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Modulation of the Nonlinear Ion Acoustic Waves in a Weakly Relativistic Warm Plasma with Nonextensively Distributed Electrons

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ABSTRACT

A reductive perturbation technique (multiple scales) is applied to a weakly relativistic warm unmagnetized adiabatic plasma system consisting of inertial ions fluid and nonextensively distributed electrons. A nonlinear Schrödinger-type (NST) equation for finite wavenumber at the second order is derived. Using the reductive perturbation technique we derived the corresponding Korteweg-de Vries (K-dV) equation. For small wavenumber limit the K-dV equation is transformed into NST equation. It is found that the coefficient of the NST equation obtained from the K-dV equation agree with the corresponding coefficients of NST equation obtained by the multiple scales. Moreover we investigated the effect of the physical parameters of the system namely temperature ratio of the ion temperature T_i to electron temperature T_e, the relativistic factor u_0/C as well as the nonextensive parameter (q) of the distribution on the stability/instability of the system. It is found that these parameters affect strongly on the stability/instability regions. Finally, the validity of our results in astrophysical plasma is briefly discussed.

Keywords

Plasma Physics, Modulation Instability, Weakly Relativistic, Nonextensive Electrons.

1. INTRODUCTION

Ion acoustic wave (IAW) is one type of longitudinal oscillations of ions and electrons in plasma systems. It is similar to the acoustic waves traveling in neutral gases. The propagation of nonlinear IAWs in a weakly dispersive medium has been investigated theoretically and experimentally[1, 2, 3, 4, 5]. Ikezi et al. [6] have reported the first experimental observation of IAWs solitons. Watanabe [7] studied the modulation instability of the monochromatic IAWs experimentally. The modulation instability of IAWs in warm non-relativistic plasma has been studied by Xue et al.[8].

The nonextensive statistic mechanics, based on the deviations from Boltzmann-Gibbs-Shannon (BGS) statistics has been studied in the last few decades. A suitable nonextensive generalization of the BGS entropy for statistical equilibrium was first investigated by Renyi [9] and afterwards suggested by Tsallis[10]. Tsallis extended the standard additivety of the entropies of the nonlinear systems. This nonadditive entropy of Tsallis and the generalized statistics have been investigated in different phenomena characterized by nonextensivity [11, 12, 13, 14, 15, 16, 17, 18, 19] through the entropic index q, characterizes the degree of nonextensivity of the considered system while the standard extensive BGS statistics is at $q = 1$. The nonextensive statistics are successfully applied to many astrophysical scenarios such as stellar polytropes, solar neutrino problem, and peculiar velocity distribution of galaxy clusters[20, 21].

If the particle velocity is much less than the velocity of light, ion waves will exhibit non-relativistic behavior, but when the particle velocity approaches that of light, relativistic effect must be considered. The modulation instability of IAWs in a weakly relativistic warm plasma for different distribution has been studied by El-Labany [22] and El-Labany et al.[23, 24]. The cold nonrelativistic modulation instability has been studied for different distributions. It has been studied using nonthermal distribution by Zhang et al.[25], q-nonextensive distribution by Bains et al. [21], and superthermal (kappa) distribution by Guo and Mei [26] and Chowdhury et al. [27]. The nonlinear evolutions in plasmas are investigated by different approximation techniques, in which one assumes small deviations for system from the equilibrium state of the linear wave. In fact such multiple scales method [23, 24], the reductive perturbation technique (RPT) [21] and Krylov-Bogoliubov-Mitropolsky method (KBM) [28] which lead to nonlinear Schrödinger-type (NST) equation. However, the system of a weakly relativistic warm unmagnetized adiabatic plasma consisting of inertial ions fluid and nonextensively distributed electrons has not been investigated; this is our goal.

The skeleton of this article is as follow:
In section 2 we present the basic system of equations representing our model and we derive the NST equation. In section 3 we derive the small wavenumber approximation Korteweg-de Vries (K-dV) equation. In section 4 we transform the K-dV equation obtained in section 3 to the NST equation and results and discussion in section 5. Section 6 is devoted conclusion.

2. BASIC EQUATIONS AND DERIVATION OF THE NST EQUATION

Consider a simple model of adiabatic unmagnetized collisionless weakly relativistic plasma that contains one warm ion species together with nonextensively distributed electrons. The one-dimensional basic equations can be written in non-dimensional form as

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)(\gamma u) + 3\sigma n\frac{\partial n}{\partial x} + \frac{\partial \Phi}{\partial x} = 0, \tag{2}$$

$$\frac{\partial^2 \Phi}{\partial x^2} = n_e - n, \tag{3}$$

$$n_e = [1 + (q-1)\Phi]^{\frac{(q+1)}{2(q-1)}} \approx 1 + \alpha_1 \varphi - \alpha_2 \varphi^2 + \alpha_3 \varphi^3 + \dots, \tag{4}$$

where,

$$\left.\begin{aligned} \alpha_1 &= \frac{q+1}{2}, \\ \alpha_2 &= \frac{(q+1)(q-3)}{8}, \\ \alpha_3 &= \frac{(q+1)(q-3)(3q-5)}{48}. \end{aligned}\right\} \tag{5}$$

and n , n_e are the densities of the ions and electrons respectively, u is the flow velocity of the ions, Φ is the electrostatic potential, x is the space coordinate, t is the time variable, $\sigma \ll 1$ is the ratio of ion temperature T_i to electron temperature T_e and the parameter q stands for the strength of nonextensively, and γ is the relativistic factor

$$\gamma = \left(1 - \frac{u^2}{C^2}\right)^{\frac{-1}{2}}$$

Assuming a weakly relativistic effect the relativistic factor can be approximated by its expansion up to the second term .i.e. [29]

