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Ion acoustic waves in the dayside of Titan ionosphere: effect of the solar wind

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ABSTRACT

We investigate the nonlinear ion-acoustic solitary waves for a plasma model in the dayside of ionosphere of Titan. The system of the plasma is composed of three positive ions, namely $HCNH^+$, $C_2H_5^+$, and CH_5^+, isothermal electrons and streaming solar wind particles (i.e. protons and electrons). The dynamics of the ion-acoustic waves is described by Korteweg de-Vries (KdV) equation. The analysis has been done by using a reductive perturbation method. The solitary wave solution is obtained and the effects of solar wind parameters on the pulses profile are examined. Furthermore, the effects of the relevant physical parameters on the profiles of soliton waves are investigated. It is found that only positive pulses are existing and there is not any negative pulses. The soliton amplitude increases with the increase of the phase velocity λ and the relative density protons-to-$HCNH^+$. Also, it was found that the pulses decrease with the increase of the relative density $C_2H_5^+$-to-$HCNH^+$ and temperature ratio protons-to-electrons.

1. Introduction

One of the major objectives of the Cassini mission is the exploration of Titan which is the largest moon of Saturn. The in-situ measurements made by Cassini spacecraft of Titan improved largely the comprehension about the complicated chemistry happening in the ionosphere of Titan. Titan has a dense extended atmosphere fundamentally composed of ∼96% molecular nitrogen, <4% methane, and <1% hydrogen [1 - 3]. The solar irradiance has a great impact on the densities of the ionosphere and the year in Titan spans over two complete solar cycles [4]. The altitude of the ionospheric peak of Titan decreases at high solar activity that indicates the neutral atmosphere was less comprehensive while the atmosphere and ionosphere of Earth extend at high solar activity. The main source of ions production on the dayside of Titan is photoionization by solar radiation. The solar irradiance reaches Titan about 100 times weaker than compared to what it is at the orbit of the Earth.

The densities of electron number in the dayside are found to peak typically at values 2000-5000 cm^{-3} in the altitude range 1000-1200 km.

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Edberg et al. indicated that the enhanced effect ionization is produced by the flux of high-energy particles when Titan was located in the shocked solar wind [5]. The enhancement of density has disappeared when Titan moved back inside the magnetopause. During the Cassini mission occurs these density variations, complicated the ability of many studies to isolate the impacts of solar cycle activity alone. The dominant ions in Titan's ionosphere are $HCNH^+$, $C_2H_5^+$, and CH_5^+. In the altitude range 1050-1200 km, $HCNH^+$ constitutes about 40-50% of the total ion population measured by Ion and Neutral Mass Spectrometer (INMS), which was capable of determining the chemical, elemental and isotopic composition of the gaseous and volatile components of the neutral particles and the low energy ions in Titan's ionosphere and atmosphere, the magnetosphere of Saturn, and environment of the ring. In fact, $C_2H_5^+$ and CH_5^+ contribute ∼15% and ∼2%, respectively [6].

The solar wind streaming electrons and protons interaction of with different planets creates an interesting phenomenon. This solar wind interaction with un-magnetized (like Titan and Venus) and magnetized (like Earth) planets or moons is quite different. The magnetic field at the Earth is enough for no direct contact between the planetary ionosphere and the solar wind. Unlike Earth, the deficiency of metals in Titan's core leads to it does not have its own magnetic field, but the motion of the super-compressed hydrogen gas inside the planet core produced the magnetosphere of Saturn. Cassini spacecraft observed the plasma environment of Titan interacted with the supersonic solar wind for the first time on 1 December 2013 [7]. It is worth noting that this manuscript is a continuation of our previous research by adding the solar wind effect and study the ion-acoustic waves that may generate as a result of this interaction at the dayside of Titan ionosphere [8]. Ion-acoustic solitary waves (IASWS) are studied first by Washimi and Taniuti (1966) in a simple plasma model by applying reductive perturbation theory to the basic equations governing the plasma dynamics. Then, IASWS studied in many experimental and theoretical studies to examine the behavior of the IASWs in many researches [9 - 12].

