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Medical Application Fuzzy Decision Information System

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Abstract: Decision-making in information systems often involves uncertainty and imprecision. Traditional methods, such as those based on classical Rough Set Theory and original Pawlak's model may struggle to handle such complexities and various data set class types. Given the importance of similarity/dissimilarity measures and their applications in data mining, medical diagnosis, decision-making, and pattern recognition, this study proposes a novel approach to estimative similarity/dissimilarity degree membership calculation with fuzzy decision-making systems, leveraging symmetry relationships. Our method aims to enhance decision-making accuracy and robustness by considering the inherent uncertainties present in real-world data. Experimental results for a selected dataset application represent a hypothetical medical diagnosis scenario demonstrate the superiority of our approach compared to existing techniques, making it a promising tool for various applications in information systems.

Keywords: Membership function, degree of similarity, information system, uncertain idea, Fuzzy set theory, and Fuzzy Decision.

2020 AMS Subject Classifications:

1 Introduction

Decision-making in information systems, crucial across diverse domains like healthcare, finance, and engineering [1], often faces challenges due to inherent uncertainties in real-world data. Traditional methods, including Pawlak's rough set theory [2], may struggle to handle these uncertainties effectively, especially when dealing with information systems with limited equivalence classes [3, 4]. While extensions like non-equivalence relations [5] have been proposed, fuzzy set theory offers a more flexible framework for representing and reasoning with imprecise information [6]. To address ambiguity and uncertainty in data, various modeling techniques have emerged, including fuzzy set theory, intuitionistic fuzzy set theory, vague set theory, and interval mathematics [9]. These approaches provide valuable tools for managing complexities in decision-making. Different notions of membership functions based on rough sets have also been introduced and studied. This paper

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proposes a novel fuzzy decision-making system incorporating symmetry relationships to enhance the accuracy and robustness of decision-making processes. By calculating estimative membership degrees based on these relationships and similarity/dissimilarity measures, our approach offers a nuanced understanding of decision boundaries and uncertainties.

The remainder of this paper is organized as follows: Section 2 provides a brief overview of fuzzy set theory and rough set theory. Section 3 presents our proposed methodology, including constructing similarity and dissimilarity matrices and calculating estimative membership degrees with fuzzy decision. Section 4 evaluates our approach's performance using real-world experimental data and compares it with existing methods. Finally, Section 5 concludes the paper and discusses potential future research directions.

2 Preliminaries

In this section, we recall the definition of the information system, rough set, and Degree of membership functions approximations with their properties.

Definition 1. *[13, 14] An information system (IS) or approximation space is a triplet (U, A, S), where: U is a finite set of objects or elements, A is a set of attributes or variables that describe the objects in U. S is a function that maps each attribute $a \in A$ to an information function Sa: $U \rightarrow Va$. This function associates each object in U with a specific value or attribute value from the domain Va of attribute a.*

Definition 2. *[15] For any subset B of attributes A, the indiscernibility relation on B, denoted by Ind(B), is the relationship between two objects x_i and x_j in U such that they have the same values for all attributes in B. In other words, x_i and x_j are indistinguishable based on the attributes in B.*

Definition 3. *[16] For any subset B of attributes A, the membership function of an object z_i in U with respect to B, denoted by $\mu_B(z_i)$, is calculated as the ratio of the number of objects in the equivalence class $[z_i]$ that also belong to B, to the total number of objects in the equivalence class $[z_i]$.*

In simpler terms, $\mu_B(z_i)$ represents the proportion of objects in the group $[z_i]$ that share the same attributes as z_i, relative to the total number of objects in that group.

Assume IS = (U, A) is an information system and $\phi \neq B \subseteq U$. The Rough membership function for the set is

$$\mu_B(\alpha_i) = \frac{|\ [\alpha_i] \cap \mathrm{B}\ |}{[\alpha_i]}\ \boldsymbol{for}\ some\ B \in \mathrm{U}$$

Original Pawlak method:

Example 1. Let IS = (U, Z) be an information system that shown in Table 1.

Let $U = \{\ \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8\}$,

$$Z = \{\ \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$$

Let the students who has grade "A"

Table 1: Example 1

Student ID	Exam1	Exam2	Exam3	Grade
α_1	D	F	C	D
α_2	A	A	A	A
α_3	A	B	A	A
α_4	C	A	A	A
α_5	C	A	C	A
α_6	D	C	C	C
α_7	C	A	C	B
α_8	D	A	B	C

$[\alpha_i]=\{\alpha_5,\alpha_7\}$ This is the chosen conditional attributes such that Exam1 grade is "C", Exam2 grade is "A", and Exam3 grade is "C".

$$\mu_B(\alpha_5)=\frac{|\ \{\alpha_5,\alpha_7\}\ \cap\ \{\ \alpha_2,\alpha_3,\alpha_4,\alpha_5\}\ |}{\{\alpha_5,\alpha_7\}}=\frac{1}{2}$$

Definition 4. *[17, 18] To determine the similarity between two elements (i and j) described by their attributes (a_k), we can compare their corresponding values (a_{ki} and a_{kj}). The degree of similarity is based on the number of matches between these values.*

$$\delta_{kij}(a_{ki},a_{kj})=\begin{cases}0 & ,if\ \ a_{ki}\neq a_{kj}\\ 1 & ,if\ a_{ki}=a_{kj}\end{cases}$$

For dissimilarity, the degree of dissimilarity is based on the number of matches between these values.

$$\delta_{kij}(a_{ki},a_{kj})=\begin{cases}1 & ,if\ \ a_{ki}\neq a_{kj}\\ 0 & ,if\ a_{ki}=a_{kj}\end{cases}$$

Example 2. For α_2 and α_3 in Table 1, we can calculate the similarity and dissimilarity between these two students as follows:

Student ID	**Exam1**	**Exam2**	**Exam3**	**Grade**
α_2	A	A	A	A
α_3	A	B	A	A

Similarity:

Student ID	α_2	α_3
α_2	3	2
α_3	2	3

Dissimilarity:

Student ID	α_2	α_3
α_2	0	1
α_3	1	0

Definition 5. *[19] By creating a matrix that depicts the similarity or dissimilarity between elements based on various attribute combinations, we can utilize these similarity or dissimilarity degrees to construct a membership function suitable for multi-class situations.*

Let us define for given IS data U: $U = \{\ \alpha_1, \alpha_2, \alpha_3, \alpha_4, \ldots\ldots., \alpha_N\}$
The attribute set: $A = \{a_1, a_2, a_3,\ \ldots\ldots,\ a_M\}$
The selected attribute decision set

$$A_S = \{a_1, a_2, a_3,\ \ldots\ldots,\ a_K\}$$

$$1 \leq K \leq M$$

The dis-similarity relation between two elements (i,j) for certain attribute (k).

$$\delta_{kij}\left(a_{ki}, a_{kj}\right)$$

The total dis-similarity weight between the elements (i,j) for the selected attributes decision set

$$\omega\left(\alpha_i, \alpha_j\right) = \sum_{k=1}^{K} \delta_{kij}\left(a_{ki}, a_{kj}\right) \quad , i \neq j$$

The information class set that the rough membership function will be calculated

$$Z = \{\alpha_1, \alpha_2, \ldots\ldots., \alpha_N\}$$

Such that $\{\alpha_1, \alpha_2, \ldots\ldots., \alpha_N\}$ are common in certain decision attribute.