$$\gamma \approx 1 + \frac{u^2}{2C^2} \tag{6}$$

All physical quantities in Eqs. (1)-(4), u, Φ, n, x and t are normalized with respect to thermal velocity $(k_B T_e/m)^{(1/2)}$, thermal potential $(k_B T_e/e)$, unperturbed ion density n_0, Debye length $\lambda_D = (k_B T_e/4\pi e^2 n_0)^{(1/2)}$ and the inverse of the ion plasma frequency $\omega_{pi}^{-1} = (4\pi e^2 n_0/m)^{(1/2)}$ respectively, where m is the ion mass, k_B is the Boltzmann constant and e is the electron charge. To derive the nonlinear Schrödinger-type equation, we employ the general method of a multiple scales. In this method we introduce the independent variables [22]

$$\tau_i = \varepsilon^i t, \;\; \xi_0 = x, \;\; \text{and} \;\; \xi_i = \varepsilon^i (x - \lambda t) \;\; (i = 1, 2, \ldots). \tag{7a}$$

Thus the time and space derivatives in Eqs. (1-4) can be written as [24]

$$\left.\begin{aligned} \frac{\partial}{\partial t} &\longrightarrow \frac{\partial}{\partial \tau_0} + \varepsilon\left(\frac{\partial}{\partial \tau_1} - \lambda \frac{\partial}{\partial \xi_1}\right) + \varepsilon^2\left(\frac{\partial}{\partial \tau_2} - \lambda \frac{\partial}{\partial \xi_2}\right) + \ldots, \\ \frac{\partial}{\partial x} &\longrightarrow \frac{\partial}{\partial \xi_0} + \varepsilon \frac{\partial}{\partial \xi_1} + \varepsilon^2 \frac{\partial}{\partial \xi_2} + \ldots, \end{aligned}\right\} \tag{7b}$$

where ε is a small dimensionless parameter representing the size of the perturbed amplitude and λ represented the group velocity ($\lambda = \frac{\partial \omega}{\partial k}$); will be determined later. Now we expand the variables n, u, Φ in terms of the expansion parameter ε as (EL-Labany 1995 [22])

$$\left.\begin{aligned} n &= 1 + \sum_{m=1}^{\infty} \varepsilon^m \sum_{l=-m}^{m} n_m^{(l)}(\tau_1, \tau_2, \ldots, \xi_1, \xi_2, \ldots) \exp[il(kx - \omega t)], \\ u &= u_0 + \sum_{m=1}^{\infty} \varepsilon^m \sum_{l=-m}^{m} u_m^{(l)}(\tau_1, \tau_2, \ldots, \xi_1, \xi_2, \ldots) \exp[il(kx - \omega t)], \\ \Phi &= \sum_{m=1}^{\infty} \varepsilon^m \sum_{l=-m}^{m} \Phi_m^{(l)}(\tau_1, \tau_2, \ldots, \xi_1, \xi_2, \ldots) \exp[il(kx - \omega t)], \end{aligned}\right\} \tag{7c}$$

where n, u and Φ are satisfied the reality condition $A_{-l}^{(m)} = A_l^{(m)*}$ and the asterisk denotes the complex conjugate. Substituting Eqs. (7) into the basic equations (1-4), we obtain to the first order of ε and

$l = 1$

$$\left.\begin{array}{c} u_1^{(1)} = \dfrac{\tilde{\omega}}{k} n_1^{(1)}, \\ \text{and} \\ \Phi_1^{(1)} = \dfrac{n_1^{(1)}}{(k^2+\alpha_1)}. \end{array}\right\} \tag{8}$$

With the linear dispersion relation and group velocity λ given respectively by

$$\tilde{\omega}^2 \gamma_1 = 3\sigma k^2 + \frac{k^2}{(k^2+\alpha_1)}, \tag{9a}$$

and

$$\lambda = u_0 + \frac{k}{\gamma_1 \tilde{\omega}} \left(3\sigma + \frac{\alpha_1}{(k^2+\alpha_1)^2}\right). \tag{9b}$$

where $\gamma_1 = 1 + \frac{3u_0^2}{2C^2}$ and $\tilde{\omega} = \omega - ku_0$.
The components of O(ε) for $l = 0$ are given by,

$$\left.\begin{array}{c} n_1^{(0)} = n_{e1}^{(0)}, \\ \text{and} \\ \Phi_1^{(0)} = \dfrac{n_1^{(0)}}{\alpha_1}, \end{array}\right\} \tag{10}$$

However the second order harmonic terms O(ε^2) of the reduced equations, with $l = 0$ are given by,

$$\left.\begin{array}{c} \dfrac{\partial n_1^{(0)}}{\partial \xi_1} = \dfrac{\partial u_1^{(0)}}{\partial \xi_1} = \dfrac{\partial \Phi_1^{(0)}}{\partial \xi_1} = 0, \\ \Phi_1^{(0)} = 0, \\ \Phi_2^{(0)} = \dfrac{(n_2^{(0)} - 2\alpha_2 |\Phi_1^{(1)}|^2)}{\alpha_1}, \end{array}\right\} \tag{11}$$

provided that

$$\gamma_1 \tilde{\lambda}^2 \neq \frac{1}{\alpha_1} + 3\sigma.$$

For $l = 1$ components,

$$\left.\begin{array}{c} \dfrac{\partial n_1^{(1)}}{\partial \tau_1} = 0, \\ u_2^{(1)} = \dfrac{\tilde{\omega}}{k} n_2^{(1)} + \dfrac{i}{k}\left(\dfrac{\tilde{\omega}}{k} - \tilde{\lambda}\right) \dfrac{\partial n_1^{(1)}}{\partial \xi_1}, \\ \Phi_2^{(1)} = \dfrac{n_2^{(1)}}{(k^2+\alpha_1)} + \dfrac{2ik}{(k^2+\alpha_1)^2} \dfrac{\partial n_1^{(1)}}{\partial \xi_1}; \end{array}\right\} \tag{12}$$