This study is carried out on the dayside of the ionosphere of Titan where the effects of solar wind (see Fig.1) are taken into consideration on the propagation characteristics of the acoustic solitary waves. Previous studies indicated that these waves are observed on Saturn and a close encounter with the moon Titan [13, 14]. Thus, we present this theoretical study to have a deep understanding on the existence of acoustic solitary waves in Titan, and we hope that this study increases our realization to the nature of plasma waves in Titan's ionosphere.

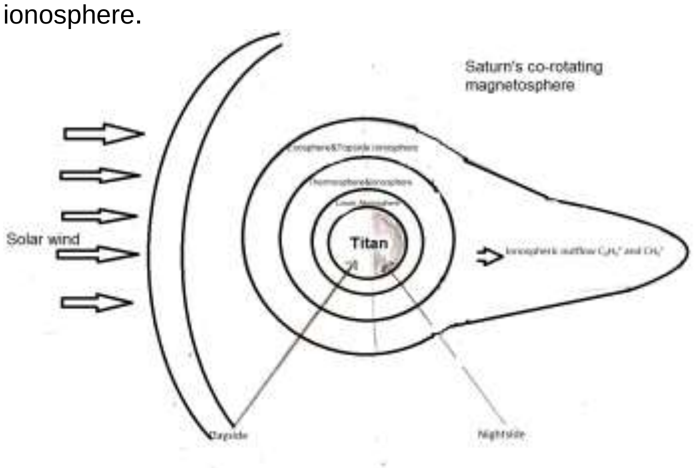

Figure 1: The effects of solar wind on the ionosphere of Titan

The outline of the paper is as follows. In Sec. 2, the basic equations for the system is presented, and we derive the Korteweg de-Vries (KdV) equation. In Sec. 3, the stationary solution of the KdV equation is obtained. The numerical analysis and the dependence of solar wind parameters on the solitary waves are discussed. The results are summarized in section 4.

Model equation and derivation of an evolution equation

Let us consider a system of an unmagnetized, collisionless plasma in the dayside of Titan's ionosphere consisting of three-positive ions, isothermal electrons interacting with solar wind protons and electrons (referred to by the subscripts 1,2,3,p, and b). The dimensionless basic equations are governed by

$$\left(\frac{\partial n_i}{\partial t}\right) + \left(\frac{\partial n_i u_i}{\partial x}\right) = 0, \qquad (1)$$

$$\left(\frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x}\right) u_i + \frac{3\sigma_i}{\mu_i} n_i \frac{\partial n_i}{\partial x} + \frac{1}{\mu_i}\frac{\partial \Phi}{\partial x} = 0. \qquad (2)$$

The electrons at Titan are described by Maxwellian-distribution

$$n_e = N_e exp(\Phi), \qquad (3)$$

and the solar wind electrons distribution reads

$$n_b = N_b exp(\Phi). \qquad (4)$$

The system is closed by Poisson equation

$$\frac{\partial^2 \Phi}{\partial x^2} = n_e + n_b - \sum_i n_i. \quad (5)$$

In equations (1)-(5), the mass ratio $\mu_i = \frac{m_i}{m_1}$ where $i = 1,2,3,p$. m_1, m_2, m_3, and m_4 are the masses of the three-positive ions $HCNH^+$, $C_2H_5^+$, and CH_5^+, and solar wind protons, respectively. The temperature ratio $\sigma_i = \frac{T_i}{T_e}$, where T_i is the temperature of the ions i and T_e is the electrons temperature. All the densities are normalized with respect to $n_1^{(0)}$, which is the unperturbed density of the first positive ions ($HCNH^+$). The variables n_j and u_j $(j = p, b, 1,2,3, e)$ are the densities and velocities of the solar wind proton and electron, the three-positive ions, and the electrons at Titan. Φ is the electrostatic potential, x is the space coordinate and t is the time variable. The distance is normalized by the positive ions Debye length $\lambda_{D1} = \left(\frac{k_B T_e}{4\pi e\, n_1^{(0)}}\right)^{(1/2)}$, and the time is normalized by the first positive ion plasma period $\omega_{p1}^{-1} = \left(\frac{m_i}{4\pi e^2 n_1^{(0)}}\right)^{(1/2)}$. Here, the electric potential is normalized by the thermal potential $\frac{k_B T_e}{e}$ and the velocities by the ion sound speed$C_{si} = \left(\frac{k_B T_e}{m_i}\right)^{(1/2)}$, where k_B is the Boltzmann constant and e the electron charge.

