

## ADVANCED MODELING OF FACTS IN NEWTON POWER FLOW

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*The latest generation FACTS controller named the convertible static compensator (CSC) was recently installed at the New York Power Authority (NYPA) Marcy substation as part of a project that will increase power transfer capability and maximize the use of the existing transmission network. Within the general framework of the CSC, two innovative FACTS controllers are used. The static synchronous series compensator (SSSC) coupled with a transformer is connected in series with a transmission line. The interline power flow controller (IPFC) combining at least two converters, can be configured. Mathematical models of the SSSC and the IPFC and their implementation in Newton power flow are reported. Numerical results based on the IEEE 30-bus are presented to demonstrate the performance of the Newton power flow algorithm with incorporation of the SSSC and IPFC.*

**KEYWORDS:** *Flexible AC transmission system (FACTS), static synchronous series compensator (SSSC), interline power flow controller (IPFC), Newton power flow.*

### 1. INTRODUCTION

The latest generation FACTS controller named the convertible static compensator (CSC) was recently installed at the NYPA's Marcy substation as part of a project that will increase power transfer capability and maximize the use of the existing transmission network [1]. Within the general conceptual framework of the CSC, two FACTS controllers, the static synchronous series compensator (SSSC) [2], the interline power flow controller (IPFC) [3, 4] are among the many possible configurations. These are significantly extended to control power flows of multi lines. In principle the CSC can be configured into many FACTS topology configurations. Among the various configurations of the CSC, two multi converter FACTS controllers, such as the IPFC combining two or more series converters, are comprehensive ones. Power flow and optimal power flow calculations are performed frequently in power system operation, planning and control. With the practical applications of FACTS in power systems, development of suitable models for FACTS controllers for power system analysis has gained growing.

## 2. MATHEMATICAL MODEL OF SSSC

### 2.1 Operation Principles of the SSSC

A SSSC usually consists of a coupling transformer, an inverter, and a capacitor. As shown in Fig. 1, the SSSC is series connected with a transmission line through the coupling transformer. It is assumed here that the transmission line is series connected with the SSSC via its bus  $j$ . The active and reactive power flow of the branch  $i - j$  entering the bus  $j$  are equal to the sending end active and reactive power flow of the transmission line, respectively. In principle, the SSSC can generate and insert a series voltage, which can be regulated to change the impedance (more precisely reactance) of the transmission line. In this way, the power flow of the transmission line or the voltage of the bus, which the SSSC is connected with, can be controlled.

