Benha Journal of Applied Sciences (BJAS) **print: ISSN 2356–9751**
Vol. (10) Issue (7) (2025), (135- 140) **online: ISSN 2356–976x**
https://bjas.journals.ekb.eg/
engineering sciences

Numerical Investigation of natural convection of hybrid nanofluid in a Γ-shaped enclosure

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Abstract

In this study, a numerical investigation of natural convection heat transfer of hybrid nanofluid inside a **Γ-shaped** enclosure is studied. The fluid flows under steady, laminar, and incompressible conditions. The simulation of flow is done for two dimensions x and y respectively. Water- Alumina- Copper hybrid nanofluid is the fluid used in this study with the base fluid taken to be water (H_2O) and the nanoparticles are Alumina (Al_2O_3) and Copper (Cu). The upper right vertical wall is kept at T_c .As for the lower right vertical and horizontal wall, they are kept at T_h. The top and bottom walls are adiabatic. The governing partial differential equations which are Navier Stokes equations as well as Energy equation are approximated and solved numerically using the finite difference method utilizing Mathematica 10 software. The streamlines and isotherms are illustrated graphically against the varied Grashof number as well as the nanoparticles volume fraction.

Keywords: Natural Convection, Hybrid nanofluid, Enclosure, Finite difference method.

1. Introduction

Over the last few years, hybrid nanofluids have gained significant interest from researchers due to their enhanced heat transfer properties. It's also worthy to mention that cooling requirements of many applications have increased significantly. The traditional working fluids failed to fulfill these requirements due to their limited thermal conductivity. To overcome these problems, the need to find enhancing additives has emerged. Hybrid nanofluids have shown improved thermophysical properties. They are composed of a base fluid in which two or more nanoparticles are dispersed[1]. Moreover, the resulting thermal conductivity enhances significantly if the dispersed nanoparticles are metallic, such as copper and silver. Hybrid nanofluids are encountered in a wide range of heat transfer applications such as industrial cooling, automotive coolants, microelectronics as well as solar collectors. Sheikholeslami et al. [2] investigated natural convection heat transfer of CuO-water nanofluid in a wavy cavity. Brownian motion effects on thermal conductivity were considered. Various parameters including Rayleigh number(Ra), Hartmann number (Ha) and nanoparticles volume fraction (∅) were examined. Kumar et al.[3] numerically examined EG-TiO_2 nanofluid flow in a square cavity experiencing thermal radiation and magnetic field. Rayleigh number (Ra), nanoparticles volume fraction (∅), Reynolds number (Re) as well as Radiation parameter (R) were varied. They concluded that heat transfer is enhanced with higher values of R while the heat transfer is hindered in case of rising values of magnetic field parameter. Desouky et al. [4] studied the effect of magnetic field applied on a micropolar fluid flow in a rectangular cavity. The study included variation of magnetic Reynolds number, Hartmann number and convective parameters and their effect on temperature and microrotation. Abbas et al.[5] used Galerkin finite element method to examine the heat transfer of Cu-water nanofluid moving inside a square cavity with two circular obstacles in the presence of a magnetic field. Relevant parameters were varied and their effect on isotherms and streamlines were presented. Suresh et al. [6] experimentally investigated the heat transfer of Al_2O_3-Cu-water hybrid nanofluid through a circular tube. Enhancement of heat transfer was observed compared with water alone. S. P. Anjali Devi and S. Suriya Uma Devi [7] performed numerical comparison between Cu-water nanofluid and Al_2O_3-Cu-water hybrid nanofluid in the presence of magnetic field. In terms of heat transfer rate, hybrid nanofluid resulted in better heat transfer. Atashafrooz et al.

[**8**] assessed the Nusselt number and distribution of dimensionless temperature resulting from Al_2O_3-CuO-water hybrid nanofluid flowing in an open trapezoidal cavity. Desouky et al. [9,[**10**] studied the heat transfer, stream function and vorticity in a T-shaped enclosure and the effect of MHD in [11]. Saleh et al. [12] tested the improvement of heat transfer in a trapezoidal enclosure using Al_2O_3-water nanofluid and Cu-water nanofluid.

