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MATHEMATICAL MODEL FOR ESTIMATING SCOUR DOWNSTREAM WEIRS

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The main objective of this research study is to simulate the scour process downstream the weirs with mathematical models for non-cohesive bed material. The basis for simulating the mobile bed is the solution of the equation for continuity of sediment by using the finite deference. Transport capacity is calculated at each cross section by using Hydraulic data obtained during the calculation of water surface profiles (e.g. , width, depth, slope, and velocity of flow).The results of the developed mathematical model proved that the technique is reliable in predicting the scour profile downstream weirs compared to the results of experimental data.

NOMENCLATURE

d_s	vertical depth of the scour hole.
d_{50}	mean particle diameter.
$Fe_1^2 = \frac{V_2^2}{gh_1}$	(Fe is the froude number).
g	Gravitational acceleration.
H_u	upstream water head on the weir
H_s	water depth over the lowest point of the scour hole.
h_1	downstream water depth at the end of apron.
L_s	scour length
P	weir height
Q	water discharge over the weir in lit/sec
T	time
v	local flow velocity at any point
V_b	bottom velocity
V_2	mean velocity downstream of the weir
x	horizontal distance of the scour hole
ρ	water density
ρ_s	soil particle density
υ	kinematics viscosity

INTRODUCTION

The problem of local scour downstream hydraulic structures is an important subject to both practical engineers and research works. There are many varieties of local-scour systems downstream of hydraulic structures, each with its own partical geometry and hence local scour mechanism. Generally scour holes are caused by non- equilibrium transport conditions. The sediment particles will deposite upstream of the construction, so that the bed level will rise, whereas downstream of the construction the sediment particales will be eroded resulting in scour. Many researchers and scientists put much effort to study these phenomena, among them are Farhoudi [1], 1982, Abo-Zied [2], 1989, Yassin [3], 1992, Ali and Hadya [4], 1994, Hoffmans [5], 1995, Bijan [6], 2003, Abo-Zied [7], 2004 and many others.

Nasr and Nagy [8] 1997, by using the fitting curve gave the following relation:

$$\frac{L_s}{h_1} = 0.33\left(\frac{H_s}{h_1}\right)^4 \quad \text{---} \quad (1)$$

Also, they developed a theoretical equation for estimating the scour hole depth downstream weir structure

$$\frac{H_s}{h_1} = \sqrt{1+2Fe_1^2\left[\beta+\left(575Fe_1^{2.33}\right)\left(\frac{V_b}{V_2}\right)^2\frac{H}{h_1}-2\left(575Fe_1^{2.33}+1\right)\frac{V_b}{V_2}+575Fe_1^{2.33}\frac{h_1}{H_s}\right]} \text{ -- } (2)$$

where:

$Fe_1^2 = \frac{V_2^2}{gh_1}$,(Fe is the Froude number),

v = the local flow velocity at any point, V_s = the surface velocity, V_b = the bottom velocity ; = $68.5\sqrt{d_{50}}\left(\frac{H_s}{d_{50}}\right)^{0.095}$, and

H_s = the water depth over the lowest point of the scour hole.

SEDIMENT TRANSPORT EQUATIONS BED LOAD

Several expressions, more or less empirical, have been suggested in the last century to compute the bed load as a function of flow characteristics and particale diameter. In this study the bed load is computed by Einstein parameters as developed by *Breusers* as mentioned in *Farhoudi [1]*. In his work on local scour adjacent to protective aprons in estuary closure structures, Breusers used an approach based on a specfic relationship between the Einstein grain mobility parameter, ψ , and the bed load parameter, Φ .