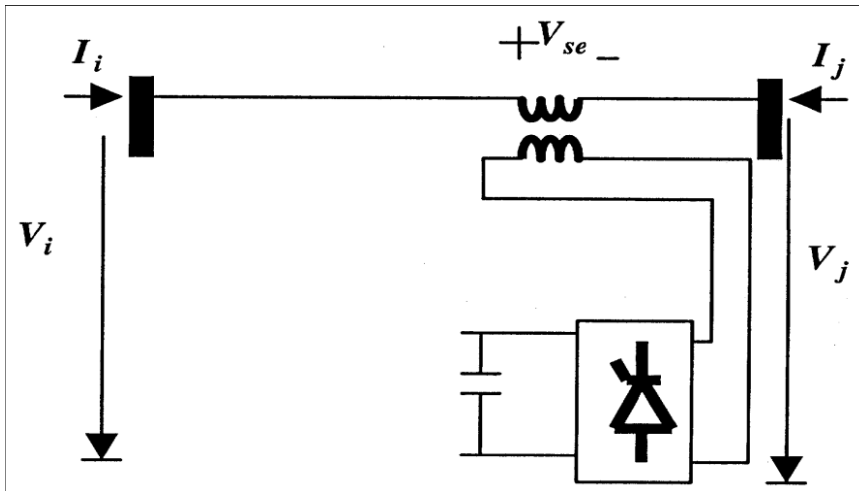


Fig. 1. SSSC operation principles

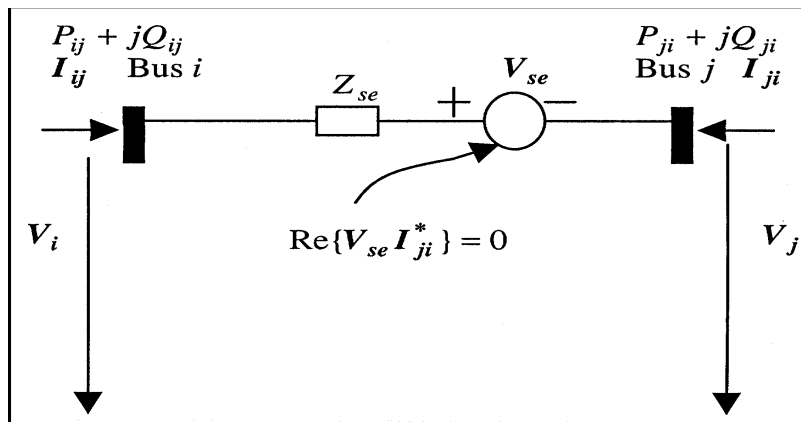


Fig. 2. SSSC equivalent circuit

### 2.2 Equivalent Circuit and Power Flow Constraint of the SSSC

An equivalent circuit of the SSSC as shown in Fig. 2 can be derived based on the operation principle of the SSSC. In the equivalent, the SSSC is represented by a voltage source  $V_{se}$  in series with a transformer's impedance. In the practical operation of the SSSC,  $V_{se}$  can be regulated to control the power flow of line  $i-j$  or voltage of bus  $i$  or  $j$ . In the equivalent circuit,  $V_{se} = V_{se} \angle \theta_{se}$ ,  $V_i = V_i \angle \theta_i$ ,  $V_j = V_j \angle \theta_j$ , then the power flow constraints of the SSSC are

$$P_{ij} = V_i^2 g_{ii} - V_i V_j (g_{ij} \cos (\theta_i - \theta_j) + b_{ij} \sin (\theta_i - \theta_j)) - V_i V_{se} (g_{ij} \cos (\theta_i - \theta_{se}) + b_{ij} \sin (\theta_i - \theta_{se})) \tag{1}$$

$$Q_{ij} = - V_i^2 b_{ii} - V_i V_j (g_{ij} \sin (\theta_i - \theta_j) - b_{ij} \cos (\theta_i - \theta_j)) - V_i V_{se} (g_{ij} \sin (\theta_i - \theta_{se}) - b_{ij} \cos (\theta_i - \theta_{se})) \tag{2}$$

$$P_{ji} = V_j^2 g_{jj} - V_i V_j (g_{ij} \cos (\theta_j - \theta_i) + b_{ij} \sin (\theta_j - \theta_i)) + V_j V_{se} (g_{ij} \cos (\theta_j - \theta_{se}) + b_{ij} \sin (\theta_j - \theta_{se})) \tag{3}$$

$$Q_{ji} = - V_j^2 b_{jj} - V_i V_j (g_{ij} \sin (\theta_j - \theta_i) - b_{ij} \cos (\theta_j - \theta_i)) + V_j V_{se} (g_{ij} \sin (\theta_j - \theta_{se}) - b_{ij} \cos (\theta_j - \theta_{se})) \tag{4}$$

where  $g_{ij} + j b_{ij} = 1 / Z_{se}$ ,  $g_{ii} = g_{ij}$ ,  $b_{ii} = b_{ij}$ ,  $g_{jj} = g_{ij}$ ,  $b_{jj} = b_{ij}$ .

The operating constraint of the SSSC (the active power exchange via the dc link) is

$$PE = Re ( V_{se} I_{se}^* ) = 0 \tag{5}$$

$$Re ( V_{se} I_{se}^* ) = - V_i V_{se} (g_{ij} \cos (\theta_i - \theta_{se}) - b_{ij} \sin (\theta_i - \theta_{se})) + V_j V_{se} (g_{ij} \sin (\theta_j - \theta_{se}) - b_{ij} \cos (\theta_j - \theta_{se}))$$

### 2.3 Two control Constraints of the SSSC

In the practical applications of the SSSC, it may be used for control of one of the following parameters: 1) the active power flow of the transmission line, 2) the reactive power flow of the transmission line. Among the two control modes, the active power flow control mode has been well recognized [2], [5], [6]. The mathematical descriptions of the two control modes of the SSSC are presented as follows.

#### Mode 1: Active Power Flow Control

The active power flow control constraint is as follows:

$$P_{ji} - P_{ji}^{Spec} = 0 \tag{6}$$

where  $P_{ji}^{Spec}$  is the specified active power flow control reference.

## Mode 2: Reactive Power Flow Control

The reactive power flow control constraint is as follows:

$$Q_{ji} - Q_{ji}^{Spec} = 0 \quad (7)$$

where  $Q_{ji}^{Spec}$  is the specified reactive power flow control reference. As mentioned,  $P_{ji}$  and  $Q_{ji}$  are the SSSC branch active and reactive power flows, respectively, leaving the SSSC bus  $j$  while the sending end active and reactive power flows of the transmission line are  $-P_{ji}$  and  $-Q_{ji}$  respectively. Equations (6)–(9) can be generally written as

$$\Delta F(x) = F(x) - F^{Spec} = 0 \quad (8)$$

where  $x = [\theta_i, V_i, \theta_j, V_j, \theta_{se}, V_{se}]^t$

### 2.4 Voltage and Current Constraints of the SSSC

The equivalent voltage injection  $V_{se}$  bound constraints are as follows:

$$0 \leq V_{se} \leq V_{se}^{Spec} \quad (9)$$

$$-\pi \leq \theta_{se} \leq \pi \quad (10)$$

where  $V_{se}^{max}$  is the voltage rating of  $V_{se}$  which may be constant, or may change slightly with changes in the dc bus voltage, depending on the inverter design.

