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Intensity Distribution of The Modified Bessel-Gauss Beams

M.A. K. Alfaramawi[1] and M.A. Alzamil[2*]

[1]*Physics Department, Faculty of Science, Alexandria University, Alexandria, Egypt*
[2]*Department of Physics and Astronomy, College of Science, King Saud University, Riyadh 11451, Saudi Arabia*

THE intensity of the modified Bessel-Gauss beam (MBGB) was studied at the waist plane and along the longitudinal coordinate (*z*). Many factors governed the intensity distribution; the radius of the circle produced by the wavevector base at the waist plane (*a*), the position vector (*r*), the initial spot size (ω_0) and the longitudinal coordinate (*z*). The MBGB intensity behavior varied according to the value of *a*. At a=ω_0 the intensity was almost constant. At $a > \omega_0$, the intensity increased with increasing *r* while at $a<\omega_0$ the intensity decreased as the distance *r* increased. The radius of curvature of the wavefront along z-axis decreased first as the coordinate increased until it reached a minimum value at z_{min} and then slowly increased. The dependences of the MBGB intensity on both *a* and *r* at various coordinate *z* were investigated. At *r* = 0.2 and 0.5 mm the MBGB intensity continue decreased by the increase of *a* until it tended to almost zero at values of *a* greater than 1.5 mm.

Keywords: Modified Bessel-Gauss beam, Beam radius, Waist plane, Intensity.

Introduction

The Bessel beam has attracted many researchers in the last decades due to its interesting and important characteristics. One of its properties is that the size of the center spot is fixed independent on the transmission distance when it propagates in free space. Moreover, the Bessel beam light intensity is very high at the center spot and it has the ability of reconstruction when it encounters obstacles [1]. These characteristics proposed that the Bessel beam can be used in various fields, such as particle trapping [2], particle optical guidance [3], statistical and atomic physics [4] optical stress and tension [5].

Durnin [6] and Durnin et al. [7] were the first who proposed the Bessel beam in the late 1980s. They initiated the diffraction-free electromagnetic field solutions to the Helmholtz equation. One of these solutions established an electromagnetic field envelope that is a Bessel function of the first kind of zero order J_0 (βr), where β is the radial component of the wavevector and *r* is the distance from the *z*-axis. This first mathematical ideal concept of the Bessel beam consists of infinite number of concentric, high energetic light rings over an infinite area [6,8].

In fact, a true experimental Bessel beam cannot be created because it is unbounded and requires infinite source of energy [9]. Only approximate Bessel beam can be utilized either by collecting a Gaussian beam with an axicon lens to generate Bessel-Gauss beam (BGB) [10] or by placing a narrow annular aperture in the far field [7].

In recent years, many new applications of BGB in both fundamental and applied physics were recorded. As an example, in optical micromanipulation, in holography and in raising the image quality and penetration in dense media. The interesting properties and applications of non-diffracting MBGB in micro-imaging have attracted many attentions in quantum communication, quantum processing systems and in optical communication [11].

The aim of the present study is to calculate the intensity of the modified Bessel-Gauss beam along the longitudinal coordinate z and at the waist.

Theoretical Model

A Gaussian beam (GB) can be represented by a propagating wave whose wavevector *k* makes an angle φ with z-axis and its projection has an angle β on the z = 0 plane. The superimposing of the GBs can produce a resultant Bessel-Gaussbeam

*Corresponding author: E-mail: alzamilm@live.com
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(BGB) whose electromagnetic field is given by [12];

$$u(r, z=0) = A\, J_0(\beta r)\, exp\left(-\frac{r^2}{\omega_o^2}\right) \quad (1)$$

where A is the amplitude, ω_o is the beam spot size at the waist plane (z = 0) and J_0 (βr) is the Bessel function of the first kind of integer order n [13]. As a special case, the modified BGB (MBGB) is initiated when the superimposed GBs have wavevectors parallel to the longitudinal z direction but their centers are located on the circumference of a circle of radius a around the z-axis. At this situation the MBGB intensity of zero order at z = 0 can be approximated as [12];

$$I(r, z=0) = exp\left(-2\frac{a^2}{\omega_o^2}\right)\left[1 - \frac{2r^2}{\omega_o^2}\left(\frac{a^2}{\omega_o^2} - 1\right)\right] \quad (2)$$