Dagtekin et al. [13] used the control volume approach to predict the entropy generation in Γ shaped cavities experiencing natural convection for Newtonian fluids. Dehnavi, R., and Rezvani, A. [14] investigated the natural convection heat transfer in Γ shaped enclosure. Isotherms and streamline as well as Nusselt number variation with Grashof number and other pertinent parameters were analyzed. Chamkha et al. [15] examined mixed convection heat transfer of Cu-water nanofluid in a porous gamma shaped cavity in addition to entropy generation in the presence of magnetic field.

As shown in the previous review, there are few studies regarding Γ shaped enclosures. Therefore, this study will be directed to investigate the natural convection of Al_2O_3-Cu-water hybrid nanofluid in a gamma shaped cavity.

2. Physical Problem and Mathematical Modeling

The schematic of the physical problem encountered in this research is presented in **Fig.** 1. The cavity through which the fluid is moving is a gamma-shaped enclosure. The flow is assumed to be steady, laminar and incompressible.

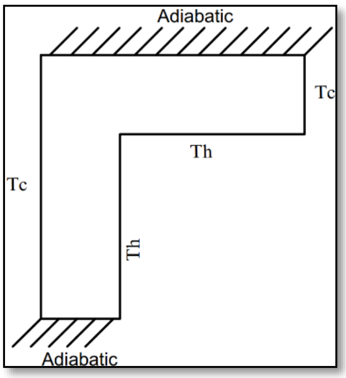


Fig. 1 "Schematic of the physical problem."

The upper right vertical wall as well as the left vertical wall are kept at T_c .As for the lower right vertical and horizontal wall, they are kept at T_h. Finally, the top and bottom walls are adiabatic. The fluid in this paper is Al_2O_3- Cu -water hybrid nanofluid.

Taking the work of Desouky et al. [9] and Reddy [16] as references, the continuity, the momentum after employing Boussinesq approximation and the energy equation for steady, laminar, incompressible, and viscid flow in the 2D conduit under consideration in dimensional form are as follows:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = \mathbf{0} \quad (1)$$

For momentum in x-direction,

$$\rho_{hnf}\left[\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right] = -\frac{\partial \bar{p}}{\partial \bar{x}} + \mu_{hnf}\left[\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}\right] \quad (2)$$

For momentum in y-direction,

$$\rho_{hnf}\left[\bar{u}\frac{\partial v}{\partial x} + \bar{v}\frac{\partial v}{\partial y}\right] = -\frac{\partial \bar{p}}{\partial \bar{y}} + \mu_{hnf}\left[\frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2}\right] + (\rho\beta_t)_{hnf}\, g(T - T_c) \quad (3)$$

For Energy,

$$(\rho c_p)_{hnf}\left[\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}}\right] = k_{hnf}\left[\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2}\right] \quad (4)$$

With the boundary conditions:

$\bar{u} = 0,\ \ \bar{v} = 0\ ,\ \ T = T_c$ at the left vertical wall and the upper right wall.

$\bar{u} = 0,\ \ \bar{v} = 0\ ,\ \ T = T_h$ at the middle vertical and horizontal walls.

$\bar{u} = 0,\ \ \bar{v} = 0\ ,\ \ \frac{\partial T}{\partial \bar{y}} = 0$ at the top and the bottom horizontal walls.

