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# The impact of rotation on wave propagation of thermoelastic medium

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Abstract: The present investigation is intended to demonstrate the effect of rotation on the thermoelastic medium. The governing equations are formulated in the context of the Lord-Shulman theory. We obtained the analytical solution using Lame's potential method and Lthe normal mode technique with appropriate boundary conditions. The field quantities were calculated analytically and displayed graphically under the rotation with the effect of time with respect to space coordinates. The numerical values of the expressions are evaluated using MATHEMATICA. The graphical results are presented to shown the effect of time and rotation. Findings indicate that rotation and the included parameters notably amplify physical responses, especially under increased time and rotation. Results are validated against existing literature. Although theoretical in nature, this work offers insights relevant to geophysics, seismology, and earthquake engineering, particularly in applications such as mining tremors and crustal drilling. The results obtained are in agreement with the previous results obtained by others research when the new parameters vanish.

Keywords: Thermoelastic half-space; rotation, Lord-Shulman theory, Normal mode technique, lame's potential method.

## 1. Introduction

In recent years, the investigation of wave propagation and memory effects in thermoelastic media has garnered significant attention due to its vast applications in geophysics, biomechanics, aerospace engineering, and advanced material science. The classical coupled thermoelasticity theory proposed by Biot [1] with the introduction of the strain-rate term in the Fourier heat conduction equation leads to a parabolic-type heat conduction equation, called the diffusion equation. This theory predicts finite propagation speed for elastic waves but an infinite speed for thermal disturbance This is physically unrealistic. In the classical theory of thermoelasticity, Fourier's heat conduction theory assumes that the thermal disturbances propagate at infinite speed, which is unrealistic from the physical point of view. Two different generalizations of the classical theory of thermoelasticity have been developed, which predict only the finite velocity of propagation of heat and displacement fields. The first one is given by Lord and Shulman [2]. The second developed a temperature rate dependent thermoelasticity by including temperature rate among the constitutive variables is given by Green and Lindsay [3] have introduced situations where very large thermal gradients or annular a high heating speed may exist on the boundaries [4]. The linear theory of elasticity is of paramount importance in the stress analysis of steel, which is the commonest engineering structural material. To a lesser extent, linear elasticity describes themechanical behavior of the other common solid materials, for example,

concrete, wood, and coal. However, the theory does not apply to the behavior of many of the new synthetic materials of the clastomer and polymer type, for example, polymethylmethacrylate (Perspex), polyethylene, and polyvinyl chloride. The linear theory of micropolar elasticity is adequate to represent the behavior of such materials. For ultrasonic waves, that is, for the case of elastic vibrations characterized by high frequencies and small wavelengths, the influence of the body microstructure becomes significant; this influence of microstructure results in the development of new type of waves that are not in the classical theory of elasticity. Metals, polymers, composites, solids, rocks, and concrete are typical media with microstructures. More generally, most of the natural and manmade materials including engineering, geological, and biological media possess a microstructure Agarwal [5, 6]. Effects of rotation and relaxation times on plane waves in generalized thermo-elasticity are studied by Roychoudhuri [7]. The classical Fourier model, which leads to an infinite propagation speed of the thermal energy, is no longer valid [8]. Ahmad and khan [9] studied the effect of rotation on thermoelastic plane waves in an isotropic medium. Some problems in thermoelastic rotating media are due to, and Schoenberg and Censor [10], Puri [11], Singh and Kumar [12], Othman [13-16], Othman and Singh [17], Abd-Alla and Abo-Dahab [18]. Also normal mode analysis is used to solve a lot of problems in thermal flexibility, such as Lotfy and Abo-Dahab [19] and Othman and Song [20].

The main purpose of the present investigation is to demonstrate the effect of rotation on the thermoelastic

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medium. The governing equations are formulated in the context of the Lord-Shulman theory in the presence of the body force, rotation and time. We obtained the analytical solution using the lame's potential method and normal mode technique with appropriate boundary conditions. The field quantities were calculated analytically and displayed graphically under the rotation and time with respect to space coordinates. The graphical results are presented to shown the effect of magnetic field, time, angular frequency and wave number. Finally, these findings were contrasted with earlier findings in the same direction, and it was discovered that the approach taken to solve the aforementioned issue. Due to existence of realistic composition of this type of model in earth's interior, it can be very useful in important structures such as body materials in the aerospace field, nuclear reactors, pressure vessels and pipes etc. In the end, the results of this study were compared in some way to the results of other studies that have been published. The variations of the considered variables are obtained and illustrated graphically. The results obtained are in agreement with the previous results obtained by others when the new parameters vanish.

