SPACE VECTOR PULSE WIDTH MODULATION OF FOUR-SWITCH VOLTAGE SOURCE INVERTER FEEDING THREE PHASE INDUCTION MOTOR

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Abstract:
An analytical model with a new space vector pulse width modulation Voltage source inverter technique method of four-switch three-phase inverter (FSVPWM) feeding induction motor is presented. This paper presents a low cost inverter employing only four switches, four diodes and a split capacitor bank in the dc-link. This work is motivated by the need of an efficient and flexible modulation method, which is optimized with respect to minimum electrical motor torque. The proposed modulation strategy for the four-switch operation has the same symmetry as in a classical six-switches space vector pulse width modulation inverter (SSVPWM). The common mode voltage generated by the four-switch space vector pulse width modulation three-phase converter is evaluated and compared to that provided by the standard six-switch space vector pulse width modulation three-phase inverter. Simulation result are presented to demonstrate the feasibility of the proposed approach.

Index Terms:
Three-phase Induction motors, FSVPWM, Classical SSVPWM, modeling, Matlab/Simulink and time-domain analysis.

Improvements in power semiconductor switching technology have significantly reduced the cost and size of ac drives and improved waveform quality. Recently there has been growing interest in low cost ac drives to meet the needs for reducing cost. A number of low cost topologies have been suggested for single-phase to three-phase[1-6] or three-phase to three-phase voltage source inverter[7-9]. There are a variety of names for these inverters, i.e. inverter system with a reduced switch count, component minimized or four-switches inverter, split capacitor dc link inverter, and four switch three-phase inverter.

Van der Broecket. al. suggested a method of generating the three-phase waveforms with two dc link voltages and discussed the harmonic effects [1-2]. The modulation strategy suggested can produce three phase
balanced sinusoidal waveforms at a reduced output voltage of 0.866 compared with the conventional six switch inverter. In another topology proposed by Enjetiet et al., the diode bridge rectifier is replaced by a single-phase current controlled rectifier employing two switches and two capacitors[4-5]. Covic et al. proposed the new voltage control scheme, which enables unity power factor and an improved dc link voltage control independent of input voltage fluctuations[6]. However, the single-phase rectifier has a limitation for high power applications because power flow is not constant, therefore, requires a much larger capacitance in the dc link, which makes system performance sluggish. G. T. Kim et al. proposed a three-phase to three-phase VSI-PWM rectifier and inverter structure with eight switches, and discussed feasibility and operational limitations of the proposed structure[7]. M. Naser Uddinet al. presented a cost effective drive system for a salient pole permanent magnet synchronous motor for high performance industrial drive system. The proposed approach utilized a 4-switch 3-phase inverter instead of a conventional 6-switch 3-phase inverter [10]. This reduced both the cost of the inverter and the computation for real-time implementation. J. Klima presented analytical investigation of an induction motor fed from four-switch VSI with a new space vector modulation strategy [11]. The analytical results of the machine are accomplished in an αβ complex plane by space vector decomposition and using mixed p-z approach. M. Azabet al. presented a control method for 4-switch three-phase inverter suitable for low power applications [12]. A suitable switching table has been derived which selects the inverter switching states to fulfill the torque and flux requirements. H. H. Lee, et al. present space vector control approach for four-switch three-phase inverter under DC-link voltage ripple imbalance in photovoltaic or fuel cell inverter technology [13]. Space vector PWM technique for FSTPI under DC-link voltage imbalance or ripples have been solved, which is based on the establishment of basic space vectors and modulation technique in similarity with six-switch three-phase inverter.

In spite of the four-switches inverter drawbacks like a higher DC-link capacitor voltage and unsymmetrical scheme exposed to the unbalanced capacitor voltage, this inverter has the following advantages over six-switches inverter[14]:

- The number of switches is reduced by a third; driving circuits are only two as only two branches are controlled.
- In spite of the switch's higher withstand-able voltage in four-switches inverter the cost is still lower thanks to the price ratio of four-switches inverter to six-switches inverter usually lower than 3/2.
- Four-switches inverter maximum common mode voltage is just 2/3 of six-switches inverter.