After propagation in free space in the z-direction, the electromagnetic field of the

$$u(r,z) = A\frac{\omega_o}{\omega(z)} exp[ikz - i\Psi(z)]\, exp[-F(z)(r^2 + a^2)]\; J_o\{2arF(z)\} \quad (3)$$

MGBB is given by [14]

where

$$\omega(z) = \omega_o\sqrt{1 + \left(\frac{Z}{Z_r}\right)^2} \quad (4\text{-}a)$$

$$z_r = \frac{\pi\omega_o}{\lambda} \quad (4\text{-}b)$$

$$F(z) = \frac{1}{\omega^2(z)} - \frac{ik}{2R(z)} \quad (4\text{-}c)$$

$$R(z) = z\left[1 + \left(\frac{z_r}{z}\right)^2\right] \quad (4\text{-}d)$$

$$\Psi(z) = tan^{-1}\left(\frac{z}{z_r}\right) \quad (4\text{-}e)$$

The intensity of the MBGB at a distance z from the source can be given by the relation [14]

$$I(r,z) = \left(\frac{\omega_o}{\omega(z)}\right)^2 exp\left(-2\frac{a^2}{\omega^2(z)}\right)[1 + 2r^2\,\xi(z)] \quad (5)$$

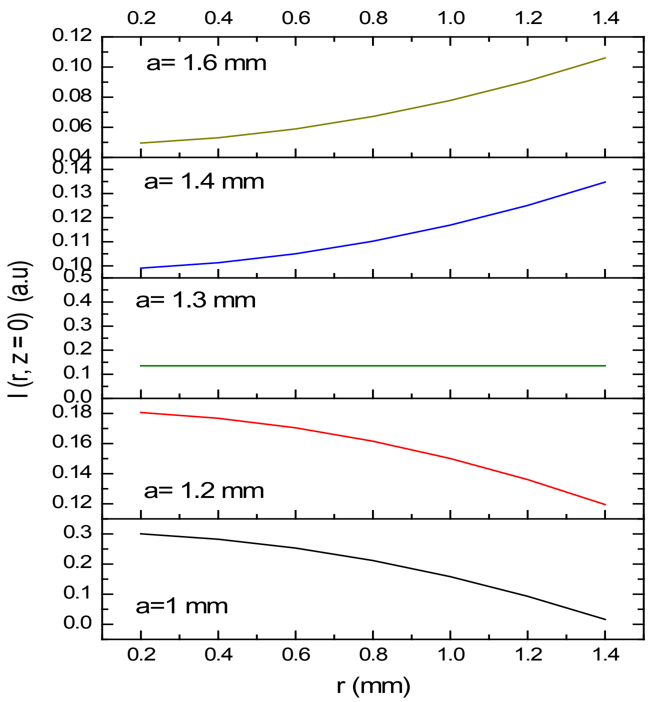

Fig. 1 .The intensity distribution of the MBGB in the waist plane as a function of r at various values of a.

where

$$\xi(z) = \left[\frac{1}{\omega^2(z)}\left(\frac{a^2}{\omega^2(z)} - 1\right)\right] - \left(\frac{ka}{2R(z)}\right)^2 \qquad (6)$$

Calculations and Discussion

The MBGB intensity was calculated at the waist and along z-axis using the model described by equations 2-6. The wavelength was kept at 623.8 nm. Figure 1 shows the intensity distribution of the MBGB in the waist plane (z = 0) as a function of $r$at various values of a.

It is noticed that the intensity behavior varied according to the value of a. At the intensity was almost constant. At the intensity increased with increasing r while at the intensity decreased as the distance r increased.

The propagating MBGB along z-direction was characterized. Figure 2 illustrates the radius of curvature of the wavefront of the MBGB as a function of the position z at two different values of . In general, the value of $R(z)$ decreased first as the distance increased until it reached a minimum value at and then slowly increased.