#### Nomenclatures

 $\lambda, \mu$  Lamè constants

t time

 $\rho$  Mass density

Ω Rotation

T Absolute temperature

Under the natural state, the temperature of the medium is  $\left| \frac{T-T_0}{T_0} \right| < 1$ 

 $\varepsilon_{ij}$  Strain tensor

 $\sigma_{ij}$  Stress tensor components

 $C_{ijkl}$  Elastic stiffness tensor

 $u_i$  component of displacement vector

e dilation

 $\tau_0$  Time of thermal relaxation

## 2. Formulation of the Problem

An infinite homogeneous thermoelastic half-space with external heat source is considered. The thermoelastic halfspace is initially at uniform reference temperature  $T_0$ . The origin of coordinate system lies on the middle surface of the half-space. The xy plane coincides with the middle surface and y axis is normal to the half-space. Let us consider that the thermoelastic medium is a half-space, subjected to rotation  $\Omega$ . The fundamental equations of the problem as follows:

The heat conduction equation is as Youssef [15]:

$$k\nabla^{2}T = \rho C_{e} \left(\frac{\partial T}{\partial t} + \tau_{O} \frac{\partial^{2}T}{\partial t^{2}}\right) + \beta \tau_{O} \frac{\partial e}{\partial t}$$
 (1)

$$e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

The equation of motion in the form

$$\rho \left( \frac{\partial^{2} \mathbf{u}}{\partial t^{2}} - \Omega^{2} \mathbf{u} - 2\Omega \dot{\mathbf{v}} \right) = (\lambda + 2\mu) \frac{\partial^{2} \mathbf{u}}{\partial x^{2}} + (\lambda + \mu) \frac{\partial^{2} \mathbf{v}}{\partial x \partial y} + \mu \frac{\partial^{2} \mathbf{u}}{\partial y^{2}} - \gamma T_{0} \frac{\partial T}{\partial x}.$$
 (2)

$$\begin{split} \rho\left(\frac{\partial^2 v}{\partial x^2} - \Omega^2 v + 2\Omega \dot{u}\right) &= (\lambda + 2\mu) \frac{\partial^2 v}{\partial y^2} + (\lambda + \mu) \frac{\partial^2 u}{\partial x \partial y} + \mu \frac{\partial^2 v}{\partial x^2} - \\ \gamma T_0 \frac{\partial T}{\partial y}. \end{split} \tag{3}$$

The constitutive equation as

$$\sigma_{xx} = (\lambda + 2\mu) \frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} - \gamma T. \tag{4}$$

$$\sigma_{yy} = (\lambda + 2\mu) \frac{\partial v}{\partial y} + \lambda \frac{\partial u}{\partial x} - \gamma T$$
 (5)

$$\tau_{xy} = \mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}) \tag{6}$$

The displacements for two-dimensional half-space have the following form

$$u_x = u(x, y, t)$$
,  $u_y = v(x, y, t)$  and The non-dimensional variables for simplifying are given as (7)

The non-dimensional variables for simplifying are given 
$$\dot{\mathbf{x}} = c_0 \eta \mathbf{x}, \, \dot{\mathbf{y}} = c_0 \eta \mathbf{y}, \, \, \dot{\mathbf{u}} = c_0 \eta \mathbf{u}, \, \dot{\mathbf{v}} = c_0 \eta \mathbf{v}, \, \, \dot{\mathbf{t}} = c_0^2 \eta \mathbf{t},$$

$$\theta = \frac{T}{T_0}, \, \dot{\mathbf{\tau}} = \frac{\tau}{2\mu + \lambda}, \, \dot{\Omega} = \frac{\Omega}{\omega^*}$$

$$\omega^* = \frac{\rho c_0 c_2^2}{\kappa}, \, \, \dot{\partial}_{ij} = \frac{\partial_{ij}}{\lambda + 2\mu}, \, \, \dot{\tau}_0 = \omega^* \tau_0$$
(8)