The definitions of the Einstein parameters are as follow:

$$\Psi = (\rho_s - \rho)\frac{gD}{\tau} \quad \text{--------------------------------} \quad (3)$$

or $$\Psi = (S_s - 1)Dg\upsilon_*^{-2} \quad \text{-----------------------------------} \quad (4)$$

where :

D = representative diameter of particles,

τ = shear stress on the particles,

ρ_s = density of particles,

$S_s = \frac{\rho_s}{\rho}$, and

υ_* = the shear velocity, $\sqrt{\frac{\tau}{\rho}}$ Also

$$\Phi = q_{s}\left(S_s-1\ \right)^{-1/2}\left(gD^3\right)^{-1/2}F^{-1} \quad \text{---------------------------} \quad (5)$$

or

$$q_s = \frac{40(v_* - v_{*c})^3}{(S_s - 1)g} \quad \text{------------------------------------} \quad (6)$$

in which $F = \sqrt{\frac{2}{3} + \frac{36\upsilon^2}{gD^3(S_s-1)}} - \sqrt{\frac{36\upsilon^2}{gD^3(S_s-1)}}$ ----------------------- (7)

Most of the previous works did not take the effect of time and particle size of bed material on the shape of the scour hole. Also, most of past researches depended on experimental results, which need to large time and effort so this research concern with mathematical model by using computer program.

THEORETICAL BASIS MASS-BALANCE EQUATION

The sediment mass- balance equation to a small control volume of the bed layer can be written as (Van Rign, 1987), as mentioned in Hoffmans [5], Fig. (1):

$$\frac{1}{1-\lambda}q_s + \left[E_0 + D_a + \frac{\partial a c_a^{'}}{\partial t}\right]dx = \frac{1}{1-\lambda}\left[q_s + dq_s\right] + \left[E_a + D_0\right]dx \quad \text{---------------} \quad (8)$$

(Input and storage) (Output)

with

$$E_0 + \frac{\partial y_b}{\partial t} = D_0 \quad \text{------------------------------------} \quad (9)$$

in which

E_a = upwards diffusive flux for Y= Yb +a

D_a = convective flux at the reference level

$c_a^{'}$ = depth – averaged concentration in the bed layer

q_s = bed load per unit width (without pores)

For steady-flow conditions the storage term is neglected. Then equation (8) reduces to

$$\frac{\partial h}{\partial t} + \left(E_a - D_a\right) + \frac{1}{1-\lambda}\frac{\partial q_s}{\partial x} = 0 \quad \text{----------------------------------} \quad (10)$$

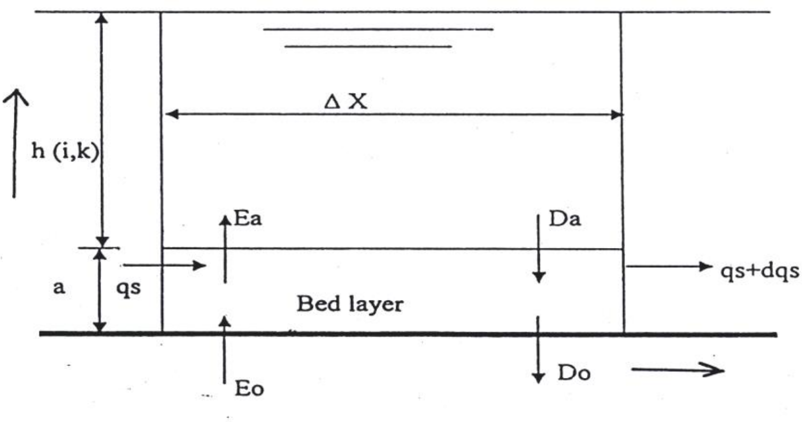


Fig (1) Schematization of the sediment exchange process

Integrating the sediment mass-balance equation over the full depth and assuming steady-flow conditions the following equation is obtained:

$$\frac{\partial h}{\partial t}+\frac{1}{1-\lambda}\frac{\partial q_s}{\partial x}=0 \qquad \text{-------------------------------------} \qquad (11)$$

This equation is the equation for continuity of sediment:
Where: λ = porosity, t = time; and x , h = the coordinates of a point on the bed of the scour hole.