i.e. no τ_1 dependent.
For $l = 2$ components,

$$\left[n_2^{(2)},\, u_2^{(2)},\, \Phi_2^{(2)}\right]^T = [A_n,\, A_u,\, A_\Phi]^T\, {n_1^{(1)}}^2 \tag{13}$$

where,

$$A_n = (k^2+\alpha_1)\left[\frac{\tilde{\omega}^2}{k^2}\left(\frac{3}{2}\gamma_1 - \frac{\tilde{\omega}}{k}\gamma_2\right) + \frac{3}{2}\sigma + A_\Phi\right],$$

$$A_u = \frac{\tilde{\omega}}{k}(A_n - 1),$$

$$A_\Phi = \frac{(k^2+\alpha_1)}{3k^2}\left[\frac{\tilde{\omega}^2}{k^2}\left(\frac{3}{2}\gamma_1 - \frac{\tilde{\omega}}{k}\gamma_2\right) + \frac{3}{2}\sigma - \frac{\alpha_2}{(k^2+\alpha_1)^3}\right],$$

$$\tilde{\lambda} = \lambda - u_0,$$

and

$$\gamma_2 = \frac{3u_0}{2C^2}.$$

The second-order quantities with zeroth harmonic are determined from $l = 0$ components of third order O(ε^3) and are given by,

$$\left[n_2^{(0)},\, u_2^{(0)},\, \Phi_2^{(0)}\right]^T = [B_n,\, B_u,\, B_\Phi]^T\, |n_1^{(1)}|^2 \tag{14}$$

where

$$B_n = \frac{1}{\tilde{\lambda}}\left[\frac{2\tilde{\omega}}{k} + B_u\right],$$

$$B_u = \frac{1}{z}\left[\frac{\tilde{\omega}^2\tilde{\lambda}^2}{k^2}\left(\frac{\gamma_1}{\tilde{\lambda}} - 2\gamma_2\right) + 3\sigma\left(\tilde{\lambda} + \frac{2\tilde{\omega}}{k}\right) + \frac{2\tilde{\omega}}{\alpha_1 k} - \frac{2\alpha_2\tilde{\lambda}}{\alpha_1(k^2+\alpha_1)^2}\right],$$

$$B_\Phi = \frac{1}{\alpha_1}\left[\frac{B_n(k^2+\alpha_1)^2 - 2\alpha_2}{(k^2+\alpha_1)^2}\right],$$

and

$$z = \gamma_1\tilde{\lambda}^2 - 3\sigma - \frac{1}{\alpha_1}$$

Finally, we obtain the NST equation from O(ε^3) for $l = 1$ components by using the above derived equations as,

$$i\frac{\partial n_1^{(1)}}{\partial \tau} + P\frac{\partial^2 n_1^{(1)}}{\partial \xi^2} + Q n_1^{(1)}|n_1^{(1)}|^2 = 0, \tag{15}$$

where

$$P = \frac{-k^2}{2\tilde{\omega}\gamma_1(k^2+\alpha_1)^3}\left[-(k^2-3\alpha_1) + \frac{(k^2+\alpha_1)^3}{k^2}\left(\frac{\tilde{\omega}^2\gamma_1}{k^2} - \frac{2\tilde{\omega}\gamma_1\tilde{\lambda}}{k} + \gamma_1\tilde{\lambda}^2\right)\right] = \frac{1}{2}\frac{\partial^2\tilde{\omega}}{\partial k^2}$$

and

$$Q = \frac{-k^2}{2\tilde{\omega}\gamma_1}\left\{\begin{array}{l}\left(\frac{\tilde{\omega}^2\gamma_1}{k^2} + 3\sigma\right)(A_n + B_n) + \frac{2\tilde{\omega}}{k}\left(\gamma_1 - \frac{\tilde{\omega}}{k}\gamma_2\right)(A_u + B_u) \\ -\frac{2\alpha_2}{(k^2+\alpha_1)^2}(A_\Phi + B_\Phi) + 2\left(\frac{\tilde{\omega}}{k}\right)^3\gamma_2 - \frac{3}{2C^2}\left(\frac{\tilde{\omega}}{k}\right)^4 - \frac{3\alpha_3}{(k^2+\alpha_1)^4}\end{array}\right\}.$$

Equation (15) satisfies the evolution of the complex amplitude of the nonlinear ion acoustic waves (IAWs) propagating in a weakly relativistic warm with nonextensively electrons on the basis of the fluid model in the finite wavenumber region.

For small wavenumber, equation (15) reduces to (appendix)

$$i\frac{\partial n_1^{(1)}}{\partial \tau} - \frac{3}{2}\frac{bk}{\alpha_1^2}\frac{\partial^2 n_1^{(1)}}{\partial \xi^2} + \frac{1}{3k}\frac{a^2\alpha_1^2}{b}n_1^{(1)}|n_1^{(1)}|^2 = 0, \tag{16}$$

where

$$a = \left[\frac{\gamma_1(3\sigma\alpha_1+1)}{\alpha_1}\right]^{\frac{-1}{2}} \left\{ \frac{(3\sigma\alpha_1+1)(3\alpha_1^2-2\alpha_2)}{2\alpha_1^3} + \frac{3}{2}\frac{\sigma(2\alpha_2+\alpha_1^2)}{\alpha_1^2} - \frac{\gamma_2}{\gamma_1^{3/2}}\left(\frac{3\sigma\alpha_1+1}{\alpha_1}\right)^{\frac{3}{2}} \right\}$$

and

$$b = \left[\frac{\gamma_1(3\sigma\alpha_1+1)}{\alpha_1}\right]^{\frac{-1}{2}}.$$

