# Comparative Study of Concrete-Encased Composite Column Strength 

 دراسة مقارنـة لمقاومة الاعمدة المركبة (لمغطاة بالخرسانةNabil S. Mahmoud ${ }^{1}$, Saad El-deen M. Abd-rabou ${ }^{2}$ and Khaled M. H. Megahed ${ }^{3}$<br>${ }^{1}$ Prof. of steel structures Mansoura University, Mansoura, Egypt<br>${ }^{2}$ Prof. of steel structures Mansoura University, Mansoura, Egypt<br>${ }^{3}$ Demonstrator in structural Engineering Department Mansoura University, Mansoura, Egypt<br>K.megahed@mans.edu.eg

الهوف من هذا البحث هو دراسة الفرق بين ثلاثة وجهات تستخدم لتصميم الأعمدة المركبة المغطاة بالخرسانة. النهج الأول "الكود الصري [1]" يستبدل العمود بقطاع مكافئا من الصلب، والثاني "الكود الاوربي 4 [2] و الكود الامريكي لتصميم المنشاءات المعدنية [3]" يعتبر العمود كقطاع مركب باستخدام توزيع الاجهادات اللانة، والثالث "الكود الامريكي [4] و الكود الكندي [5] لتصميم المنشاءات الخرسانية " يشبه النهج الثاني مع استخدام توزيع الاجهادات المرن. تثبر اللاراسة المقارنة إلى أن الوجهة الثانية و الثالثة تعطي نتائج جيدة لتحمل الأعمدة المركبة مقارنة مع الوجهة الأولى وفقا لنتائج الاختبارات التي قام باحثون سابقون. كما اقترحت هذا البحث صيغة مقترحة لمعادلة التفاعل للأعمدة المركبة.


#### Abstract

The objective of this study is to investigate the difference between three approaches used for the design of concrete encased composite columns. The first approach "ECP-SCLRFD [1]" considers the composite column as an equivalent to steel section, the second "Eurocode 4 [2] and AISC [3]" considers the composite column as composite section using rigid plastic stress distribution, and the third "ACI [4] and CSA [5]" is similar to second approach with elastic plastic stress distribution. The comparative study indicates that the second and third approach give good results for composite columns strength compared with the first approach according to physical test results done by previous researchers. Also, this paper suggested a proposed formula for interaction equation for composite columns.


## Keywords:

Concrete encased composite column; test results; code provision; design approach; second order effect; interaction diagram.

## 1. Introduction

The composite concrete and steel structural system combines the rigidity and formability of reinforced concrete with the strength,
ductility and speed of construction of structural steel to produce an economic structure (Griffis 1986) [31]. It also protects it from fire damage and local buckling failure.

The design for composite members were studied and modified by a lot of researchers over the past decades. Different approaches for design of composite members are used in Egyptian Code of Practice for Steel Construction (LRFD) (ECP-SCLRFD) [1], Eurocode 4 (CEN2004) [2], AISC-Load and Resistance Factor (LRFD) (AISC) [3], American Concrete Institute ACI318-14 [4], and Canadian Standard Association Code (CSA) [5].

The codes provisions for the design of concrete-encased composite column follow three approaches. The first approach "ECPSCLRFD" considers the composite column as an equivalent steel section, the second "Eurocode 4 and AISC" considers the composite column as composite section using rigid plastic stress distribution, and the third "ACI and CSA" use elastic plastic stress distribution.

The objectives of this study are to: (i) investigate the differences between these approaches in determining the strength ratio ( $\mathrm{P}_{\text {test }} / \mathrm{P}_{\text {calc. }}$ ), (ii) study some variables and their effect on strength ratio were studied such as structural steel ratio, end eccentricity ratio, and slenderness ratio through statistical comparisons, and (iii) suggest a proposed formula for axial-bending moment interaction diagram.

This study presents a comparison between the calculated strength according to applied codes approaches and 399 physical tested composite symmetric rectangular columns which are pin-end at both ends [6 to 30].The previous tests have been divided according to the applied straining actions as the following: (i) 161 tests are subjected to pure axial load, (ii) 156 tests with combined axial load and major bending, (iii) 64 tests with combined axial load and minor bending, and (iv) 18 tests with pure bending. Table 1 includes summary of different limitations between applied codes. Tables (2-a) and (2-b) include comparison between various codes and proposed equation for calculating column strengths ratio ( $\mathrm{P}_{\text {test }} / \mathrm{P}_{\text {calc }}$ ). $\mathrm{P}_{\text {calc }}$ means the calculated column strength computed by applied code and presents nominal axial load strength except that
for 18 tests with pure bending, it presents nominal moment strength for the column.

## 2. International Codes Specifications:

## [1] Egyptian Code of Practice of steel construction (ECP-SCLRFD) [1]:

In this approach, the composite columns are converted to an equivalent steel section with modified mechanical properties.

### 2.1.1. Axial compressive strength

The design strength, $\emptyset_{c} P_{n}$, for symmetric axially loaded composite columns shall be computed on the steel section area using a modified radius of gyration $r_{m}$, yield stress $\mathrm{F}_{\mathrm{ym}}$ and Young's modulus $\mathrm{E}_{\mathrm{m}}$ to take into account the composite behavior. [ECP-SCLRFD [1] Clause 12.2, 12.3]
$\mathrm{P}_{\mathrm{u}}=\emptyset_{\mathrm{c}} \mathrm{p}_{\mathrm{n}}=\emptyset_{\mathrm{c}} \mathrm{A}_{\mathrm{s}} \mathrm{F}_{\mathrm{cr}}$
$[1-1]$
For $\lambda_{m} \leq 1.1 \quad F_{\text {cr }}=\left(1-0.348 \lambda_{m}{ }^{2}\right) F_{y m}$
[1-2]
For $\lambda_{\mathrm{m}} \geq 1.1 \quad \mathrm{~F}_{\mathrm{cr}}=0.648 \mathrm{~F}_{\mathrm{ym}} / \lambda_{\mathrm{m}}{ }^{2}$
[1-3]
Where $F_{y m}=F_{y}+c_{1} F_{y r}\left(A_{r} / A_{s}\right)+c_{2} F_{c u}$
$\left(\mathrm{A}_{\mathrm{c}} / \mathrm{A}_{\mathrm{s}}\right)[1-4] \quad \mathrm{E}_{\mathrm{m}}=\mathrm{E}_{\mathrm{s}}+\mathrm{c}_{3} \mathrm{E}_{\mathrm{c}}$
$\left(\mathrm{A}_{\mathrm{c}} / \mathrm{A}_{\mathrm{s}}\right)$ [1-5]
$\lambda_{\mathrm{m}}=\mathrm{KL}\left(\mathrm{F}_{\mathrm{ym}} / \mathrm{E}_{\mathrm{m}}\right)^{1 / 2} / \pi \mathrm{r}_{\mathrm{m}}$
[1-6]
$\mathrm{c}_{1}=0.7, \mathrm{c}_{2}=0.48$, and $\mathrm{c}_{3}=0.2$
[1-7]

### 2.1.2. Second order effect

Second order P- $\delta$ effects shall be considered in the design of pin-ended members subjected to combined axial load and symmetrical single curvature bending by multiplying first order elastic analysis moment by the moment magnifier $\delta$ expressed as:

$$
\begin{align*}
& \delta=\frac{1}{1-\frac{P_{u}{ }^{*}(K L)^{2}}{\pi^{2} E I}} \geq 1  \tag{1-8}\\
& \text { or } \delta=\frac{1}{1-\frac{P_{u}}{0.648 F_{y m} / \lambda_{m}{ }^{2} A_{s}}} \geq 1 \tag{1-9}
\end{align*}
$$

Negative value for moment magnifier means unstable column ( $P>P_{c r}$ ).