To study the electrostatic perturbations of the ion-acoustic waves for small, but finite, amplitude, we employ the reductive perturbation method [15, 16] we introduce the stretching space-time coordinates

$$\zeta = \varepsilon^{1/2}(x - \lambda t) \; and \; \tau = \varepsilon^{3/2} t, \quad (6)$$

where λ is the phase velocity and ε is a small parameter $(\varepsilon < 1)$. Furthermore, we expand all the dependent physical quantities in equations (1)-(5) as follows

$$n_i = N_i + \varepsilon n_i^{(1)} + \varepsilon^2 n_i^{(2)} + \varepsilon^3 n_i^{(3)} + \cdots, \qquad (7a)$$

$$u_i = \varepsilon u_i^{(1)} + \varepsilon^2 u_i^{(2)} + \varepsilon u_i^{(3)} + \cdots, \qquad (7b)$$

$$\Phi = \varepsilon \Phi^{(1)} + \varepsilon^2 \Phi^{(2)} + \varepsilon^3 \Phi^{(3)} + \cdots. \qquad (7c)$$

At equilibrium, the quasi-neutrality condition reads

$$N_e + N_b - N_2 - N_3 - N_p = 1, \quad (8)$$

Where $N_e = n_e{}^{(0)}/n_1{}^{(0)}$, $N_b = n_b{}^{(0)}/n_1{}^{(0)}$, $N_2 = n_2{}^{(0)}/n_1{}^{(0)}$, $N_3 = n_3{}^{(0)}/n_1{}^{(0)}$, and $N_p = n_p{}^{(0)}/n_1{}^{(0)}$.

Substituting equations (6) and (7) into equations (1)-(5), we obtain to the lowest-order in ε the following equations

$$n_i^{(1)} = \left(\frac{N_i}{\mu_i z_i}\right)\Phi^{(1)}, u_i^{(1)} = \left(\frac{\lambda}{\mu_i z_i}\right)\Phi^{(1)}, \qquad (9a)$$

$$N_e + N_b = \sum_i \left(\frac{N_i}{\mu_i z_i}\right), \qquad (9b)$$

Where $z_i = \lambda^2 - 3\sigma_i \frac{N_i^2}{\mu_i}$. The next-order in ε gives

$$\lambda\left(\frac{\partial n_i^{(2)}}{\partial \zeta}\right) = \left(\frac{\partial n_i^{(1)}}{\partial \tau}\right) + \left(\frac{\partial}{\partial \zeta}\right)\left(N_i^{(0)} u_i^{(2)}\right) + \left(\frac{\partial}{\partial \zeta}\right)\left(n_i^{(1)} u_i^{(1)}\right), \qquad (10a)$$

$$\lambda\left(\frac{\partial u_i^{(2)}}{\partial \zeta}\right) = \left(\frac{\partial u_i^{(1)}}{\partial \tau}\right) + u_i^{(1)}\left(\frac{\partial}{\partial \zeta}\right)u_i^{(1)} + \left(\frac{3\sigma_i}{\mu_i}\right)N_i^{(0)}\left(\frac{\partial n_i^{(2)}}{\partial \zeta}\right) + \left(\frac{3\sigma_i}{\mu_i}\right)n_i^{(1)}\left(\frac{\partial n_i^{(1)}}{\partial \zeta}\right) + \left(\frac{1}{\mu_i}\right)\left(\frac{\partial \Phi^{(2)}}{\partial \zeta}\right), \qquad (10b)$$

$$\frac{\partial^2 \Phi^{(1)}}{\partial \zeta^2} + \sum_i n_i^{(2)} = n_e^{(2)} + n_b^{(2)}. \qquad (10c)$$