3. DERIVATION K-DV EQUATION FOR THE SYSTEM

If we apply the reductive perturbation theory, we can show that the amplitude of the perturbed ion density in a weakly relativistic warm plasma and nonextensively distributed electrons in the small wavenumber limit is governed by the K-dV equation. This equation can be derived by introducing the stretched variables τ and ξ as [30, 31]

$$\xi = \mu^{\frac{1}{2}}(x-\overline{\lambda}t) \quad \text{and} \quad \tau = \mu^{\frac{3}{2}}t. \tag{17a}$$

Thus,

$$\left.\begin{aligned} \frac{\partial}{\partial x} &= \mu^{\frac{1}{2}}\frac{\partial}{\partial \xi}, \\ \frac{\partial}{\partial t} &= \mu^{\frac{1}{2}}\left(-\overline{\lambda}\frac{\partial}{\partial \xi}+\mu\frac{\partial}{\partial \tau}\right), \end{aligned}\right\} \tag{17b}$$

and we expand the dependent variables as[29],

$$\left.\begin{aligned} n &= 1+\mu\tilde{n}_1+\mu^2\tilde{n}_2+\mu^3\tilde{n}_3+\ldots \\ u &= u_0+\mu u_1+\mu^2 u_2+\mu^3 u_3+\ldots \\ \Phi &= \mu\Phi_1+\mu^2\Phi_2+\mu^3\Phi_3+\ldots \end{aligned}\right\} \tag{17c}$$

where $\tilde{n}$ is the perturbed ion density and μ is the ordering parameter and is a measure of the size of the wavenumber k; that is, k=$O(\mu^{1/2}$). Using Eqs. (17) in basic set of Eqs. (1)-(3) and equating the similar power coefficients, the lowest order terms of Îij are written as

$$\left.\begin{aligned} \tilde{n} &= \alpha_1\Phi_1, \\ u_1 &= \grave{\lambda}\tilde{n}_1, \\ &\text{and} \\ \Phi_1 &= \frac{\tilde{n}_1}{\alpha_1}, \end{aligned}\right\} \tag{18a}$$

where

$$\grave{\lambda} = \overline{\lambda} - u_0 \tag{18b}$$

Poisson's equation gives the compatibility condition

$$(\grave{\lambda}^2\gamma_1 - 3\sigma)\alpha_1 = 1. \tag{18c}$$

The next order of μ gives,

$$\frac{\partial \tilde{n}_1}{\partial \tau} - (\overline{\lambda}-u_0)\frac{\partial \tilde{n}_2}{\partial \xi} + \frac{\partial u_2}{\partial \xi} + \grave{\lambda}\frac{\partial \tilde{n}_1^2}{\partial \xi} = 0, \tag{19a}$$

$$\gamma_1\dot{\lambda}\frac{\partial\tilde{n}_1}{\partial\tau}-(\overline{\lambda}-u_0)\frac{\partial}{\partial\xi}(\gamma_1 u_2+\gamma_2\dot{\lambda}^2\tilde{n}_1^2)+\gamma_1\dot{\lambda}^2\tilde{n}_1\frac{\partial\tilde{n}_1}{\partial\xi}+3\sigma\left[\frac{\partial\tilde{n}_2}{\partial\xi}+\tilde{n}_1\frac{\partial\tilde{n}_1}{\partial\xi}\right]+\frac{\partial\Phi_2}{\partial\xi}=0, \tag{19b}$$

and

$$\frac{\partial^2\Phi_1}{\partial\xi^2}=\alpha_1\Phi_2+\alpha_2\frac{\tilde{n}_1^2}{\alpha_1^2}-\tilde{n}_2. \tag{19c}$$

Eliminating the second order perturbed quantities and using the results of the pervious order with some algebraic manipulations we obtain the K-dV equation, which describes the evolution of the nonlinear ion acoustic waves,

$$\frac{\partial\tilde{n}}{\partial\tau}+a\tilde{n}\frac{\partial\tilde{n}}{\partial\xi}+\frac{b}{2\alpha_1^2}\frac{\partial^3\tilde{n}}{\partial\xi^3}=0, \tag{20}$$

where a and b are written as

$$a=\left[\frac{\gamma_1(3\sigma\alpha_1+1)}{\alpha_1}\right]^{\frac{-1}{2}}\left\{\frac{(3\sigma\alpha_1+1)(3\alpha_1^2-2\alpha_2)}{2\alpha_1^3}+\frac{3}{2}\frac{\sigma(2\alpha_2+\alpha_1^2)}{\alpha_1^2}-\frac{\gamma_2}{\gamma_1^{3/2}}\left(\frac{3\sigma\alpha_1+1}{\alpha_1}\right)^{\frac{3}{2}}\right\}$$

and

$$b=\left[\frac{\gamma_1(3\sigma\alpha_1+1)}{\alpha_1}\right]^{\frac{-1}{2}}.$$

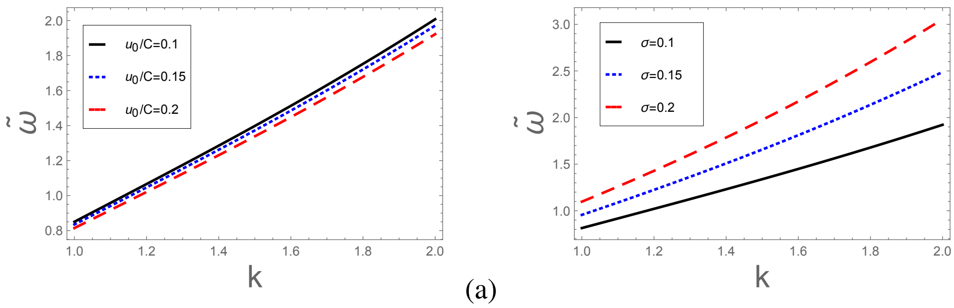

Figure 1: The variation of the angular frequency ($\tilde{\omega}$) with wavenumber (k): (a) for different values of u_0/C and $\sigma=0.1$, (b) for different values of σ and $u_0/C=0.2$. Here the nonextensive parameter $q=0.55$.