The current through the series converter of the SSSC should be within its current rating

$$I_{se} \leq I_{se}^{max} \quad (11)$$

where  $I_{se}^{max}$  is the current rating of the series converter of the SSSC while  $I_{se}$  is the practical current through the series converter.

## 3. IMPLEMENTATION OF THE TWO CONTROL FUNCTIONAL MODEL OF THE SSSC IN NEWTON POWER FLOW

### 3.1 The Two control Functional Model of the SSSC in Newton Power Flow

For the SSSC, the power mismatches, at its buses  $i, j$ , respectively, should hold

$$\Delta P_i = P_{gi} - P_{di} - P_i = 0 \quad (12)$$

$$\Delta Q_i = Q_{gi} - Q_{di} - Q_i = 0 \quad (13)$$

$$\Delta P_j = P_{gj} - P_{dj} - P_j = 0 \quad (14)$$

$$\Delta Q_j = Q_{gj} - Q_{dj} - Q_j = 0 \quad (15)$$

For the SSSC, it has only one control degree of freedom since the active power exchange with the dc link should be zero at any time. So each SSSC may be used to

control of one of the following parameters: (a) the active power flow on the transmission line, (b) the reactive power flow on the transmission line.

A Newton power flow algorithm with simultaneous solution of power flow constraints and power flow control constraints of the SSSC may be represented by (16) as follows:

$$\begin{pmatrix}
 \frac{\partial F}{\partial \theta_{se}} & \frac{\partial F}{\partial V_{se}} & \frac{\partial F}{\partial \theta_i} & \frac{\partial F}{\partial V_i} & \frac{\partial F}{\partial \theta_j} & \frac{\partial F}{\partial V_j} \\
 \frac{\partial PE}{\partial \theta_{se}} & \frac{\partial PE}{\partial V_{se}} & \frac{\partial PE}{\partial \theta_i} & \frac{\partial PE}{\partial V_i} & \frac{\partial PE}{\partial \theta_j} & \frac{\partial PE}{\partial V_j} \\
 \frac{\partial P_i}{\partial \theta_{se}} & \frac{\partial P_i}{\partial V_{se}} & \frac{\partial P_i}{\partial \theta_i} & \frac{\partial P_i}{\partial V_i} & \frac{\partial P_i}{\partial \theta_j} & \frac{\partial P_i}{\partial V_j} \\
 \frac{\partial Q_i}{\partial \theta_{se}} & \frac{\partial Q_i}{\partial V_{se}} & \frac{\partial Q_i}{\partial \theta_i} & \frac{\partial Q_i}{\partial V_i} & \frac{\partial Q_i}{\partial \theta_j} & \frac{\partial Q_i}{\partial V_j} \\
 \frac{\partial P_j}{\partial \theta_{se}} & \frac{\partial P_j}{\partial V_{se}} & \frac{\partial P_j}{\partial \theta_i} & \frac{\partial P_j}{\partial V_i} & \frac{\partial P_j}{\partial \theta_j} & \frac{\partial P_j}{\partial V_j} \\
 \frac{\partial Q_j}{\partial \theta_{se}} & \frac{\partial Q_j}{\partial V_{se}} & \frac{\partial Q_j}{\partial \theta_i} & \frac{\partial Q_j}{\partial V_i} & \frac{\partial Q_j}{\partial \theta_j} & \frac{\partial Q_j}{\partial V_j}
 \end{pmatrix}
 \begin{pmatrix}
 \Delta \theta_{se} \\
 \Delta V_{se} \\
 \Delta \theta_i \\
 \Delta V_i \\
 \Delta \theta_j \\
 \Delta V_j
 \end{pmatrix}
 =
 \begin{pmatrix}
 -\Delta F \\
 -\Delta PE \\
 -\Delta P_i \\
 -\Delta Q_i \\
 -\Delta P_j \\
 -\Delta Q_j
 \end{pmatrix}
 \tag{16}$$

In equation (16), the system Jacobian matrix is split into four blocks by the dotted lines, the bottom diagonal block has the same structure as that of the system Jacobian matrix of conventional power flow [7] though the terms of former should consider the contributions from the SSSC. The other three blocks of the system Jacobian matrix in (16) are related with SSSC. It should be pointed herein that the Newton formulation of (16), which is similar to the formulation presented in [8], is different from the conventional Newton formulation proposed in [7]. For the latter, the voltage correction vector is  $\Delta V/V$  and corresponding voltage-related Jacobian terms are  $V(\partial P/\partial V)$ ,  $V(\partial Q/\partial V)$  while for the present formulation of (16), the  $\Delta V$  and corresponding voltage related Jacobian terms are  $(\partial P/\partial V)$ ,  $(\partial Q/\partial V)$ .