Properties of the hybrid nanofluid are related to the base fluid properties as described below,

$$\mu_{hnf} = \frac{\mu_f}{(1-\emptyset_1)^{2.5}(1-\emptyset_2)^{2.5}}\ ,\ A_1 = \frac{1}{(1-\emptyset_1)^{2.5}(1-\emptyset_2)^{2.5}}\ ,\ \mu_{hnf} = A_1\mu_f \quad (5)$$

$$\rho_{hnf} = (1-\emptyset_2)\left[(1-\emptyset_1)\rho_f + \emptyset_1\rho_{s1}\right] + \emptyset_2\rho_{s2}\ ,A_2 = \left[(1-\emptyset_1)(1-\emptyset_2) + \frac{\emptyset_1\rho_{s1}(1-\emptyset_2)}{\rho_f} + \frac{\emptyset_2\rho_{s2}}{\rho_f}\right]\ ,\rho_{hnf} = A_2\rho_f \quad (6)$$

$$\sigma_{hnf} = \sigma_{nf}\left[\frac{\sigma_{s2}(1+2\emptyset_2)+2\sigma_{nf}(1-\emptyset_2)}{\sigma_{s2}(1-\emptyset_2)+\sigma_{nf}(2+\emptyset_2)}\right],\quad \sigma_{nf} = \sigma_f\left[\frac{\sigma_{s1}(1+2\emptyset_1)+2\sigma_f(1-\emptyset_1)}{\sigma_{s1}(1-\emptyset_1)+\sigma_f(2+\emptyset_1)}\right] \quad (7)$$

$$(\rho\beta_t)_{hnf} = (1-\emptyset_2)[(1-\emptyset_1)(\rho\beta_t)_f + \emptyset_1(\rho\beta_t)_{s1}] + \emptyset_2(\rho\beta_t)_{s2}\ ,$$

$$A_3 = \left[(1-\emptyset_1)(1-\emptyset_2) + \frac{\emptyset_1(\rho\beta_t)_{s1}(1-\emptyset_2)}{(\rho\beta_t)_f} + \frac{\emptyset_2(\rho\beta_t)_{s2}}{(\rho\beta_t)_f}\right],$$

$$(\rho\beta_t)_{hnf} = A_3(\rho\beta_t)_f \quad (8)$$

$$(\rho c_P)_{hnf} = (1-\emptyset_2)\left[(1-\emptyset_1)(\rho c_P)_f + \emptyset_1(\rho c_P)_{s1}\right] + \emptyset_2(\rho c_P)_{s2}$$

$$,\ A_4 = \left[(1-\emptyset_1)(1-\emptyset_2) + \frac{\emptyset_1\rho c_P|_{s1}(1-\emptyset_2)}{\rho c_P|_f} + \frac{\emptyset_2\rho c_P|_{s2}}{\rho c_P|_f}\right]$$

$$,(\rho c_P)_{hnf} = A_4(\rho c_P)_f \quad (9)$$

$$k_{hnf} = k_{nf}\left[\frac{k_{s2}+2k_{nf}-2\emptyset_2(k_{nf}-k_{s2})}{k_{s2}+2k_{nf}+\emptyset_2(k_{nf}-k_{s2})}\right],$$

$$k_{nf} = k_f\left[\frac{k_{s1}+2k_f-2\emptyset_1(k_f-k_{s1})}{k_{s1}+2k_f+\emptyset_1(k_f-k_{s1})}\right] \quad (10)$$

The non-dimensional parameters are introduced as:

$$x = \frac{\bar{x}}{L_0}\ ,\ y = \frac{\bar{y}}{L_0},\ u = \frac{\bar{u}}{U_0},\ v = \frac{\bar{v}}{U_0},\ P = \frac{\bar{P}}{\rho_{hnf}U_0^2}\ ,\ \theta = \frac{T-T_c}{T_h-T_c}$$

, $\Pr = \frac{\mu_f c_{Pf}}{k_f}$, $\mathrm{Re} = \frac{\rho_f U_0 L_0}{\mu_f}$, $G_r = \frac{g B_{tf}(T_h - T_c) L_0^3 \rho_f^2}{\mu_f^2}$ (11)