4. DERIVATION OF NST EQUATION FROM K-DV EQUATION (SMALL WAVENUMBER APPROXIMATION)

To obtain the NST equation from K-dV equation (20) we follow the work by recently by El-Labany et al.[24] and we reach at

$$i\frac{\partial\tilde{n}_1^{(1)}}{\partial\sigma}-\frac{3}{2}\frac{bk}{\alpha_1^2}\frac{\partial^2\tilde{n}_1^{(1)}}{\partial\chi^2}+\frac{1}{3k}\frac{a^2\alpha_1^2}{b}\tilde{n}_1^{(1)}|\tilde{n}_1^{(1)}|^2=0. \tag{21}$$

which similar with the small wavenumber limit of equation (15) i.e. equation (16).

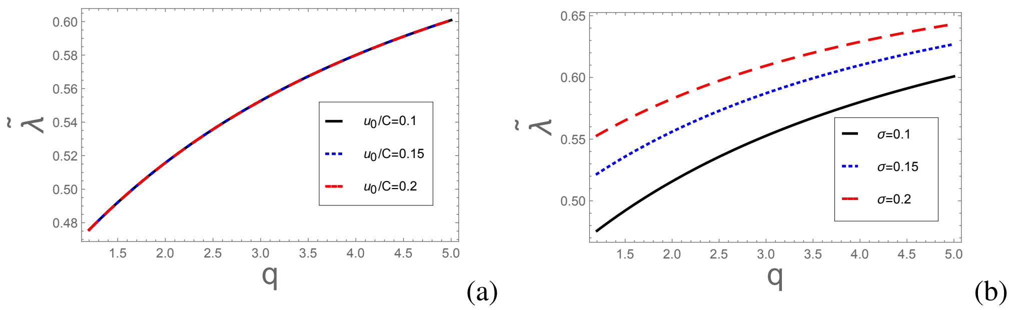

Figure 2: The variation of the group velocity ($\tilde{\lambda}$) with nonextensive parameter q: (a) for different values of u_0/C and $\sigma = 0.1$, (b) for different values of σ and $u_0/C = 0.2$. Here $k = 1.4$.

5. RESULTS AND DISCUSSION

Figures (1.a) and (1.b) show the numerical analysis of Eq. (9a) to examine the linear properties of the IAWs for different values of relativistic factor (u_0/C), temperatures ratio σ ($= T_i/T_e$) with the value of the nonextensive parameter $q = 0.55$. These figures show that, the phase velocity decreases with increasing u_0/C and is enhanced with increasing σ. Also, figures (2.a) and (2.b) show the group velocity properties for different values of u_0/C and σ, which are given in Eq. (9b). The group velocity is independent on the variation of u_0/C but varies with the nonextensive parameter q, and increases with increasing σ.

On the other hand, we investigated the variation of the critical wavenumbers (higher and lower wavenumber) with u_0/C for different values of σ ($\sigma = 0.1$ and $\sigma = 0.2$) as shown in figures (3.a) and (3.b). These figures show that the upper critical wavenumber decreases as σ increases while the lower wavenumber remains constant, where $\sigma (= T_i/T_e)$ is always very low $\simeq 0.2$. Figures (4.a) and (4.b) show the variation of the critical wavenumbers with σ for different values of the nonextensive parameter q. We notice that, the upper critical wavenumber increases with increasing the nonextensive parameter q and we have only one lower wavenumber which decreases as σ increase.

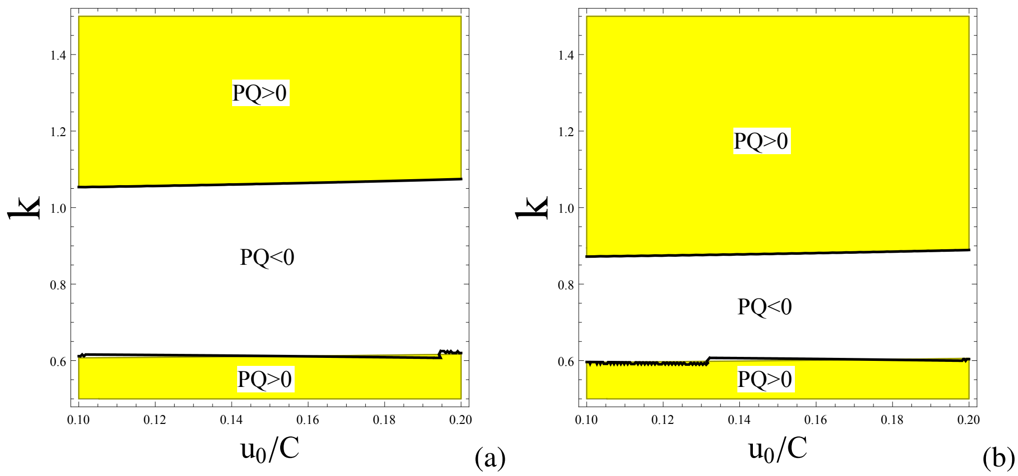

Figure 3: Contour plot of the product $PQ = 0$, depicted against k and u_0/C: (a) for $\sigma = 0.1$, (b) for $\sigma = 0.2$. Here the nonextensive parameter $q = 0.55$, where the (white) yellow region represents the (stability) instability region.

