The Egyptian International Journal of Engineering Sciences and Technology

Vol. 31 (2020) 1–9

https://eijest.journals.ekb.eg/

EIJEST
ISSN:1687-1006

Bridge-Vehicle Dynamic Interaction Modeling and Solution - An Overview

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ARTICLE INFO

Keywords:

Train-bridge interaction
High speed trains
Dynamic analysis
Railway bridges

ABSTRACT

The dynamic interaction between the bridge and the passing vehicle or train is considered a point of interest concerning the railway bridge design and maintenance, and the interest becomes greater for the bridges serving as links on high-speed lines. In this paper, the interaction problem is presented through the different models used to describe the phenomenon and the different techniques adopted to solve the non-linear interaction problem. The models describing the problem vary greatly from very simple 2D models with moving loads over beams to complex 3D models with multiple degrees of freedom (DOFs) for both the bridge and vehicle and with precise definition of various parts and parameters affecting the response such as the type of bridge element, the track structure and the bridge elastic supports. The solution algorithms of the non-linear interaction problem also vary from simple analytic solutions and non-direct techniques to more sophisticated iterative techniques in finite element (FE) domains.

1. Introduction

Modern railway lines appeared with the development of steam locomotives and go back to the beginning of the 19th century in England. The first railway line witnessed a collapse over a bridge link, which aroused the question of dynamic interaction and impact effects of the train passage over the bridge and the debate about those effects led to some experimental works to give some estimations, and since then, railway dynamics became a subject of interest [1]. The first works that deal with vibration problem date back to half of 19th century by Willis [2] in England and since then, many investigation works were introduced. Timoshenko [3] studied the vibration problem of beam being traversed by moving constant force at constant speed and derived formulas for the transverse deflection, and other problems related to the vibration of bridges were also studied including the case of force with reciprocating nature resulting from the unbalance in the locomotive and the case of smoothly passing mass over the beam. Analytical solutions for various cases and models of the railway bridges passed by moving systems were studied, e.g., moving forces on beams, uniformly distributed moving loads, smoothly moving masses, and many other more realistic models and different boundary conditions [4]. The simple cases considering the train or vehicle as moving forces have been widely adopted to study the bridge vibration but, as any simple method, a limitation of this type of analysis was made to cases

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for which the train weight is considerably small compared with that of bridge provided that the train response is not important. The dynamic effects of trains on railway bridges have usually been considered in design codes through the dynamic factor which magnifies the static load, for example, EN 1991-2 [5] applies the dynamic factor according to the maintenance level as a function of the determinant length. The high-speed trains' era started in the second half of the 20th century with the *Shinkansen* or *bullet trains* in japan in 1964. Tokaido Shinkansen line was started by the Japanese national railways (JNR) to travel from Tokyo to Osaka at speed 210 km/h which was increased later and the line was extended to a bigger network of high speed rail (HSR). The HSR in Europe started in France with the introduction of the TGV trains which started in 1981 with speeds now exceeding 300 km/h, and then the HSR spread to many other European countries. In the 21st century many countries started their high speed lines such as Turkey, South Korea and China which is now giving a great share in the rail traffic with high speed [6]. With the operation of the first high speed railway line in France, some bridges passed by the high speed trains started to show track distortions. Destabilization problems for the ballast appeared due to the vertical acceleration of bridge deck that exceeded 0.7 g [7]. Resonance effects caused by the high speeds of trains with uniformly spaced axles on railway bridges became an interest and the magnified static response turned out to be insufficient for the assessment. Dynamic response - especially vibrations - obtained from dynamic analysis became a necessity to make the right assessment for different limitations such as deck acceleration and riding comfort of passengers [8].

2. Load modeling for dynamic analysis

There are many ways for load modeling which depend on the solution method and the required accuracy such as the moving load, train signature method, lumped mass with spring-dashpot unit, full 2D model including the car body with bogies and two suspension layers and full 3D model for the car body. Fig. 1 shows the evolution of 2D modeling of trains/vehicles.