Substituting from equation , $G_r = \frac{g B_{tf}(T_h - T_c) L_0^3 \rho_f^2}{\mu_f^2}$

$$\boldsymbol{u}\frac{\partial u}{\partial x} + \boldsymbol{v}\frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{A_1}{A_2}\frac{1}{Re}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] \qquad (12)$$

$$\boldsymbol{u}\frac{\partial v}{\partial x} + \boldsymbol{v}\frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \frac{A_1}{A_2}\frac{1}{Re}\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] + \frac{Gr}{Re^2}\frac{A_3}{A_2}\boldsymbol{\theta} \qquad (13)$$

$$\boldsymbol{u}\frac{\partial \theta}{\partial x} + \boldsymbol{v}\frac{\partial \theta}{\partial x} = \frac{k_{hnf}}{k_f A_4}\frac{1}{Re}\frac{1}{Pr}\left[\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right] + \frac{R}{A_4}\frac{1}{Re}\frac{1}{Pr}\left[\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right] \qquad (14)$$

With the following boundary conditions:

$u = 0, \quad v = 0, \quad \theta = 0$ at the left vertical wall and the upper right wall.

$u = 0, \quad v = 0, \quad \theta = 1$ at the middle vertical and horizontal walls.

$u = 0, \quad v = 0, \quad \theta_{i,j} = \theta_{i,j-1}$ at the top wall

$u = 0, \quad v = 0, \quad \theta_{i,j} = \theta_{i,j+1}$ at the bottom wall

Knowing that,

$$\omega = -\nabla^2\psi, \; u = \frac{\partial\psi}{\partial y}, \; v = -\frac{\partial\psi}{\partial x}, \; \omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \qquad (15)$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y} = \frac{A_1}{A_2}\frac{1}{Re}\left[\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right] + \frac{Gr}{Re^2}\frac{A_3}{A_2}\frac{\partial\theta}{\partial x} \qquad (16)$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y} = \frac{k_{hnf}}{k_f A_4}\frac{1}{Re}\frac{1}{Pr}\left[\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right] \qquad (17)$$

Finite difference method is used to solve the governing partial differential equations in this study through utilizing Mathematica 10 software. The used grid used is composed of 51 nodes in both the x-direction and the y-direction. The inner area ranges from 1 to N_x and from 1 to N_y for the x-direction and the y-direction respectively. The grid spacing between the points is equal and defined by $\frac{A}{N_x-1}$ and $\frac{B}{N_y-1}$ for the x-direction and the y-direction respectively.

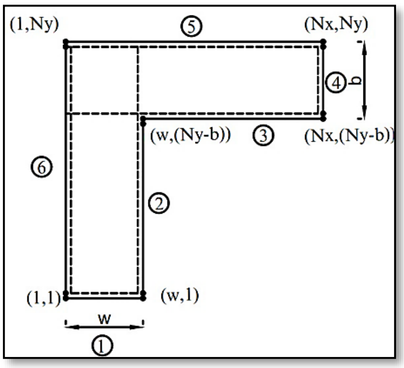
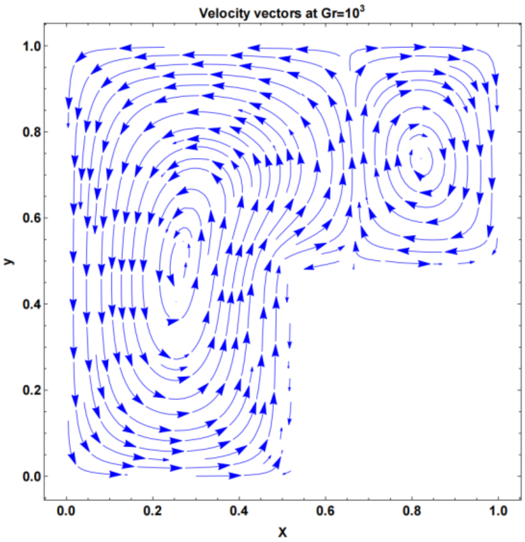