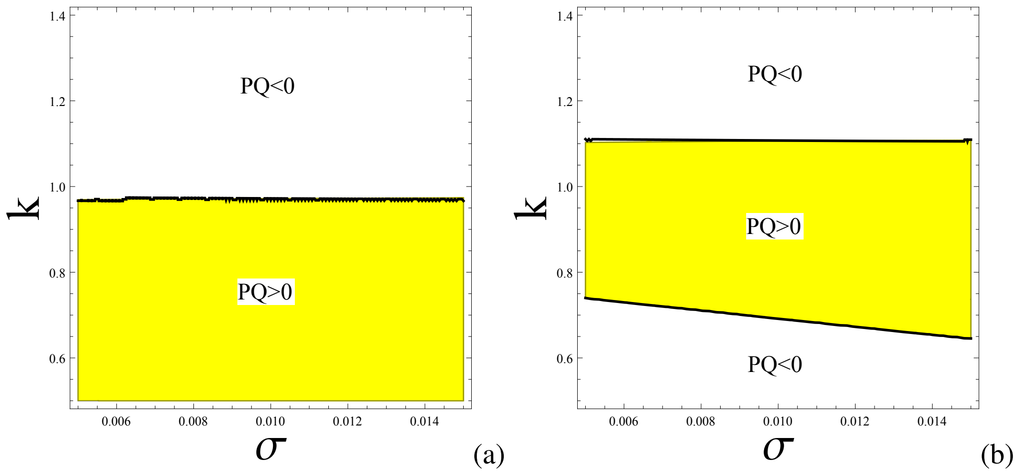

Figure 4: Contour plot of the product $PQ = 0$, depicted against k and σ: (a) for the nonextensive parameter $q = 2.87$, (b) for the nonextensive parameter $q = 4$. Here $u_0/C = 0.2$, where the (white) yellow region represents the (stability) instability region.

Figure 5 shows that, the effect of the q-nonextensively parameter on the stability and instability domains. We find that the increases of the q parameter increase the lower and higher critical wavenumbers. Also this figure shows that when q parameter increase the stable region becomes more narrow. The stable region at $\sigma = 0.1$ (figure (5.a)) is larger than the stable region at $\sigma = 0.2$ (figure (5.b)) this mean that the system gains more energy and becomes more unstable when the ion temperature T_i increases.

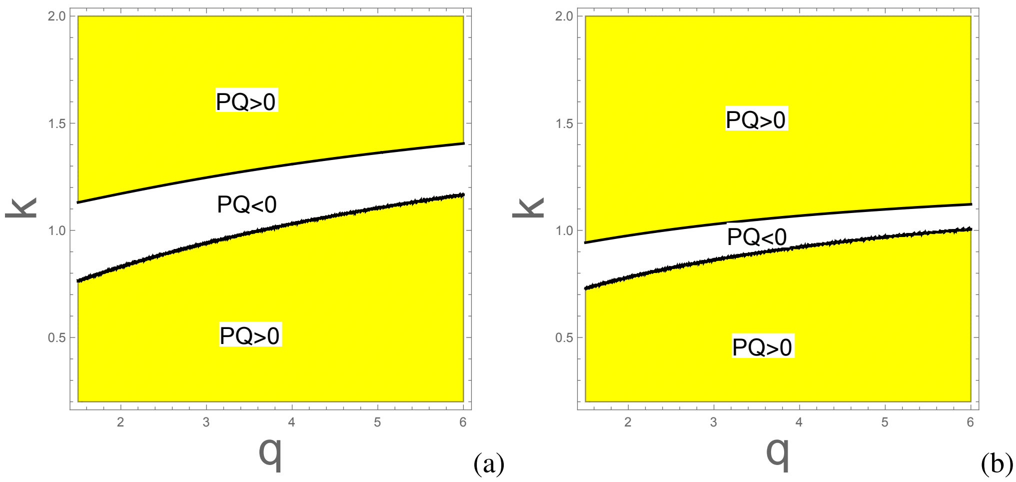

Figure 5: Contour plot of the product $PQ = 0$, depicted against k and q: (a) for $\sigma = 0.1$, (b) for $\sigma = 0.2$. Here $u_0/C = 0.1$, where the (white) yellow region represents the (stability) instability region.

The comparison between the results obtained by nonextensively and Maxwellian (at the limit of $q \longrightarrow 1$) distributed electrons are displayed in figures (6.a) and (6.b). It is obvious from these figures that, the unstable region for nonextensively distributed electrons is larger than the unstable region for Maxwellian. This means that the nonextensive particles have high energy than the Maxwellian ones which increase the instability of the system. So, from these results we find that the Maxwellian distribution is inadequate for explain the highly energetic particles vice the nonextensive distribution.