Eliminating the second-order perturbed quantities in (10) and making use of the first-order equation (9), we derive the Korteweg-de Vries (KdV) equation

$$\frac{\partial \Phi^{(1)}}{\partial \tau} + AB\Phi^{(1)}\frac{\partial \Phi^{(1)}}{\partial \zeta} + \frac{1}{2}A\frac{\partial^3 \Phi^{(1)}}{\partial \zeta^3} = 0, \qquad (11)$$

where A and B are given by

$$A = \left[\sum_i \left(\frac{\lambda N_i^{(0)}}{\mu_i z_i^2}\right)\right]^{-1},$$

$$B = \frac{3}{2}\left[\frac{-1}{3}(N_e + N_b) + \sum_i \frac{N_i^{(0)}}{\mu_i^2 z_i^3}\left(z_i + \frac{4\sigma_i N_i^{(0)2}}{\mu_i}\right)\right].$$

Numerical results and discussion

To obtain the stationary solution of equation (11), we use the transformation $X = \zeta - U\tau$ into equation (11) and integrate, we thus obtain

$$\Phi^{(1)} = \Phi^{(0)} sech^2\left(\frac{X}{W}\right), \qquad (12)$$

where the amplitude $\Phi^{(0)} = \frac{3U}{AB}$ and the width $W = (\frac{2A}{U})^{1/2}$.

Since A is always positive. We will examine the sign of the coefficient B depending on the solar wind physical parameters. Keep in mind that we have to use the neutrality condition (8) and the compatibility condition (9b) into the coefficient B. It clears from Figure 2 that the regions where B may change its polarity between the phase velocity λ and its dependence on the relative density N_p. It is noted that B is positive in the red color regions except for small yellow regions which have a negative sign. Nevertheless, the yellow regions do not fulfill the presence of λ, because λ appears at the red region only and it does not pass through the yellow zone. It indicates that the yellow (negative) B does not satisfy the compatibility condition (9b). This condition means that we have only positive (compression) wave, as we will present below.

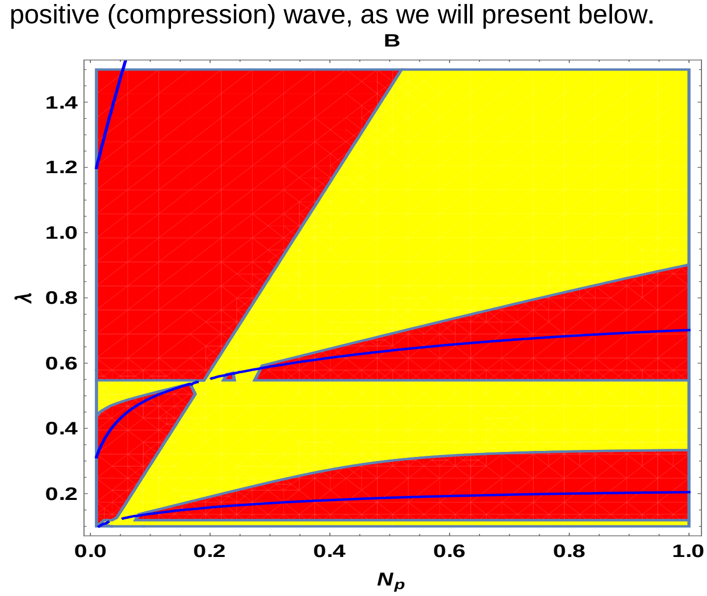

Figure 2: (Color online) The polarity of the nonlinear term B where the phase velocity λ versus the relative density N_p. The red region refers to $B > 0$, the yellow regions refer to $B < 0$, and the blue lines refer to λ, since $\mu_2 = 1.03$, $\mu_3 = 0.6$, $\mu_4 = 0.036$, $\sigma_i = 0.1$, $N_2 = 0.22$, and $N_3 = 0.044$.