### 2.1.3. Flexural and axial load (interaction diagram)

The interaction of axial compression and flexural for doubly symmetric composite members shall be limited by the following:

$$
\begin{aligned}
& \frac{P_{u}}{\phi_{c} P_{n}}+\frac{8 M_{u}}{9 \phi_{b} M_{n}} \leq 1.0 \text { for } P_{u} \geq 0.2 \phi_{c} P_{n} \\
& \text { and } \frac{P_{u}}{2 \phi_{c} P_{n}}+\frac{M_{u}}{\phi_{b} M_{n}} \leq 1.0 \text { for } P_{u}<0.2 \phi_{c} P_{n} \\
& \quad[1-10]
\end{aligned}
$$

Where, $P_{u}$ factored axial load, $M_{u}$ factored moment magnified with moment magnifier $\delta$, $P_{n}$ nominal axial compressive capacity, $\mathrm{M}_{\mathrm{n}}$ nominal flexural capacity, where $\phi_{c}=0.8, \phi_{b}$ $=0.85$ for ECP-SCLRFD.

## [2] Eurocode 4 approach [2]:

Eurocode 4 Clause 6.7 is concerning the design of the composite columns using the concepts of limit state using fully plasticized structural steel section and reinforcing steel bars in tension and compression with stress ordinates equal to their yield strengths and a rectangular stress block for concrete compressive stress distribution, having a magnitude of $0.85 f_{c}$.

In this Code, General Method and Simplified Method are used for calculating column strength. Simplified Method is used for this study which is limited to columns of doubly symmetrical cross-section and with uniform
section over the length. These two methods are both based on the following assumptions:

- There is full interaction between the steel and concrete sections until failure occurs.
- Geometric imperfections and residual stresses are taken into account in the calculation, although this is usually done by using an equivalent initial out-of-straightness.
- Plane sections remain plane while the column deforms.


### 2.2.1. Axial compressive strength

The characteristic value of the plastic resistance to compression force, $P_{o}$, equal to

$$
P_{o}=0.85 f c A c+F y r A r+F y A \mathrm{~s}
$$

[2-1]
While the design value of the plastic resistance to compression force, $P_{n}$, of a composite column

$$
\begin{equation*}
P_{n}=0.85 f c / \gamma_{c} A c+F \mathrm{yr} / \gamma_{r} A \mathrm{r}+F \mathrm{y} / \gamma_{s} A \mathrm{~s} \tag{2-2}
\end{equation*}
$$

The effect of buckling reduces section resistance to the design axial loading $P_{r}$, $P_{r}=\chi P_{n}$, in which the value of $\chi$, the strength reduction factor in the plane of buckling considered, is a function of the relative slenderness $\bar{\lambda}=\sqrt{P_{o} / P_{c r}}$ and the appropriate European buckling curve.

$$
\begin{equation*}
\chi=\frac{1}{\phi+\left[\phi^{2}-\bar{\lambda}^{2}\right]^{1 / 2}} \leq 1, \phi=0.5\left[1+\alpha(\bar{\lambda}-0.2)+\bar{\lambda}^{2}\right] \tag{2-3}
\end{equation*}
$$

Elastic critical buckling, $P_{c r}=\frac{\pi^{2}(E I)_{e f f l l}}{(K L)^{2}}$, where K
is buckling length factor, $E I_{\text {eff, }}$ I represent the effective flexural stiffness $E I_{\text {eff }} . I=0.6 E_{c m} I_{c}$ $+E_{a} I_{a}+E_{s} I_{s}$ in which :
$I a$, Ic and $I s$ are the respective second moments of area, for the bending plane considered, of the steel section, the un-cracked concrete section and the reinforcement; $E a$ and $E s$ are the
respective elastic moduli of the steel of the structural section and of the reinforcement; Ecm is the elastic secant modulus of the concrete; 0.6 is a correction factor for cracking of concrete; and $\alpha$ is a generalized imperfection parameter which takes into account the initial out-of-straightness and residual stresses and equal to 0.49 for major bending and 0.34 for minor bending.

### 2.2.2. Flexural and axial load (interaction diagram)

Here, the axial-bending moment interaction diagram of the cross section is approximated by four-point polygon ABCD for major bending and five-point polygon ABCDE for minor bending as shown in Figure 1.

- Point A represents pure axial load strength $\mathrm{P}_{\mathrm{o}}(\mathrm{e} / \mathrm{h}=0)$.
- Point B represents the plastic resistance of cross section subjected to $\mathrm{M}_{\mathrm{o}}$ (pure moment $-\mathrm{e} / \mathrm{h}=\infty$ ).
- Point C corresponds to the axial load strength carried by the concrete only with moment equals $\mathrm{M}_{\mathrm{o}}$.
- Point D corresponds to the axial load equals to the half axial load of point C $\left(1 / 2 p_{c}\right)$ resulting in approximately the maximum bending moment $\mathrm{M}_{\text {max }}$.
- Point E is an arbitrary point located between point C and point A computed at the half of the eccentricity of point C


### 2.2.3. Second order effect

The second order effects can be taken into account multiplying the first order moment with imperfections effect by $\delta$ defined as:

$$
\begin{equation*}
\delta=\frac{1.1}{1-P_{u} / P_{\text {creff }}} \tag{2-4}
\end{equation*}
$$

$P_{c r, e f f}$ is based on the design value of effective flexural stiffness (EI $)_{\text {eff,II }}$ of the second order expressed as:

$$
\begin{equation*}
E I_{e f f, I I}=0.9\left(E_{r} I_{r}+E_{s} I_{s}+0.5 E_{c m} I_{c}\right) \tag{2-5}
\end{equation*}
$$

Creep effect should be taken into account by replacing $\mathrm{E}_{\mathrm{cm}}$ by $\mathrm{E}_{\mathrm{c}, \text { eff }}$ if applicable. If $\mathrm{P}_{\mathrm{cr}, \text { eff }}$ / $\mathrm{P}_{\mathrm{u}} \geq 10$, the second order can be neglected. The influence of geometric imperfections shall be considered by the equivalent global imperfections equal to $\mathrm{L} / 200$ for major bending and $\mathrm{L} / 150$ for minor bending where L is the buckling length of the column. This imperfection eccentricity is added to load eccentricity during design.

For given value of axial load, get $\boldsymbol{\mu} \mathrm{M}_{\mathrm{o}}$ and the first order moment for the column shall not exceed $\alpha_{M} \boldsymbol{\mu} \mathrm{M}_{0} / \delta$, where $\alpha_{M}$ accounts for the simplifications in the calculation method, and equal to 0.9 for the range of S235 to S355 steels, and 0.8 for steels range from S420 and S460.

## [3] AISC approach [3]:

AISC Specification Section I-5 permits the use of a strain compatibility or plastic stress distribution method (used in this study).