This paper presents a new technique to generate space-vector pulse width modulation SVPWM signals for control of the four-switch, three-phase voltage source inverter based on principal of SVPWM. The analysis of this system under this technique will be discussed. The comparison between the two techniques (four switches and six switches) of SVPWM will carried out.

The proposed structure uses four diodes for rectifier, four power switches (IGBT) for inverter and two split-capacitor dc high voltage link as shown in Fig. 1. The four-switch inverter provide two of the inverter output phases. The third phase is fed by the dc link from the center of a split-capacitor bank.

II. Analysis of space voltage pulse width modulation

With respect to the circuit of Fig. 1, the phase and line voltages at the three-phase load terminals depend on the conduction states of the power switches. When the switches status are set to "1" when the power switch is closed and "0" when open. In addition the switches in one inverter branch are controlled complementary, therefore:

\[ S_1+S_2=1 \] and \[ S_1+S_4=1 \] (1)

The phase voltage between a, b and c to the point "0" are given by:

\[ v_{a0} = v_{an} - v_{cn}, v_{b0} = v_{bn} - v_{cn} \text{ and } v_{c0} = 0 \]

\[ v_{an} = (2S_1 - 1) \frac{V_d}{2} \text{ and } v_{bn} = (2S_3 - 1) \frac{V_d}{2} \] (2)

The phase voltage across motor , between a, b and c to the point "n" are given by:

\[ v_{an} = \frac{V_d}{6} [4S_1 - 2S_3 - 1], \text{ } v_{bn} = \frac{V_d}{6} [4S_3 - 2S_1 - 1] \text{ and } \]
The resultant space vector of the inverter output voltage is calculated using the following equations:

\[ v_i = \frac{2}{3} (v_{an} + av_{bn} + a^2v_{cn}) = \frac{2}{3} (v_a + jv_b)a = e^{-j2\pi/3}a \text{ and } a^2 = e^{j2\pi/3} \]  

(4)

Thus the orthogonal components voltage is given by:

\[ v_\alpha = \frac{2}{3} [v_{2n} - 0.5v_{bn} - 0.5v_{cn}] \]  

and

\[ v_\beta = \frac{1}{\sqrt{3}} [v_{bn} - v_{cn}] \]  

(5)

Table I shows the simplified of switching status, phase-zero voltage, phase voltage to neutral and the component \(v_\alpha\) and \(v_\beta\), space vector voltage \(v_i\) and its angle \(\theta_i\), where \(i=1,2,3\) and 4.

Table I shows the Simplified of switching status, phase-zero voltage, phase voltage to neutral and the component \(v_\alpha\) and \(v_\beta\), space vector voltage \(v_i\) and its angle \(\theta_i\), where \(i=1,2,3\) and 4.

<table>
<thead>
<tr>
<th>(S_3)</th>
<th>(S_2)</th>
<th>(v_{\alpha})</th>
<th>(v_{\beta})</th>
<th>(v_{\alpha})</th>
<th>(v_{\beta})</th>
<th>(v_{\alpha})</th>
<th>(v_{\beta})</th>
<th>(v_{\alpha})</th>
<th>(v_{\beta})</th>
<th>(\theta_i)</th>
<th>(\text{angle})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>360</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>150</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>60</td>
</tr>
</tbody>
</table>

Table II shows the relationship and similarity between FSVPWM and the conventional SVPWM.

\[ \bar{\varphi} = \int v_i(t)\,dt \]  

(6)

When the four-switch inverter fed the motor, the flux linkage vector becomes:

\[ \bar{\varphi}_i = t_i v_i + \bar{\varphi}_0 \]  

(7)

where \(i=1\) to 4; and \(t_i\) is the duration of \(v_i\). If the switching algorithms can ensure the best approximation by minimizing the discrepancy between vector loci \(\bar{\varphi}\) and \(\bar{\varphi^*}\), the stator voltage performance will be optimize.