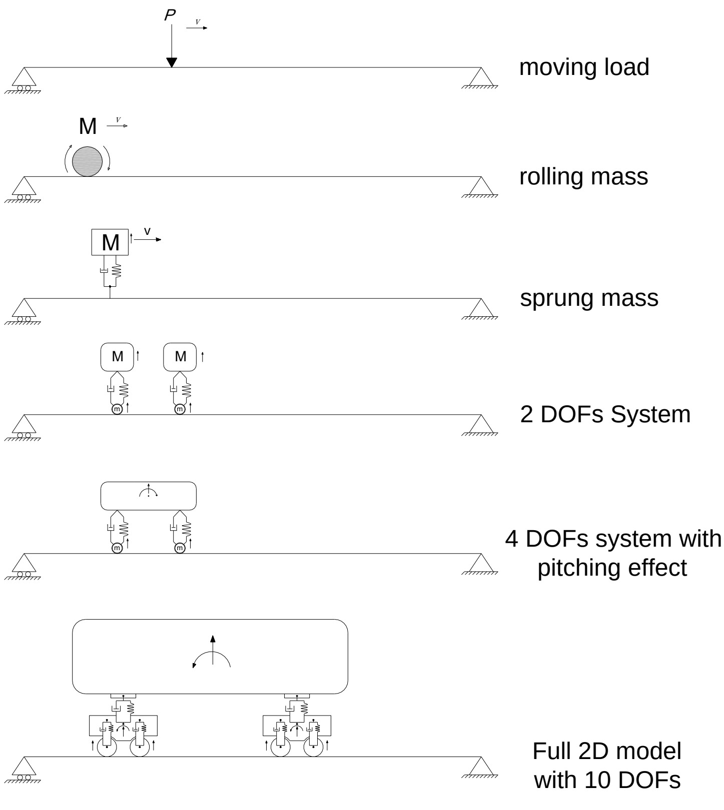

Fig. 1. Evolution of 2D load modeling

2.1. Moving load models

The train load has been widely represented by a pattern of moving loads with specified axle loads spaced at defined distances. This level of modeling cannot consider some external sources of dynamic excitation such as the rail irregularities but it still the simplest and fastest approach in handling the train loads. When the mass ratio between the train and the bridge is very small, the inertial effects of train can be ignored [9]. In the preliminary stage of bridge design or in the quick assessment of existing bridges, this model can be the most convenient. Various applications of this model with both analytical and FE approaches were widely discussed [4,10–14]. In the EN 1991-2 [5], the dynamic analysis is performed using the so-called HSLM (high speed load model) which represents a series of moving forces and it is stated that the dynamic analysis should be conducted using the real specified train in a specific project (bridges on local lines) but concerning the international lines for which the interoperability criteria are applied, the HSLM should be used to ensure an envelope response of all current real and prospective high-speed train loads.

The HSLM includes two sets of train loads, HSLM-A and HSLM-B and contains a number of train loads with varying coach length, coach number, bogie axle spacing and axle load. HSLM-A family is intended for the dynamic analysis of continuous and complex bridges and simple bridges with spans equal to or greater than 7 m while HSLM-B family is intended for simple bridges with spans less than 7 m.

Fig. 2 illustrates the configuration of HSLM-A which represents an articulated train configuration (one bogie for each two coaches) with power car and end coaches.

2.2. Moving mass models

The moving mass model comprises a mass rolling smoothly over the beam with no consideration for any jumps or impact due to any irregularities. The beam response under the passage of moving masses was studied by Stanišić and Hardin [15] and the equation of motion was solved using Fourier transforms. The conclusion was that the resonance frequency becomes lower when including the inertial effect in comparison to the moving load model which ignores such effect. A numerical-analytical technique was applied by Akin and Mofid [16] through transforming the governing equation into ordinary differential equation series to solve the moving mass over a beam problem with various boundary conditions. As in the moving load model, the response of the moving train/vehicle cannot be obtained.