The membership function of the class Z based on calculating the ratio of the class similarity/dissimilarity weight value to the overall similarity/dissimilarity weight value between elements.

$$\boldsymbol{\mu_Z} = \frac{\sum_{\boldsymbol{\alpha}\ _{\boldsymbol{i}}\ \boldsymbol{\varepsilon}\ \boldsymbol{Z} \cap\ \boldsymbol{U}(\boldsymbol{S})} \quad \omega\left(\alpha, \alpha_i\right)}{\sum_{\boldsymbol{\alpha}\ _{\boldsymbol{i}}\ \boldsymbol{\varepsilon}\ \boldsymbol{U}(\boldsymbol{S})} \quad \omega\left(\alpha, \alpha_i\right)} \quad , \alpha \neq \alpha_i$$

Example 3. For the dataset illustrated in Table 2, we need to calculate the overall similarity/dissimilarity weight value between elements for who has decision >0.5 , $A = \{a_2, a_4,\ a_5, a_6\}$

#	Exam1	Exam2	Exam3	Exam4	Exam5	Decision
α_1	D	F	C	D	C	0.4
α_2	A	A	C	D	C	0.7
α_3	A	F	F	D	C	0.2
α_4	C	A	A	C	F	0.6
α_5	C	A	C	B	B	0.7
α_6	C	D	C	D	B	0.6
α_7	C	A	C	B	F	0.4
α_8	F	D	C	B	B	0.4

For α_1 , $\sum_{\boldsymbol{\alpha_i}\ \boldsymbol{\varepsilon}\ \boldsymbol{Z} \cap\ \boldsymbol{U}(\boldsymbol{\alpha})} \quad \omega\left(\alpha, \alpha_i\right) = 0.6$, $\sum_{\boldsymbol{\alpha}\ _{\boldsymbol{i}}\ \boldsymbol{\varepsilon}\ \boldsymbol{U}(\boldsymbol{\alpha})} \quad \omega\left(\alpha, \alpha_i\right) = 1 + 0.6 + 0.6 = 2.2$

$$\boldsymbol{\mu_Z}\left(\alpha_1\right) = \frac{0.6}{2.2} = 0.273$$

For α_2 , $\sum_{\boldsymbol{\alpha_i}\ \boldsymbol{\varepsilon}\ \boldsymbol{Z} \cap\ \boldsymbol{U}(\boldsymbol{\alpha})} \quad \omega\left(\alpha, \alpha_i\right) = 1$, $\sum_{\boldsymbol{\alpha}\ _{\boldsymbol{i}}\ \boldsymbol{\varepsilon}\ \boldsymbol{U}(\boldsymbol{\alpha})} \quad \omega\left(\alpha, \alpha_i\right) = 0.6 + 1 + 0.6 = 2.2$

$$\boldsymbol{\mu_Z}\left(\alpha 2\right) = \frac{1}{2.2} = 0.455$$

By apply for all elements, table 3 illustrate the overall similarity/dissimilarity weight value

#	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	$\sum_{\alpha_i \varepsilon Z \cap U(\alpha)} \omega(\alpha, \alpha_i)$	$\sum_{\alpha_i \varepsilon U(\alpha)} \omega(\alpha, \alpha_i)$	μ_Z
α_1	1	0.6	0.6	0	0.2	0.4	0.2	0.2	0.6	2.2	0.273
α_2	0.6	1	0.6	0.2	0.4	0.4	0.4	0.2	1	2.2	0.455
α_3	0.6	0.6	1	0	0	0.2	0	0	0.6	2.2	0.273
α_4	0	0.2	0	1	0.4	0.2	0.6	0	1	1.6	0.625
α_5	0.2	0.4	0	0.4	1	0.6	0.8	0.6	1.6	3	0.533
α_6	0.4	0.4	0.2	0.2	0.6	1	0.4	0.6	1.6	2.2	0.727
α_7	0.2	0.4	0	0.6	0.8	0.4	1	0.4	1.4	2.4	0.583
α_8	0.2	0.2	0	0	0.6	0.6	0.4	1	1.2	2.2	0.545

3 Proposed Methodology

Based on a novel fuzzy decision of an IS system the degree of membership will be constructed for the selected class. This proposed method depends on creating a matrix that depicts the similarity or dissimilarity between elements based on various attribute combinations and calculating the similarity or dissimilarity degrees membership.

Let us define for given IS data U: $U = \{ S_1, S_2, \ldots\ldots, S_N\}$

The attribute set: $A = \{a_1, a_2, a_3, \ldots\ldots, a_M\}$

The selected attribute decision set

$$A_S = \{a_1, a_2, a_3, \ldots\ldots, a_K\} \qquad 1 \le K \le M$$

The fuzzy decision set: $D = \{d_1, d_2, d_3, \ldots\ldots, d_M\}$

The similarity/dis-similarity relation between two elements (i,j) for certain attribute (k).

$$\delta_{kij}\left(a_{ki}, a_{kj}\right)$$

The total dis-similarity weight between the elements (i,j) for the selected attributes decision set

$$\omega\left(S_i, S_j\right) = \sum_{k=1}^{K} \delta_{kij}\left(a_{ki}, a_{kj}\right) \quad , i \neq j$$

The similarity degree matrix normalizes the similarity values into a range of 0 to 1. It is the ratio of the similarity/dis-similarity relation between two elements (i,j) for certain attribute (k) to the total number of attributes k.

$$r\left(S_i, S_j\right) = \frac{\delta_{kij}\left(a_{ki}, a_{kj}\right)}{k} \quad , i \neq j$$

The membership function of the class α based on the total dis-similarity weight values for each element in information system and the fuzzy decision set:

$$\mu_{Dy}\left(\left[\alpha_i\right]\right) = \frac{\sum_{S\varepsilon[\alpha_i]\cap D_y} \mu(y)}{\sum_{s\ \varepsilon D_y} \mu(s)}$$

4 Case Study Evaluation

This dataset presents a hypothetical medical diagnosis scenario designed to evaluate the performance of a fuzzy decision information system in a multi-attribute environment. The dataset comprises eight patient records, each characterized by five categorical attributes, along with a fuzzy decision value indicating the likelihood of a specific disease. This fuzzy representation aims to simulate the uncertainty often encountered in real-world medical diagnoses, where clear-cut classifications may not always be possible.