Where 
$$\eta = \frac{\rho c_e}{K}$$
,  $c_2^2 = \frac{\mu}{\rho}$ , and  $c_0^2 = \frac{2\mu + \lambda}{\rho}$ 

By dropping the dashed for convenience and substituting from Eq (1), Eqs (2) and (3) in the non-dimensional form take the

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = \delta 1 \frac{\partial \theta}{\partial t} + \delta 2 \frac{\partial^2 \theta}{\partial t^2} + \delta 3 \frac{\partial}{\partial t} (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})$$
(9)

$$\frac{\partial^{2} u}{\partial t^{2}} - \delta_{5} u - \delta_{6} \frac{\partial v}{\partial t} = \frac{\partial u}{\partial x} + \delta_{7} \frac{\partial^{2} v}{\partial x \partial y} + \delta_{8} \frac{\partial^{2} u}{\partial y^{2}} - \delta_{9} \frac{\partial \theta}{\partial x}$$
(10)

$$\frac{\partial^2 v}{\partial t^2} - \delta_{10} v + \delta_{11} \frac{\partial u}{\partial t} = \delta_{12} \frac{\partial v}{\partial y} + \delta_{13} \frac{\partial^2 v}{\partial x \partial y} + \delta_{14} \frac{\partial^2 v}{\partial x^2} - \delta_{15} \frac{\partial \theta}{\partial y}$$
(11)

here 
$$\varepsilon = \frac{\gamma}{\rho C_F}$$
 and  $\beta = a\eta^2 c_0^2$ 

By using Lame potentional

$$u = \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y} \quad , v = \frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x}$$
 (12)

From equations (9-11) and 12 we get

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = \delta_1 \frac{\partial \theta}{\partial t} + \delta_2 \frac{\partial^2 \theta}{\partial t^2} + (\delta_3 \frac{\partial}{\partial t} + \delta_4 \frac{\partial^2}{\partial t^2}) (\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2})$$
(13)

$$\frac{\partial^2 \varphi}{\partial t^2} - \delta_5 \varphi = \frac{\partial^2 \varphi}{\partial x^2} + \delta_7 \frac{\partial^2 \varphi}{\partial y^2} + \delta_8 \frac{\partial^2 \varphi}{\partial y^2} - \delta_9 \theta \tag{14}$$

$$-\frac{\partial^2 \psi}{\partial t^2} + \delta_{10} \psi = -\delta_{12} \frac{\partial \psi}{\partial y} + \delta_{13} \frac{\partial^2 \psi}{\partial y^2} - \delta_{14} \frac{\partial^2 \psi}{\partial x^2}$$
 (15)

#### 3. The solution of the proplam

In this section, normal mode analysis (NMA) is used to obtain the analytical expressions for displacement components, stress components and temperature. The assumed solution of physical variables is decomposed into normal modes to obtain the (20)

analytical solutions of the physical quantities. It is assumed that all the functions are sufficiently smooth such that the NMA of these functions exist. The solution of the equations( 13-15) can be decomposed in terms of the normal mode technique in the form

$$[\psi, \varphi, \theta, \sigma_{ij}](x, y, t) =$$
  
 $[\psi^*(x), \varphi^*(x), \theta^*(x), \sigma_{ij}^*(x)] \exp(\omega t + iby)$  (16)  
where  $i = \sqrt{-1}$ ,  $b$  is a wave number,  $\omega$  is the time constant, and  $u^*(x), \varphi^*(x), \theta^*(x)$  and  $\sigma_{ij}^*(x)$  are the amplitudes of the physical field quantities.  
Using Eqs. (16), into Eqs. (13) and (14)-(15), we obtain