### 3.2 Initialization of the SSSC in the Newton Power Flow

In the present implementation, for control modes 1, 2 the initial values of the voltage angle and magnitude of a SSSC may be set as follows:

$$\theta_{se}^0 = -\frac{\theta}{2} \tag{17}$$

$$V_{se}^0 = 0.1 \tag{18}$$

## 4. MATHEMATICAL MODEL OF IPFC

### 4.1 Equivalent Circuit

By combining two or more series-connected converters working together the IPFC [3, 4] extends the concepts of voltage and power flow control beyond what is achievable by the one-converter FACTS controller, SSSC. The simplest IPFC consists of two back-to-back DC-to-AC converter, which in a substation are connected in series with two transmission lines via transformers and the DC terminals of the converters are connected together via a common DC link. Basically the IPFC can control three power system quantities: three independent power flows of two lines. Such an IPFC, which is shown in Fig. 3, is used to show the basic operation principles. The mathematical derivation is applicable to an IPFC with any number of series converters. In Fig. 3, there are three FACTS buses while two transmission lines are connected with bus  $j$  and  $k$  although the transmission lines are not shown. In Fig. 3,  $V_i, V_j, V_k$  are complex voltages at buses  $i, j, k$ , respectively. They can be further defined as  $V_l = V_l \angle \theta_l$ , ( $l=i, j, k$ ) where  $V_l, \theta_l$  are the magnitude and angle of  $V_l$  respectively.  $V_{se_{in}}$  is the complex controllable series injected voltage source, which represents the series compensation of the series converter.  $V_{se_{in}}$  is defined as  $V_{se_{in}} = V_{se_{in}} \angle \theta_{se_{in}}$  ( $n=j, k, \dots$ ), and  $V_{se_{in}}, \theta_{se_{in}}$  are the magnitude and angle of  $V_{se_{in}}$  respectively.

The equivalent circuit of the IPFC with two converters being represented by two controllable series-injected voltage sources is shown in Fig. 4. Real power can be exchanged between two or among three or more series converters via the common DC link, and the sum of the real power exchange should be zero.  $Z_{se_{in}}$  in Fig. 4 is the series transformer impedance.  $P_{se_{in}}$  ( $n=j, k, \dots$ ) is the active power exchange of each converter via the common DC link.  $P_i, Q_i$  are the sum of the active and reactive power flows leaving the bus  $i$ .  $P_{ji}, Q_{ji}$  are the IPFC branch active and reactive power flows leaving bus  $j$ , respectively.  $P_{ki}, Q_{ki}$  are branch active and reactive power flows leaving bus  $k$ , respectively.  $I_{ji}, I_{ki}$  are the IPFC branch currents of branch  $j-i, k-i$  leaving bus  $j$  and  $k$ , respectively.

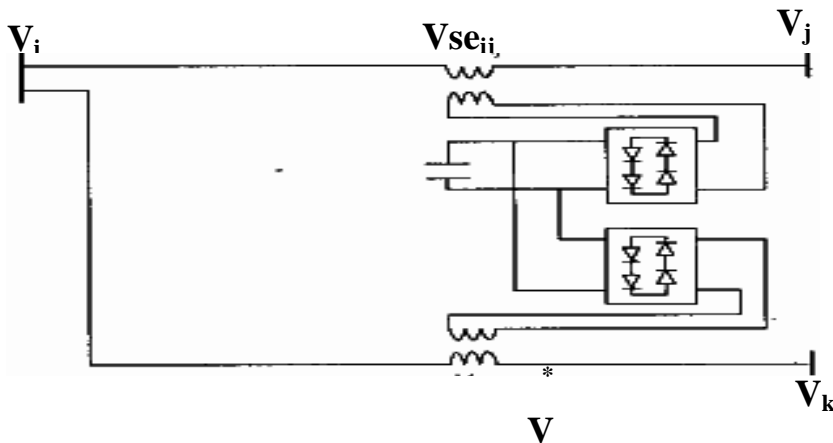


Fig 3 Operational principle of two converters IPFC

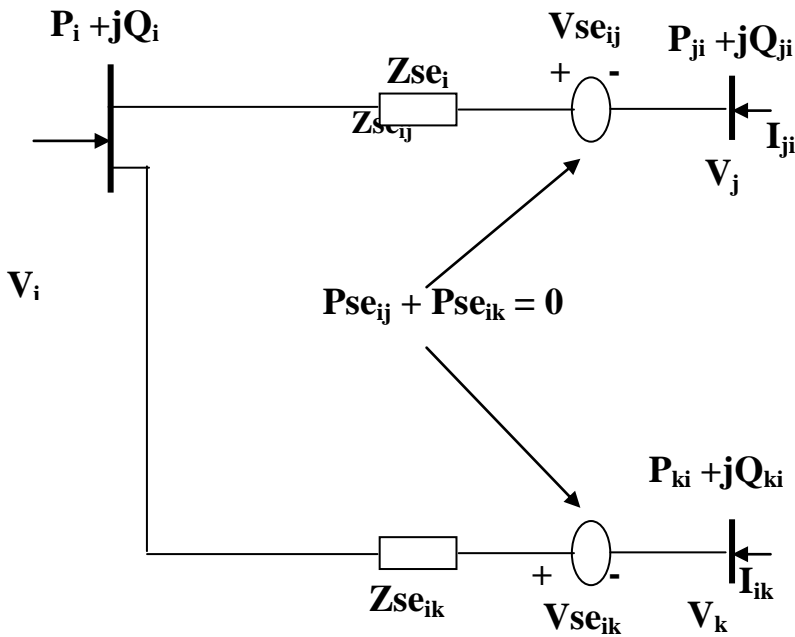


Fig 4 Equivalent circuit of two converters IPFC

### 4.2 Power Flow Balance Constraints of the IPFC

According to the equivalent circuit of the IPFC shown in Fig. 4, the power flow equations can be derived as