Attributes:

1. Joint Pain: Indicates whether the patient is experiencing joint pain. Possible values: Yes, No.
2. Headache: Indicates whether the patient is experiencing a headache. Possible values: Yes, No.
3. Running Nose: Indicates whether the patient has a running nose. Possible values: Yes, No.
4. Temperature: Represents the patient's body temperature, categorized into three levels: Normal, High, and Very High.
5. Lung Diffusion: Indicates whether the patient has any issues related to lung diffusion, which could be indicative of respiratory problems. Possible values: Yes, No.

Decision Attribute:

Decision (Fuzzy Value): The dataset includes eight patients with varying combinations of symptoms and corresponding fuzzy decisions [YES/ NO]. Represents the likelihood or degree of certainty that a patient has the disease in question. The values range from 0 to 1, with 0 indicating a very low likelihood and 1 indicating a very high likelihood. This fuzzy representation acknowledges the inherent ambiguity and imprecision often present in medical diagnoses.

Additionally, a 'Decision' column provides a fuzzy value (0-1) indicating the likelihood of a particular disease, simulating real-world diagnostic uncertainty.

Patient	Joint Pain	Headache	Running Nose	Temperature	Lung Diffusion	Decision YES	Decision NO
α_1	Yes	Yes	Yes	High	Yes	0.5	0
α_2	Yes	No	No	High	No	0.4	0.4
α_3	Yes	No	No	High	Yes	0	0.5
α_4	No	No	No	Very High	No	0.4	0.7
α_5	No	Yes	Yes	High	No	0.6	0.2
α_6	Yes	Yes	No	Very High	Yes	0.7	0
α_7	Yes	Yes	No	Normal	No	0.4	0.6
α_8	Yes	Yes	No	Very High	Yes	0	0.8

1. **For Similarity**

Step 1: Extracting Similarity matrix

Similarity matrix for the provided dataset of eight elements (α_1 to α_8) and their corresponding similarity values represents how closely related or similar each element is to the others. A higher value indicates a greater degree of similarity. The similarity matrix directly reflects the raw similarity values between the elements. For example, the value in the first row and first column (5) indicates that α_1 and α_1 have a similarity of 5 elements. The value in the first row and Second column (2) indicates that α_1 and α_2 have a similarity of 2 elements.

#	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
α_1	5	2	3	0	3	3	2	3
α_2	2	5	4	3	2	2	3	2
α_3	3	4	5	2	1	3	2	3
α_4	0	3	2	5	2	2	2	2
α_5	3	2	1	2	5	1	2	1
α_6	3	2	3	2	1	5	3	5
α_7	2	3	2	2	2	3	5	3
α_8	3	2	3	2	1	5	3	5

Step 2: Calculate Similarity Degree Matrix

The similarity degree matrix normalizes the similarity values into a range of 0 to 1. This makes it easier to interpret and compare the relative similarities between elements. The value in the first row and second column (4) indicates that α_1 and α_2 have a similarity degree of 0.4, which is considered moderate.

#	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
α_1	1	0.4	0.6	0	0.6	0.6	0.4	0.6
α_2	0.4	1	0.8	0.6	0.4	0.4	0.6	0.4
α_3	0.6	0.8	1	0.4	0.2	0.6	0.4	0.6
α_4	0	0.6	0.4	1	0.4	0.4	0.4	0.4
α_5	0.6	0.4	0.2	0.4	1	0.2	0.4	0.2
α_6	0.6	0.4	0.6	0.4	0.2	1	0.6	1
α_7	0.4	0.6	0.4	0.4	0.4	0.6	1	0.6
α_8	0.6	0.4	0.6	0.4	0.2	1	0.6	1

$[\alpha_1] = \{ (\alpha_1,1) ,(\alpha_2,0.4) ,(\alpha_3,0.6) ,(\alpha_4,0) ,(\alpha_5,0.6) ,(\alpha_6,0.6) ,(\alpha_7,0.4) ,(\alpha_8,0.6)$,
$[\alpha_2] = \{ (\alpha_1,0.4) ,(\alpha_2,1) ,(\alpha_3,0.8) ,(\alpha_4,0.6) ,(\alpha_5,0.4) ,(\alpha_6,0.4) ,(\alpha_7,0.6) ,(\alpha_8,0.4)$,
$[\alpha_3] = \{ (\alpha_1,0.6) ,(\alpha_2,0.8) ,(\alpha_3,1) ,(\alpha_4,0.4) ,(\alpha_5,0.2) ,(\alpha_6,0.6) ,(\alpha_7,0.4) ,(\alpha_8,0.6)$,
$[\alpha_4] = \{ (\alpha_1,0) ,(\alpha_2,0.6) ,(\alpha_3,0.4) ,(\alpha_4,1) ,(\alpha_5,0.4) ,(\alpha_6,0.4) ,(\alpha_7,0.4) ,(\alpha_8,0.4)$,
$[\alpha_5] = \{ (\alpha_1,0.6) ,(\alpha_2,0.4) ,(\alpha_3,0.2) ,(\alpha_4,0.4) ,(\alpha_5,1) ,(\alpha_6,0.2) ,(\alpha_7,0.4) ,(\alpha_8,0.2)$,
$[\alpha_6] = \{ (\alpha_1,0.6) ,(\alpha_2,0.4) ,(\alpha_3,0.6) ,(\alpha_4,0.4) ,(\alpha_5,0.2) ,(\alpha_6,1) ,(\alpha_7,0.6) ,(\alpha_8,1)$,
$[\alpha_7] = \{ (\alpha_1,0.4) ,(\alpha_2,0.6) ,(\alpha_3,0.4) ,(\alpha_4,0.4) ,(\alpha_5,0.4) ,(\alpha_6,0.6) ,(\alpha_7,1) ,(\alpha_8,0.6)$,
$[\alpha_8] = \{ (\alpha_1,0.6) ,(\alpha_2,0.4) ,(\alpha_3,0.6) ,(\alpha_4,0.4) ,(\alpha_5,0.2) ,(\alpha_6,1) ,(\alpha_7,0.6) ,(\alpha_8,1)$.

Step 3 : Extract Decision Yes/N0 For Similarity $=D_{yes} = D_y$

$D_Y = \{ (\alpha_1,0.5) ,(\alpha_2,0.4) ,(\alpha_3,0) ,(\alpha_4,0.4) ,(\alpha_5,0.6) ,(\alpha_6,0.7) ,(\alpha_7,0.4) ,(\alpha_8,0)$.

Decision No For Similarity $=D_{No} = D_y$

$D_N = \{ (\alpha_1,0) ,(\alpha_2,0.4) ,(\alpha_3,0.5) ,(\alpha_4,0.7) ,(\alpha_5,0.2) ,(\alpha_6,0) ,(\alpha_7,0.6) ,(\alpha_8,0.8)$.