### 2.3.1.Axial compressive strength [AISC

 Spec. Section I-5]$\mathrm{P}_{\mathrm{o}}=\mathrm{A}_{\mathrm{s}}{ }^{*} \mathrm{~F}_{\mathrm{y}}+\mathrm{A}_{\mathrm{sr}} \mathrm{F}_{\mathrm{yr}}+0.85 \mathrm{Ac}_{\mathrm{c}} \mathrm{f}_{\mathrm{c}}{ }^{\prime}$
$E \mathrm{I}_{\text {eff }}=\mathrm{E}_{\mathrm{s}} \mathrm{I}_{\mathrm{s}}+0.5 \mathrm{E}_{\mathrm{r}} \mathrm{I}_{\mathrm{sr}}+\mathrm{C}_{1} \mathrm{E}_{\mathrm{c}} \mathrm{I}_{\mathrm{c}}$
$\mathrm{C}_{1}=0.1+2\left(\mathrm{~A}_{s}\left(\mathrm{~A}_{\mathrm{c}}+\mathrm{A}_{\mathrm{s}}\right)\right) \leq 0.3$
$\lambda=\left(\mathrm{P}_{\mathrm{o}} / \mathrm{Pe}_{\mathrm{e}}\right)$
Where $E_{c}=43 \mathrm{w}^{1.5}\left(\mathrm{f}_{\mathrm{c}}^{`}\right)^{0.5} \quad \mathrm{E}_{\mathrm{c}}$ in MPa
[3-6]
w weight of concrete in $\mathrm{KN} / \mathrm{m}^{3}$, $\mathrm{f}_{\mathrm{c}}{ }^{`}$ is in Mpa
$P_{n} / P_{o}=0.658^{\lambda}$ for $\lambda \leq 2.25$
$P_{n} / P_{o}=0.877 / \lambda$
The design axial compression resistance $\mathrm{P}_{\mathrm{d}}=$ $\phi_{c} \mathrm{P}_{\mathrm{n}}$

Where $\phi_{\mathrm{c}}=0.75, \phi_{\mathrm{b}}=0.9$ (LRFD)

### 2.3.2.Flexural and axial load (interaction diagram)

It is similar to Eurocode 4 in using an approximate polygonal ABCDE for interaction diagram but with reduced capacity due to slenderness effect $\lambda c$. The axial capacity was reduced from point i to point $\mathrm{i} \times \mathrm{P}_{\mathrm{n}} / \mathrm{P}_{\mathrm{o}}$ ( $\mathrm{i}=\mathrm{A}$ to E ) as shown in figure 1. Finally, the resistance factor, $\phi c$, was applied for axial capacity and $\phi \mathrm{b}$ for the flexural capacity. AISC Provision always gives unsafe zone (shaded area) which makes it less accurate as shown in Figure 1.


Figure 1: Eurocode 4, AISC, ECP-SCLRFD approaches $v s$. proposed method

## [4] ACI approach (ACI318-14) [4]:

Composite columns are treated like the concrete reinforced column. The strain compatibility method is used in which the flexural and axial strength of a member calculated by the strength design method of the Code requires that two basic conditions be satisfied: 1) equilibrium; and 2 ) compatibility of strains.

### 2.4.1. Axial compressive strength

ACI318-14 specifies that the maximum (nominal) axial load acting on composite column shall be limited to $0.8 \phi P 0$, where $\mathrm{P}_{\mathrm{o}}$ is the
cross section pure axial strengths and limited to the critical buckling load. [Clause 22.4].
$P_{o}=0.85 f \mathrm{c} A \mathrm{c}+F \mathrm{yr} A \mathrm{r}+F \mathrm{y} A \mathrm{~s} \leq P_{c r}=\pi^{2} E I /$ $(K L)^{2}$ [4-1]

Where:
$f c \quad$ compressive strength of concrete
Ac net area of concrete
Fyr yield strength of longitudinal reinforcement
Ar area of longitudinal reinforcement
Fy yield strength of steel shape
As area of steel shape
$\phi \quad$ resistance factor for compression $=$ 0.65

EI flexural stiffness for short-term loading is taken as $0.4 \mathrm{E}_{\mathrm{c}} \mathrm{I}_{\mathrm{g}}$ or $0.2 \mathrm{E}_{\mathrm{c}} \mathrm{I}_{\mathrm{g}}+\mathrm{E}_{\mathrm{s}} \mathrm{I}_{\mathrm{s}}$
[Clause 6.6.4.4],
$\mathrm{E}_{\mathrm{c}} \quad$ concrete elastic modulus
$\mathrm{E}_{\mathrm{s}} \quad$ steel elastic modulus
$\mathrm{I}_{\mathrm{g}} \quad$ gross section moment of inertia
$I_{s} \quad$ includes the section moment of inertia for steel and reinforcing bar
The nominal axial compressive strength $P \mathrm{n}$ for an encased composite column is limited to $0.8 P 0$ owing to a minimum eccentricity under axial load and limited to critical buckling load. EI seems to be conservative to account for the variations in stiffness due to cracking, creep and nonlinearity of concrete [Mirza] [33].

### 2.4.2. Second order effect

The ACI318-14 approach uses moment magnification approach and moment magnifier $\delta$ [Clause 6.6.4.5.2]. The column (slenderness) moment strength is equal to cross-section (material) strength divided by $\delta$ Eqn. [1-8] with stiffness reduction factor equal $=0.75$ as shown in Figure 1. The column is considered short neglecting second-order effect if the column
slenderness ratio, KL/r $\leq 34+12$ (M1/M2), where $\mathrm{M}_{1} / \mathrm{M}_{2} \leq 0.5$ [Clause 6.6.4.5.2].

Where $r=\sqrt{\frac{0.2 E_{c} I_{c}+E_{s} I_{s s}}{0.2 E_{c} A_{c}+E_{s} A_{s s}}}$

### 2.4.3. Flexural and axial load (interaction diagram)

Third approach is based on a strain compatibility analysis and static equilibrium at the limit state to develop a thrust-moment ( $\mathrm{P}-\mathrm{M}$ ) interaction relation with using the following assumptions [Clause 22.2]:

- Plane section remains plane.
- Rectangular stress block for concrete compressive stress distribution, having a magnitude of $0.85 f \mathrm{c}$, is used for the concrete with ( $a=\beta_{1} c$ ) height measured from the fiber of maximum compressive strain, Where $\mathrm{c}=$ distance from the extreme fiber to the neutral axis location and $\beta_{1}=0.85-0.05\left(\mathrm{f}_{\mathrm{c}} `-27.6\right)$ and limited to $0.65 \leq \beta_{1} \leq 0.85$.
- Tensile strength of the concrete is neglected.
- Strain hardening of steel shape and rebar is neglected.
- To get the interaction diagram for the column we use 0.003 as maximum strain for extreme compression fibers and change neutral axis position over the section, then get resistance of the section to moment and normal force
- Relative slip between structural steel and concrete is ignored
- The area of compressed concrete displaced by steel structure or reinforcing bars was removed in strength calculation.


## [5] CSA approach [5]

It is similar to ACI approach in most provisions except for:

- Nominal compressive strength, $\mathrm{P}_{\mathrm{n}}$ [Clause 10.10]:
$P_{n}=0.8\left(\alpha_{1} \phi_{c} f c A c+\phi_{r} F y r A r+\phi_{s} F y A \mathrm{~s}\right)$ [5-1]
Where, $\alpha_{1}=0.85-0.0015 f_{c}{ }^{`} \geq 0.67$ (height of concrete rectangular stress block), resistance factor $\phi_{c}=0.65, \phi_{s}=0.9$, and $\phi_{r}=0.8$.
- The second-order effect can be neglected if the column slenderness ratio, $\mathrm{KL} / \mathrm{r} \leq[25-$ $10(\mathrm{M} 1 / \mathrm{M} 2)] /\left[\mathrm{Pf}_{\mathrm{f}} / \mathrm{f}_{\mathrm{c}}{ }^{\prime} \mathrm{Ag}_{\mathrm{g}}\right.$ ] ${ }^{\wedge} 0.5$ [Clause 10.15].
- The Whitney stress block, having a magnitude of $\alpha_{1} f \mathrm{c}$, is used for the concrete with height $\mathrm{a}=\beta_{1} \mathrm{c}$ measured from the fiber of maximum compressive strain which limited to 0.0035 [Clause 10.1]. Where $\beta_{I}=0.97-0.0025 * f_{c}{ }^{`} \geq 0.67$ (fc` in Mpa).