Figures (3)-(6) show the behavior of the electrostatic potential Φ [given by Eq. (12)] for different values of N_2, N_p, σ_p, and λ. It is noticed that from Fig. 3, the amplitude increases with increasing the relative density N_2 but the width has not change. Figure 4 clears that the amplitude and the width decrease with the increase of the density ratio N_p. The same behavior is noticed when the relative density N_3 increases, so we did not include this figure here.

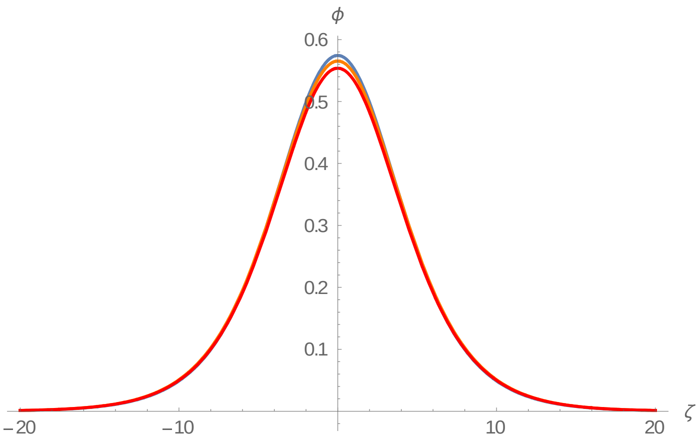

Figure 3: (Color online) The soliton profile of the electrostatic potential Φ against ζ where $U = 0.09$, $\mu_2 = 1.03$, $\mu_3 = 0.6$, $\mu_4 = 0.036$, $\sigma_i = 0.1$, $\lambda = 1.4$, $N_3 = 0.044$, $N_p = 0.0189$, and $N_2 = 0.22$ (blue curve), $N_2 = 0.15$ (orange curve), and $N_2 = 0.07$ (red curve).

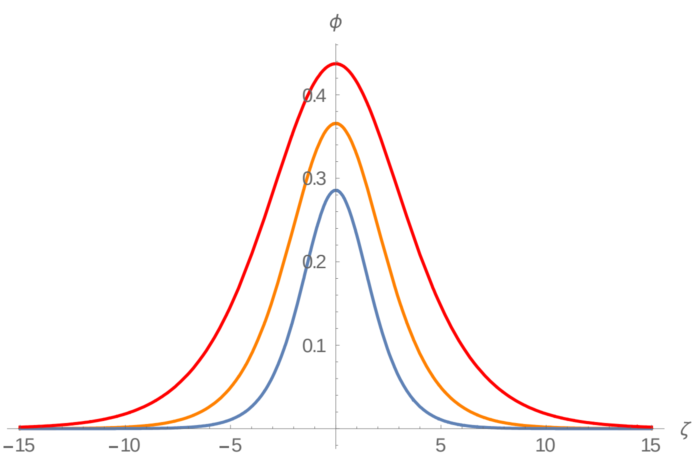

Figure 4: (Color online) The solitary wave profile is depicted for different values of relative density N_p where $U = 0.09$, $\mu_2 = 1.03$, $\mu_3 = 0.6$, $\mu_4 = 0.036$, $\sigma_i = 0.1$, $\lambda = 1.3$, $N_2 = 0.22$, $N_3 = 0.044$, and $N_p = 0.189$ (blue curve), $N_p = 0.1$ (orange curve), and $N_p = 0.0189$ (red curve)