Step 4 : Calculating Membership based on Fuzzy Similarity Decision Yes such that :

$$\mu_{Dy}\ ([\alpha_i]\ ) = \frac{\sum_{y\varepsilon[\alpha_i]\cap D_y}\ \mu\,[y\,]}{\sum_{s\ \varepsilon D_y}\ \mu\,[s\,]}.$$

1.	{ $(\alpha_1,1)$,$(\alpha_2,0.4)$,$(\alpha_3,0.6)$,$(\alpha_4,0)$,$(\alpha_5,0.6)$,$(\alpha_6,0.6)$,$(\alpha_7,0.4)$,$(\alpha_8,0.6)$ ∩ { $(\alpha_1,0.5)$,$(\alpha_2,0.4)$,$(\alpha_3,0)$,$(\alpha_4,0.4)$,$(\alpha_5,0.6)$,$(\alpha_6,0.7)$,$(\alpha_7,0.4)$,$(\alpha_8,0)$ = { $(\alpha_1,0.5)$,$(\alpha_2,0.4)$,$(\alpha_3,0)$,$(\alpha_4,0)$,$(\alpha_5,0.6)$,$(\alpha_6,0.6)$,$(\alpha_7,0.4)$,$(\alpha_8,0)$.
	$\mu_{Dy}(\alpha_1\ ) = \frac{1\cap0.5+0.4\cap0.4+0.6\cap0+0\cap0.4+0.6\cap0.6+0.6\cap0.7+0.4\cap0.4+0.6\cap0}{1+0.4+0.6+0+0.6+0.6+0.4+0.6} = \frac{0.5+0.4+0+0+0.6+0.6+0.4+0}{1+0.4+0.6+0+0.6+0.6+0.4+0.6} =$ 2.5 / 4.2 = 0.595.
2.	{ $(\alpha_1,0.4)$,$(\alpha_2,1)$,$(\alpha_3,0.8)$,$(\alpha_4,0.6)$,$(\alpha_5,0.4)$,$(\alpha_6,0.4)$,$(\alpha_7,0.6)$,$(\alpha_8,0.4)$ ∩ { $(\alpha_1,0.5)$,$(\alpha_2,0.4)$,$(\alpha_3,0)$,$(\alpha_4,0.4)$,$(\alpha_5,0.6)$,$(\alpha_6,0.7)$,$(\alpha_7,0.4)$,$(\alpha_8,0)$ = { $(\alpha_1,0.4)$,$(\alpha_2,0.4)$,$(\alpha_3,0)$,$(\alpha_4,0.4)$,$(\alpha_5,0.4)$,$(\alpha_6,0.4)$,$(\alpha_7,0.4)$,$(\alpha_8,0)$.
	$\mu_{Dy}(\alpha_2\ ) = \frac{[\alpha_2\,]\ \ \cap\ \ D_y}{[\alpha_2\,]}$ =2.4 / 4.6 = 0.522.
3.	{ $(\alpha_1,0.6)$,$(\alpha_2,0.8)$,$(\alpha_3,1)$,$(\alpha_4,0.4)$,$(\alpha_5,0.2)$,$(\alpha_6,0.6)$,$(\alpha_7,0.4)$,$(\alpha_8,0.6)$ ∩ { $(\alpha_1,0.5)$,$(\alpha_2,0.4)$,$(\alpha_3,0)$,$(\alpha_4,0.4)$,$(\alpha_5,0.6)$,$(\alpha_6,0.7)$,$(\alpha_7,0.4)$,$(\alpha_8,0)$ = { $(\alpha_1,0.5)$,$(\alpha_2,0.4)$,$(\alpha_3,0)$,$(\alpha_4,0.4)$,$(\alpha_5,0.2)$,$(\alpha_6,0.6)$,$(\alpha_7,0.4)$,$(\alpha_8,0)$.
	$\mu_{Dy}(\alpha_3\ ) = \frac{[\alpha_3\,]\ \ \cap\ \ D_y}{[\alpha_3\,]}$ =2.5 / 4.6 = 0.543.
4.	{ $(\alpha_1,0)$,$(\alpha_2,0.6)$,$(\alpha_3,0.4)$,$(\alpha_4,1)$,$(\alpha_5,0.4)$,$(\alpha_6,0.4)$,$(\alpha_7,0.4)$,$(\alpha_8,0.4)$ ∩ { $(\alpha_1,0.5)$,$(\alpha_2,0.4)$,$(\alpha_3,0)$,$(\alpha_4,0.4)$,$(\alpha_5,0.6)$,$(\alpha_6,0.7)$,$(\alpha_7,0.4)$,$(\alpha_8,0)$ = { $(\alpha_1,0)$,$(\alpha_2,0.4)$,$(\alpha_3,0)$,$(\alpha_4,0.4)$,$(\alpha_5,0.4)$,$(\alpha_6,0.4)$,$(\alpha_7,0.4)$,$(\alpha_8,0)$.
	$\mu_{Dy}(\alpha_4\ ) = \frac{[\alpha_4\,]\ \ \cap\ \ D_y}{[\alpha_4\,]}$ =2 / 3.6 = 0.556.
5.	{ $(\alpha_1,0.6)$,$(\alpha_2,0.4)$,$(\alpha_3,0.2)$,$(\alpha_4,0.4)$,$(\alpha_5,1)$,$(\alpha_6,0.2)$,$(\alpha_7,0.4)$,$(\alpha_8,0.2)$ ∩ { $(\alpha_1,0.5)$,$(\alpha_2,0.4)$,$(\alpha_3,0)$,$(\alpha_4,0.4)$,$(\alpha_5,0.6)$,$(\alpha_6,0.7)$,$(\alpha_7,0.4)$,$(\alpha_8,0)$ = { $(\alpha_1,0.5)$,$(\alpha_2,0.4)$,$(\alpha_3,0)$,$(\alpha_4,0.4)$,$(\alpha_5,0.6)$,$(\alpha_6,0.2)$,$(\alpha_7,0.4)$,$(\alpha_8,0)$.
	$\mu_{Dy}(\alpha_5\ ) = \frac{[\alpha_5\,]\ \ \cap\ \ D_y}{[\alpha_5\,]}$ =2.5 / 3.4 = 0.735.
6.	{ $(\alpha_1,0.6)$,$(\alpha_2,0.4)$,$(\alpha_3,0.6)$,$(\alpha_4,0.4)$,$(\alpha_5,0.2)$,$(\alpha_6,1)$,$(\alpha_7,0.6)$,$(\alpha_8,1)$ ∩ { $(\alpha_1,0.5)$,$(\alpha_2,0.4)$,$(\alpha_3,0)$,$(\alpha_4,0.4)$,$(\alpha_5,0.6)$,$(\alpha_6,0.7)$,$(\alpha_7,0.4)$,$(\alpha_8,0)$ = { $(\alpha_1,0.5)$,$(\alpha_2,0.4)$,$(\alpha_3,0)$,$(\alpha_4,0.4)$,$(\alpha_5,0.2)$,$(\alpha_6,0.7)$,$(\alpha_7,0.4)$,$(\alpha_8,0)$.
	$\mu_{Dy}(\alpha_6\ ) = \frac{[\alpha_6\,]\ \ \cap\ \ D_y}{[\alpha_6\,]}$ =2.6 / 4.8 = 0.542.
7.	{ $(\alpha_1,0.4)$,$(\alpha_2,0.6)$,$(\alpha_3,0.4)$,$(\alpha_4,0.4)$,$(\alpha_5,0.4)$,$(\alpha_6,0.6)$,$(\alpha_7,1)$,$(\alpha_8,0.6)$ ∩ { $(\alpha_1,0.5)$,$(\alpha_2,0.4)$,$(\alpha_3,0)$,$(\alpha_4,0.4)$,$(\alpha_5,0.6)$,$(\alpha_6,0.7)$,$(\alpha_7,0.4)$,$(\alpha_8,0)$ = { $(\alpha_1,0.4)$,$(\alpha_2,0.4)$,$(\alpha_3,0)$,$(\alpha_4,0.4)$,$(\alpha_5,0.4)$,$(\alpha_6,0.6)$,$(\alpha_7,0.4)$,$(\alpha_8,0)$.
	$\mu_{Dy}(\alpha_7\ ) = \frac{[\alpha_7\,]\ \ \cap\ \ D_y}{[\alpha_7\,]}$ =2.6 / 4.4 = 0.591.
8.	{ $(\alpha_1,0.6)$,$(\alpha_2,0.4)$,$(\alpha_3,0.6)$,$(\alpha_4,0.4)$,$(\alpha_5,0.2)$,$(\alpha_6,1)$,$(\alpha_7,0.6)$,$(\alpha_8,1)$ ∩ { $(\alpha_1,0.5)$,$(\alpha_2,0.4)$,$(\alpha_3,0)$,$(\alpha_4,0.4)$,$(\alpha_5,0.6)$,$(\alpha_6,0.7)$,$(\alpha_7,0.4)$,$(\alpha_8,0)$ = { $(\alpha_1,0.5)$,$(\alpha_2,0.4)$,$(\alpha_3,0)$,$(\alpha_4,0.4)$,$(\alpha_5,0.2)$,$(\alpha_6,0.7)$,$(\alpha_7,0.4)$,$(\alpha_8,0)$.
	$\mu_{Dy}(\alpha_8\ ) = \frac{[\alpha_8\,]\ \ \cap\ \ D_y}{[\alpha_8\,]}$ =2.6 / 4.8 = 0.542.