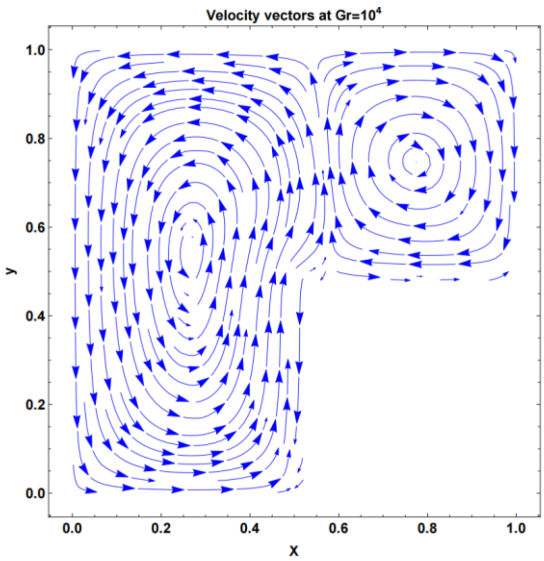
Fig. 2 "The gird"

3. Results and Discussion

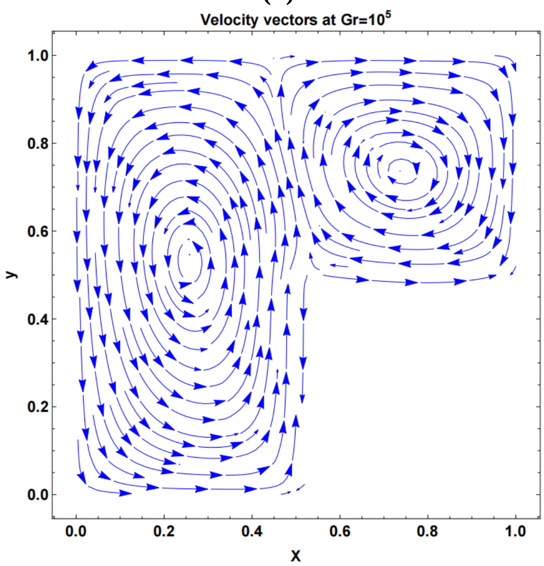
This paper is focused on solving Navier-stokes equations in addition to the energy equation for steady, incompressible, laminar and viscid hybrid nanofluid in a Γ-shaped cavity. **Fig.** 3 **s**hows the velocity vectors in the enclosure for various Grashof number $\mathrm{Gr} = 10^3, \mathrm{Gr} = 10^4, \mathrm{Gr} = 10^5$ respectively. The three figures present two vortices around which the fluid rotates, but in opposite directions.



(a)



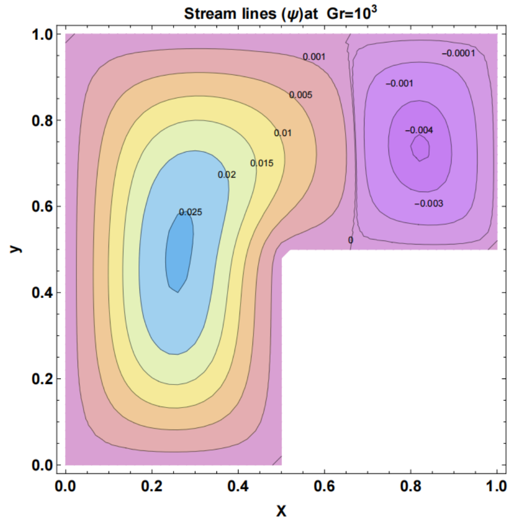
(b)