$$\begin{array}{ll} (D^2-b^2-\delta_1\omega-\delta_2\omega^2)\theta^*(x)-(B_4D^2-\delta_3\omega b^2-\delta_4\omega^2b^2)\varphi^*(x) & (17)\\ (\omega^2-\delta_5-D^2+\delta_7b^2+\delta_8b^2)\varphi^*(x)+\delta_9\theta^*(x) & (18)\\ [D^2-K_{11}]\psi^*=0 & (19)\\ \text{where } K_{11}=\frac{\omega}{\delta_{14}}-\frac{\delta_{10}}{\delta_{14}}-\frac{\delta_{12}}{\delta_{14}}\text{ib}-\frac{\delta_{13}}{\delta_{14}}b^2\,,\quad D=\frac{d}{dx}\\ \text{Solving Eqs. (17) and (18) and by eliminating }\theta^*(x),\psi^*(x)\,,\\ \text{and }\varphi^*(x)\,,\text{ we obtain the partial differential equation satisfied}\\ \text{by }\theta^*(x)\\ (D^4-D^2(b^2-\beta_5+\beta_4\delta_9+\delta_1\omega+\delta_2\omega^2)-(b^2\beta_5+\beta_5\delta_1\omega-b^2\delta_3\delta_9\omega+\beta_5\delta_2\omega^2-\frac{1}{2})\\ \end{array}$$

where 
$$\beta_5 = \omega^2 - \delta_5 + \delta_7 b^2 + \delta_8 b^2, \gamma_1 = b^2 - \beta_5 + \beta_4 \delta_9 + \delta_1 \omega + \delta_2 \omega^2, \gamma_2 = b^2 \beta_5 + \beta_5 \delta_1 \omega - b^2 \delta_3 \delta_9 \omega + \beta_5 \delta_2 \omega^2 - b^2 \delta_4 \delta_9 \omega^2$$

 $b^2 \delta_4 \delta_9 \omega^2)(\varphi^*(x), \psi^*(x), \theta^*(x)) = 0$ 

which can be factorized to 
$$(D^4 - \gamma_1 D^2 - \gamma_2)(\varphi^*(x), \theta^*(x)) = 0$$
 (21)

where  $k_n^2$  (n = 1,2) are the roots of the characteristic equation.

$$k^4 - \gamma_1 k^2 + \gamma_2 = 0 (22)$$

as  $x \to \infty$ , the solution of Eq. (21) is given by

$$\theta^*(x) = \sum_{n=1}^2 A_n \exp(-k_n x)$$
 Similarly, (23)

$$\phi^*(x) = \sum_{n=1}^{2} \hat{A}_n \exp(-k_n x)$$
 (24)

From equation (17), (18), (23) and (24), we get

$$\phi^*(x) = H_{11}A_1 \exp(-\lambda_1 x) + H_{12}A_2 \exp(-\lambda_2 x)$$
 (25)

The solution of equation (19) take the form

$$\psi^*(x) = M_3 e^{-K_{11}x} \tag{26}$$

From equations (25),(26), (12) and (16)

$$\mathbf{u}(\mathbf{x}) = (\frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y}) \exp(\omega t + iby)$$

$$\mathbf{v}(\mathbf{x}) = (\frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x}) \exp(\omega t + iby)$$
(27)

$$\mathbf{v}(\mathbf{x}) = \left(\frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x}\right) \exp(\omega t + iby) \tag{28}$$

$$u(x) = [-H_{11}\lambda_1 A_1 \exp(-\lambda_1 x) - H_{12}\lambda_2 A_2 \exp(-\lambda_2 x) + ibB_2 \exp(\sqrt{-K_{11}x})] \exp(\omega t + iby)$$
(29)

$$v(x) = [ibH_{11}A_1 \exp(-\lambda_1 x) + ibH_{12}A_2 \exp(-\lambda_2 x) + B_2\sqrt{K_{11}} \exp(\sqrt{-K_{11}x})] \exp(\omega t + iby)$$
(30)