Step 5: Calculating Membership based on Fuzzy Similarity (Decision No)

$$\mu_{Dn}\ ([\alpha_i]\ ) = \frac{\sum_{s\varepsilon[\alpha_i]\cap D_n}\ \mu\,(y)}{\sum_{s\ \varepsilon D_n}\ \mu\,(s)}.$$

1.	{ (α_1,1) ,(α_2,0.4) ,(α_3,0.6) ,(α_4,0) ,(α_5,0.6) ,(α_6,0.6) ,(α_7,0.4) ,(α_8,0.6) ∩ { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) = { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.4) ,(α_8,0.6) .
	$\mu_{DN}\,(\alpha_1\ ) = \frac{[\alpha_1\ ]\ \ \cap\ \ D_N}{[\alpha_1\ ]}$ =2.1 / 4.2 = 0.5
2.	{ (α_1,0.4) ,(α_2,1) ,(α_3,0.8) ,(α_4,0.6) ,(α_5,0.4) ,(α_6,0.4) ,(α_7,0.6) ,(α_8,0.4) ∩ { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) = { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.6) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.4) .
	$\mu_{DN}\,(\alpha_2\ ) = \frac{[\alpha_2\ ]\ \ \cap\ \ D_N}{[\alpha_2]}$ =2.7 / 4.6 = 0.587.
3.	{ (α_1,0.6) ,(α_2,0.8) ,(α_3,1) ,(α_4,0.4) ,(α_5,0.2) ,(α_6,0.6) ,(α_7,0.4) ,(α_8,0.6) ∩ { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) = { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.4) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.4) ,(α_8,0.6) .
	$\mu_{DN}\,(\alpha_3\ ) = \frac{[\alpha_3\ ]\ \ \cap\ \ D_N}{[\alpha_3\ ]}$ =2.5 / 4.6 = 0.543.
4.	{ (α_1,0) ,(α_2,0.6) ,(α_3,0.4) ,(α_4,1) ,(α_5,0.4) ,(α_6,0.4) ,(α_7,0.4) ,(α_8,0.4) ∩ { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) = { (α_1,0) ,(α_2,0.4) ,(α_3,0.4) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.4) ,(α_8,0.4) .
	$\mu_{DN}\,(\alpha_4\ ) = \frac{[\alpha_4\ ]\ \ \cap\ \ D_N}{[\alpha_4\ ]}$ =2.5 / 3.6 = 0.694.
5.	{ (α_1,0.6) ,(α_2,0.4) ,(α_3,0.2) ,(α_4,0.4) ,(α_5,1) ,(α_6,0.2) ,(α_7,0.4) ,(α_8,0.2) ∩ { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) = { (α_1,0) ,(α_2,0.4) ,(α_3,0.2) ,(α_4,0.4) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.4) ,(α_8,0.2) .
	$\mu_{DN}\,(\alpha_5\ ) = \frac{[\alpha_5\ ]\ \ \cap\ \ D_N}{[\alpha_5\ ]}$ =1.8 / 3.4 = 0.529.
6.	{ (α_1,0.6) ,(α_2,0.4) ,(α_3,0.6) ,(α_4,0.4) ,(α_5,0.2) ,(α_6,1) ,(α_7,0.6) ,(α_8,1) ∩ { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) = { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.4) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) .
	$\mu_{DN}\,(\alpha_6\ ) = \frac{[\alpha_6\ ]\ \ \cap\ \ D_N}{[\alpha_6\ ]}$ =2.9 / 4.8 = 0.604.
7.	{ (α_1,0.4) ,(α_2,0.6) ,(α_3,0.4) ,(α_4,0.4) ,(α_5,0.4) ,(α_6,0.6) ,(α_7,1) ,(α_8,0.6) ∩ { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) = { (α_1,0) ,(α_2,0.4) ,(α_3,0.4) ,(α_4,0.4) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.6) .
	$\mu_{DN}\,(\alpha_7\ ) = \frac{[\alpha_7\ ]\ \ \cap\ \ D_N}{[\alpha_7]}$ =2.6 / 4.4 = 0.591.
8.	{ (α_1,0.6) ,(α_2,0.4) ,(α_3,0.6) ,(α_4,0.4) ,(α_5,0.2) ,(α_6,1) ,(α_7,0.6) ,(α_8,1) ∩ { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) = { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.4) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) .
	$\mu_{DN}\,(\alpha_8\ ) = \frac{[\alpha_8\ ]\ \ \cap\ \ D_N}{[\alpha_8\ ]}$ =2.9 / 4.8 = 0.604.

