-R_n(z-h_s)}]H(z-h_s) \tag{17}$$

$$\tilde{c}_y(s,z) = c_y(s,0)\cosh R_nz - \frac{Q}{R_n}\sinh R_n(z-h_s)*H(z-h_s) \tag{18}$$

Using the boundary condition (iii) one gets: $k_z\frac{\partial}{\partial z}\tilde{c}_y(s,z) = 0$ at $z = z_i$ then:

$$\frac{\partial}{\partial z}\tilde{c}_y(s,z) = R_nc_y(s,0)\sinh R_nz - \frac{Q}{R_n}R_n\cosh R_n(z-h_s)H(z-h_s) - \frac{Q}{R_n}\sinh R_n(z-h_s)\frac{\partial}{\partial z}H(z-h_s) \tag{19}$$

$$c_y(s,0)\sinh(R_nz_i) = \frac{Q}{R_n}\cosh(R_n(z_i-h_s))H(z_i-h_s) \tag{20}$$

$$c_y(s,0) = \frac{Q}{R_n}\frac{\cosh R_n(z_i-h_s)}{\sinh(R_nz_i)}$$

$$\therefore c_y(s,0) = \frac{Q}{\frac{\sigma_w}{su}}\frac{\cosh\frac{\sigma_w}{su}(z_i-h_s)}{\sinh\frac{\sigma_w}{su}z_i} \tag{21}$$

Substituting from Eq. (21) in Eq. (18) one gets:

$$\tilde{c}_y(s,z) = \frac{Q}{\frac{\sigma_w}{su}}\frac{\cosh\frac{\sigma_w}{su}(z_i-h_s)}{\sinh\frac{\sigma_w}{su}z_i}\cosh\left(\frac{\sigma_w}{su}z\right) - \frac{Q}{\frac{\sigma_w}{su}}\sinh\frac{\sigma_w}{su}(z-h_s)*H(z-h_s) \tag{22}$$

At ground level (i.e. z = 0), H (z-h_s) = 0, the crosswind integrated concentration can be written as follows:

$$\tilde{c}_y(s,0) = \frac{Q}{\frac{\sigma_w}{su}}\frac{\cosh\frac{\sigma_w}{su}(z_i-h_s)}{\sinh\frac{\sigma_w}{su}z_i} \quad \text{at z = 0}$$

By using Gaussian quadrature formulas then:

$$\frac{c_y(x,0)}{Q} = \sum_{i=1}^{N=8} a_i\left(\frac{s_i}{x}\right)\frac{1}{\frac{us_i}{x\sigma_w}}\frac{\cosh\frac{us_i}{x\sigma_w}(z_i-h_s)}{\sinh\frac{us_i}{x\sigma_w}z_i} \tag{23}$$

where N is the number of quadrature points. $a_i ands_i$ is the Gaussian quadrature parameters.

2.2. Second Model

In the second model the eddy diffusivity is influenced by the vertical height (z), and then Eq. (6) can be written as:

$$u\frac{\partial c_{y_n(x,z)}}{\partial x} = k_n(z)\frac{\partial^2 c_{y_n(x,z)}}{\partial z^2} \quad (24)$$

Advection–diffusion equation for non-homogeneous turbulence can be solved according to the dependence of eddy diffusivity "k" and wind speed profile "u" on the height variable (z). Therefore, to solve this problem by the Laplace transform technique, a stepwise approximation have been performed of these coefficients discretizing the height z_i of the PBL into N sub-intervals in a manner of inside each sub-region, k (z) and u (z), assuming the following average values:

$$k_n = \frac{1}{z_{n+1}-z_n}\int_{z_n}^{z_{n+1}} k_n(z)dz$$

$$u_n = \frac{1}{z_{n+1}-z_n}\int_{z_n}^{z_{n+1}} u(z)dz$$

for n = 1:N.