### III. NEW TECHNIQUE OF FSVPWM

FSVPWM technique proposed in this paper is based on the principle of similarity of four-switch inverter, where \(\alpha\beta\)-plan is divided into six sectors, instead of four sector, and the formation of \(V_{ref}\) is done similarly as six-switch. Fig. 3 shows the new vectors with aiding the four vector of Fig. 2.

The new vectors: 

\(V_{23}\) is resultant of \(V_2\) and \(V_3\) as shown in Fig. 3, \(V_{34}\) is resultant of \(V_3\) and \(V_4\), \(V_{14}\) is the resultant of \(V_1\) and \(V_4\) and 
\(V_{13}\) is the resultant of \(V_1\) and \(V_3\) gives the new space vector of four-switch FSVPWM.

The zero vector \(V_0\) can be obtained by resulting \(V_4\) and \(V_1\) or by resulting \(V_3\) and \(V_2\). Those new vectors like the conventional six-switch SVPWM that shown in Fig. 4. Where new vectors 
\(V_{24m}=1/2 V_{24}\) as shown in both Fig. 3 and Fig. 4, 
\(V_{34m}=1/2 V_{34}\), 
\(V_{13m}=1/2 V_{13}\), 
\(V_{12m}=1/2 V_{12}\) and 
\(V_{0m}=(V_4+V_1)/2\) or \((V_3+V_2)/2\).

Table II shows the relationship and similarity between FSVPWM and the conventional SVPWM.

![Fig. 2 Space vector voltage in the plan αβ.](image)

The flux linkage vector of three-phase induction motor can be represented as [14]:

![Fig. 3 SVPWM technique of 4-switch on the principle similar to 6-switch.](image)
The calculation of the switching states in six-switch and four-switch are as follows for $T_s/2$ [15-16], where $T_s$ is the switching frequency.

\[
\begin{align*}
t_1 &= \frac{\sqrt{3}}{\pi} M T_s \sin\left(\frac{\pi}{3} - \alpha\right), \\
t_2 &= \frac{\sqrt{3}}{\pi} M T_s \sin(\alpha) \quad \text{and} \\
t_0 &= \frac{T_s}{2} - t_1 - t_2
\end{align*}
\]

where $t_1$ is the duration for vector $V_1$, $t_2$ is the duration for vector $V_2$ and $t_0$ is the duration of vector $V_0$. $M$ is the modulation index = $V_{\text{ref}}$/peak voltage of six step voltage and $V_{\text{ref}}$ is required amplitude of voltage vector.

For example, in sector I (Fig.3), the effective vectors $V_{24m}$, $V_4$ and $V_0$ are defined as equation (8):

Time duration of vector $V_{24m}$: $T_1= t_1$

Time duration of vector $V_4$: $T_2= t_2$

Time duration of Vector $V_0$: $T_0= T_s/2 - T_1-T_2$.

Now the timings of the switching pattern is shown in Fig. 5 can be calculated as:

- $T_a = T_s/2$ that is equivalent to the switching state [0 0].
- $T_b = T_s/2$ that is equivalent to the switching state [1 0].
- $T_c = T_s/2 + T_s/2 + T_0/2$ that is equivalent to the switching state [1 1].

Where $T_a$, $T_b$, and $T_c$ represent the time duration of base vector $V_1$, $V_2$, and $V_5$. Similarly, we can calculate the space vector modulation for the other sectors. The calculation results and the switching states are shown in Table III.

**III. Simulation results of the proposed system and comparison with conventional type of SVPWM**

Simulation induction motor drive based on FSVPWM topology are carried out using Matlab/Simulink package. Moreover, to evaluate the performance of the proposed system, induction motor drive by conventional SSVPWM topology is also simulated to make a comparison between both schemes.