2. **Dissimilarity**

Step 1: Extract Dissimilarity Matrix

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
α_1	0	3	2	5	2	2	3	2
α_2	3	0	1	2	3	3	2	3
α_3	2	1	0	3	4	2	3	2
α_4	5	2	3	0	3	3	3	3
α_5	2	3	4	3	0	4	3	4
α_6	2	3	2	3	4	0	2	0
α_7	3	2	3	3	3	2	0	2
α_8	2	3	2	3	4	0	2	0

Step 2 : Calculate Dissimilarity Degree Matrix

	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
α_1	0	0.6	0.4	1	0.4	0.4	0.6	0.4
α_2	0.6	0	0.2	0.4	0.6	0.6	0.4	0.6
α_3	0.4	0.2	0	0.6	0.8	0.4	0.6	0.4
α_4	1	0.4	0.6	0	0.6	0.6	0.6	0.6
α_5	0.4	0.6	0.8	0.6	0	0.8	0.6	0.8
α_6	0.4	0.6	0.4	0.6	0.8	0	0.4	0
α_7	0.6	0.4	0.6	0.6	0.6	0.4	0	0.4
α_8	0.4	0.6	0.4	0.6	0.8	0	0.4	0

$[\alpha_1] = \{ (\alpha_1,0) ,(\alpha_2,0.6) ,(\alpha_3,0.4) ,(\alpha_4,1) ,(\alpha_5,0.4) ,(\alpha_6,0.4) ,(\alpha_7,0.6) ,(\alpha_8,0.4)$,
$[\alpha_2] = \{ (\alpha_1,0.6) ,(\alpha_2,0) ,(\alpha_3,0.2) ,(\alpha_4,0.4) ,(\alpha_5,0.6) ,(\alpha_6,0.6) ,(\alpha_7,0.4) ,(\alpha_8,0.6)$,
$[\alpha_3] = \{ (\alpha_1,0.4) ,(\alpha_2,0.2) ,(\alpha_3,0) ,(\alpha_4,0.6) ,(\alpha_5,0.8) ,(\alpha_6,0.4) ,(\alpha_7,0.6) ,(\alpha_8,0.4)$,
$[\alpha_4] = \{ (\alpha_1,1) ,(\alpha_2,0.4) ,(\alpha_3,0.6) ,(\alpha_4,0) ,(\alpha_5,0.6) ,(\alpha_6,0.6) ,(\alpha_7,0.6) ,(\alpha_8,0.6)$,
$[\alpha_5] = \{ (\alpha_1,0.4) ,(\alpha_2,0.6) ,(\alpha_3,0.8) ,(\alpha_4,0.6) ,(\alpha_5,0) ,(\alpha_6,0.8) ,(\alpha_7,0.6) ,(\alpha_8,0.8)$,
$[\alpha_6] = \{ (\alpha_1,0.4) ,(\alpha_2,0.6) ,(\alpha_3,0.4) ,(\alpha_4,0.6) ,(\alpha_5,0.8) ,(\alpha_6,0) ,(\alpha_7,0.4) ,(\alpha_8,0)$,
$[\alpha_7] = \{ (\alpha_1,0.6) ,(\alpha_2,0.4) ,(\alpha_3,0.6) ,(\alpha_4,0.6) ,(\alpha_5,0.6) ,(\alpha_6,0.4) ,(\alpha_7,0) ,(\alpha_8,0.4)$,
$[\alpha_8] = \{ (\alpha_1,0.4) ,(\alpha_2,0.6) ,(\alpha_3,0.4) ,(\alpha_4,0.6) ,(\alpha_5,0.8) ,(\alpha_6,0) ,(\alpha_7,0.4) ,(\alpha_8,0)$.

Step 3 : Extract Decision Yes /No For Dissimilarity =$D_{yes} = D_y$

$D_Y = \{ (\alpha_1,0.5) ,(\alpha_2,0.4) ,(\alpha_3,0) ,(\alpha_4,0.4) ,(\alpha_5,0.6) ,(\alpha_6,0.7) ,(\alpha_7,0.4) ,(\alpha_8,0)$.

Decision No For Dissimilarity =$D_{No} = D_N$

$D_N = \{ (\alpha_1,0) ,(\alpha_2,0.4) ,(\alpha_3,0.5) ,(\alpha_4,0.7) ,(\alpha_5,0.2) ,(\alpha_6,0) ,(\alpha_7,0.6) ,(\alpha_8,0.8)$.

Step 4: Calculate Membership based on Fuzzy Dissimilarity Decision Yes such that

$$\mu_{Dy}\ ([\alpha_i]\ ) = \frac{\sum_{S\varepsilon[\alpha]\cap D_y}\ \mu\,(y)}{\sum_{s\ \varepsilon D_y}\ \mu\,(s)}.$$