Figure 2: Material and column strength interaction diagrams.

## [6] Proposed Interaction Equation:

Hsu [37] proposed an equation for $\mathrm{N}-\mathrm{M}$ interaction diagram solved with numerical procedures for composite columns:

$$
\begin{align*}
& \left(\frac{P_{n}-P_{n b x}}{P_{o}-P_{n b x}}\right)^{\alpha_{x}}+\left(\frac{M_{o x}}{M_{r x}}\right)=1 ; \quad M_{r x}=M_{n b x} \\
& \left(\frac{P_{n}-P_{n b y}}{P_{o}-P_{n b y}}\right)^{\alpha_{y}}+\left(\frac{M_{o y}}{M_{r y}}\right)=1 ; \quad M_{r y}=M_{n b y} \tag{6-1}
\end{align*}
$$

where $\alpha_{x}$ and $\alpha_{y}=$ numerical coefficients for shape of uniaxial interaction diagrams in $\mathrm{M}_{\mathrm{x}^{-}}$and $\mathrm{M}_{\mathrm{y}}$-axis, respectively; $\mathrm{P}_{\mathrm{nbx}}, \mathrm{P}_{\mathrm{nby}}$ are nominal balanced load for bending about x axis and y -axis, respectively; and $\mathrm{M}_{\mathrm{rx}}$ and $\mathrm{M}_{\mathrm{ry}}$ $=$ reference nominal bending moments that take different values depending on location of nominal axial load $\mathrm{P}_{\mathrm{n}}$

In the present study, the proposed formula for N -M interaction diagram for composite columns is similar to Hsu [37] equation as following:
$\left(\frac{\left|P_{u}-\frac{1}{2} P_{c}\right|}{P_{o}-\frac{1}{2} P_{c}}\right)^{\alpha}+\left(\frac{M_{u}}{\beta M_{\max }}\right)=1$
(Proposed Eq.) [6-2]
$\alpha$ is determined from interaction diagram points ABCD used by Eurocode 4 procedure.
at point $\mathrm{B}, \mathrm{M}_{n}=\mathrm{M}_{0}, \mathrm{P}_{n}=0.5 \mathrm{P}_{c}=0.5 A_{c} * 0.67 f_{c u} / \gamma_{c}$ then $\alpha=\frac{\ln \left(1-\frac{M_{o}}{M_{\text {max }}}\right)}{\ln \left(\frac{0.5 P_{c}}{P_{o}-0.5 P_{c}}\right)}$

Parameter $\beta$ accounts for the simplifications in the calculation method, and equal to 1.0 for major bending, 0.9 for minor bending. $\beta$ values are chosen to give the best fit with test results From the statistical analysis, the proposed equation gives a better agreement with Eurocode4.

Note that: $\left|P_{u}-\frac{1}{2} P_{c}\right|$ is a positive value for all cases.

At the end of this paper, a solved example using this equation was presented.

## 3. Comparison between test results and predicted strengths.

The tests strength of $\mathbf{3 9 9}$ specimens have been compared with the predicted one using the different codes, Eurocode 4, ECPSCLRFD, AISC, ACI, and CSA. The computed strength used for various codes is unfactored strength by using material resistance factor (partially safety factor) equal to 1.0 to study the reliability of each approach. For ACI provision, stiffness reduction factor $\left(\phi_{K}=0.75\right)$ was not included in these analyses when compared with the test results. The coefficient $\boldsymbol{\alpha}_{\mathbf{M}}$ ]used in Eurocode 4 was neglected.
The concrete strength $\mathrm{f}_{\mathrm{c}}{ }^{\prime}$ was defined as the strength obtained from the standard 150 mm diameter by 300 mm high cylinder tests. For some physical tests, the cube strengths were used. These cube strengths were converted to the equivalent standard cylinder strengths using following L'Hermite [38] equation [1]:

$$
\begin{equation*}
f_{c}=f_{c u}\left[0.76+0.2 \log _{10}\left(\frac{f_{c u}}{19.6}\right)\right] \tag{1}
\end{equation*}
$$

For ECP-SCLRFD, AISC and Eurocode 4, the evaluation of expected strength is by interpolation while, for ACI and CSA material interaction diagram is divided by $\delta$ for each point to get column interaction diagram and evaluation of expected strength by interpolation on column interaction diagram as shown in figure 2.

Figure 3 shows the normal frequency distribution curve for strength ratio prepared for all of $\mathbf{3 9 9}$ column tests results used.

Figure 3 (b, e) indicate the least scatter about the line of equality with a nearly bell-shaped normal distribution for Eurocode 4 and proposed equation. While, for ECP-SCLRFD and AISC plotted in figure 4(c, d) indicate the widest spread distribution. The mathematical verification of these observations is shown by the strength ratio statistics given in tables 2 (a-b), particularly by the coefficients of variation (COV. ${ }^{++}$). Lower coefficient of variation indicates a more reliable computational method. The coefficient of variation of the strength ratios ranges from the highest value of 0.428 obtained for AISC to the lowest value of 0.186, $\mathbf{0 . 1 7 6}$ obtained by Eurocode 4 and proposed equation respectively. Figure 4 shows comparison of strength ratios between different codes and proposed equation. This statistical observation provides valuable information on the accuracy and reliability of the proposed equation for Eurocode 4 in the prediction of the strength of concrete encased composite columns.

## Based on the above comparative results,

 the following observations are obtained:1. With an average strength ratio of $\mathbf{1 . 0 7 6}$ and a coefficient of variation of $\mathbf{0 . 1 7 4}$, proposed equation gives a better agreement than the polygon ABCDE interaction diagram for Eurocode 4.
2. Figure 4 shows the comparison between proposed equation with Eurocode4, ECPSCLRFD, and AISC using statistical distribution of strength ratio using various approaches against frequency percent of tested specimens. The proposed equation gives a good agreement with test results as compared with AISC and ECP-SCLRFD.
3. AISC and ECP-SCLRFD compute the column strengths least accurately of others codes procedures studied.
4. Without doubt, Reinforced concrete codes, ACI and CSA, give a good agreement in calculating strength, while Eurocode 4 gives easy hand calculation with better accuracy especially using the proposed equation.
5. The ECP-SCLRFD approach treats the design of composite columns as a steel columns using equations [1-1 to $1-10$ ] which created for column strength influenced by 'residual stress' and 'initial out-of straightness'. However, for the composite column, these two parameters play a minor role because the reinforced concrete portion of the composite column is much less sensitive to these parameters. These observations show why the ECP-SCLRFD approach gives less accurate strength predictions.
6. Figure 5 shows the distribution of the strength ratios of codes approaches and effect of some parameters such as slenderness ratio $\mathrm{kL} / \mathrm{r}$, end eccentricity ratio $\mathrm{e} / \mathrm{h}$ and steel ratio $\left[\rho_{s}=A_{s} / A_{c}\right]$ on strength ratio for different applied codes. Using the curve fitting for strength ratio test results, an equation; $y(x)=a . x^{2}+b . x+c$; was fitted for each figure and presented in dotted line. Less (a) and (b) values indicate a less effect of this parameter on strength ratio for the applied code. As shown in figure 5, Eurocode 4 using polygon or proposed interaction diagrams give the less effect of column parameters on strength ratio.
7. Figure 5 shows the distribution of the strength ratios of codes approaches with respect to the steel ratios of the 399 tested specimens. The steel ratios shown in this figure range from $2 \%$ to $14.6 \%$. For the
specimens with steel ratio under $4 \%$, it is observed that the average strength ratios of various codes increase especially for ECPSCLRFD. These observations indicated that the limitation of a minimum $4 \%$ of steel ratio in an encased composite section is essential for the ECP-SCLRFD strength provisions.
8. As shown in Figure 6, the Axial-moment interaction diagrams of the Eurocode 4 approach with using polygon, and proposed interaction equation are constructed to compare some test results (Mirza - Yokoo) with Eurocode 4 provision using ABCD polygon and proposed equation for interaction diagram. The tests were chosen with the same properties as possible for each interaction diagram. It is shown that the proposed equation gives a good agreement as compared to the column test data (denoted as hollow circular symbol).
9. It is observed that all approaches give conservative estimates of the column strengths, especially ECP-SCLRFD.
10. The proposed method for Eurocode 4 is recommended because the development of $\mathrm{M}-\mathrm{N}$ interaction curve is direct and this enables easy hand calculation to be done.
11.It was found that ECP-SCLRFD gives the least accurate results compared with other provisions. In addition, ECP-SCLRFD rules specially require at least $\mathbf{4 \%}$ steel ratio of the composite section. However, the other codes rules have no such limitation on steel ratio.