The minimum distance can be calculated alternatively from the equation [8]

$$z_{min} = \frac{\omega_o}{2\tan\theta} \quad , \quad \theta = (n-1)\delta \qquad (7)$$

where n is the refractive index of the axicon material and is the opening angle of the axicon. Taking n = 1.51 and = 0.01 radian, the calculated values of was found as 5.6 m at = 1 mm and 7.3 at = 1.3 mm respectively. These values coincide well with those deduced from Fig. 2.

The dependences of the MBGB intensity on both a and r at various coordinate z are depicted in Fig. 3. The initial spot size was kept at 1.3 mm.

The initial spot size was kept at 1 mm in the all cases. The general trend of the MBGB was the same in all cases. The behavior of the intensity depends strongly on the value of r. At r = 0.2 and 0.5 mm the MBGB intensity continue decreased by the increase of a until it tends to almost zero at values of a greater than 1.5 mm. The case was completely different at r = 0.7 mm. For the positions z = 0 , 2 and 4 m the intensity first increased by a until it reaches a maximum at $a \sim$ 0.7 mm and then decreased again to reach zero at large values of a while at z = 6 m the intensity has almost a plateau at small values of a and then decreased smoothly to zero at large values of a.

More investigation of the dependence of the MBGB intensity on a at r = 0.7 mm at various values of z is shown in Fig. 4. This behavior is clearly noticed. The peak of the MBGB intensity was found at values of a near the value of r (0.7 mm or less) at distances z = 0, 2 and 4 m. It is expected that for longer z values the MBGB intensity will remain constant.

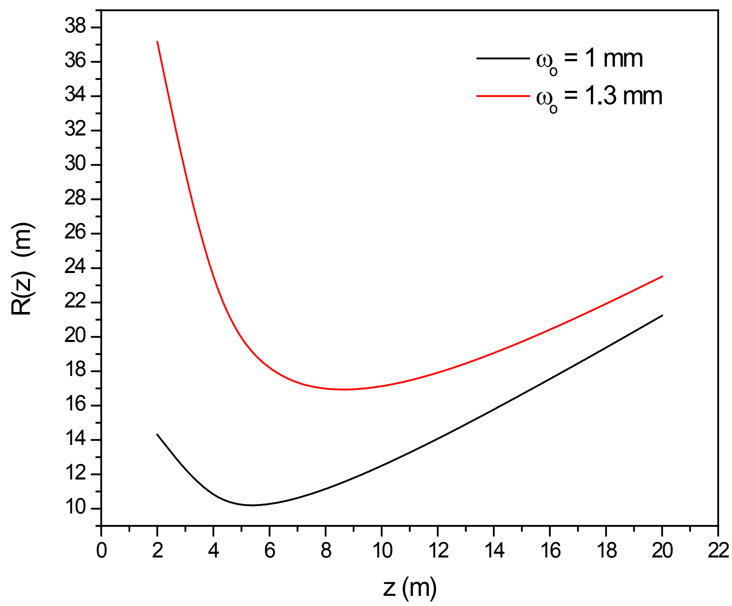

Fig. 2. The radius of curvature of the wavefront of MBGB as a function of the position z.