Fig. 6 to Fig. 8 shows the simulation results of induction motor using FSVPWM topology. The parameters of motor are listed in Appendix I. The applied ac voltage is 600V, frequency is 50Hz and modulation index is 0.9. Fig. 6 shows the performance of motor (the three phase motor current, the voltage across phase “a”, the speed of motor, motor torque and load torque). The motor is tested with constant load torque equal 10Nm. Fig. 7 shows the flux and the relationship between the flux in d- and q- axes. Fig. 8 shows the spectrum analysis for the stator voltage and current. It is clear that, the current is sinusoidal and the phase shift is 120°.

The results show voltage THD is 5.374% and current THD is 4.2396%, those values are acceptable for IEEE stander.
Table III The calculation results of time duration of vectors and the corresponding their switching states.

<table>
<thead>
<tr>
<th>Sector I (0°-60°)</th>
<th>Sector IV (180°-240°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1 = \frac{\sqrt{3}}{\pi} MT_s \sin \left(\frac{\pi}{3} - \alpha\right)$</td>
<td>$T_1 = \frac{\sqrt{3}}{\pi} MT_s \sin \left(\frac{\pi}{3} - \alpha\right)$</td>
</tr>
<tr>
<td>$T_2 = \frac{\sqrt{3}}{\pi} MT_s \sin(\alpha)$ and</td>
<td>$T_2 = \frac{\sqrt{3}}{\pi} MT_s \sin(\alpha)$ and</td>
</tr>
<tr>
<td>$T_0 = \frac{T_2}{2} - T_1 - T_2$</td>
<td>$T_0 = \frac{T_2}{2} - T_1 - T_2$</td>
</tr>
<tr>
<td>The duration states and switching states are:</td>
<td>The duration states and switching states are:</td>
</tr>
<tr>
<td>$T_a = T_0 \rightarrow [0\ 0]$</td>
<td>$T_a = \left(T_1 + \frac{T_2}{2} + \frac{T_0}{2}\right) \rightarrow [0\ 0]$</td>
</tr>
<tr>
<td>$T_b = \frac{T_2}{2} \rightarrow [1\ 0]$</td>
<td>$T_b = \left(T_1 + \frac{T_2}{2}\right) \rightarrow [0\ 1]$</td>
</tr>
<tr>
<td>$T_c = \left(T_2 + \frac{T_1}{2} + \frac{T_0}{2}\right) \rightarrow [1\ 1]$</td>
<td>$T_c = \frac{T_0}{2} \rightarrow [1\ 1]$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector II (60°-120°)</th>
<th>Sector V (240°-300°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1 = \frac{\sqrt{3}}{\pi} MT_s \sin \left(\frac{\pi}{3} - \alpha\right)$</td>
<td>$T_1 = \frac{\sqrt{3}}{\pi} MT_s \sin \left(\frac{\pi}{3} - \alpha\right)$</td>
</tr>
<tr>
<td>$T_2 = \frac{\sqrt{3}}{\pi} MT_s \sin(\alpha)$ and</td>
<td>$T_2 = \frac{\sqrt{3}}{\pi} MT_s \sin(\alpha)$ and</td>
</tr>
<tr>
<td>$T_0 = \frac{T_2}{2} - T_1 - T_2$</td>
<td>$T_0 = \frac{T_2}{2} - T_1 - T_2$</td>
</tr>
<tr>
<td>The duration states and switching states are:</td>
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</tr>
<tr>
<td>$T_a = T_0 \rightarrow [0\ 0]$</td>
<td>$T_a = \left(T_1 + \frac{T_2}{2} + \frac{T_0}{2}\right) \rightarrow [0\ 0]$</td>
</tr>
<tr>
<td>$T_b = \frac{T_2}{2} \rightarrow [1\ 0]$</td>
<td>$T_b = \left(T_1 + \frac{T_2}{2}\right) \rightarrow [0\ 1]$</td>
</tr>
<tr>
<td>$T_c = \left(T_2 + \frac{T_1}{2} + \frac{T_0}{2}\right) \rightarrow [1\ 1]$</td>
<td>$T_c = \frac{T_0}{2} \rightarrow [1\ 1]$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector III (120°-180°)</th>
<th>Sector VI (300°-360°)</th>
</tr>
</thead>
</table>