## 4. Conclusions and Recommendations

1. The proposed equation enables easy hand calculation instead of interpolation used by
polygon ABCDE interaction diagram. It states:-

$$
\left(\frac{\left|P_{u}-\frac{1}{2} P_{c}\right|}{P_{o}-\frac{1}{2} P_{c}}\right)^{\alpha}+\left(\frac{M_{u}}{\beta M_{\max }}\right)=1
$$

(Proposed Eq.) [6-2]
2. Eurocode 4 gives the best agreement with test results for composite column design. While, ECP-SCLRFD [1] gives conservative values for strength ratio.
3. ACI and CSA give a good agreement in calculating strength, while Eurocode 4 gives a simple calculation with better accuracy.
4. From the studied parameters in figure 5, slenderness ratio $\mathrm{kL} / \mathrm{r}$, end eccentricity ratio $\mathrm{e} / \mathrm{h}$ and steel ratio $\left[\rho_{s}=\mathrm{A}_{s} / \mathrm{A}_{\mathrm{c}}\right.$ ], Eurocode 4 gives a good curve fitting for strength ratio than other codes.

## 5. Symbols

Ac area of concrete.
Ar longitudinal reinforcement.
As area of steel shape.
$B \quad$ width of section
$\mathrm{b}_{\mathrm{f}} \quad$ steel section flange width
$c 1, c 2, c 3$ numerical coefficients,
$c 1=0.7, c 2=0.6$ and $c 3=0.2$ for encased composite columns.
$e / h \quad$ eccentricity ratio.
$e \quad$ eccentricity of axial load at column ends about major axis
Ec Young's modulus of concrete
Em modified modulus of elasticity
$E_{s} \quad$ Young's modulus of steel
$f c^{\prime} \quad$ compressive strength of concrete

Fcr critical stress of the composite column

Fmy modified yield stress
$H$ geometrical height of section
$h_{s} \quad$ steel section height
KL Buckling length of column
$K L / r$ slenderness ratio
Mo nominal moment capacity without axial load
Mu factored moment
M1, M2 the smaller and the larger required moments applied at both ends of the column, respectively
$\mathrm{M}_{1} / \mathrm{M}_{2}$ end moment ratio (smaller to larger moment ratio)
$P$ test experimental strength of column
$P$ calc calculated strength of column by using the proposed code method.
P0 composite column capacity under uniaxial compression
Pcr critical load of column
$P n \quad$ nominal axial compressive capacity
$P \mathrm{u}$ factored axial load
$\mathrm{P}_{\mathrm{c}}$ nominal concrete section capacity ( $0.85 f_{c}{ }^{`} / \gamma_{c}{ }^{*} \mathrm{~A}_{\mathrm{c}}$ )
$r$ radius of gyration.
$r_{m} \quad$ radius of gyration of the steel shape, it shall not be less than 0.3 times the overall width of the composite column in the plane of the bending.
$\mathrm{t}_{\mathrm{w}} \quad$ steel section web thickness
$\mathrm{t}_{\mathrm{f}} \quad$ flange thickness
$\delta \quad$ moment magnifier
$\phi b \quad$ resistance factor for bending, taken as
$\phi c \quad$ resistance factor for compression, tak-
en as 0.85
$\lambda c \quad$ slenderness parameter
$\lambda^{`} \quad$ relative slenderness
$\delta^{\text {` }} \quad$ Steel contribution factor
$\rho_{s} \quad$ steel section ratio $\mathrm{A}_{s} / \mathrm{A}_{\mathrm{c}}$
$\rho_{r} \quad$ longitudinal bars ratio $\mathrm{A}_{\mathrm{s}} / \mathrm{A}_{\mathrm{c}}$
$\gamma_{c} \quad$ partially safety factor for concrete
$\gamma_{\mathrm{s}} \quad$ partially safety factor for steel section
$\gamma_{\mathrm{r}} \quad$ partially safety factor for longitudinal bars.
$\alpha_{m}$
$\alpha \quad$ interaction diagram shape factor (proposed method)
$\beta \quad$ reduction factor for moment (proposed method).

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Example: check on the safety of this column Column properties: $k l=3000 \mathrm{~mm}, B=280 \mathrm{~mm}$, $H=280 \mathrm{~mm}, h s=200 \mathrm{~mm}, b f=200 \mathrm{~mm}, t f=9 \mathrm{~mm}$,
$t w=6 \mathrm{~mm}, \mathrm{fcu}=51.8 \mathrm{~N} / \mathrm{mm}^{2}, f y r=420 \mathrm{~N} / \mathrm{mm}^{2}$, $f y s=248 \mathrm{~N} / \mathrm{mm}^{2}, \quad A_{r}=603 \mathrm{~mm}^{2}$ (4ф14) under these straining actions:

1. $P_{u}=3000 \mathrm{KN}, M_{u}=0.0$
(case 1)
$2 . P_{u}=1000 \mathrm{KN}, M_{u}=120.0 \mathrm{KN} . \mathrm{m}$ (major bending) (case 2)