2.3. Sprung mass model

The simplest interaction model is the sprung mass model which comprises a lumped mass representing a mass part of train supported by spring-dashpot unit. Pesterev et al. [17] studied the asymptotic behavior associated with the problem of the moving oscillator over a simple beam in a general manner applying the non-zero initial conditions for the beam and held a comparison between the moving oscillator and the other two cases of the moving force and moving mass to show that setting the spring stiffness to small values leads to the moving force case but setting the stiffness to infinite value can be equivalent to the results of the moving mass case regarding beam displacements but not beam stresses.

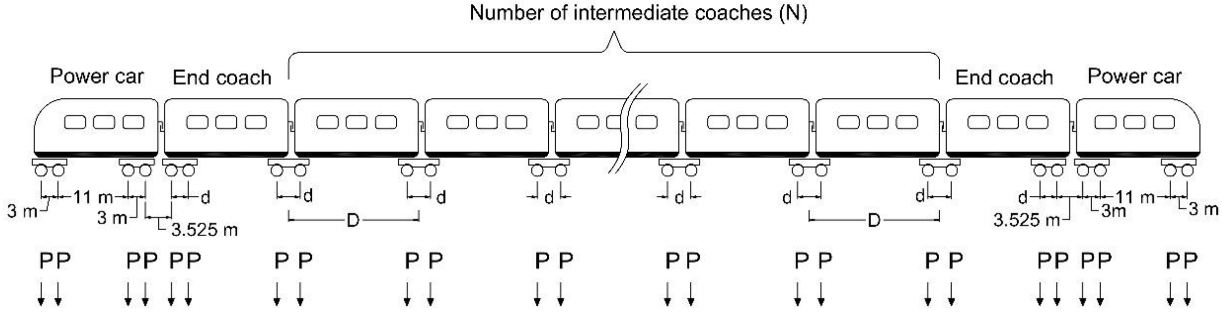

Fig. 2. HSLM-A configuration

Biggs [10] introduced a semi-analytical solution for the problem and the solution could be accomplished in a numerical straightforward procedure. In order to obtain the fundamental frequency of the bridge from the response of the passing vehicle, a study had been conducted applying the moving sprung mass model with an idea of considering the dynamic response of vehicle as a *message carrier* [18]. Later, the previous study was compared with contact point response which turned out to be outperforming regarding the bridge information extraction [19].

2.4. Two DOFs and four DOFs moving system

The moving systems assure a more realistic modeling and behavior for the interaction problem. The 2 DOFs system includes a lumped mass representing the half mass of the car body connected to a mass wheel and the train axles are represented by a series of this system. The two DOFs are the vertical translational DOFs of the lumped train mass and the wheel. This model is widely used to simply represent the car body and the suspensions. Frýba [4] made an analytical formulation for the problem. Other researchers adopted this model to study the interaction between the vehicle/train and the bridge [20–25].

This 2 DOFs simple model was further modified by Yang et al. [26] to include the pitching effect with a 4 DOFs system. The 4 DOFs are the translational DOFs of the two wheels and the center of the car body in addition to the rotational DOF of the car body. Yang and Fonder [27] also proposed this model but applied their solution algorithm on the 2 DOFs system.

2.5. Full 2D model

The full 2D model representing the conventional train coach includes 10 DOFs. This model comprises two layers of suspensions; the primary suspension and the secondary suspension with the DOFs are the translational DOFs of the two wheels, the two bogies and the car body in addition to the rotational DOFs of the two bogies and the car body. Wu and Yang [28] applied this model to study the steady-state response of simple bridges and to assess the riding comfort. Nour and Issa [29] studied the train-track-bridge interaction problem for short high-speed railway bridges, they studied the effect of the type of the bridge elements such as Bernoulli and Timoshenko beam type and the effect of supports flexibility. Museros et al. [30] adopted this model to investigate the response of short span bridges on high-speed lines.

2.6. 3D models of trains

Complex 3D models had also been given interest. Zhang et al. [31] proposed a 3D model for the train coach with 20 DOFs while Majka and Hartnett [32] adopted a reduced 3D model with unconstrained 15 DOFs. The interaction between a railway bridge and a high-speed articulated train system was studied by Xia et al. [33], the model of the train comprised vertical and transverse connections between car bodies and the two suspension layers with the whole train system containing 115 DOFs as illustrated in Fig. 3. Wu et al. [34] applied 3D model with 27 DOFs to study the spatial behavior of the train-bridge interaction as shown in Fig. 4.