FINITE DEFERENCE TECHNIQUE TO CONVERT DIFFERENTIAL EQUATION INTO ALGEBRAIC EQUATION

This equation is solved by using the finite difference from using the notation shown in Fig. (2), in which I-1, I, I+1 refer to cross sections along the model and k refer to time.

$$\frac{\left(h(I,k+1)-h(I,k)\right)}{\Delta t}+B\frac{\left(q_s(I+1,K)-q_S(I,k)\right)}{\Delta x}=0 \qquad \text{----------------} \qquad (12)$$

Assuming no sediment inflow to the scour hole

$$h(I,k+1)=h(I,K)+\Delta s \qquad \text{-------------------------------------} \qquad (13)$$

if $\tau \succ \tau_c$

$$h(I,k+1)=h(I,K)-\Delta s \qquad \text{---------------------------------} \qquad (14)$$

if $\tau \leq \tau_c$

Where:

$$\Delta s=\frac{\Delta t \Delta q}{\Delta x(1-\lambda)} \qquad \text{-------------------- ------------} \qquad (15)$$

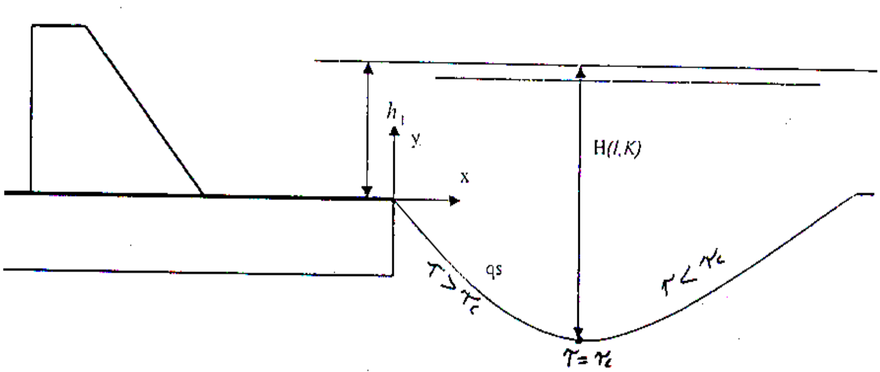


Fig.(2) Definition sketch showing the geometry of the scour hole.

Substituting Eq. (6) into Eq. (15) finally gives

$$\Delta s = \frac{40\Delta t}{(1-\lambda)\Delta x(S_s - 1)g}\left(\upsilon_* - \upsilon_{*c}\right)^3 \quad \text{------------------------} \qquad (16)$$

in which Δs = the deformation in the bed and the new bed profile is finally calculated by solving Eq. (13) and Eq. (14) with an explicit computation schem.

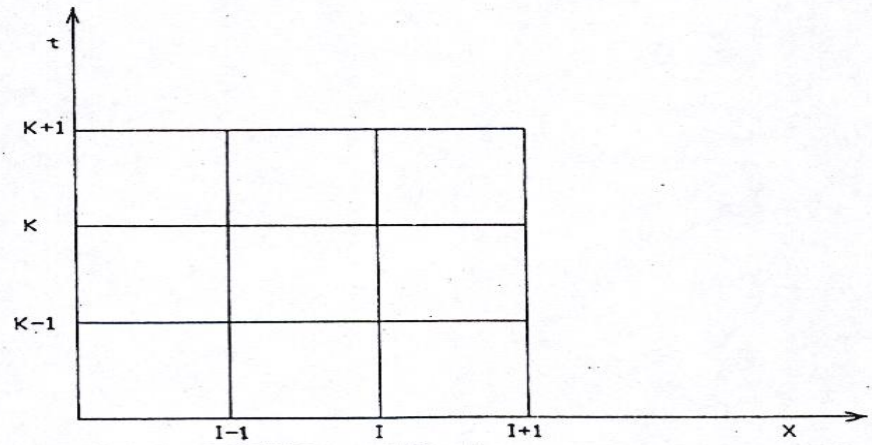


Fig. (3) Computation Net

EXPERIMENTAL ARRANGEMENTS

The present experiments were carried out in a horizontal channel of trapezoidal cross-sections, 18.50 m. long, 0.84 m. bottom width and side slope 1 : 1. The model sand is sieved into the following sizes; 0.47, 0.82 and 1.75 mm. The channel was located in a hall of irrigation and hydraulics laboratory in Civil engineering department, Assiut University. Steel models for the weirs were inserted into the flume with height equal 25 cm. Depths of flow were measured by a point gauge. The discharges passing over the weirs were measured by a v-notch weir.