Applying the Laplace transform on x under the boundary condition:

a- $c_{yn}(0,z_n) = \frac{Q}{u}\delta(z_n-h_s)$

b- $k_n(z)\frac{\partial c_{yn}(x,z)}{\partial z} = 0$ at $z_n = 0,z_i$

Then the Eq. (23) can be written as:

$$\int_0^\infty u\frac{\partial c_{y_n}}{\partial x}e^{-sx}dx = k_n\int_0^\infty \frac{\partial^2 c_{y_n}}{\partial z_n^2}e^{-sx}dx \quad (25)$$

Integrating and substituting in the Eq. (25), one gets:

$$-uc_{y_n}(0,z) + sc_{y_n}\tilde{c}(s,z) = k_n(z)\frac{\partial^2 \tilde{c}_{y_n}(s,z)}{\partial z_n^2} \quad (26)$$

Applying the boundary condition (a) one gets:

$$\frac{\partial^2\tilde{c}_{y_n}(s,z)}{\partial z_n^2} - \frac{su}{k_n}\tilde{c}_{y_n}(s,z) = -\frac{Q}{k_n}\delta(z_n-h_s) \quad (27)$$

Now applying Laplace transform on z then:

$$p^2\tilde{\tilde{c}}_{y_n}(s,p) - pc_{y_n}(s,0) - \frac{\partial\tilde{c}_{y_n}(s,0)}{\partial z} - \frac{us}{k_n}\tilde{\tilde{c}}_{y_n}(s,p) = -\frac{Q}{k_n}e^{-ph_s} \quad (28)$$

Substituting the condition (b), Eq. (28) becomes:

$$\tilde{\tilde{c}}_{y_n}(s,p) = \frac{c_{y_n}(s,0)p}{(p^2-\frac{us}{k_n})} - \frac{Qe^{-ph_s}}{k_n(p^2-\frac{us}{k_n})} \quad (29)$$

$$\tilde{\tilde{c}}_{y_n}(s,p) = c_{y_n}(s,0)F(s,p) - \frac{Q}{k_n}e^{-ph_s}G(s,p) \quad (30)$$

where $F(s,p) = \frac{p}{(p^2-\frac{us}{k_n})}$ and $G(s,p) = \frac{1}{(p^2-\frac{us}{k})}$

Taking the inverse of Laplace transform on "z" i.e. $L^{-1}\{(\tilde{\tilde{c}}_y(s,p),z\} = \tilde{c}_{y_n}(s,z)$

$$\tilde{c}_{y_n}(s,z) = \frac{c_{y_n}(s,0)}{2}\left[e^{\sqrt{\frac{su}{k_n}}z} + e^{-\sqrt{\frac{su}{k_n}}z}\right] - \frac{Q}{2k_n}\sqrt{\frac{k_n}{su}}\left[e^{\sqrt{\frac{su}{k_n}}(z-h_s)} - e^{-\sqrt{\frac{su}{k_n}}(z-h_s)}\right]H(z-h_s) \quad (31)$$

Let $R_n = \sqrt{\frac{su}{k_n}}$ and $R_a = \sqrt{suk_n}$

$$\tilde{c}_{y_n}(s,z) = \frac{c_{y_n}(s,0)}{2}[e^{R_nz} + e^{-R_nz}] - \frac{Q}{2R_a}[e^{R_n(z-h_s)} - e^{-R_n(z-h_s)}]H(z-h_s) \quad (32)$$

$$\tilde{c}_{y_n}(s,z) = c_{y_n}(s,0)\cosh R_nz - \frac{Q}{R_a}\sinh R_n(z-h_s)*H(z-h_s) \quad (33)$$

Applying the boundary condition (b) one gets:

$k_n(z)\frac{\partial}{\partial z}\tilde{c}_{y_n}(s,z) = 0$ at $z = z_i$ then:

$$\frac{\partial}{\partial z}\tilde{c}_{y_n}(s,z) = R_nc_{y_n}(s,0)\sinh R_nz - \frac{Q}{R_a}R_n\cosh R_n(z-h_s)H(z-h_s) - \frac{Q}{R_a}R_n\sinh R_n(z-h_s)\frac{\partial}{\partial z}H(z-h_s) \quad (34)$$

$$c_{y_n}(s,0)\sinh(R_nz_i) = \frac{Q}{R_a}\cosh(R_n(z_i-h_s))H(z_i-h_s) \quad (35)$$

$$c_{y_n}(s,0) = \frac{Q}{R_a}\frac{\cosh R_n(z_i-h_s)}{\sinh(R_nz_i)}$$

$$\therefore c_{y_n}(s,0) = \frac{Q}{\sqrt{suk_n}}\frac{\cosh\sqrt{\frac{su}{k_n}}(z_i-h_s)}{\sinh\sqrt{\frac{su}{k_n}}z_i} \quad (36)$$