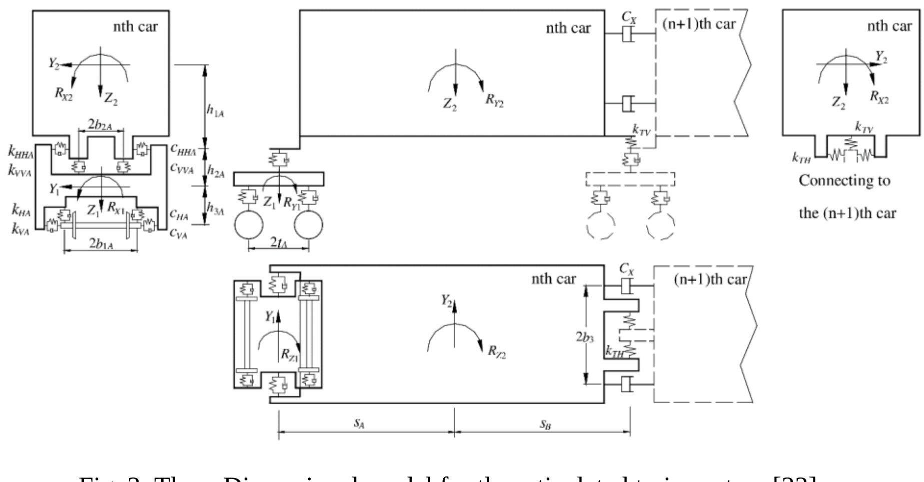

Fig. 3. Three Dimensional model for the articulated train system [33]

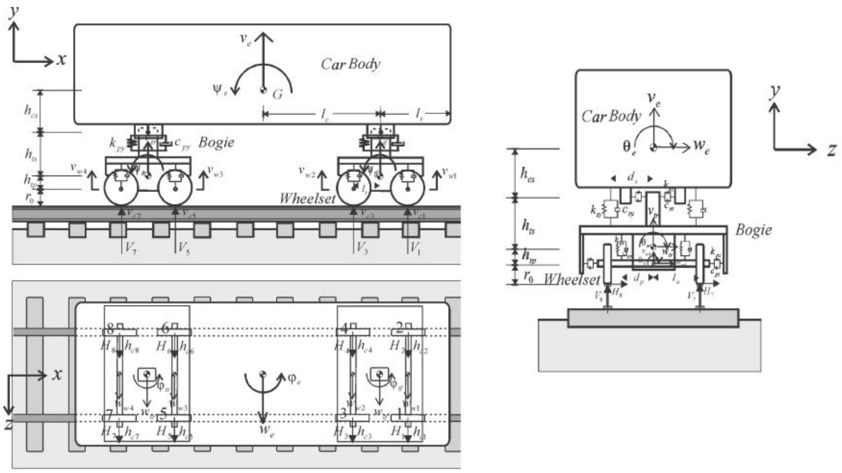

Fig. 4. Three Dimensional model for the moving train system [34]

3. Bridge and track modeling

3.1. Bridge element type and supports

Bridge elements have been usually modeled as 2D or 3D Bernoulli beam type ignoring shear deformation. This simplification, in some cases, is not acceptable and the Timoshenko beam type is preferred to handle rotary inertia and shear deformation [32]. The effect of modeling type of bridge elements on the bridge and train response was studied by Nour and Issa [29], and some other researchers applied Timoshenko beam modeling [22,35]. The bridge supports are usually modeled as rigid supports but in some cases elastic supports are introduced to mitigate the earthquake-induced forces from ground to bridge and can be efficient this way. An analytical approach for handling the case of simple elastically supported beam traversed by moving load, as illustrated in Fig. 5, can be found in Yang et al. [9] with a conclusion that elastic supports have an adverse effect on the response of bridge due to moving loads. Elastic supports are also inserted to account for the soft soil under supports. Nour and Issa [29] also studied the elasticity of supports concluding that soft supports along with stiff bridge girder may lead to response amplification of bridge.