$$P_i = V_i^2 g_{ii} - \sum_n V_i V_n (g_{in} \cos(\theta_i - \theta_n) + b_{in} \sin(\theta_i - \theta_n)) - \sum_n V_i V_{se_{in}} (g_{in} \cos(\theta_i - \theta_{se_{in}}) + b_{in} \sin(\theta_i - \theta_{se_{in}})) \tag{19}$$

$$Q_i = -V_i^2 b_{ii} - \sum_n V_i V_n (g_{in} \sin(\theta_i - \theta_n) - b_{in} \cos(\theta_i - \theta_n)) - \sum_n V_i V_{se_{in}} (g_{in} \sin(\theta_i - \theta_{se_{in}}) - b_{in} \cos(\theta_i - \theta_{se_{in}})) \tag{20}$$

$$P_{ni} = V_n^2 g_{nn} - V_i V_n (g_{in} \cos(\theta_n - \theta_i) + b_{in} \sin(\theta_n - \theta_i)) + V_j V_{se_{in}} (g_{in} \cos(\theta_j - \theta_{se_{in}}) + b_{in} \sin(\theta_j - \theta_{se_{in}})) \tag{21}$$

$$Q_{ni} = -V_n^2 b_{nn} - V_i V_j (g_{in} \sin(\theta_n - \theta_i) - b_{in} \cos(\theta_n - \theta_i)) + V_n V_{se_{in}} (g_{in} \sin(\theta_n - \theta_{se_{in}}) - b_{in} \cos(\theta_n - \theta_{se_{in}})) \tag{22}$$

where  $g_{in} + j b_{in} = 1 / Z_{se_{in}}$ ,  $g_{ii} = \sum_n g_{in}$ ,  $b_{ii} = \sum_n b_{in}$ ,  $g_{nn} + j b_{nn} = 1 / Z_{se_{in}} (n = j, k, \dots)$ .

### 4.3 Operating Constraints of IPFC

The equivalent controllable injected voltage source magnitude and angle of the series converter are constrained by

$$V_{se_{in}}^{\min} \leq V_{se_{in}} \leq V_{se_{in}}^{\max} \quad (23)$$

$$-\pi \leq \theta_{se_{in}} \leq \pi \quad (24)$$

where  $n = j, k$ ,  $V_{se_{in}}^{\max}$ ,  $V_{se_{in}}^{\min}$  ... are the maximal and minimal voltage limits of  $V_{se_{in}}$  respectively. According to the operating principle of the IPFC, the operating constraint representing the active power exchange between or among the converters via the common DC link is given by

$$PE = \sum_n P_{se_{in}} = 0 \quad (n = j, k, \dots) \quad (25)$$

where

$$P_{se_{in}} = \text{Re} ( V_{se_{in}} I_{se_{in}}^* ) \quad (n = j, k, \dots) \quad (26)$$

where  $I_{se_{in}}^*$  is conjugate of  $I_{se_{in}}$

### 4.4 Power Flow Control Of IPFC

The IPFC shown in Figs.3 and 4 can control active and reactive power flows of line  $i$ - $j$  and active power flow or reactive power flow of line  $i$ - $k$ . As assumed in the previous section, the sending ends of two lines are connected with bus  $j$  and  $k$ . Then the active and reactive power flows of the two lines at the sending ends are  $P_{ni}$ ,  $Q_{ni}$  ( $n = j, k$ ). The active and reactive power flow control constraints of the IPFC are

$$P_{ni} - P_{ni}^{\text{Spec}} = 0 \quad (27)$$

$$Q_{ni} - Q_{ni}^{\text{Spec}} = 0 \quad (28)$$

where  $n = j, k, \dots$ ,  $P_{ni}^{\text{Spec}}$ ,  $Q_{ni}^{\text{Spec}}$  are the specified active and reactive power flow control references, respectively, and

$$P_{ni} = \text{Re} ( V_n I_{ni} ) , Q_{ni} = \text{Im} ( V_n I_{ni}^* ) .$$

### 4.5 Implementation of IPFC in Newton Power Flow

For the sake of simplicity, the IPFC with two series converters shown in Figs. 3 and 4 is used to show the basic principle of implementation of the IPFC power flow control model in Newton power flow algorithm. Suppose for the FACTS branch  $i$ ,  $j$  active and reactive power flows  $P_{ni}$  and  $Q_{ni}$  can be controlled to power flow control references  $P_{ni}^{\text{Spec}}$ ,  $Q_{ni}^{\text{Spec}}$  by the series converter  $i$ - $j$  while for the FACTS branch  $i$ - $k$  only one of the active power flow and reactive power flow can be controlled by the series converter  $i$ - $k$ , and in the mean time the active power exchange between the two series converters should be balanced. In addition, active and reactive power balance at