Fig. 9 to Fig. 11 shows the simulation results of induction motor using SSVPWM topology. The applied ac voltage is 450V, frequency is 50 Hz and modulation index is 0.9. The parameters of motor are the same for previous simulation. Fig. 9 shows the performance of motor (the three phase load current, the voltage across phase "a", the speed of motor, motor torque and load torque) under constant load torque equal 10Nm. Fig. 10 shows the flux and the relationship between the flux in d- and q- axes. Fig. 11 shows the spectrum analysis for the stator voltage and current. The results show voltage THD is 4.1543% and current THD is 3.624%, those values are acceptable for IEEE stander.

Fig. 6 The performance of motor with load torque is 10Nm for FSVPWM topology.
The power consumed

Flux in q-axis

50

450

550

0.43

0.4

0.2

150

250

0.3

0.41

50

Fig. 7 The stator flux of motor with load torque 10Nm for FSVPWM topology.

Table IV The Comparison between SSVPWM and FSVPWM.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>SSVPWM</th>
<th>FSVPWM</th>
<th>FSVPWM/SSVPWM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applied ac V</td>
<td>450V</td>
<td>600</td>
<td>0.75</td>
</tr>
<tr>
<td>Motor rms V</td>
<td>200V</td>
<td>170V</td>
<td>0.85</td>
</tr>
<tr>
<td>Motor rms I</td>
<td>8.45A</td>
<td>7.84A</td>
<td>0.92</td>
</tr>
<tr>
<td>Speed</td>
<td>1600 rpm</td>
<td>1477rpm</td>
<td>0.9</td>
</tr>
<tr>
<td>Output power</td>
<td>1675.5W</td>
<td>1546.7W</td>
<td>0.923</td>
</tr>
<tr>
<td>THD_V</td>
<td>4.1543%</td>
<td>5.374%</td>
<td>1.293</td>
</tr>
<tr>
<td>THD_I</td>
<td>3.624%</td>
<td>4.239%</td>
<td>1.17</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

This paper presented a modified PWM using only four switches that can used in low power applications. The proposed system shows good agreement between the SSVPWM and FSVPWM topologies. Where the voltage and current THD for SSVPWM are 4.1543% and 3.624% respectively and voltage and current THD for SSVPWM are 5.374% and 4.239%. Those values of THD are acceptable. The power consumed of the proposed system is about 0.923 as compared with SSVPWM.
REFERENCES


Appendix I

The parameter of Induction motor:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>P</td>
<td>4kW</td>
</tr>
<tr>
<td>Rated voltage</td>
<td>V</td>
<td>400V</td>
</tr>
<tr>
<td>Rated frequency</td>
<td>F</td>
<td>50Hz</td>
</tr>
<tr>
<td>Rated speed</td>
<td>N</td>
<td>1430rpm</td>
</tr>
<tr>
<td>No. of poles</td>
<td>p</td>
<td>4pole</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>Rs</td>
<td>1.405Ω</td>
</tr>
<tr>
<td>Stator leakage inductance</td>
<td>Ls</td>
<td>0.005839H</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>Rs</td>
<td>1.395Ω</td>
</tr>
<tr>
<td>Rotor leakage inductance</td>
<td>Lr</td>
<td>0.005839H</td>
</tr>
<tr>
<td>Mutual inductance</td>
<td>Lm</td>
<td>0.1722H</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>J</td>
<td>0.13kgm²</td>
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<tr>
<td>Friction coefficient</td>
<td>B</td>
<td>0.002985Nms</td>
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