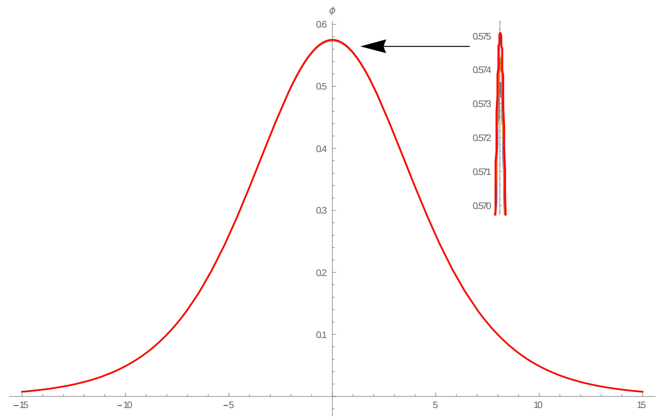

Figure 5: (Color online) The profile of solitary wave is depicted for different values of relative temperature σ_p where $U = 0.09$, $\mu_2 = 1.03, \mu_3 = 0.6$, $\mu_4 = 0.036$, $N_2 = 0.22$, $N_3 = 0.044$, $N_p = 0.0189$, $\sigma_1 = \sigma_2 = \sigma_3 = 0.1$, and $\sigma_p = 0.2$ (blue curve), $\sigma_p = 0.1$ (orange curve), and $\sigma_p = 0.001$ (red curve).

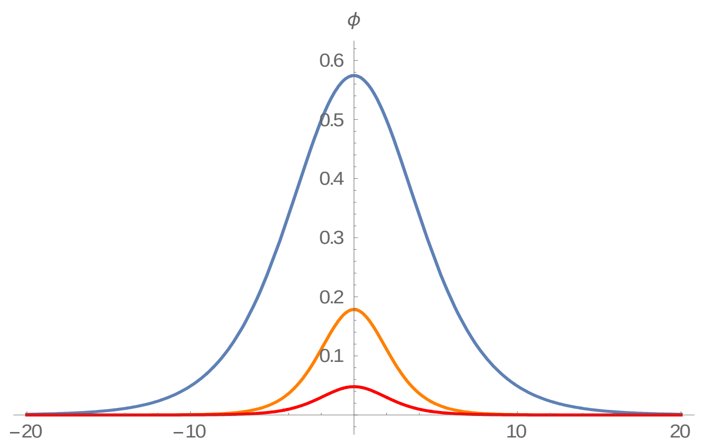

Figure 6: (Color online) The solitary wave profile is depicted for different values of phase velocity λ where $U = 0.09$, $\mu_2 = 1.03$, $\mu_3 = 0.6$, $\mu_4 = 0.036$, $\sigma_i = 0.1$, $N_2 = 0.22$, $N_3 = 0.044$, $N_p = 0.0189$, and $\lambda = 1.4$ (blue curve), $\lambda = 1$ (orange curve), and $\lambda = 0.7$ (red curve).

It is seen from Fig. 5, the amplitude decreases slightly with the increase of temperature ratio σ_p but the width does not change. The effect of the temperature ratio σ_i on the behavior of the electrostatic potential Φ (width and amplitude) is approximately as in Fig. 5, so we did not include this Figure.

Finally, Fig. 6 shows that the increase of the phase velocity λ leads to increase both the amplitude and the width. Physically, in Figure 3 the increase of N_2 would lead to enhance the bumped energy in the system which increases the nonlinearity and therefore the amplitude becomes taller. Figures 4 and 5 clear that for lower values of N_p and σ_p the amplitude is taller than for higher values of N_p *and* σ_p. This occurred since for lower N_p and σ_p would lead to intensify more energy through the plasma that increases the nonlinearity and makes the amplitude taller.

Conclusion

In this paper, a theoretical model was adopted to describe the nonlinear ion-acoustic solitary waves at the dayside of the ionosphere of Titan at altitude 1000 to 1200 km. This physical model compressing of three-positive ions (namely $HCNH^+$, $C_2H_5^+$, and CH_5^+), isothermal electrons, as well as streaming solar wind protons and electrons. The reductive perturbation method has been applied to decrease the basic equations to one evolution equation called the KdV equation. We obtained the soliton solution of the KdV equation. The effects of the relevant physical parameters on the profiles of soliton waves are investigated. It is found that the propagating pulse is generally positive and there is no negative pulse exists. The soliton amplitude increases with the increase of relative density protons-to-$HCNH^+$ and the phase velocity λ. However, it decreases with the increase of the relative density $C_2H_5^+$-to-$HCNH^+$ and temperature ratio protons-to-electrons.

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