Case 1: (axial capacity) [using Eurocode 4 provision]
$A_{c}=B^{*} H-A_{s}-A_{r}=69937 \mathrm{~mm}^{2}$.
$P_{o}=0.67 f c u A c+F y r A r+F y A s=4611 \mathrm{KN}$
$P_{n}=0.67 f c u l \gamma_{c} A c+F y r / \gamma_{r} A \mathrm{r}+F \mathrm{y} / \gamma_{s} A \mathrm{~s}=$ 3592 KN
$E I_{x}=0.6 \mathrm{E}_{\mathrm{c}} \mathrm{I}_{\mathrm{cx}}+\mathrm{E}_{\mathrm{s}} \mathrm{I}_{\mathrm{sx}}+\mathrm{E}_{\mathrm{r}} \mathrm{I}_{\mathrm{rx}}=2397 \mathrm{KN} . \mathrm{m}^{2}, \mathrm{EI}_{\mathrm{y}}$ $=1700 \mathrm{KN} . \mathrm{m}^{2}$
$\mathrm{P}_{\text {crx }}=\pi^{2} \mathrm{EI} /(\mathrm{KL})^{2}=26310 \mathrm{KN}, \mathrm{P}_{\text {cry }}=18658$
KN. $\quad \lambda_{\mathrm{x}}{ }^{`}=\sqrt{ }\left(\mathrm{po} / \mathrm{P}_{\text {crx }}\right)=0.419, \lambda_{\mathrm{y}}{ }^{`}=\sqrt{ }\left(\mathrm{po}_{\mathrm{o}} / \mathrm{P}_{\text {cry }}\right)$ $=0.497$.
$\phi_{\mathrm{x}}=0.5\left[1+0.49\left(\lambda_{\mathrm{x}}{ }^{`}-0.2\right)+\lambda_{\mathrm{x}}{ }^{`}{ }^{\wedge} 2\right]=0.64$.
$\phi_{y}=0.5\left[1+0.34\left(\lambda_{y}{ }^{\prime}-0.2\right)+\lambda_{y}{ }^{\wedge}{ }^{\wedge} 2\right]=0.614$.
$\chi_{x}=1 /\left[\phi_{x}+\left(\phi_{x}{ }^{2}-\lambda_{x}{ }^{-2}\right)^{\wedge} 0.5\right]=0.887 \leq 1.0, \chi_{y}=$ 0.885 .
$P_{u}=3000 \leq \chi_{\text {minimum }} \mathrm{P}_{\mathrm{n}}=0.885 * 3592=3180$ KN. (Safe)
Case 2: (axial capacity with major bending [proposed interaction diagram])
a. Neutral axis position is located at $\mathrm{h}_{\mathrm{n}}=$ 78.21 mm from the middle of the section, giving $\mathrm{M}_{\mathrm{o}}=191.8 \mathrm{KN} . \mathrm{m}$ (material stress
used are design strength $\left[0.67 f_{c u} / \gamma_{c}, f_{y} / \gamma_{s}\right.$, $\left.f_{y r} / \gamma_{r}\right]$ for concrete, steel, longitudinal reinforcement, respectively.
b. $\mathrm{M}_{\max }($ maximum moment can be carried $)=$ 222.5 at which axial load capacity $=$ $0.5 P_{c}=0.5 * 0.67 f c u / \gamma_{c} A c=1613 \mathrm{KN}$
$\left(\frac{\left|P_{u}-\frac{1}{2} P_{c}\right|}{P_{o}-\frac{1}{2} P_{c}}\right)^{\alpha}+\left(\frac{M_{u}}{\beta M_{\max }}\right)=1$ where $\alpha=\frac{\ln \left(1-\frac{M_{o}}{M_{\max }}\right)}{\ln \left(\frac{0.5 P_{c}}{P_{o}-0.5 P_{c}}\right)}=1.59$
While $\boldsymbol{\beta}=\mathbf{1 . 0}$ for major bending, $\beta=0.9$ for minor bending.
c. Second order effect
(EI) eff .II $x=0.9(0.5 E c I c x+$ Er Ir + Es Is $)=$
2017 KN.m ${ }^{2}$.
$P_{c r x, I I}=22134$
$M_{\text {design }}=\left[M_{u}+e_{i m p} P_{u}\right]\left(\frac{1}{1-\frac{P u}{p c r}}\right)=148.4 \mathrm{KN} . \mathrm{m}$
While $\boldsymbol{e}_{\text {imp }}=\boldsymbol{L} / 200$ (major bending), L/150 (minor bending).
$\left(\frac{\left|P_{u}-\frac{1}{2} P_{c}\right|}{P_{o}-\frac{1}{2} P_{c}}\right)^{\alpha}+\left(\frac{M_{u}}{\beta M_{\max }}\right)=0.16+0.65=0.81 \leq 1.0$

Note that this example is solved using Eurocode 4 provisions with using proposed interaction equation.

Table 1: Summary of limitations between applied codes:

| Code | Ec | fc ${ }^{\prime}$ | fy | pr | ps | A slenderness parameter |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ECPSCLRF D | $\begin{gathered} \text { Ec =22000 -31000 for fcu }=25-50 \\ \text { Mpa, respectively. } \end{gathered}$ | $\begin{gathered} 25 \leq \mathrm{fcu} \leq 50 \mathrm{M} \\ \mathrm{~Pa} \end{gathered}$ | fy $\leq 350 \mathrm{MPa}$ | No limit | $\rho \mathrm{s} \geq 4 \%$ | No limit |
| $\begin{aligned} & \hline \text { CEN } \\ & 2004 \\ & \hline \end{aligned}$ | 22000 * ((fc`+ 8) / 10) \({ }^{\wedge} 0.3\) & \[ \begin{gathered} 20 \leq f c` \leq 50 \mathrm{MP} |  |  |  |  |  |
| a |  |  |  |  |  |  |
| \end{gathered} \] | $\begin{gathered} 235 \leq f y \leq \\ 460 \mathrm{MPa} \\ \hline \end{gathered}$ | $\mathrm{pr} \leq 6 \%$ | $\begin{gathered} 0.2 \leq \delta \leq 0 . \\ 9 \end{gathered}$ | $\bar{\lambda} \leq 2.0$ |  |  |
| ACI | $4700 \sqrt{f_{c}}$ | $\mathrm{fc}^{\prime} \geq 17 \mathrm{MPa}$ | fy $\leq 350 \mathrm{MPa}$ | $1 \% \leq \rho r \leq 8 \%$ | No limit | $\begin{gathered} \text { Short column } \\ \mathrm{KL} / \mathrm{r} \leq[34+12(\mathrm{M} 1 / \mathrm{M} 2] \end{gathered}$ |
| AISC | $\begin{gathered} 43 * \mathbf{W}^{1.5} \sqrt{f_{c}} \text { fc (Mpa), W } \\ (\mathbf{K N / m 3}) \end{gathered}$ | $\begin{gathered} 21 \leq \\ \mathrm{fc}^{\prime} \leq 69 \mathrm{MPa} \end{gathered}$ | $\mathrm{fy} \leq 517 \mathrm{MPa}$ | $0.4 \% \leq \rho r$ | $\rho \mathrm{s} \geq 1 \%$ | No limit |


| CSA | $\left(3300 \sqrt{f_{c u}}+6900\right)\left(\frac{\gamma_{c}}{23}\right)^{1.5}$ <br> fc (Mpa), W (KN/m3) | No limit | fy $\leq 350 \mathrm{MPa}$ | 1\% $\leq \mathrm{pr} \leq 8 \%$ | No limit | $\begin{aligned} & \text { Short column KL/r } \\ & \leq[25-10(\mathrm{M} 1 / \mathrm{M} 2)] /[\mathrm{Pf} \\ & / \mathrm{fc} \mathrm{c} \mathrm{Ag}]^{\wedge} 0.5 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |

Table 2-a: Comparison between various codes and proposed equation for calculating columns strength ratio, (Ptest/Pcalc.).