Substituting Eqs. (29) and (30) into Eqs. (5)-(7), we obtain

$$\sigma_{xx} = [H_{11}\lambda_{1}^{2} - b^{2}B_{1}H_{11} - B_{2}]A_{1}\exp(-\lambda_{1}x) + [H_{12}\lambda_{2}^{2} - b^{2}B_{1}H_{12} - B_{2}]A_{2}\exp(-\lambda_{2}x) + B_{2}\exp(-\sqrt{K_{11}x})]\exp(\omega t + iby) \quad (31)$$

$$\sigma_{yy} = [-b^{2}H_{11} + \lambda_{1}^{2}B_{1}H_{11} - B_{2}]A_{1}\exp(-\lambda_{1}x) + [-b^{2}H_{12} + \lambda_{2}^{2}B_{1}H_{12} - B_{2}]A_{2}\exp(-\lambda_{2}x) + [ib\sqrt{K_{11}} - B_{1}ib\sqrt{K_{11}}]B_{2}\exp(-\sqrt{K_{11}x})]\exp(\omega t + iby) \quad (32)$$

$$\tau_{xy} = [-2ibH_{11}\lambda_{1}\beta_{3}]A_{1}\exp(-\lambda_{1}x) + [-2ibH_{12}\lambda_{2}\beta_{3} - ib]A_{2}\exp(-\lambda_{2}x) + [-\beta_{3}b^{2} - \beta_{3}K_{11}]B_{2}\exp(-\sqrt{K_{11}x})]\exp(\omega t + iby) \quad (33)$$

# 4. Applications

The problem under study is related to thermoelastic half-space with rotation at the boundary surface are:

1) The boundary conditions for the thermal at the surface under the thermal shock

$$\theta = \frac{Q_0}{T_0} \,, \tag{34}$$

2) The boundary condition for the mechanical at the surface under the initial stress

$$\sigma_{xx}(0, y, t) = 0, \tau_{xy}(0, y, t) = 0$$
 (35)

Substituting into the above boundary conditions in the physical quantities, we obtain

$$\begin{array}{lll} A_1 + A_2 &= \frac{\varphi_0}{T_0} & (36) \\ [H_{11} \ \lambda_1^2 - b^2 \beta_1 H_{11} - \beta_2] A_1 + [H_{12} \ \lambda_2^2 - b^2 \beta_1 H_{12} - \beta_2] A_2 + \\ [-ib\sqrt{K_{11}} + \beta_1 ib\sqrt{K_{11}}] B_2 &= 0 & (37) \\ [-2ibH_{11} \ \lambda_1 \beta_3] A_1 + [-2ibH_{12} \lambda_2 \beta_3] A_2 + [-\beta_3 b^2 - \beta_3 K_{11}] B_2 &= 0 & (38) \\ [-3] & \text{In the context of the boundary conditions in Eqs. (36)-(38) at the surface } \ \mathbf{X} = \mathbf{0} \ , \ \text{we get a system of three Algebraic} \end{array}$$

equations. We apply the inverse matrix method and get the three constants A<sub>1</sub>, A<sub>2</sub> and B<sub>2</sub>. After that, we substitute into the main expressions to obtain the displacements, temperature, and other physical quantities.

Appling the Kramer 's on the Algebraic equations on Eqs. (36)-(38), we obtain the arbitrary  $A_1$ ,  $A_2$  and  $B_2$ , which it defined in appendix I.

## 5. Numerical results and discussions

In this section, we delve into the intriguing aspects of wave propagation in a thermoelastic medium. Our approach involves numerical simulations to analyze crucial fields, including displacement components, temperature and stress components. Furthermore, Mathematica software has been used in order to evaluate the numerical values of the field quantities. To illustrate the analytical variable obtained earlier, we consider a numerical example and consider the copper material. The results display the variation of temperature, and stress components in the context of LS the  $\lambda = 7.59 \times 10^9 \ N/m^2$   $\mu = 3.86 \times 10^{10} \ kg/ms^2$ 

$$C_E = 383.1 \quad J/(kgk), \ \varepsilon = 0.0168$$
  
 $\alpha = -1.28 \times 10^9 \ N/m^2, \ \rho = 7800 \ kg/m^2$ 

 $K = 386 \quad N/Ks , \tau_0 = 0.02, \alpha_t = 1.78 \times 10^{-5} \, N/m^2,$   $a = 1, \quad T_0 = 293 \, \mathrm{K}, \quad \omega = \omega_0 + i \xi, \quad \omega_0 = 2, \quad \xi = 1,$   $\eta = 8886.73 \, m/s^2, \quad Q_0 = 0.5$ 