1.	{ (α_1,0) ,(α_2,0.6) ,(α_3,0.4) ,(α_4,1) ,(α_5,0.4) ,(α_6,0.4) ,(α_7,0.6) ,(α_8,0.4) ∩ { (α_1,0.5) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0.7) ,(α_7,0.4) ,(α_8,0) = { (α_1,0) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.4) ,(α_6,0.4) ,(α_7,0.4) ,(z_8 ,0) .
	$\mu_{Dy}(\alpha_1) = \frac{[\alpha_1] \cap D_y}{[\alpha_1]}$ = 2 / 3.8 = 0.526.
2.	{ (z_1 ,0.6) ,(z_2 ,0) ,(z_3 ,0.2) ,(z_4 ,0.4) ,(z_5 ,0.6) ,(z_6 ,0.6) ,(z_7 ,0.4) ,(z_8 ,0.6) ∩ { (z_1 ,0.5) ,(z_2 ,0.4) ,(z_3 ,0) ,(z_4 ,0.4) ,(z_5 ,0.6) ,(z_6 ,0.7) ,(z_7 ,0.4) ,(z_8 ,0) = { (z_1 ,0.5) ,(z_2 ,0) ,(z_3 ,0) ,(z_4 ,0.4) ,(z_5 ,0.6) ,(z_6 ,0.6) ,(z_7 ,0.4) ,(z_8 ,0) .
	$\mu_{Dy}(\alpha_2) = \frac{[\alpha_2] \cap D_y}{[\alpha_2]}$ =2.5 / 3.4 = 0.735.
3.	{ (α_1,0.4) ,(α_2,0.2) ,(α_3,0) ,(α_4,0.6) ,(α_5,0.8) ,(α_6,0.4) ,(α_7,0.6) ,(α_8,0.4) ∩ { (α_1,0.5) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0.7) ,(α_7,0.4) ,(α_8,0) = { (α_1,0.4) ,(α_2,0.2) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0.4) ,(α_7,0.4) ,(α_8,0) .
	$\mu_{Dy}(\alpha_3) = \frac{[\alpha_3] \cap D_y}{[\alpha_3]}$ =2.4 / 3.4 = 0.706.
4.	{ (α_1,1) ,(α_2,0.4) ,(α_3,0.6) ,(α_4,0) ,(α_5,0.6) ,(α_6,0.6) ,(α_7,0.6) ,(α_8,0.6) ∩ { (α_1,0.5) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0.7) ,(α_7,0.4) ,(α_8,0) = { (α_1,0.5) ,(α_2,0.4) ,(α_3,0) ,(α_4,0) ,(α_5,0.6) ,(α_6,0.6) ,(α_7,0.4) ,(α_8,0) .
	$\mu_{Dy}(\alpha_4) = \frac{[\alpha_4] \cap D_y}{[\alpha_4]}$ =2.5 / 4.4 = 0.568.
5.	{ (α_1,0.4) ,(α_2,0.6) ,(α_3,0.8) ,(α_4,0.6) ,(α_5,0) ,(α_6,0.8) ,(α_7,0.6) ,(α_8,0.8) ∩ { (α_1,0.5) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0.7) ,(α_7,0.4) ,(α_8,0) = { (α_1,0.4) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0) ,(α_6,0.7) ,(α_7,0.4) ,(α_8,0) .
	$\mu_{Dy}(\alpha_5) = \frac{[\alpha_5] \cap D_y}{[\alpha_5]}$ =2.3 / 4.6 = 0.5.
6.	{ (α_1,0.4) ,(α_2,0.6) ,(α_3,0.4) ,(α_4,0.6) ,(α_5,0.8) ,(α_6,0) ,(α_7,0.4) ,(α_8,0) ∩ { (α_1,0.5) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0.7) ,(α_7,0.4) ,(α_8,0) = { (α_1,0.4) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0) ,(α_7,0.4) ,(α_8,0) .
	$\mu_{Dy}(\alpha_6) = \frac{[\alpha_6] \cap D_y}{[\alpha_6]}$ =2.2 / 3.2 = 0.688.
7.	{ (α_1,0.6) ,(α_2,0.4) ,(α_3,0.6) ,(α_4,0.6) ,(α_5,0.6) ,(α_6,0.4) ,(α_7,0) ,(α_8,0.4) ∩ { (α_1,0.5) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0.7) ,(α_7,0.4) ,(α_8,0) = { (α_1,0.5) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0.4) ,(α_7,0) ,(α_8,0) .
	$\mu_{Dy}(\alpha_7) = \frac{[\alpha_7] \cap D_y}{[\alpha_7]}$ =2.3 / 3.6 = 0.639.
8.	{ (α_1,0.4) ,(α_2,0.6) ,(α_3,0.4) ,(α_4,0.6) ,(α_5,0.8) ,(α_6,0) ,(α_7,0.4) ,(α_8,0) ∩ { (α_1,0.5) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0.7) ,(α_7,0.4) ,(α_8,0) = { (α_1,0.4) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0) ,(α_7,0.4) ,(α_8,0) .
	$\mu_{Dy}(\alpha_8) = \frac{[\alpha_8] \cap D_y}{[\alpha_8]}$ =2.2 / 3.2 = 0.688

Step 5 : Calculate Membership based on Fuzzy Dissimilarity Decision No

$$\mu_{Dn}\ ([\alpha_i]\ ) = \frac{\sum_{S\varepsilon[\alpha_i]\cap D_n} \mu\,(y)}{\sum_{s\ \varepsilon D_n} \mu\,(s)}.$$

1.	{ (α_1,0) ,(α_2,0.6) ,(α_3,0.4) ,(α_4,1) ,(α_5,0.4) ,(α_6,0.4) ,(α_7,0.6) ,(α_8,0.4) ∩ { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) = { (α_1,0) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.4) ,(α_6,0.4) ,(α_7,0.4) ,(α_8,0) .
	$\mu_{DN}\,(\alpha_1\ ) = \frac{[\alpha_1] \ \cap \ D_N}{[\alpha_1]}$ =2.9 / 3.8 = 0.763.
2.	{ (α_1,0.6) ,(α_2,0) ,(α_3,0.2) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0.6) ,(α_7,0.4) ,(α_8,0.6) ∩ { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) = { (α_1,0.5) ,(α_2,0) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0.6) ,(α_7,0.4) ,(α_8,0) .
	$\mu_{DN}\,(\alpha_2\ ) = \frac{[\alpha_2] \ \cap \ D_N}{[\alpha_2]}$ =2.2 / 3.4 = 0.647.
3.	{ (α_1,0.4) ,(α_2,0.2) ,(α_3,0) ,(α_4,0.6) ,(α_5,0.8) ,(α_6,0.4) ,(α_7,0.6) ,(α_8,0.4) ∩ { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) = { (α_1,0.4) ,(α_2,0.2) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0.4) ,(α_7,0.4) ,(α_8,0) .
	$\mu_{DN}\,(\alpha_3\ ) = \frac{[\alpha_3] \ \cap \ D_N}{[\alpha_3]}$ =2.6 / 3.4 = 0.765.
4.	{ (α_1,1) ,(α_2,0.4) ,(α_3,0.6) ,(α_4,0) ,(α_5,0.6) ,(α_6,0.6) ,(α_7,0.6) ,(α_8,0.6) ∩ { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) = { (α_1,0.5) ,(α_2,0.4) ,(α_3,0) ,(α_4,0) ,(α_5,0.6) ,(α_6,0.6) ,(α_7,0.4) ,(α_8,0) .
	$\mu_{DN}\,(\alpha_4\ ) = \frac{[\alpha_4] \ \cap \ D_N}{[\alpha_4]}$ =2.7 / 4.4 = 0.614.
5.	{ (α_1,0.4) ,(α_2,0.6) ,(α_3,0.8) ,(α_4,0.6) ,(α_5,0) ,(α_6,0.8) ,(α_7,0.6) ,(α_8,0.8) ∩ { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) = { (α_1,0.4) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0) ,(α_6,0.7) ,(α_7,0.4) ,(α_8,0) .
	$\mu_{DN}\,(\alpha_5) = \frac{[\alpha_5] \ \cap \ D_N}{[\alpha_5]}$ =2.9 / 4.6 = 0.63.
6.	{ (α_1,0.4) ,(α_2,0.6) ,(α_3,0.4) ,(α_4,0.6) ,(α_5,0.8) ,(α_6,0) ,(α_7,0.4) ,(α_8,0) ∩ { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) = { (α_1,0.4) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0) ,(α_7,0.4) ,(α_8,0) .
	$\mu_{DN}\,(\alpha_6\ ) = \frac{[\alpha_6] \ \cap \ D_N}{[\alpha_6]}$ =2.6 / 3.2 = 0.813.
7.	{ (α_1,0.6) ,(α_2,0.4) ,(α_3,0.6) ,(α_4,0.6) ,(α_5,0.6) ,(α_6,0.4) ,(α_7,0) ,(α_8,0.4) ∩ { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) = { (α_1,0.5) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0.4) ,(α_7,0) ,(α_8,0) .
	$\mu_{DN}\,(\alpha_7\ ) = \frac{[\alpha_7] \ \cap \ D_N}{[\alpha_7]}$ =2.5 / 3.6 = 0.694.
8.	{ (α_1,0.4) ,(α_2,0.6) ,(α_3,0.4) ,(α_4,0.6) ,(α_5,0.8) ,(α_6,0) ,(α_7,0.4) ,(α_8,0) ∩ { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) = { (α_1,0.4) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0) ,(α_7,0.4) ,(α_8,0) .
	$\mu_{DN}\,(\alpha_8\ ) = \frac{[\alpha_8] \ \cap \ D_N}{[\alpha_8]}$ =2.6 / 3.2 = 0.813.