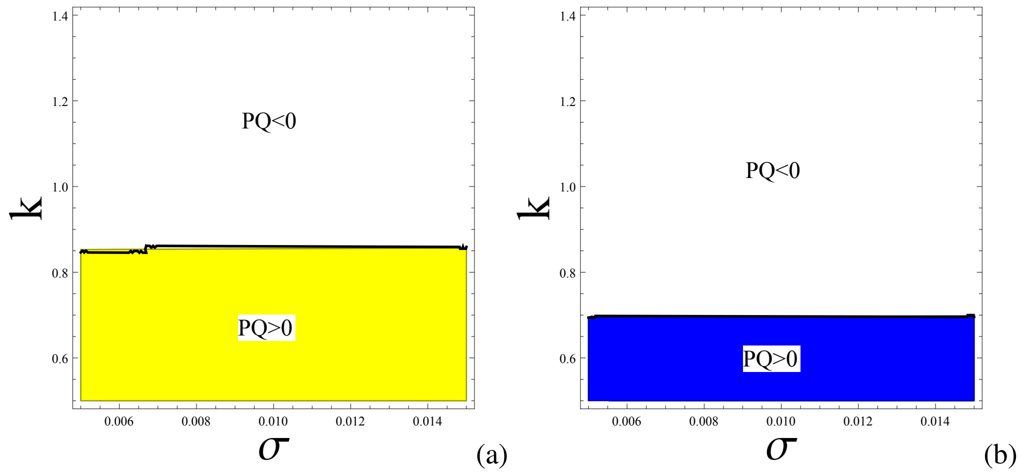

Figure 6: Comparison between the contour plot of the product $PQ = 0$, depicted against k and σ obtained by the nonextensively distributed electrons in (a) at the nonextensive parameter $q = 2$ and the Maxwellian distributed electrons in (b), where the (white) blue region represents the (stability) instability region.

6. CONCLUSION

In the present work, we employed the nonlinear hydrodynamic equations of a weakly relativistic unmagnetized adiabatic plasma model including warm ions and nonextensively distributed electrons. This system of equations is reduced to the NST equation for finite wavenumber by using the multiple scales method in which the coefficients of this equation are strongly dependent on both ion temperature σ and the ion streaming velocity (γ_1, γ_2). We also found good agreement between the small-wavenumber limit of the NST equation obtained by using multiple scales method and the NST equation obtained from K-dV equation obtained by using the reductive perturbation method. It is well known that the stability condition for the NST equation is $PQ < 0$. Since P is always negative for $\omega > ku_0$, one has to determine the value of the critical wavenumber k_C at which Q vanishes. Then, for all values of $k > k_C$, the wave has modulation instability, while modulation stability for all values of $k > k_C$. Moreover we investigated the dependence of the stability/instability regions on the physical parameters σ, u_0/C and q characterizing the system. A comparison between nonextensivelly and Maxwellian distributed electrons is carried out and the validity of our results in astrophysical and finement fusion plasma. Finally, To show the validity of our results, we consider the cold nonrelativistic limit ($\sigma = 0$ and $u_0/C = 0$) of the present work which is found to agree with the work done previously by Bains et al. [21].

A large number of observations clearly reveal the existence of a weakly relativistic warm plasma and nonextensively distributed electrons in astrophysics environments such as solar neutrino problem, stellar polytropes and galaxy cluster as well as finement fusion plasma [20, 21].

Appendix

To estimate the coefficients of the NST equation for small wavenumber (k), firstly we calculate the different terms appearing in these coefficients.

From Eqs.(9), as $k \longrightarrow 0$, we have

$$\frac{\tilde{\omega}}{k} \approx \tilde{\lambda} \approx \left[\frac{1+3\sigma\alpha_1}{\gamma_1\alpha_1}\right]^{1/2},$$

thus

$$\left[\frac{1}{z}\right] = \left[\frac{\alpha_1(k^2+\alpha_1)}{k^2}\left\{k^2\alpha_1(3\gamma_1\sigma(k^2+\alpha_1)+1)-(k^2+3\alpha_1)(k^2+\alpha_1)\right\}^{-1}\right] = \frac{-\alpha_1^2}{3k^2}$$

where the square-bracket notation indicates the quantity is estimated at small wave number ($k \longrightarrow 0$). Also

$$\begin{aligned}
A_n &= (k^2+\alpha_1)\left\{\frac{\tilde{\omega}^2}{k^2}\left(\frac{3}{2}\gamma_1-\frac{\tilde{\omega}}{k}\gamma_2\right)+\frac{3}{2}\sigma+\frac{(k^2+\alpha_1)}{3k^2}\left[\frac{\tilde{\omega}^2}{k^2}\left(\frac{3}{2}\gamma_1-\frac{\tilde{\omega}}{k}\gamma_2\right)+\frac{3}{2}\sigma-\frac{\alpha_2}{(k^2+\alpha_1)^3}\right]\right\}\\
&\approx \frac{\alpha_1^2}{3k^2}\left[\frac{3}{2}\left(\frac{1+3\sigma\alpha_1}{\alpha_1}\right)-\frac{\gamma_2}{\gamma_1^{3/2}}\left(\frac{1+3\sigma\alpha_1}{\alpha_1}\right)^{3/2}+\frac{3}{2}\sigma-\frac{\alpha_2}{\alpha_1^3}\right],
\end{aligned}$$

$$\begin{aligned}
A_u &= \frac{\tilde{\omega}}{k}(A_n-1)\\
&\approx \frac{\alpha_1^2}{3k^2}\left[\frac{1+3\sigma\alpha_1}{\gamma_1\alpha_1}\right]^{1/2}\left[\frac{3}{2}\left(\frac{1+3\sigma\alpha_1}{\alpha_1}\right)-\frac{\gamma_2}{\gamma_1^{3/2}}\left(\frac{1+3\sigma\alpha_1}{\alpha_1}\right)^{3/2}+\frac{3}{2}\sigma-\frac{\alpha_2}{\alpha_1^3}\right],
\end{aligned}$$

$$\begin{aligned}
A_\Phi &= \frac{(k^2+\alpha_1)}{3k^2}\left[\frac{\tilde{\omega}^2}{k^2}\left(\frac{3}{2}\gamma_1-\frac{\tilde{\omega}}{k}\gamma_2\right)+\frac{3}{2}\sigma-\frac{\alpha_2}{(k^2+\alpha_1)^3}\right],\\
&\approx \frac{\alpha_1^2}{3k^2}\left[\frac{3}{2}\left(\frac{1+3\sigma\alpha_1}{\alpha_1}\right)-\frac{\gamma_2}{\gamma_1^{3/2}}\left(\frac{1+3\sigma\alpha_1}{\alpha_1}\right)^{3/2}+\frac{3}{2}\sigma-\frac{\alpha_2}{\alpha_1^3}\right],
\end{aligned}$$