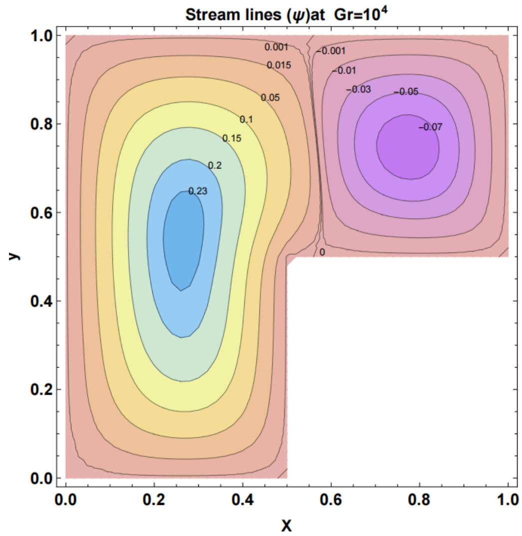
(c)

Fig. 3 "Non-dimensional velocity vector at (a)Gr=10^3, (b)Gr=10^4, (c)Gr=10^5"

Fig. 4 show streamlines contours for the various Grashof number. It's noted that the right streamlines have a negative sign and rotate in the clockwise direction, while the left streamlines rotate in the opposite (counterclockwise) direction.



(a)



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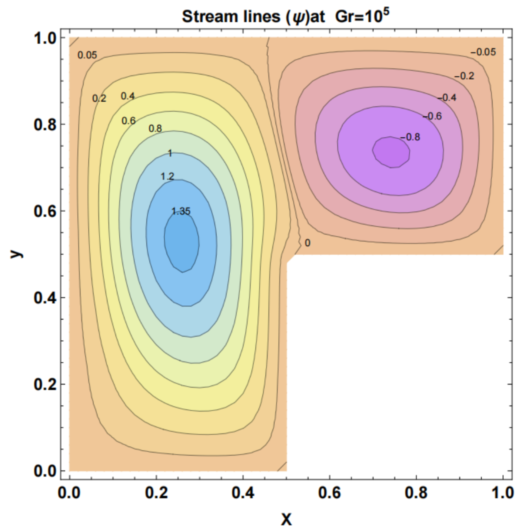
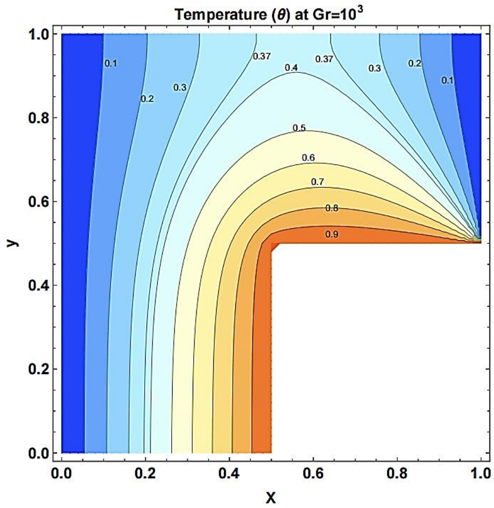


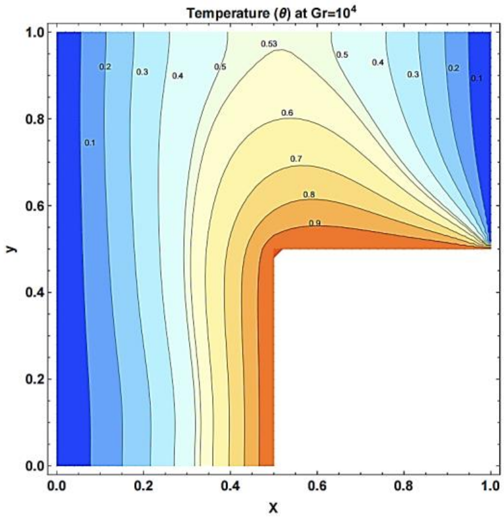
(c)

Fig. 4 "Non-dimensional streamlines ψ at (a)Gr=10^3, (b)Gr=10^4, (c)Gr=10^5"