DESIGN OF COMPUTER PROGRAM

In concept, digital modeling utilizes the digital computer to simulate, with respect to time, the behavior of a physical system or process by applying the necessary theory to describe the physical process in terms of dependent and independent variables and by specifying the necessary functional realationships to describe the physical system. The program is written in standard Fortran so that it is portable from one computer to another. By entering the initial depth of water at the end of apron, changes are calculated with respect both to time and to distance along the channel.

DISCUSSION OF RESULTS

Figures (4a), (4b), (4c), and (4d) show the comparison between results of both the mathematical model and the experimental data after 5, 120, 300, 480 min. It is clear that the agreement is compatible. Also, from Fig. (5) it is clear that the depth of scour increase with the increase of the time but the shape of the scour hole remains the same. Figure (6) gives the longitudinal sections along the centerline of the scour hole for d50 = 0.47, 0.82, and 1.75mm. at Fe = 0.2533 for all tested weirs. From these figures, it is clear that, as the particles size increase the scour hole dimensions decrease. This means the force required by the sediment particle to be lifted up and be moved higher in case of large sediment size than the small one. This finding is in agreement with the finding of others, as obtained by *Abou-Zeid* [7], 2004.

CONCLUSIONS

The use of finite difference method for solving the partial differential equation which governs the scour downstream the weirs is more useful for estimating the scour profile. The following conclusions may be considered in the design of such structure.

1- The variation of scour profile strongly depends on the discharge passes than the sediments particle size.
2- 98% of scour occurs after 6 houres of the beginning.
3- The depth of scour increase more with time in the initial phase
4- In the equilibrium phase (after 8 hours) the dimension of the scour no longer changes significantly.

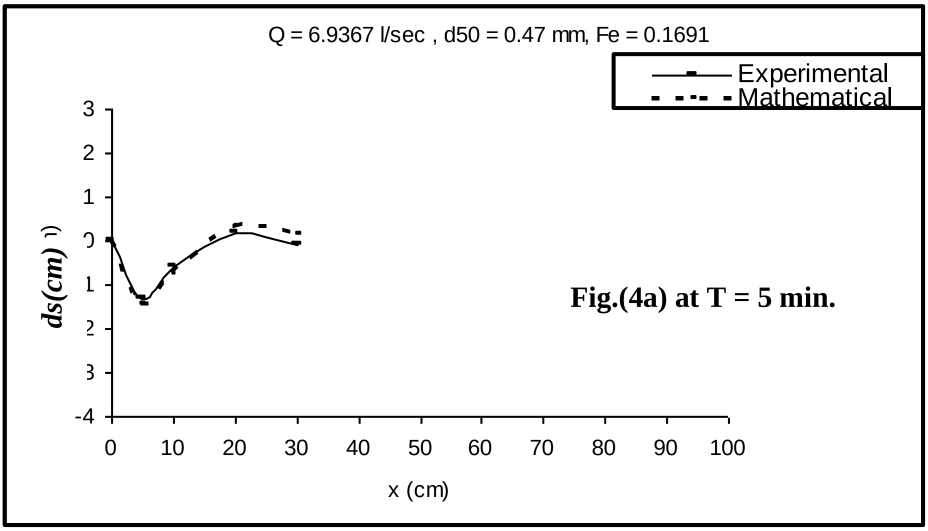


Fig.(4a) at T = 5 min.