| Summary |  | ACI |  | CSA |  | Eurocode 4 |  | AISC |  | $\begin{gathered} \text { ECP- } \\ \text { SCLRFD } \end{gathered}$ |  | Proposed Eqn. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { Typ } \\ \text { e } \end{gathered}$ | n* | Mean | $\text { COV. }{ }^{+}$ | mean | $\mathrm{COV}$ | mean | $\mathrm{COV}$ | mean | $\mathrm{COV}$ | mean | $\mathrm{COV}$ | mean <br> $+$ | $\mathrm{COV}$ |
| A $^{* *}$ | $\begin{gathered} 16 \\ 1 \end{gathered}$ | 0.928 | 0.208 | 0.94 | 0.158 | 1.1 | 0.168 | 1.367 | 0.481 | 1.296 | 0.181 | 1.1 | 0.168 |
| B** | 64 | 1.189 | 0.204 | 1.129 | 0.16 | 1.074 | 0.16 | 0.945 | 0.201 | 1.08 | 0.192 | 1.096 | 0.18 |
| C*** | $\begin{gathered} 15 \\ 6 \end{gathered}$ | 1.089 | 0.314 | 1.082 | 0.237 | 1.093 | 0.215 | 1.008 | 0.248 | 1.431 | 0.256 | 1.051 | 0.184 |
| $\mathrm{D}^{* *}$ | 18 | 0.997 | 0.096 | 1.362 | 0.114 | 0.997 | 0.096 | 0.997 | 0.096 | 0.997 | 0.096 | 0.997 | 0.096 |
| All ${ }^{* *}$ | 39 9 | 1.036 | 0.273 | 1.045 | 0.217 | 1.089 | 0.186 | 1.142 | 0.428 | 1.3 | 0.244 | 1.076 | 0.176 |

Where: $\mathrm{n}^{*} \quad$ indicates number of tested sample.
$\mathrm{A}^{* *} \quad$ indicates tested column with uniaxial load.
$\mathrm{B}^{* *} \quad$ indicates tested column axial load with bending about the minor axis.
C** indicates tested column axial load with bending about the major axis.
$\mathrm{D}^{* *} \quad$ indicates tested column pure bending.
All ${ }^{* *}$ indicates tested column stands for all beam-columns tests.
Mean ${ }^{+}$indicates the average of strength ratio $\left(\mathrm{P}_{\text {test }} / \mathrm{P}_{\text {calc }}\right)$ for tests. (Mean for $\mathrm{All}^{* *}=\boldsymbol{\Sigma}\left[\mathbf{n}^{*}\right.$ x mean ${ }^{+} / \boldsymbol{\Sigma}^{\mathbf{n}} \mathbf{n}^{*}$ )

COV. ${ }^{++}$is Coefficient of variance $=\left[\right.$standard deviation $/$mean $\left.^{+}\right]$.

Table 2-b: Summary of comparison between various codes and proposed equation for calculating columns strength ratio

| $\underset{\text { exp }}{\text { typ }}$ |  |  | ACI |  | CSA |  | Eurocode 4 |  | AISC |  | $\begin{gathered} \text { ECP- } \\ \text { SCLRFD } \end{gathered}$ |  | $\begin{gathered} \text { Proposed } \\ \text { eqn. } \\ \hline \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{A}^{*}$ | $\begin{aligned} & \text { Research- } \\ & \text { ers } \end{aligned}$ | $\mathbf{n}^{*}$ | Mea | cov. | Me an | $\begin{gathered} \text { CO } \\ \text { V. } \end{gathered}$ | Me an | $\begin{gathered} \text { CO } \\ \text { v. } \end{gathered}$ | Me an | $\begin{gathered} \text { CO } \\ \text { v. } \end{gathered}$ | $\underset{\mathrm{n}}{\text { Mea }}$ | cov | $\begin{gathered} \mathrm{Me} \\ \text { an } \end{gathered}$ | $\begin{gathered} \text { CO } \\ \text { V. } \end{gathered}$ |
|  | $\begin{gathered} \text { Janss[13\&2 } \\ \text { 3] } \\ \hline \end{gathered}$ | 30 | 0.86 | 0.11 | 0.86 | 0.1 | 1 | 0.11 | 1.22 | 0.21 | 1.18 | 0.13 | 1 | 0.11 |
|  | Ste- vens[19\&23 $]$ | 39 | 1 | 0.2 | 1 | 0.19 | 1.26 | 0.17 | 1.78 | 0.65 | 1.34 | 0.24 | 1.26 | 0.17 |
|  | $\begin{gathered} \text { Chen- } \\ \text { Astan.[30] } \end{gathered}$ | 13 | 0.84 | 0.13 | 0.89 | 0.12 | 0.97 | 0.1 | 1.08 | 0.13 | 1.21 | 0.12 | 0.97 | 0.1 |