Fig. 5. Elastically supported beam

3.2. Track structure

Museros et al. [30] studied the results of concentrated loads modeling versus the distributed load modeling and concluded that a reduction in the response is associated with the moving distributed load model in short bridges and this reduction becomes negligible for bridges with spans of 10 m or greater. In his master thesis, Rashid [22] concluded that the track structure should not be ignored in dealing with short span bridges while the response of long span bridges are not sensitive to the presence of track.

Regarding the track modeling, many models can be found in the literature which vary in their complexity. The simplest model for the track is the continuous elastic property between the bridge element and the train wheel. This track model, as shown of Fig. 6, has been adopted by many researchers [4,24,31].

Adding the damping property in the track parts; rails, sleepers and ballast, a more realistic model is reached as shown in Fig. 7. This model is a single-layer model that had been used for studying the interaction of the train, track and the bridge [29,35–37]. A modification for the last track model was introduced by Yang et al. [9] to consider the friction effects between the wheel and the rail by adding an equivalent horizontal springs and dampers to the track structure as illustrated in Fig. 8. This planar single layer continuous modeling for the track structure – which represents infinite beam on viscoelastic Winkler foundation – cannot consider the response of the individual parts of the track structure; rail, sleeper and the ballast.

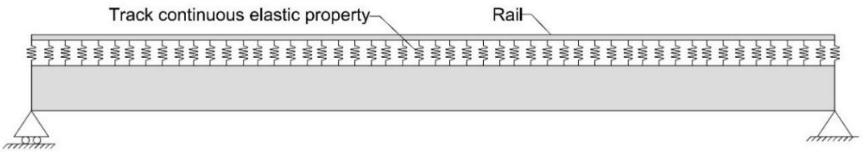

Fig. 6. Track model as continuous elastic property

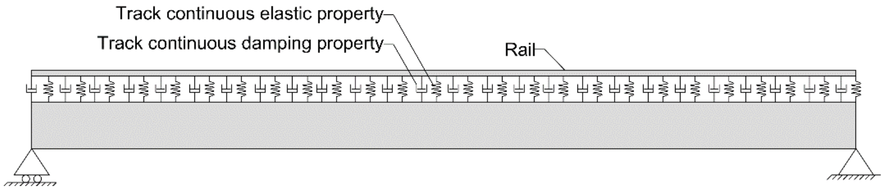

Fig. 7. Track model with continuous elastic and damping properties

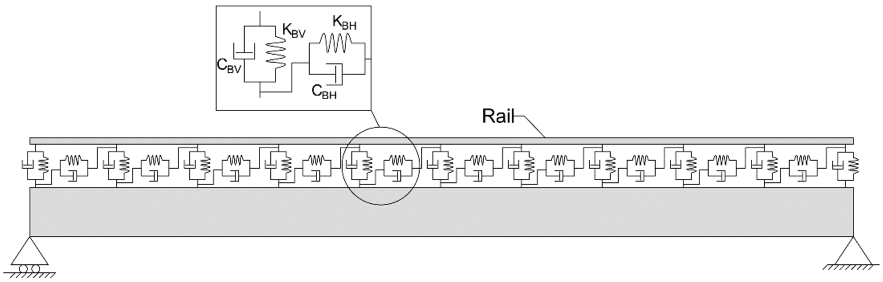

Fig. 8. Track model with horizontal and vertical continuous elastic and damping properties

Therefore, the multi-layer models provide a more realistic and detailed representation that allows to analyze the response of individual parts. The track can be modeled with number of layers up to four with these models being discrete models that comprise finite beam elements (rails) resting on spaced viscoelastic supports (sleepers) and are distinguished according to the finite element (FE) modeling to two types; mass-spring-dashpot models and solid models with rigid bodies [38]. Generally, multi-layer models are discrete models. Fig. 9 illustrates multi-layer models.