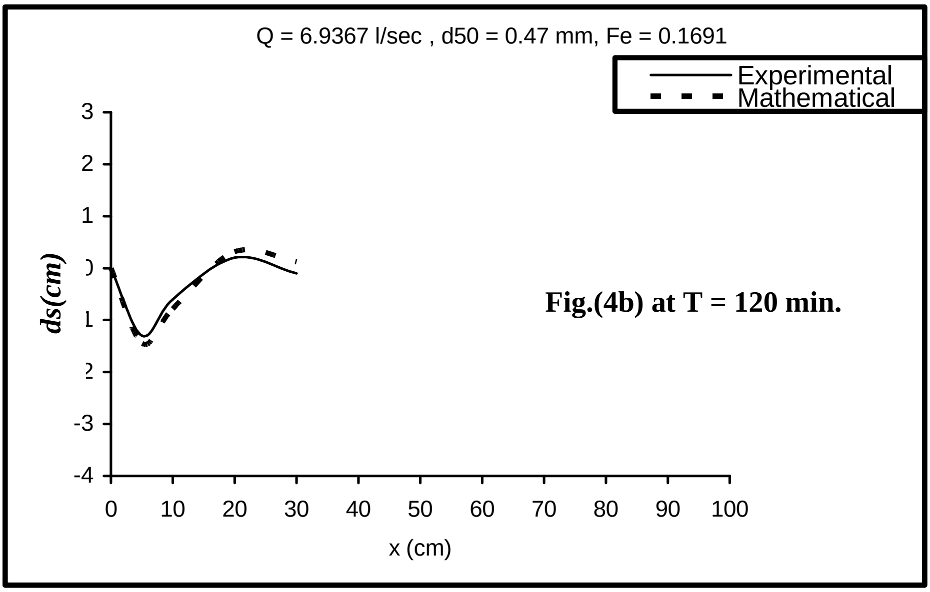
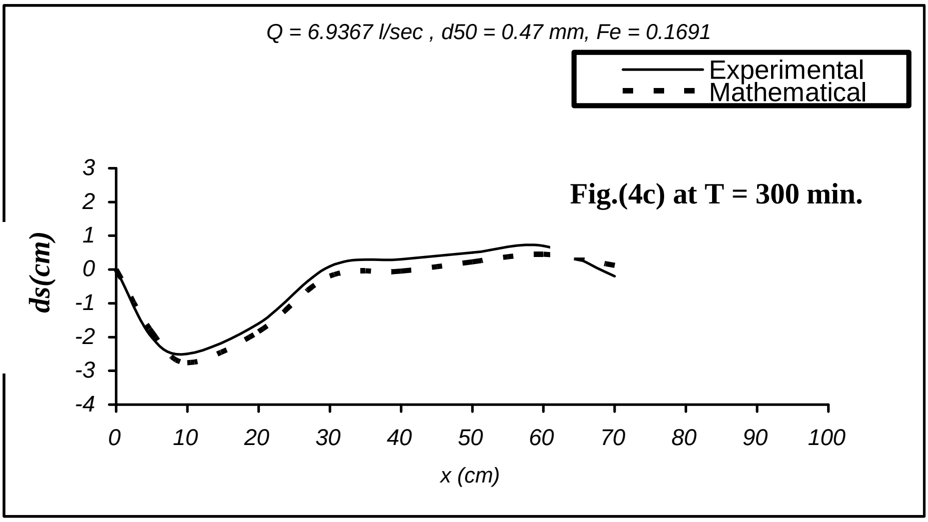


Fig.(4b) at T = 120 min.