buses  $i, j, k$  should also be maintained. Taking into account all these power flow control constraints and bus power mismatch constraints, the compact form is

$$\begin{pmatrix}
 \frac{\partial P_{ji}}{\partial \theta_{se_{ji}}} & \frac{\partial P_{ji}}{\partial V_{se_{ji}}} & 0 & 0 & \frac{\partial P_{ji}}{\partial \theta_i} & \frac{\partial P_{ji}}{\partial V_i} & \frac{\partial P_{ji}}{\partial \theta_j} & \frac{\partial P_{ji}}{\partial V_j} & 0 & 0 \\
 \frac{\partial Q_{ji}}{\partial \theta_{se_{ji}}} & \frac{\partial Q_{ji}}{\partial V_{se_{ji}}} & 0 & 0 & \frac{\partial Q_{ji}}{\partial \theta_i} & \frac{\partial Q_{ji}}{\partial V_i} & \frac{\partial Q_{ji}}{\partial \theta_j} & \frac{\partial Q_{ji}}{\partial V_j} & 0 & 0 \\
 0 & 0 & \frac{\partial P_{ki}}{\partial \theta_{se_{ki}}} & \frac{\partial P_{ki}}{\partial V_{se_{ki}}} & \frac{\partial P_{ki}}{\partial \theta_i} & \frac{\partial P_{ki}}{\partial V_i} & 0 & 0 & \frac{\partial P_{ki}}{\partial \theta_k} & \frac{\partial P_{ki}}{\partial V_k} \\
 \frac{\partial PE}{\partial \theta_{se_{ji}}} & \frac{\partial PE}{\partial V_{se_{ji}}} & \frac{\partial PE}{\partial \theta_{se_{ki}}} & \frac{\partial PE}{\partial V_{se_{ki}}} & \frac{\partial PE}{\partial \theta_i} & \frac{\partial PE}{\partial V_i} & \frac{\partial PE}{\partial \theta_j} & \frac{\partial PE}{\partial V_j} & \frac{\partial PE}{\partial \theta_k} & \frac{\partial PE}{\partial V_k} \\
 \frac{\partial P_i}{\partial \theta_{se_{ji}}} & \frac{\partial P_i}{\partial V_{se_{ji}}} & \frac{\partial P_i}{\partial \theta_{se_{ki}}} & \frac{\partial P_i}{\partial V_{se_{ki}}} & \frac{\partial P_i}{\partial \theta_i} & \frac{\partial P_i}{\partial V_i} & \frac{\partial P_i}{\partial \theta_j} & \frac{\partial P_i}{\partial V_j} & \frac{\partial P_i}{\partial \theta_k} & \frac{\partial P_i}{\partial V_k} \\
 \frac{\partial Q_i}{\partial \theta_{se_{ji}}} & \frac{\partial Q_i}{\partial V_{se_{ji}}} & \frac{\partial Q_i}{\partial \theta_{se_{ki}}} & \frac{\partial Q_i}{\partial V_{se_{ki}}} & \frac{\partial Q_i}{\partial \theta_i} & \frac{\partial Q_i}{\partial V_i} & \frac{\partial Q_i}{\partial \theta_j} & \frac{\partial Q_i}{\partial V_j} & \frac{\partial Q_i}{\partial \theta_k} & \frac{\partial Q_i}{\partial V_k} \\
 \frac{\partial P_j}{\partial \theta_{se_{ji}}} & \frac{\partial P_j}{\partial V_{se_{ji}}} & 0 & 0 & \frac{\partial P_j}{\partial \theta_i} & \frac{\partial P_j}{\partial V_i} & \frac{\partial P_j}{\partial \theta_j} & \frac{\partial P_j}{\partial V_j} & 0 & 0 \\
 \frac{\partial Q_j}{\partial \theta_{se_{ji}}} & \frac{\partial Q_j}{\partial V_{se_{ji}}} & 0 & 0 & \frac{\partial Q_j}{\partial \theta_i} & \frac{\partial Q_j}{\partial V_i} & \frac{\partial Q_j}{\partial \theta_j} & \frac{\partial Q_j}{\partial V_j} & 0 & 0 \\
 0 & 0 & \frac{\partial P_k}{\partial \theta_{se_{ki}}} & \frac{\partial P_k}{\partial V_{se_{ki}}} & \frac{\partial P_k}{\partial \theta_i} & \frac{\partial P_k}{\partial V_i} & 0 & 0 & \frac{\partial P_k}{\partial \theta_k} & \frac{\partial P_k}{\partial V_k} \\
 0 & 0 & \frac{\partial Q_k}{\partial \theta_{se_{ki}}} & \frac{\partial Q_k}{\partial V_{se_{ki}}} & \frac{\partial Q_k}{\partial \theta_i} & \frac{\partial Q_k}{\partial V_i} & 0 & 0 & \frac{\partial Q_k}{\partial \theta_k} & \frac{\partial Q_k}{\partial V_k}
 \end{pmatrix}$$