We take the constants y=-1, b=0.25,  $H=10^5$ ,  $P=10^{10}$ ,  $\beta=0.1$ ,  $\tau=0.1$  for all computations, and we use for the real part of the displacement u, v, strain e and the stresses ( $\sigma_{xx}$ ,  $\sigma_{yy}$ ,  $\sigma_{xy}$ ), thermal temperature  $\theta$ , and conductive

temperature  $\phi$ . All field quantities do not depend only on space x and time, t, but they also depend on the relaxation time  $\tau$  and take the dimensionless form:

The output is plotted in Figs. 1-6. The variations of all the quantities are shown in Figures 1-6 to show the effect of different time t, and rotation  $\Omega$ . Figure 1: shows the variations of the temperature  $\theta$  with respect to x-axis for different values time t. It is observed that the temperature increases with increasing of time t, while it decreases with increasing of axial x. It is noticed that the due to the time effect, the elastic waves (described by temperature  $\theta$ ) on the surface are generated with a positive amplitude, which starts increasing when moving away from the source. After that, the elastic waves start showing periodic nature. This result is in a good agreement with the results obtained by [20].

Figure 2: displays the variations of the shear stress  $\tau_{xy}$  with respect to x-axis for different values time t. It is observed that the temperature increases with increasing of time t, while it oscillation in the whole range of x-axis, as well the shear stress satisfies the boundary connotation. It is clearly observed that the mechanical waves are highly sensitive towards the characteristic rotation  $\Omega$ . This result is in a good agreement with the results obtained by [15].

Figure 3: displays the variations of the normal stress  $\sigma_{yy}$  with respect to x-axis for different values time t .It is observed that the temperature increases with increasing of time t ,while it decrees with increasing of axial x. When comparing the magnitude of the physical quantities for three different values of  $\Omega$ , we found the fact that the effect of rotation corresponds to the term signifying positive forces that tend to accelerate the metal particles. This result is in a good agreement with the results obtained by [20].

Figure 4: Figure demonstrated the variations of the temperature  $\theta$  with respect to x-axis for different values of rotation  $\Omega$ . It is observed that the temperature increases with increasing of rotation  $\Omega$ , while it decreases with increasing of axial  $^{\mathcal{X}}$ . It is also observed that as value of  $^{\mathcal{X}}$  increases, the magnitude of  $\theta$  decreases rapidly and beyond a certain point in the region which agrees with the Lord-Shulman theory of thermoelasticity. This result is in a good agreement with the results obtained by [15]. Figure 5: displays the variations of the shear stress  $\tau_{xy}$  with respect to x-axis for different values of rotation  $\Omega$ . It is observed that the shear stress increases with

increasing of of rotation  $\Omega$ , while it oscillation in the whole range of x-axis, as well the shear stress satisfies the boundary connotation. Due to the rotation effect, the elastic waves on the surface are generated with positive amplitude, which starts increasing when moving away from the source. After that, the elastic waves start showing periodic nature. Both the physical quantity viz.  $\tau_{xy}$  shows similar sensitivity towards  $\Omega$ . Starting from a positive value, then showing an oscillatory nature with increasing amplitude as x keeps increasing. Thus, it can be said that for higher values of x, we can find the same values of  $\Omega$ , which can keep the amplitude in a controlled range. Varying  $\Omega$  in a system can significantly influence its behavior and performance, making sensitivity analysis a crucial tool for understanding the underlying dynamics. This result is in a good agreement with the results obtained by [15]. Figure 6: displays the variations of the normal stress  $\sigma_{vv}$ 

with respect to x-axis for different values of rotation x. It is observed that the normal stress decreases with increasing of rotation x, while it oscillation in whole rang x-axis. Understanding the relationship between parameters and outputs is essential for optimization, control, and decision-making processes, as it provides insight into which electric permittivity most significantly impact the system's behavior. Additionally, recognizing points of high sensitivity can guide the design of more robust systems that perform consistently across a range of operating conditions, which frequently aligns [17].

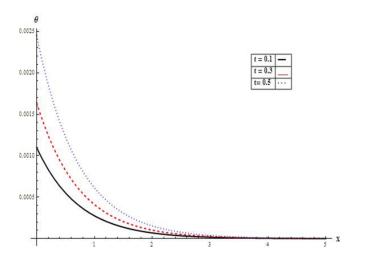


Figure 1: Variations of the temperature  $\theta$  with respect to x-axis for different value of time t.