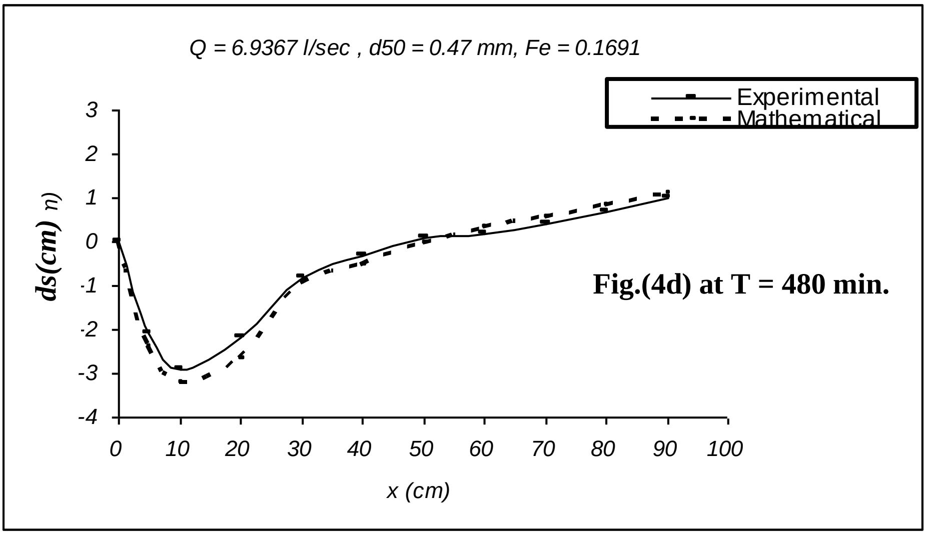


Fig. (4) Comparison between mathematical model and experimental data.

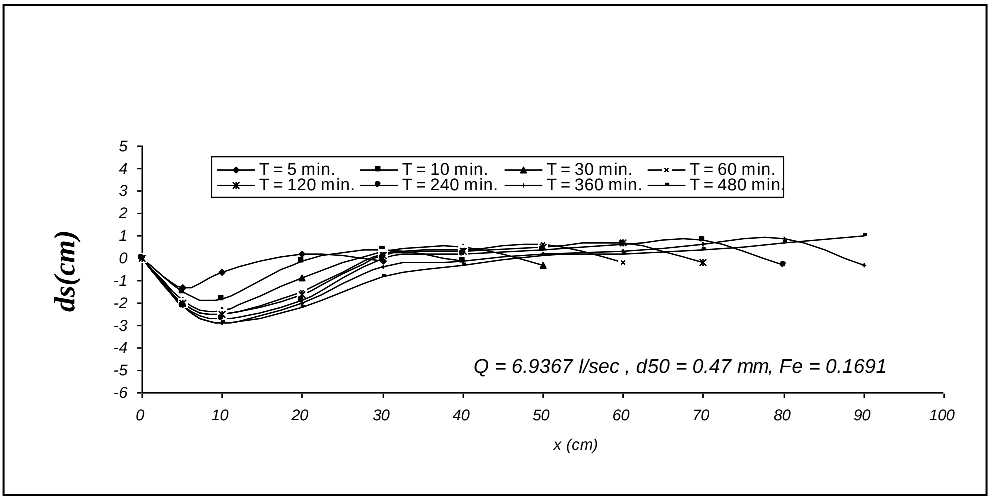


Fig.(5) Variation of bed profile with time.

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نموذج رياضي لتحديد النحر خلف الهدارات

الهدارات من أقدم المنشآت المائية التي يستخدمها مهندسي الري والهيدروليكا في قياس التصرف وحيث أن وجود أي منشأ مائي يعمل على اضطراب السريان وهذا الاضطراب يسبب النحر خلف هذه المنشآت ممم يكلف الدول ملايين الدولارات لمنع انهيار المنشأ ونحن نحاول في هذه الدراسة باستخدام طريقة رياضية معرفة أبعاد النحر في اى زمن في المستقبل وذلك لمنع حدوثه وأمكن التوصل إلى النتائج الآتية

1- تبين من الدراسة ان عمق وطول النحر خلف الهدارات يعتمد اعتمادا كلياً على عمق المياه في الخلف ، قطر حبيبات القاع ، رقم فرويد والزمن
2- تبين إن عمق النحر خلف الهدارات يزداد زيادة ملحوظة في بداية الزمن إلى أن يصل إلى حوالي 80 % من أقصى عمق للنحر ثم يقل معدل الزيادة بعد ذلك.
3- وجد ان بزيادة قطر حبيبات الرمل تقل أبعاد حفرة النحر .
4- وجد ان عمق وطول النحر يزداد مع زيادة رقم فرويد خلف الهدارات