$$\mathbf{X} \begin{pmatrix} \Delta \theta_{se_{ji}} \\ \Delta V_{se_{ji}} \\ \Delta \theta_{se_{ki}} \\ \Delta V_{se_{ki}} \\ \Delta \theta_i \\ \Delta V_i \\ \Delta \theta_j \\ \Delta V_j \\ \Delta \theta_k \\ \Delta V_k \end{pmatrix} = \begin{pmatrix} P_{ji}^{spec} - P_{ji} \\ Q_{ji}^{spec} - Q_{ji} \\ P_{ki}^{spec} - P_{ki} \\ PE \\ \Delta P_i \\ \Delta Q_i \\ \Delta P_j \\ \Delta Q_j \\ \Delta P_k \\ \Delta Q_k \end{pmatrix} \tag{29}$$

In this formulation, the terms of the first four rows of the system jacobian matrix correspond to the IPFC power flow control and active power exchange balance constraints (27), (28) and (25) while the terms of the last six rows of the system jacobian matrix relate directly to the power mismatch equations at buses  $i, j, k$ . In the current formulation of (29). for the second series converter, it is assumed that the active power flow control constraint is used while the reactive power flow constraint is relaxed. Alternatively, the reactive power flow control constraint can be used in the formulation while the active power flow constraint needs to be relaxed. In principle, either scheme can be implemented in the Newton power flow.

## 5. NUMERICAL EXAMPLES

### 5.1 Test Systems

Test cases are carried out on the IEEE 30-bus shown in Fig. 5

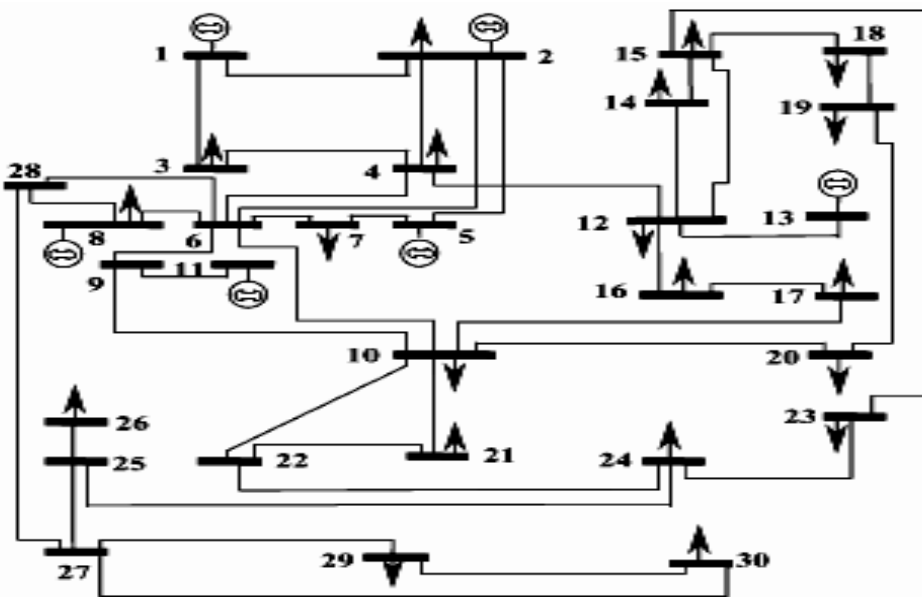


Fig. 5. Single-line circuit diagram of the IEEE 30-bus system

For all cases, the convergence tolerance is  $1.0 \times 10^{-12}$  p.u. (or  $1 \times 10^{-12}$  MW/MVAR) for maximal absolute bus power mismatches and power flow control mismatches. To show the power flow control capability and performance of the Newton power flow algorithm proposed seven cases based on the IEEE 30-bus system are carried out.

**Case 1:** This is a base case IEEE 30-bus system.

**Case 2:** This is similar to case 1 except that there is a SSSC installed for control of active power flow of line 27–29. The active power flow control reference is set to  $-5$  MW.

**Case 3:** This is similar to case 2 except that the SSSC is used to control the reactive power flow of line 27–29. The reactive power flow control reference is  $-0.8$  MVAR.

**Case 4:** This is similar to case 1 except that there are two SSSCs installed on lines 27–29 and 24–23 respectively. These SSSCs are used to control the active power flow of line 27–29 and the reactive power flow of line 24–23 respectively. The control references are - 5 MW and 2 MVAR respectively.

**Case 5:** Similar to case I except that there is an IPFC installed for control of the active and reactive power flows of line 27–25 and active power flow of line 27–29. The control references are - 5 MW, - 1.6 MVAR and - 5 MW respectively.