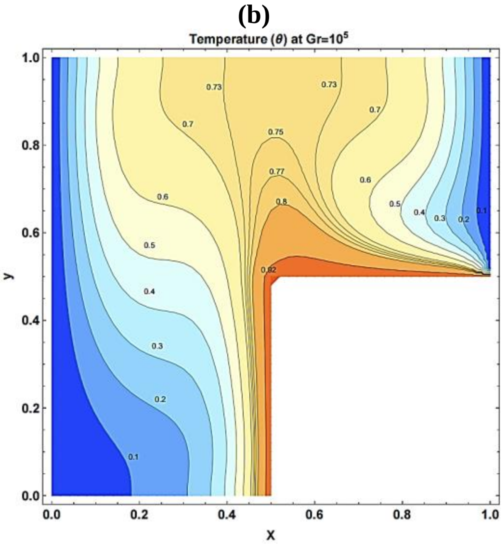
Fig. 5 shows the temperature distributions against Grashof number. Temperature contours exhibit a smooth and curved appearance, which means that the distribution of temperature follows a continuous and gradual change rather than sudden shifts.



(a)



(b)



(c)

Fig. 5 “Non-dimensional Temperature θ at (a)Gr=10^3, (b)Gr=10^4, (c)Gr=10^5”

Fig. 6 shows the temperature distribution between the two vertical walls (2 and 6) for various volume fractions of nanoparticles. The figure indicates that heat transfer between the two walls increases with increasing nanoparticle volume fraction.

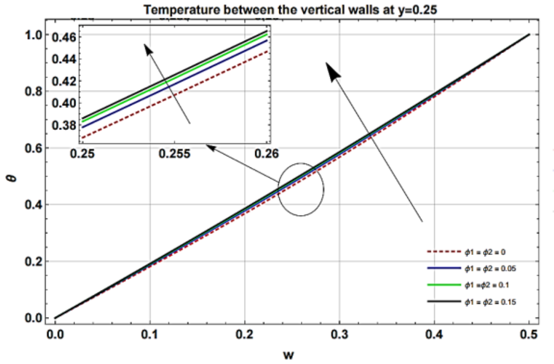
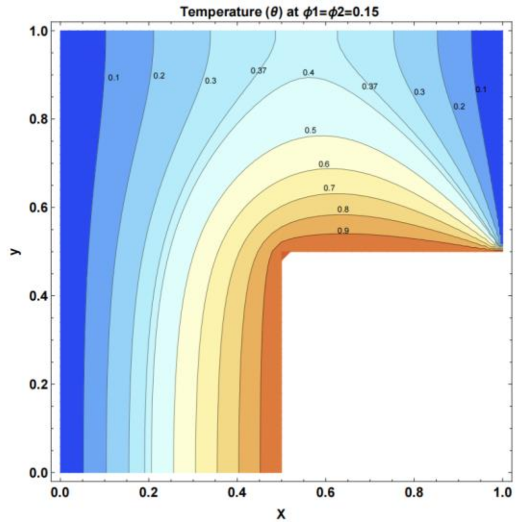


Fig. 6 “Non-dimensional Temperature θ variation between the vertical walls”

Fig. 7 shows the influence of nanofluids volume fraction on the isotherms with Grashof number 10^3 and volume fractions of the two nanoparticles 0.15.



(c)

Fig. 7 “Non-dimensional Temperature θ at $\phi_1, \phi_2 = 0.15$”

4. Conclusion

In this paper, the influence of changing volume fraction of nanofluids on the temperature distribution of a Γ-shaped enclosure is investigated. Steady, incompressible, laminar and viscid flow is assumed. The impact of Grashof number variation on the resulting streamlines and isotherms is also addressed.

Based on the data presented in the preceding figures, it’s evident that,

- Increasing Grashof number enhances the convection heat transfer from the hot regions to the cold one as the buoyancy effects are dominant.

- The more Grashof number is, the stronger the buoyancy which leads to the formation of larger eddies and recirculation zones and higher velocities beside the hot walls.
- As the volume fraction of the nanofluids increases, the temperature distribution tends to be more uniform due to enhanced heat transfer characteristics of theses fluids.

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