The 2-layer model had been utilized by Lou et al. [39] to study the train-track-bridge interaction and compared the cases in which the rail element and the bridge element are equal in length with the cases in which they are different, and compared the single layer track with the 2-layer one regarding the effect on response results. Rigueiro et al. [23] investigated the dynamic response of railway viaducts with medium span making use of a 3-layer model for the track.

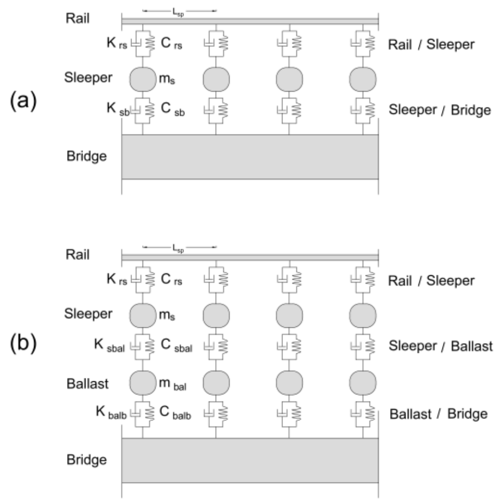

Fig. 9. Multi-layer track models (mass-spring-dashpot): (a) two-layer model, (b) three-layer model

Conclusions were made that different track models have insignificant influence on the bridge response [20,23,40]. Lou et al. [39] concluded that the sleeper mass is crucial in the 2-layer track model and that the 2-layer model is more accurate than the single layer model.

4. Techniques of solution of the interaction problem

Analytical solutions are usually limited to simple cases of bridges in which the first mode of vibration is only included and hence, a reduction in the problem leads to dealing with single degree of freedom problem. The interaction between the passing train and the bridge is considered a non-linear problem of two coupled systems. The exerted forces by the train upon the bridge depend on both the weights of the train axles and the response of the train –especially the train vertical acceleration– which in turn depends on the bridge response, thus, making the complicated non-linear problem.

As a non-direct dynamic analysis method for the simply supported bridges, the interest is only in the upper limit of the response which may be the deflection or the acceleration. This method is intended for simple bridges for which the dynamic representation can be limited to one mode with harmonic vibration. *Train Signature,* which is a characteristic for the train depending only on the train axles distribution and the damping ratio of the bridge, is used to get the maximum response of a bridge to avoid the complete dynamic analysis which is time consuming. Many techniques make use of this concept and some of them were developed by the D214 committee of the European Rail Research Institute (ERRI), one of these techniques is the simplified method based on the Residual Influence Line (LIR) which gives the maximum response as a product of three terms representing the contribution of structure and train separately as in Eq. 1 for the acceleration at mid span (Γ) [41].

$$\Gamma = C_a . A(K) . G(\lambda) \qquad (1)$$

where $C_a = 1/M$, $K = \lambda/2L$ and $\lambda = v/f_0$.

M is the total mass of bridge, λ is the wavelength, L is the span of bridge, v is the speed of train and f_0 is the frequency of the 1st eigen mode of vibration (Hz).

The term $A(K)$ is defined as:

$$A(K) = \frac{K}{1-K^2}\sqrt{e^{-2\zeta\frac{\pi}{K}} + 1 + 2\cos\left(\frac{\pi}{K}\right)e^{-\zeta\frac{\pi}{K}}} \qquad (2)$$

G(λ) is the dynamic signature of the train and defined as:

$$G(\lambda) = \max_{i=1}^{N}\sqrt{\left[\sum_{x_1}^{x_i} F_i \cos(2\pi\delta_i)e^{-2\pi\zeta\delta_i}\right]^2 + \left[\sum_{x_1}^{x_i} F_i \sin(2\pi\delta_i)e^{-2\pi\zeta\delta_i}\right]^2} \qquad (3)$$

where ζ is the damping ratio of bridge and N is the number of axles of train. x_i is the distance of axle number i from the leading axle while F_i is the load of axle i and $\delta_i = \left(x_i - x_1\right) / \lambda$.