$$\begin{aligned}
B_n &= \frac{1}{\tilde{\lambda}}\left\{\frac{2\tilde{\omega}}{k}+\frac{1}{z}\left[\frac{\tilde{\omega}^2\tilde{\lambda}^2}{k^2}\left(\frac{\gamma_1}{\tilde{\lambda}}-2\gamma_2\right)+3\sigma\left(\tilde{\lambda}+\frac{2\tilde{\omega}}{k}\right)+\frac{2\tilde{\omega}}{\alpha_1 k}-\frac{2\alpha_2\tilde{\lambda}}{\alpha_1(k^2+\alpha_1)^2}\right]\right\}\\
&\approx -\frac{\alpha_1^2}{3k^2}\left[\gamma_1\left(\frac{1+3\sigma\alpha_1}{\gamma_1\alpha_1}\right)-2\gamma_2\left(\frac{1+3\sigma\alpha_1}{\gamma_1\alpha_1}\right)^{3/2}+9\sigma+\frac{2}{\alpha_1}-\frac{2\alpha_2}{\alpha_1^3}\right],
\end{aligned}$$

$$\begin{aligned}
B_u &= \frac{1}{z}\left[\frac{\tilde{\omega}^2\tilde{\lambda}^2}{k^2}\left(\frac{\gamma_1}{\tilde{\lambda}}-2\gamma_2\right)+3\sigma\left(\tilde{\lambda}+\frac{2\tilde{\omega}}{k}\right)+\frac{2\tilde{\omega}}{\alpha_1 k}-\frac{2\alpha_2\tilde{\lambda}}{\alpha_1(k^2+\alpha_1)^2}\right]\\
&\approx -\frac{\alpha_1^2}{3k^2}\left(\frac{1+3\sigma\alpha_1}{\gamma_1\alpha_1}\right)^{1/2}\left[\gamma_1\left(\frac{1+3\sigma\alpha_1}{\gamma_1\alpha_1}\right)-2\gamma_2\left(\frac{1+3\sigma\alpha_1}{\gamma_1\alpha_1}\right)^{3/2}+9\sigma+\frac{2}{\alpha_1}-\frac{2\alpha_2}{\alpha_1^3}\right],
\end{aligned}$$

and

$$\begin{aligned}
B_\Phi &= \frac{1}{\alpha_1}\left[\frac{B_n(k^2+\alpha_1)^2-2\alpha_2}{(k^2+\alpha_1)^2}\right]\\
&\approx -\frac{\alpha_1}{3k^2}\left[\gamma_1\left(\frac{1+3\sigma\alpha_1}{\gamma_1\alpha_1}\right)-2\gamma_2\left(\frac{1+3\sigma\alpha_1}{\gamma_1\alpha_1}\right)^{3/2}+9\sigma+\frac{2}{\alpha_1}-\frac{2\alpha_2}{\alpha_1^3}\right],
\end{aligned}$$

Thus the coefficients P and Q are given by

$$P = \frac{-k^2}{2\tilde{\omega}\gamma_1(k^2+\alpha_1)^3}\left[-(k^2-3\alpha_1)+\frac{(k^2+\alpha_1)^3}{k^2}\left(\frac{\tilde{\omega}^2\gamma_1}{k^2}-\frac{2\tilde{\omega}\gamma_1\tilde{\lambda}}{k}+\gamma_1\tilde{\lambda}^2\right)\right]$$
$$\approx -\frac{1}{2}\frac{k}{\gamma_1\alpha_1^3}\left[\frac{1+3\sigma\alpha_1}{\gamma_1\alpha_1}\right]^{-1/2}[3\alpha_1] = -\frac{3}{2}\frac{k}{\alpha_1^2}\left[\frac{1+3\sigma\alpha_1}{\gamma_1\alpha_1}\right]^{-1/2} = -\frac{3}{2}\frac{b}{\alpha_1^2}k$$

and

$$Q = \frac{-k^2}{2\tilde{\omega}\gamma_1}\left\{\begin{array}{l}\left(\frac{\tilde{\omega}^2\gamma_1}{k^2}+3\sigma\right)(A_n+B_n)+\frac{2\tilde{\omega}}{k}\left(\gamma_1-\frac{\tilde{\omega}}{k}\gamma_2\right)(A_u+B_u)\\ -\frac{2\alpha_2}{(k^2+\alpha_1)^2}(A_\Phi+B_\Phi)+2\left(\frac{\tilde{\omega}}{k}\right)^3\gamma_2-\frac{3}{2C^2}\left(\frac{\tilde{\omega}}{k}\right)^4-\frac{3\alpha_3}{(k^2+\alpha_1)^4}\end{array}\right\}$$

$$\approx \frac{\alpha_1^2}{3k}\left[\frac{\gamma_1(1+3\sigma\alpha_1)}{\alpha_1}\right]^{-1/2}\left[\frac{(3\sigma\alpha_1+1)(3\alpha_1^2-2\alpha_2)}{2\alpha_1^3}+\frac{3}{2}\frac{\sigma(2\alpha_2+\alpha_1^2)}{\alpha_1^2}-\frac{\gamma_2}{\gamma_1^{\frac{3}{2}}}\left(\frac{\gamma_1(1+3\sigma\alpha_1)}{\alpha_1}\right)^{\frac{3}{2}}\right]$$
$$= \frac{1}{3k}\frac{a^2\alpha_1^2}{b}$$

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