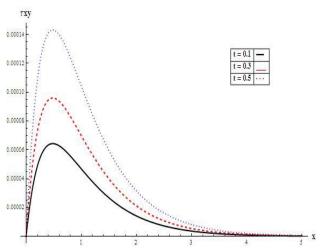


Figure 2: Variations of shear stress  $\tau_{xy}$  with respect to x-axis for different value of time t.

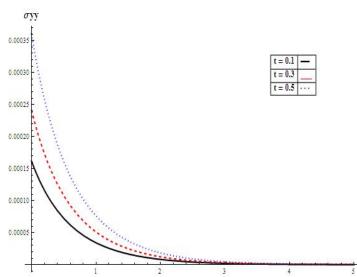


Figure 3: Variations of normal stress  $\sigma_{yy}$  with respect to x-axis for different value of time t.

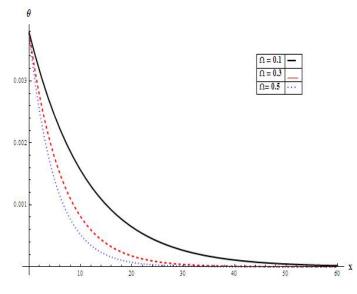


Figure 4: Variations of the temperature  $\theta$  with respect to x-axis for different value of rotation  $\Omega$ .

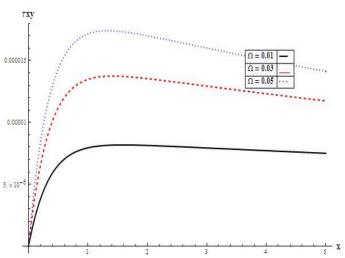


Figure 5: Variations of shear stress  $\tau_{xy}$  with respect to x-axis for different value of rotation  $\Omega$ .

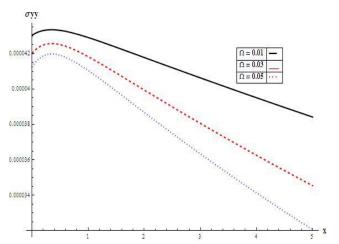


Figure 6: Variations of normal stress  $\sigma_{yy}$  with respect to x-axis for different value of rotation  $\Omega$ .

#### 6. Conclusions

From the results obtained analytically and from the graphs, the effect of the rotation and time on plane waves thermoelasticity generalization and these conclusions is presented in the following points.

- All the physical quantities satisfy the boundary conditions and obtain nonzero values only in the bounded region of space which supports the notion of Lord-Shulman theory of thermoelasticity.
- Numerical results and analysis show that there is a significant effect of time and rotation on the distribution of various enhances the temperature, displacement components and stress components.

- 3. The normal mode analysis of the problem of thermoelastic in solid has been applied and developed.
- 4. The generalized thermoelasticity with rotation can be described by characteristics by the fourth-order equation.

  5. The role of rotation and time is shown strongly in the physical quantities depending on the nature of the medium, as well as the horizontal and the vertical distances x and y, respectively.
- 6. The temperature converge to zero with increasing the distance x.
- 7. The nature of the force applied as well as the type of boundary conditions deformation are illustrated.

Finally, it is concluded that all the external parameters affect strongly the physical quantities of the phenomenon, which has more applications, especially in engineering, geophysics, astronomy, acoustics, industry, structure, and other related topics.

## **CRediT** authorship contribution statement:

Author Contributions: For research articles with several authors, a short paragraph specifying their individual contributions must be provided. The following statements should be used "Conceptualization, A. M. A. and S. M. A.; methodology, A. F. A. and S. B. M.; software, S. B. M.; validation, M. R. A., S. M. A. and A. M. A; formal analysis, M. R. A.; investigation, A. M. A.; resources, A. M. A.; data curation, A. F. A..; writing—original draft preparation, A. M. A.; writing—review and editing, S. B. M.; visualization, S. M. A.; supervision, M. R. A.; project administration, M. R. A.; funding acquisition, S. B. M. All authors have read and agreed to the published version of the manuscript."

## Data availability statement

The data used to support the findings of this study are available from the corresponding author upon request.

#### **Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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