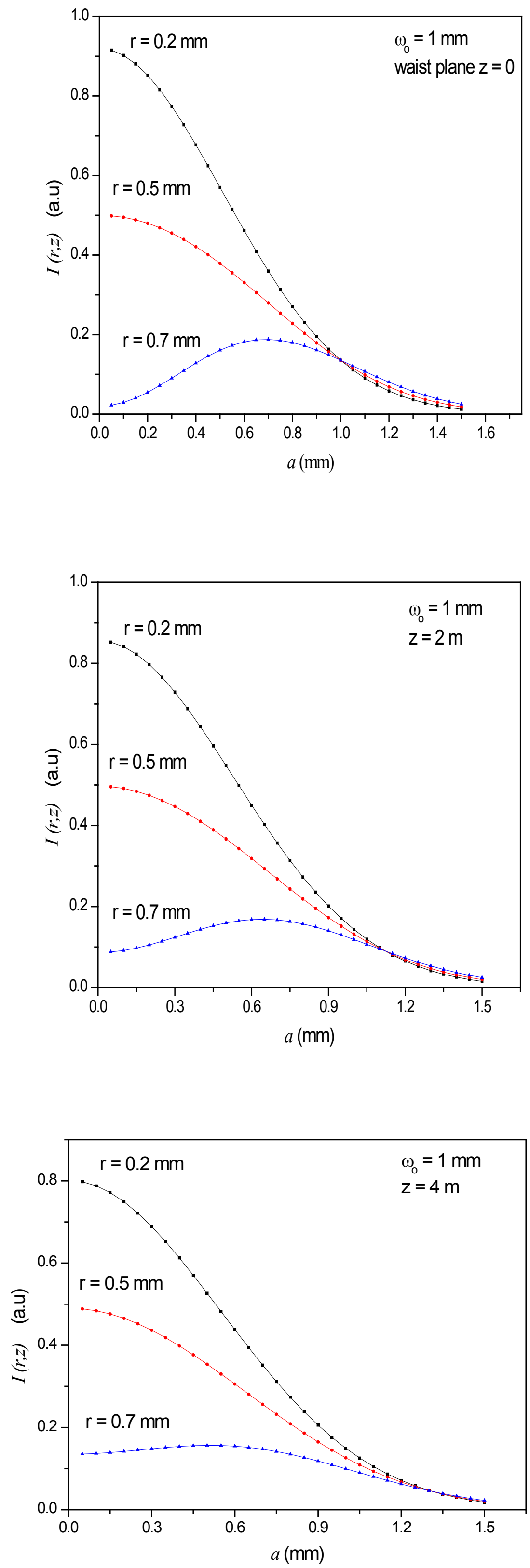

Fig. 3. The dependences of the MBGB intensity on both a and r at various longitudinal coordinate z (0, 2, 4 m). The initial spot size was kept at = 1 mm in the all cases.

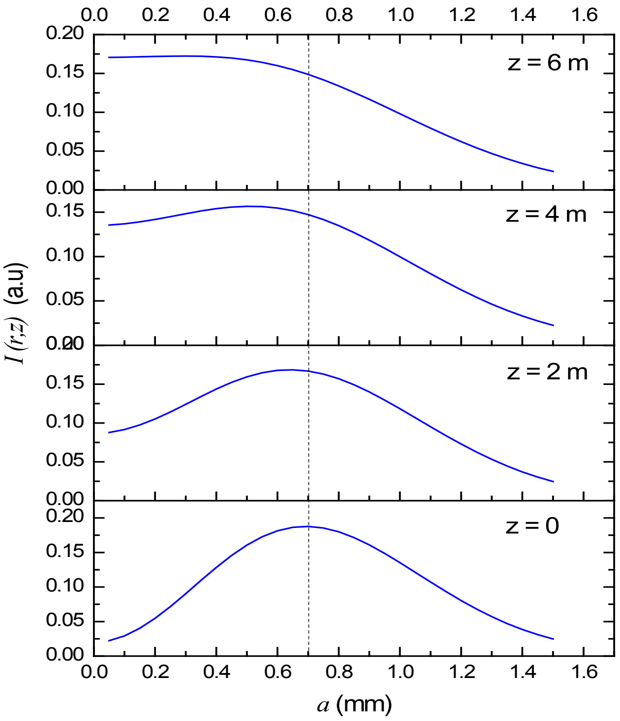

Fig. 4. The dependence of the MBGB on a at r = 0.7 mm at various values of z. The initial spot size was kept at = 1 mm in the all cases.