Substituting from Eq. (36) in Eq. (33) then:

$$\tilde{c}_{y_n}(s,z) = \frac{Q}{\sqrt{suk_n}}\frac{\cosh\sqrt{\frac{su}{k_n}}(z_i-h_s)}{\sinh\sqrt{\frac{su}{k_n}}z_i}\cosh R_nz - \frac{Q}{R_a}\sinh R_n(z-h_s)*H(z-h_s)$$

Then at the ground level (i.e. z = 0), the following equation describes the crosswind integrated concentration as:

$$\tilde{c}_{y_n}(s,0) = \frac{Q}{\sqrt{suk_n}}\frac{\cosh\sqrt{\frac{su}{k_n}}(z_i-h_s)}{\sinh\sqrt{\frac{su}{k_n}}z_i} \quad \text{at} \quad z=0 \quad (37)$$

By using Gaussian quadrature formulas (eight root), one gets:

$$\frac{c_{y_n}(x,z)}{Q} = \sum_{i=1}^{8} a_i\left(\frac{s_i}{x}\right)\frac{1}{\sqrt{\frac{uk_n(z)s_i}{x}}}\frac{\cosh\sqrt{\frac{s_iu}{xk_n}}(z_i-h_s)}{\sinh\sqrt{\frac{s_iu}{xk_n}}z_i} \quad (38)$$

Taking the eddy diffusivity function of (z) in the form (Degrazia et al., 1997):

$$\frac{k_n(z)}{w_*h} = 0.22\left(\frac{z}{z_i}\right)^{\frac{1}{3}}\left(1-\frac{z}{z_i}\right)^{\frac{1}{3}}\left[1-\exp\left(-\frac{4z}{z_i}\right)-0.0003\exp\left(\frac{8z}{z_i}\right)\right] \quad (39)$$

where w_* is the convective velocity scale, z_i is the mixing height and z is the vertical height.

3. Results and discussion

The used data was obtained from experiments carried out under unstable condition at the Northern part of Copenhagen, Denmark, (Gryning and Lyck, 1984, Gryning et al. 1987). Table 1 shows comparisons between observed and predicted normalized integrated crosswind concentrations under unstable conditions using our predicted models (1 & 2), previous work (Sharan and Modani 2006) and Gaussian model. Results show that our predicted two models performance well with the observed data with different degrees of accuracy (see Table 2).

Fig. 1 Shows that the normalized crosswind integrated concentration values for predicted one and two, previous work (Sharan and Modani, 2006) and Gaussian predicted models and the observed via downwind distance.

Also Fig. 2 shows that the proposed normalized crosswind integrated concentrations values of the predicted (1) and (2), previous work (Sharan and Modani, 2006) and Gaussian models via the observed values. From these two figures, one finds that there is agreement between the predicted (2) of normalized crosswind integrated concentrations with the observed values than predicted (1), Gaussian and Sharan model because the predicted (2) depends on the vertical height while the predicted (1) depends on the downwind distance only. Also the, Gaussian and previous work (Sharan and Modani 2006) are inside a factor of two

4. Model evaluation statistics

Comparisons between predicted and observed results were carried out using statistical method (Hanna, 1989). The standard statistical measures that characterize the agreement between prediction ($C_p = C_{pred}/Q$) and observations ($C_o = C_{obs}/Q$):

Table 1
Comparison between predicted and observed crosswind-integrated normalized concentration with the emission source rate at different boundary layer height, downwind distance, wind speed, scaling convection velocity and distance for different runs.