|  | Loke[16] | 5 | 0.8 | 0.07 | 0.82 | 0.08 | 1.15 | 0.12 | 1.65 | 0.22 | 1.37 | 0.09 | 1.15 | 0.12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | RoikSchw.[11] | 1 | 0.8 | --- | 0.82 | --- | 1.15 | --- | 1.65 | --- | 1.37 | --- | 1.15 | --- |
|  | Han[20] | 6 | 0.95 | 0.04 | 0.99 | 0.03 | 1 | 0.14 | 1.01 | 0.14 | 1.26 | 0.13 | 1 | 0.14 |
|  | $\begin{gathered} \text { Han- } \\ \text { Kim[29] } \\ \hline \end{gathered}$ | 5 | 0.88 | 0.09 | 0.91 | 0.09 | 1.05 | 0.06 | 1.13 | 0.09 | 1.27 | 0.08 | 1.05 | 0.06 |
|  | $\begin{gathered} \text { Janss[13,23 } \\ \text { ] } \\ \hline \end{gathered}$ | 23 | 0.87 | 0.1 | 0.89 | 0.1 | 1 | 0.14 | 1.27 | 0.26 | 1.24 | 0.15 | 1 | 0.14 |
|  | $\begin{gathered} \text { Ste- } \\ \text { vens[19\&23 } \\ ] \\ \hline \end{gathered}$ | 6 | 0.95 | 0.02 | 0.98 | 0.02 | 1.32 | 0.06 | 1.36 | 0.06 | 1.73 | 0.09 | 1.32 | 0.06 |
|  | $\begin{gathered} \text { Janss[13\&2 } \\ \text { 3] } \end{gathered}$ | 5 | 0.86 | 0.11 | 0.88 | 0.11 | 1.22 | 0.09 | 1.39 | 0.13 | 1.44 | 0.17 | 1.22 | 0.09 |
|  | Chen[18] | 4 | 1.07 | 0.21 | 1.08 | 0.2 | 1.11 | 0.11 | 1.5 | 0.29 | 1.27 | 0.11 | 1.11 | 0.11 |
|  | Proctor[9] | 7 | 0.84 | 0.15 | 0.87 | 0.16 | 1.1 | 0.11 | 1.3 | 0.24 | 1.38 | 0.1 | 1.1 | 0.11 |
|  | Suzuki[12] | 17 | 1.1 | 0.34 | 1.07 | 0.13 | 1.05 | 0.08 | 1.15 | 0.25 | 1.29 | 0.12 | 1.05 | 0.08 |
| B* | $\begin{gathered} \text { Stevens } \\ {[19 \& 23]} \end{gathered}$ | 23 | 1.16 | 0.14 | 1.12 | 0.08 | 1.16 | 0.12 | 0.95 | 0.15 | 1.07 | 0.14 | 1.16 | 0.15 |
|  | Johnson[7] | 1 | 0.98 | --- | 0.99 | --- | 1.03 | --- | 0.77 | --- | 1.06 | --- | 0.98 | --- |
|  | $\begin{gathered} \text { Janss[13\&2 } \\ \text { 3] } \\ \hline \end{gathered}$ | 9 | 1.18 | 0.05 | 1.2 | 0.05 | 1.03 | 0.11 | 0.88 | 0.11 | 1.11 | 0.18 | 1.01 | 0.14 |
|  | Loke[16] | 11 | 0.92 | 0.24 | 0.88 | 0.09 | 1.02 | 0.15 | 0.99 | 0.27 | 1.07 | 0.17 | 1.03 | 0.24 |
|  | Roik- <br> Man.[10] | 6 | 1.35 | 0.15 | 1.42 | 0.13 | 1.19 | 0.24 | 1.19 | 0.18 | 1.41 | 0.19 | 1.28 | 0.21 |
|  | Yokoo[28] | 3 | 1.55 | 0.12 | 1.07 | 0.13 | 0.93 | 0.11 | 0.86 | 0.12 | 1.05 | 0.05 | 1.03 | 0.13 |
|  | Morino[8] | 8 | 1.41 | 0.11 | 1.28 | 0.08 | 1.01 | 0.08 | 0.91 | 0.14 | 0.97 | 0.18 | 1.08 | 0.06 |
|  | Bon- dale[15] | 3 | 1.17 | 0.39 | 1.02 | 0.11 | 0.83 | 0.1 | 0.7 | 0.16 | 0.79 | 0.08 | 0.89 | 0.11 |
| C** | Jahnson[7] | 3 | 0.98 | 0.2 | 1 | 0.2 | 1.17 | 0.22 | 1.09 | 0.23 | 1.42 | 0.17 | 1.11 | 0.2 |
|  | Loke[16] | 2 | 0.96 | 0.01 | 0.98 | 0.01 | 1.15 | 0.01 | 1.09 | 0.02 | 1.4 | 0.03 | 1.14 | 0.01 |
|  | RoikSchw.[11] | 17 | 0.98 | 0.11 | 0.99 | 0.12 | 1.03 | 0.32 | 0.86 | 0.1 | 1.32 | 0.14 | 0.93 | 0.09 |
|  | $\begin{gathered} \text { Roik- } \\ \text { Man.[10] } \end{gathered}$ | 8 | 0.95 | 0.15 | 0.96 | 0.14 | 1.11 | 0.19 | 0.94 | 0.18 | 1.28 | 0.16 | 1.09 | 0.2 |
|  | Mirza[21] | 16 | 1.23 | 0.13 | 1.25 | 0.13 | 1.03 | 0.12 | 0.9 | 0.11 | 1.21 | 0.12 | 1.01 | 0.11 |
|  | Roik- <br> Diek.[22] | 6 | 1.06 | 0.07 | 1.09 | 0.07 | 1.19 | 0.08 | 1.08 | 0.07 | 1.65 | 0.13 | 1.17 | 0.07 |
|  | Han[20] | 4 | 0.98 | 0.06 | 1.03 | 0.05 | 1.08 | 0.07 | 1.04 | 0.08 | 1.69 | 0.11 | 1.05 | 0.06 |
|  | $\begin{aligned} & \text { Han- } \\ & \text { Kim[29] } \\ & \hline \end{aligned}$ | 15 | 0.89 | 0.06 | 0.91 | 0.07 | 1 | 0.08 | 0.91 | 0.09 | 1.37 | 0.12 | 0.98 | 0.07 |
|  | $\begin{gathered} \text { Ku- } \\ \text { ramato[17] } \end{gathered}$ | 17 | 1.29 | 0.27 | 1.25 | 0.15 | 1.24 | 0.13 | 1.18 | 0.12 | 1.78 | 0.11 | 1.16 | 0.1 |
|  | $\begin{gathered} \text { Kim- } \\ \text { Park[14] } \end{gathered}$ | 1 | 0.83 | --- | 0.89 | --- | 0.55 | --- | 0.5 | --- | 0.71 | --- | 0.54 | --- |
|  | Morino[8] | 8 | 0.93 | 0.12 | 0.92 | 0.1 | 1.02 | 0.16 | 0.99 | 0.18 | 1.16 | 0.16 | 1 | 0.13 |
|  | ZHAO[6] | 10 | 1.02 | 0.12 | 1.03 | 0.11 | 1.03 | 0.18 | 0.88 | 0.24 | 1.44 | 0.19 | 1 | 0.19 |
|  | Rickles [24] | 8 | 1.27 | 0.04 | 1.28 | 0.05 | 1.16 | 0.05 | 1.12 | 0.05 | 1.33 | 0.06 | 1.13 | 0.06 |
|  | $\begin{aligned} & \text { Yama- } \\ & \text { da[25] } \end{aligned}$ | 5 | 2.39 | 0.29 | 2 | 0.19 | 1.9 | 0.08 | 1.76 | 0.09 | 2.69 | 0.14 | 1.77 | 0.07 |
|  | Naka[26] | 3 | 1.03 | 0.03 | 1.04 | 0.02 | 0.96 | 0.03 | 0.94 | 0.03 | 1.17 | 0.09 | 0.95 | 0.04 |
|  | Waka.[27] | 3 | 0.8 | 0.07 | 0.91 | 0.16 | 0.95 | 0.04 | 0.92 | 0.05 | 1.28 | 0.07 | 0.94 | 0.04 |


|  | Yokoo[28] | 16 | 0.99 | 0.28 | 0.95 | 0.24 | 0.99 | 0.08 | 0.97 | 0.09 | 1.17 | 0.09 | 0.97 | 0.1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { Bon- } \\ \text { dale[15] } \end{gathered}$ | 3 | 0.97 | 0.06 | 0.98 | 0.06 | 1.19 | 0.08 | 1.65 | 0.54 | 1.98 | 0.27 | 1.18 | 0.09 |
|  | Proctor[9] | 9 | 0.94 | 0.18 | 0.98 | 0.1 | 1.06 | 0.09 | 0.89 | 0.08 | 1.54 | 0.14 | 1.02 | 0.09 |
|  | Suzuki[12] | 2 | 1.12 | 0.13 | 1.12 | 0.13 | 0.91 | 0.13 | 0.88 | 0.13 | 1.11 | 0.11 | 0.91 | 0.13 |
| D** | Naka[26] | 1 | 0.91 | --- | 1.16 | --- | 0.91 | --- | 0.91 | --- | 0.91 | --- | 0.91 | --- |
|  | Waka.[27] | 1 | 1.45 | --- | 1.37 | --- | 1.13 | --- | 1.12 | --- | 1.24 | --- | 1.12 | --- |
|  | Suzuki[12] | 16 | 1 | 0.1 | 1.37 | 0.11 | 1 | 0.1 | 1 | 0.1 | 1 | 0.1 | 1 | 0.1 |



Figure 3: Statistical distribution of strength ratio using various approaches against frequency percent of tested specimens ( $n=399$ ).


Figure 4: Comparison between proposed equation with Eurocode 4, ECP-SCLRFD, and AISC using statistical distribution of strength ratio using various approaches against frequency percent of tested specimens ( $n=399$ ).

c) $\quad C S A$

d) CSA



Figure 5: Effect of some parameters such as slenderness ratio KL/r, end eccentricity ratio e/h and steel ratio $[\rho s=A s / A c] \%$ on strength ratio from different code provisions; (a) ACI; (b) CSA; (c) Eurocode 4; (d) AISC; (e) ECP-SCLRFD; and (f) proposed method ( $n=399$ ).

Note that: using statistical analysis, $y(x)=a \cdot x^{2}+b \cdot x+c$ indicates strength ratio as a function of ( x ) parameters slenderness ratio, end eccentricity ratio and steel ratio that presented in dotted line in the figures. $\mathrm{R}^{2}$ indicates R -squared value.


Figure 6: Comparison between nominal strengths predicted by Eurocode 4, using polygon interaction diagram and proposed equation, and test results for specimens with similar properties tested by Mirza [21] and Yokoo [28].