**Case 6:** Similar to case 5 except that there is an IPFC installed for control of the active and reactive power flows of line 27–25 and reactive power flow of line 27–29. The control references are - 5 MW, - 1.6 MVAR and - 2.2 MVAR respectively.

**Case 7:** This is similar to case 1 except that there is a SSSC installed for control of active power flow of line 27–29 and an IPFC installed for control of the active and reactive power flows of line 24–23 and active power flow of line 24–25. The control references are -5 MW, 1.8 MW, 3 MVAR and 1 MW respectively.

The results of cases 1–7 summarized in Table I. In these cases above and the following discussions, the control references of active and reactive power flows are referred to  $P_{ji}^{Spec}$ ,  $Q_{ji}^{Spec}$  which are at the sending end of a transmission line. Active power flow and reactive power flows at the sending end of the line are referred to  $-P_{ji}$ ,  $-Q_{ji}$  since the sending end of the line is connected to the FACTS bus j.

Table 1 Results of the IEEE 30-bus system

Case	Solution of the FACTS parameters	Number of iteration
Case 1	None	4
Case 2	$\Theta_{se(27\_29)} = 53.6165$ $V_{se(27\_29)} = 0.0146$	6
Case 3	$\Theta_{se(27\_29)} = 251.4319$ $V_{se(27\_29)} = 0.0516$	10
Case 4	$\theta_{se(27\_29)} = 53.5223$ $V_{se(27\_29)} = 0.0147$ $\Theta_{se(24\_23)} = 181.2058$ $V_{se(24\_23)} = 0.0184$	10
Case 5	$\theta_{se(27\_25)} = 311.5575$ $V_{se(27\_25)} = 0.0126$ $\Theta_{se(27\_29)} = 85.6704$ $V_{se(27\_29)} = 0.0190$	9
Case 6	$\theta_{se(27\_25)} = 308.5505$ $V_{se(27\_25)} = 0.0121$ $\Theta_{se(27\_29)} = 59.4172$ $V_{se(27\_29)} = 0.0429$	10
Case 7	$\theta_{se(27\_29)} = 53.7409$ $V_{se(27\_29)} = 0.0146$ $\theta_{se(24\_23)} = 168.3731$ $V_{se(24\_23)} = 0.0032$ $\Theta_{se(24\_25)} = 260.0572$ $V_{se(24\_25)} = 0.0080$	11

## 6. CONCLUSION

Mathematical models of static synchronous series compensator (SSSC) and the interline power flow controller (IPFC) and their implementation in Newton power flow have been presented. A two control functional model for the SSSC and IPFC suitable for power flow analysis is proposed. The model has explored the two control options of the SSSC and IPFC such as 1) the active power flow on the transmission line; 2) the reactive power flow on the line transmission. Furthermore, within the model, the operating voltage and current constraints of the SSSC have been fully considered. Detailed implementation of the novel two control functional models in the Newton power flow algorithm has been presented. A special consideration of the initialization of the variables of the SSSC is also proposed.

Numerical results on the IEEE 30-bus system with single SSSC, two SSSCs, single IPFC and the two FACTS in the same system have demonstrated the feasibility and effectiveness of the established two control functional model of SSSC and IPFC model. The Newton power flow algorithm with incorporation of the SSSCs and IPFC demonstrates the quadratic convergence characteristics though number of iterations of the cases with the SSSCs and IPFC is slightly higher than that of base case power flow studies. The Newton power flow algorithm proposed is a very useful tool to understand and explore the operation and control capabilities of the SSSCs in practical power systems. The Newton power flow program with the modeling of the IPFC is a useful tool for power system planning, operational planning, and operational control of large-scale power systems. The strong multiline control capability of the IPFC with controlling multiline power flows will be likely to play an important role in solving many of the potential problems facing electric utilities in the deregulated electricity market environment.

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## نمذجة حديثه لخطوط النقل المتقاربة المطواعة لتنفيذها في خوارزمية نيوتن رافسون لسريان القدرة والتحكم في السريان الداخلي (SSSC)

في هذا البحث تم شرح نماذج رياضية للمعوض المتوالي المتزامن الثابت و (SSSC) و تم تنفيذها في خوارزمية نيوتن رافسون لسريان القدرة كما تم شرح طريقتين ل (IPFC) للقدرة الفعالة وغير الفعالة في خطوط نقل القدرة و تم إجراء للتحكم في سريان القدرة و هم سريان (IPFC) وذلك لاختبار هاتين الطريقتين وقد أتضح أن خوارزمية نيوتن IEEE-30 bus عدد من المحاكات باستخدام في أنظمة القدرة (SSSC) رافسون لسريان القدرة سوف تكون أداة مفيدة جدا لفهم و شرح القدرات التحكمية في برنامج نيوتن لسريان القدرة أداة مفيدة جدا في التحكم و التخطيط العملي (IPFC) العملية و كذلك نمذجة لأنظمة القدرة الكبيرة