Run No.	Date	PG Stability	z_i (m)	w[a] (ms^{-1})	U_{10} (ms^{-1})	Distance (m)	C_y /Q (10^{-4} sm^{-2}) Model assessment				
							Observed	Predicted (1)	Predicted (2)	Previous work[a]	Gaussian
1	20-9-78	A	1980	0.83	2.1	1900	6.48	4.6	9.64	4.62	5.16
1	20-9-78	A	1980	0.83	2.1	3700	2.31	5.5	1.57	2.30	2.52
2	26-9-78	C	1920	1.07	4.9	2100	5.38	1.04	3.64	3.18	2.29
2	26-9-78	C	1920	1.07	4.9	4200	2.95	2.9	4.12	1.58	1.18
3	19-10-78	B	1120	0.68	2.4	1900	8.20	2.03	8.09	5.66	4.51
3	19-10-78	B	1120	0.68	2.4	3700	6.22	3.91	4.03	2.88	2.65
3	19-10-78	B	1120	0.68	2.4	5400	4.30	3.91	7.82	2.21	2.58
5	9-11-78	C	820	0.71	3.1	2100	6.72	1.25	5.02	4.81	3.63
5	9-11-78	C	820	0.71	3.1	4200	5.84	3.08	6.19	2.43	2.44
5	9-11-78	C	820	0.71	3.1	6100	4.97	3.22	6.51	2.00	2.41
6	30-4-78	C	1300	1.33	7.2	2000	3.96	0.37	2.27	2.63	1.63
6	30-4-78	C	1300	1.33	7.2	4200	2.22	2.19	2.74	1.28	0.82
6	30-4-78	C	1300	1.33	7.2	5900	1.83	2.52	2.82	0.90	0.68
7	27-6-78	B	1850	0.87	4.1	2000	6.70	0.78	4.71	4.16	2.51
7	27-6-78	B	1850	0.87	4.1	4100	3.25	2.72	2.58	2.03	1.17
7	27-6-78	B	1850	0.87	4.1	5300	2.23	2.97	2.66	1.56	0.79
8	6-7-78	D	810	0.72	4.2	1900	4.16	0.16	4.61	4.87	4.20
8	6-7-78	D	810	0.72	4.2	3600	2.02	1.57	2.0	2.74	2.80
8	6-7-78	D	810	0.72	4.2	5300	1.52	2.25	2.12	1.84	2.18
9	19-7-78	C	2090	0.98	5.1	2100	4.58	0.58	3.49	3.44	2.20
9	19-7-78	C	2090	0.98	5.1	4200	3.11	2.35	3.96	1.74	1.13
9	19-7-78	C	2090	0.98	5.1	6000	2.59	2.69	1.83	1.19	0.81

[a] Sharan and Modani (2006).

Table 2
Comparison between predicted (1), predicted (2), Sharan and Gaussian models according to standard statistical Performance measure.

Models	NMSE	FB	COR	FAC_2
Predicted 1	0.94	0.55	−0.01	0.72
Predicted 2	0.13	0.00	0.72	1.03
Previous work[a]	0.30	0.42	0.80	0.69
Gaussian model	0.56	0.58	0.69	0.59

[a] Sharan and Modanim (2006).

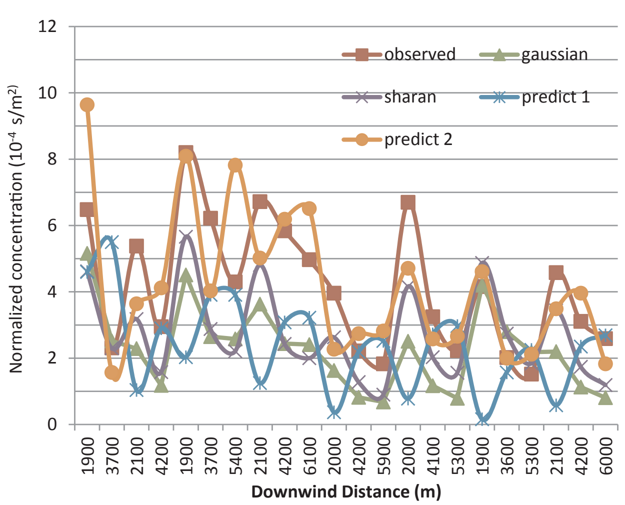

Fig. 1. The variation of the predicted and observed models via downwind distances.