The dynamic signature based analysis was further used to prepare the EN 1991-2, for the interoperability in all European high-speed railway lines, high-speed train models had been developed with the characteristic of including the dynamic signature of both the running high-speed trains and the future ones. The High Speed Load Model (HSLM), which is series of concentrated moving loads, was then developed to make a limitation for the number of required dynamic analyses in the case of many different high-speed trains are supposed to operate on the same lines [7]. The iterative solutions are the most widely used solutions which are based on the constraint equations and convergence criteria. In the literature, many iterative algorithms had been developed [20,27,29,32,42,43]. Yang and Fonder [27] applied an iterative algorithm to solve two coupled systems; the bridge and a simplified vehicle model with 2 DOFs. They applied acceleration techniques as relaxation and Aitken procedures to attain a good convergence. In the previous algorithm, the transferred forces between the two systems were considered as two components; response independent forces and response dependent forces. They also applied the convergence criterion on the bridge response. A related algorithm was introduced by Delgado and Santos [42] with a convergence criterion based on the dynamically transferred forces.

An algorithm based on the dynamic condensation has been adopted by Yang and Yau [24]. They applied a finite element referred to as vehicle-bridge interaction (VBI) element to study the interaction between the bridge and the train which is modeled as a series of lumped sprung masses. In this method the degrees of freedom of the sprung masses are condensed to the degrees of freedom of the bridge element with which they are contact. Later, Yang et al. [26] developed the last technique, based on the dynamic condensation, to include the pitching effect with a 4 DOFs model for the car body.

Lou and Zeng [35] adopted the stationary value of the total potential energy principle for the dynamic system of the bridge and the train. In this method of solution, the contact forces between the wheels and the rails are considered as internal forces of the whole system. They applied two types of train modeling; spring-damper unit with 2 DOFs and a full 2D model with 10 DOFs.

A new non-iterative procedure was presented by Neves et al. [44] in which a new single system formed up of two components; the equations of motion of the two interacting systems and the constraint equations between them, and this single system is solved directly. The constraint equations conform to the compatibility between the wheel response and the response of the element with which the wheel is in contact with a no-separation criterion. In this method, the equation describing the complete single system is given in the matrix form as in Eq. (4).

$$\begin{bmatrix} \overline{K}_{FF} & \overline{G}_{FX} \\ \overline{H}_{XF} & 0 \end{bmatrix} \begin{bmatrix} u_F^c \\ X_X^c \end{bmatrix} = \begin{bmatrix} \overline{F}_F \\ \overline{r}_X \end{bmatrix} \tag{4}$$

In the last equation, the term $\overline{K}_{FF}$ is the effective stiffness matrix of the interacting systems (vehicle and bridge structure) and stays constant while the other blocks $\overline{G}_{FX}$ and $\overline{H}_{XF}$ are modified during the linear analysis. The last two blocks are transformation matrices while u_F^c, X_X^c, $\overline{F}_F$, and $\overline{r}_x$ are the current nodal displacements, current contact forces, load vector and the track irregularities respectively. Neves et al. [45], later, modified the direct method to allow the wheel separation in the dynamic analysis and to detect which elements are in contact with wheel and which are not.

Generally, the equations of motion of the coupled systems are solved in time domain adopting various time integration schemes such as Newmark scheme [46], Hilber, Hughes, and Taylor scheme (HHT-α scheme) [47], and Wilson scheme [48].

5. Conclusions

The interaction problem between the interacting parts: train/vehicle; track; bridge, can be formulated with various levels of modeling. The more complex models can assure a more realistic definition for the problem. The complexity is associated with more degrees of freedom and more computation efforts but gives a more reliable response. Complex models allow for the deep investigation of the response of the different parts of the interacting systems. The multi-layer track models can predict the vibration effects on the track parts leading to a better understanding of track behavior and precise computation of the transferred contact forces between the moving system the track. Hence, precautions and maintenance can be more efficient with realistic models. The better understanding of the train/vehicle system response is important for the right assessment of the running safety and riding comfort. Many solution algorithms have been found in the literature to handle the nonlinear interaction problem but still the iterative solutions with constraint equations are the widely used ones.

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