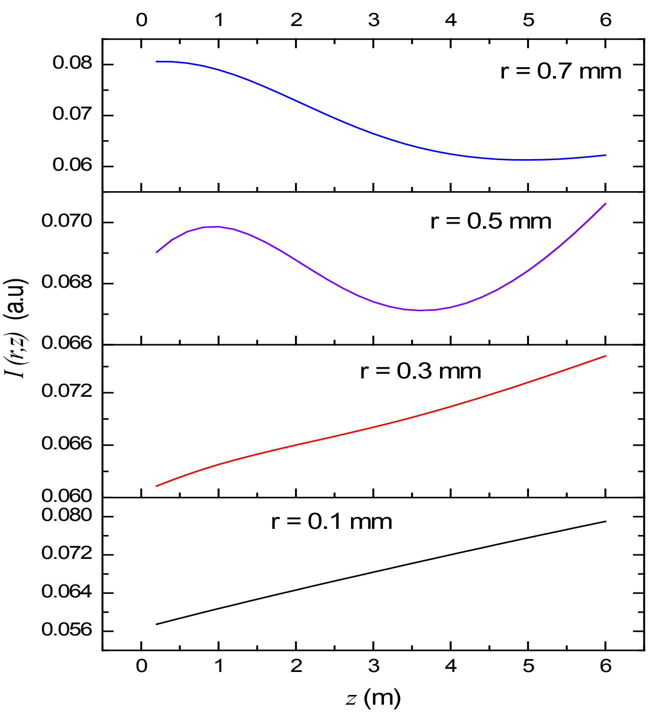

Fig. 5. The dependence of the MBGB intensity on the longitudinal coordinate z at various values of r. The initial spot size was kept at = 1 mm in the all cases.

The dependence of the MBGB intensity on the longitudinal coordinate at various values of *r* is studied in Fig. 5. For values of *r* = 0.1 and 0.3 mm the MBGB intensity was continuously increased by the increase of *z*. At *r* = 0.5 mm the intensity possessed maximum and then minimum at *z* = 1 and 3.6 m respectively. At *r* = 0.7 mm the MBGB intensity was continuously decreased along the longitudinal coordinate *z*.

Conclusions

The intensity of the modified Bessel-Gauss beam was found to vary with the following parameters: the radius of the circle produced by the wavevector base at the waist plane, the distance from the z-coordinate, the spot size at the waist and the position along the longitudinal z-coordinate. The change of any parameter at fixed the other parameters lead to a large variation of the Bessel-Gauss bean intensity. The radius of curvature of the wavefront along z-axis decreased as the coordinate increased until it reached a minimum value at and then slowly increased.

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توزيع شدة أشعة بيسل – جاوس المعدلة

خالد محمد الفرماوي١ و محمد عبد العزيز الزامل٢

١ قسم الفيزياء – كلية العلوم – جامعة الاسكندرية – الاسكندرية – مصر
٢ قسم الفيزياء والفلك ، كلية العلوم ، جامعة الملك سعود ، الرياض – المملكة العربية السعودية

تمت دراسة توزيع شدة أشعة بيسل - جاوس المعدلة عند مستوى الخصر وعلى طول الإحداثيات الطولية (z). هناك العديد من العوامل التي تحكم توزيع شدة الأشعة: نصف قطر الدائرة التي يكونها متجه موجة القاعدة عند مستوى الخصر (a)، ومتجه الموضع (r) ، وحجم البقعة الأولي (ω_0) والإحداثيات الطولية (z).

ويختلف سلوك شدة شعاع بيسل - جاوس المعدل وفقًا لقيمة a. عندما يكون $a=\omega_0$، فإن الشدة تكون تقريبًا ثابتة. وعندما يكون $a > \omega_0$، تزداد شدة الأشعة بزيادة r بينما تقل الشدة مع زيادة $a<\omega_0$.

وتثبت النتائج أن نصف قطر انحناء جبهة الموجة على طول المحور z يقل أولاً مع زيادة الإحداثيات حتى يصل إلى أدنى قيمة له عند z_{min}ثم يزداد ببطء. تمت دراسة اعتماد شدة أشعة بيسل – جاوس المعدلة على كل منa و rعند إحداثيات مختلفة. عند قيم r تساوي 0.2 و 0.5 مم ، تستمر شدة الأشعة في الانخفاض بزيادة a حتى تؤل إلى الصفر تقريبًا عند قيم أكبر من ١.٥ مم.