$$\text{Fraction Bias(FB)} = \frac{(\overline{C_o} - \overline{C_p})}{[0.5(\overline{C_o} + \overline{C_p})]}$$

$$\text{Normalized Mean Square Error(NMSE)} = \frac{\overline{(C_p - C_o)^2}}{\overline{(C_p C_o)}}$$

$$\text{Correlation Coefficient(COR)} = \frac{1}{N_m}\sum_{i=1}^{N_m}(C_{pi} - \overline{C_p}) \times \frac{(C_{oi} - \overline{C_o})}{(\sigma_p \sigma_o}$$

$$\text{Factor of Two(FAC2)} = 0.5 \leqslant \frac{C_p}{C_o} \leqslant 2.0$$

where σ_p and σ_o are the standard deviations of predicted $\overline{C_p}$ and observed $\overline{C_o}$ normalized crosswind integrated concentration respectively. Over bars indicate the average over all measurements. For a perfect model NMSE must be = FB = 0 and COR = FAC_2 = 1.

From the statistical method, one finds that all models are inside a factor of two with observed data. The correlation of predicted model (2), Sharan model and Gaussian model and Gaussian model equal (0.72, 0.80 and 0.69 respectively) and predicted model (1) equals (−0.01). Regarding to NMSE and FB, predicted model (2) is performance well with observed data than the predicted (1), Gaussian and Sharan models in unstable condition. One concludes that the predicted model (2) is performance well with observed normalized crosswind integrated concentration than predicted (1), Gaussian and previous work (Sharan and Modani, 2006).

5. Conclusions

The predicted models (1 & 2) normalized crosswind integrated concentration of air pollutants was obtained by solving diffusion equation in two dimensions using Laplace technique then, using Gaussian quadrature formulas. Considering that the eddy diffusivity is a function in downwind distance (proposed 1) and depends on the vertical distance (predicted 2) in unstable case. One finds that there is a good agreement between predicted model (2) and the observed concentrations than predicted (1), Gaussian and. previous work (Sharan and Modani, 2006).

From the statistical method, one finds that all models are inside a factor of two with observed data. Regarding to NMSE and FB, predicted models (2) is performance well with observed data than predicted (1), the Gaussian and Sharan models in unstable condition. One can conclude that, our predicted model (2) is performance well with the observed concentrations than predicted (1), the previous work (Sharan

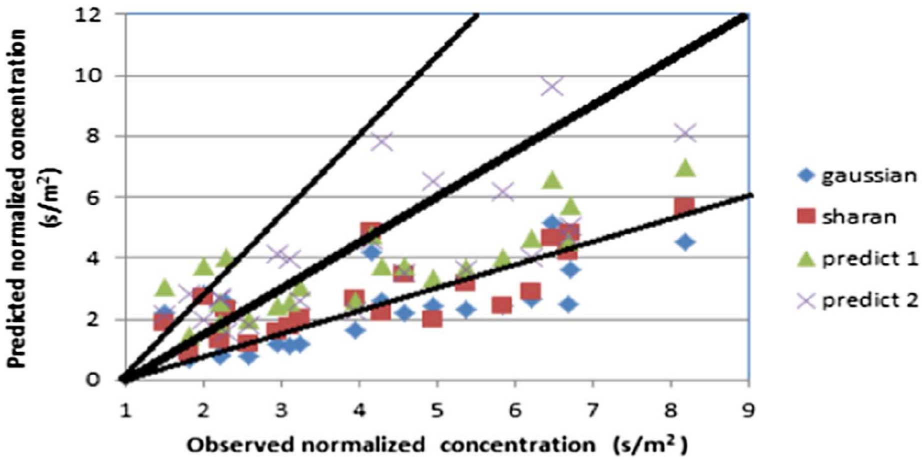

Fig. 2. The variation of the four predicted models via observed concentrations.

and Modani, 2006) and Gaussian model.

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