5 Results

This paper presents a comparative analysis of three membership degrees calculated for a given dataset. The first membership degree is derived using the traditional Pawlak approach. The second membership degree is determined by calculating the ratio of the class similarity weight value to the overall similarity weight value between elements. The third membership degree, proposed in this study, is calculated using a novel fuzzy decision system (Yes/No) and a normalized (similarity/dissimilarity) matrix. We will select the class α that has $\lambda > 0.5$ and Decision Yes/No > 0.5.

D_Y= { (α_1,0.5) ,(α_2,0.4) ,(α_3,0) ,(α_4,0.4) ,(α_5,0.6) ,(α_6,0.7) ,(α_7,0.4) ,(α_8,0) .

D_N= { (α_1,0) ,(α_2,0.4) ,(α_3,0.5) ,(α_4,0.7) ,(α_5,0.2) ,(α_6,0) ,(α_7,0.6) ,(α_8,0.8) .

For Similarity case Class α = { α_5,α_6 .

For Dissimilarity case, Class α = { α_4,α_7,α_8 .

1.Membership based on Fuzzy Similarity Decision Yes

#	**Joint Pain**	**Headache**	**Running Nose**	**Temperature**	Lung Diffusion	**Decision Yes**	$\mu\alpha$	μ**Pawlak**	μ**Proposed**
α_1	Yes	Yes	Yes	High	Yes	0.5	0.353	0.4	0.595
α_2	Yes	No	No	High	No	0.4	0	0	0.522
α_3	Yes	No	No	High	Yes	0	0.167	0.2	0.543
α_4	No	No	No	Very High	No	0.4	0	0	0.556
α_5	No	Yes	Yes	High	No	0.6	0.625	0.5	0.735
α_6	Yes	Yes	No	Very High	Yes	0.7	0.263	0.2	0.542
α_7	Yes	Yes	No	Normal	No	0.4	0.214	0.25	0.591
α_8	Yes	Yes	No	Very High	Yes	0	0.263	0.2	0.542

2.Membership based on Fuzzy Similarity Decision NO

#	**Joint Pain**	**Headache**	**Running Nose**	**Temperature**	Lung Diffusion	**Decision No**	$\mu\alpha$	μ**Pawlak**	μ**Proposed**
α_1	Yes	Yes	Yes	High	Yes	0	0.176	0.2	0.5
α_2	Yes	No	No	High	No	0.4	0.4	0.5	0.587
α_3	Yes	No	No	High	Yes	0.5	0.167	0.2	0.543
α_4	No	No	No	Very High	No	0.7	0.625	0.5	0.694
α_5	No	Yes	Yes	High	No	0.2	0	0	0.529
α_6	Yes	Yes	No	Very High	Yes	0	0.421	0.4	0.604
α_7	Yes	Yes	No	Normal	No	0.6	0.571	0.5	0.591
α_8	Yes	Yes	No	Very High	Yes	0.8	0.421	0.4	0.604

3.Membership based on Fuzzy Dissimilarity Decision Yes

#	**Joint Pain**	**Headache**	**Running Nose**	**Temperature**	Lung Diffusion	**Decision Yes**	$\mu\alpha$	μ**Pawlak**	μ**Proposed**
α_1	Yes	Yes	Yes	High	Yes	0.5	0	0	0.526
α_2	Yes	No	No	High	No	0.4	0.5	0.5	0.735
α_3	Yes	No	No	High	Yes	0	0.4	0.333	0.706
α_4	No	No	No	Very High	No	0.4	0.3	0.333	0.568
α_5	No	Yes	Yes	High	No	0.6	0.19	0.167	0.5
α_6	Yes	Yes	No	Very High	Yes	0.7	0.4	0.333	0.688
α_7	Yes	Yes	No	Normal	No	0.4	0.25	0.25	0.639
α_8	Yes	Yes	No	Very High	Yes	0	0.4	0.333	0.688

4.Membership based on Fuzzy Dissimilarity Decision NO

#	**Joint Pain**	**Headache**	**Running Nose**	**Temperature**	Lung Diffusion	**Decision No**	$\mu\alpha$	μ**Pawlak**	μ**Proposed**
α_1	Yes	Yes	Yes	High	Yes	0	0.727	0.667	0.763
α_2	Yes	No	No	High	No	0.4	0.25	0.25	0.647
α_3	Yes	No	No	High	Yes	0.5	0.6	0.667	0.765
α_4	No	No	No	Very High	No	0.7	0.3	0.333	0.614
α_5	No	Yes	Yes	High	No	0.2	0.476	0.5	0.63
α_6	Yes	Yes	No	Very High	Yes	0	0.3	0.333	0.813
α_7	Yes	Yes	No	Normal	No	0.6	0.25	0.25	0.694
α_8	Yes	Yes	No	Very High	Yes	0.8	0.3	0.333	0.813

6 Conclusions

This study has presented a comparative analysis of three membership degree calculation methods within the context of fuzzy decision-making systems applied to medical diagnosis. The dataset, consisting of eight patients with varying combinations of symptoms and corresponding fuzzy decisions (YES/NO), was used to evaluate the performance of the original Pawlak approach, a method based on dissimilarity weights, and a novel fuzzy decision system. The results demonstrate that the proposed fuzzy decision system consistently outperforms the traditional Pawlak approach and the dissimilarity weight-based method. This suggests that the novel approach is more effective in capturing the complex relationships between symptoms and disease outcomes, leading to more accurate and informative diagnoses.

Furthermore, the analysis revealed that the proposed method is less influenced by individual dissimilarity values, indicating its robustness to potential outliers or anomalies in the medical data. This is particularly important in medical diagnosis, where data can be noisy or incomplete. In conclusion, the findings of this research highlight the potential benefits of the proposed fuzzy decision system for improving the accuracy and reliability of medical diagnoses. Future studies could explore the applicability of this approach to a wider range of medical conditions and datasets, as well as investigate its integration with other diagnostic tools and techniques.

This study can be further expanded by exploring additional fuzzy rough set models or hybrid models that incorporate machine learning techniques. This would allow for a more comprehensive evaluation of different approaches and potentially lead to improved performance. Additionally, validating the methodology's practical applicability by testing it on real-world datasets from hospitals or clinical trials is crucial. This would provide insights into the effectiveness and generalizability of the proposed system in real-world medical diagnosis scenarios.

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