Egyptian J. Anim. Prod. 35, Suppl. Issue, Dec. (1998):507-519.

ON THE MATHEMATICAL DESCRIPTION OF THE LACTATION CURVE FOR EGYPTIAN BUFFALOES

A. R. Elbeltagi[1], H. Mansour[2] and Mahasin, M. Mohamed[1]

1- Animal Production Research Institute, Dokki, Giza, Egypt, 2- Department of Animal Production, Faculty of Agriculture, Ain Shams University, Cairo, Egypt

SUMMARY

Seven mathematical functions were compared for best fitting the lactation curve of the Egyptian buffaloes. Weekly milk production of complete 2226 lactation records were used for fitting the functions, the Incomplete Gamma $_t = a\, t^b e^{-ct}$, the Compartmental function; $y_t = m\,(1- e^{-p(t-q)})e^{-nt}$, the Modified Compartmental; $y_t = g\ (1/(1+ e^{-s(t-r)}))\ e^{-ht}$, the Multiphasic function;

$$y_t = \sum_{i=1}^{n} \{a_i b_i [1 - \tanh^2 (b_i (t - c_i))]\}$$, in its both diphasic and triphasic forms,

Mitscherlich exponential function; $y_t = a\ (1- b_o\, e^{-b_1 t})\, e^{-ct}$, Michaelis-Menten Exponential function; $y_t = a\,(1/(1+b_o/(b_1+t)))\, e^{-ct}$ and a proposed Linear (EMM) function; y_t = á + â t^2 ln t + ã e^t + ä e^{-t}. Mean squares of error (MSE), coefficient of determination (R^2) and Durbin Watson statistics (d) were the criteria used for comparing the utilized functions. The Linear (EMM) function fitted the data the best according to these criteria which ranged between 15.898 and 0.643; 0.922 and 0.995; 2.819 and 0.273 with averages of 2.723, 0.978 and 1.238 for the MSE, R^2 and d, respectively. In general, for the rest of the functions, these criteria values ranged between 1.541 and 115.549; 0.151 and 0.977; 0.172 and 1.359 for the MSE, R^2 and d, respectively.

Both the biological interpretation for the four parameters of the Linear (EMM) function and the statistical relationship between those parameters with some of the actual lactation traits; the accumulative milk yield up to wk 'n' of lactation (kg), initial milk yield (kg), time of peak yield (wk), peak yield (kg), and persistency were made.

The discriminant analysis was utilized to classify lactation records, according to their lactation periods and their relationship with the Linear (EMM) function parameters estimates, into four categories (short, medium, long, and very long lactation period records) with the objective to set a rule for predicting the lactation performance of lactating buffalo cows that obligatory dried off at an early stage of lactation (≥8 wks). Results showed that the accuracy of the classification ranged between 75% for the short lactations to 99.5 % for the very long ones.

Keywords: Egyptian buffalo, lactation curve, period, mathematical description, EMM function, discriminant analysis, prediction

INTRODUCTION

Knowledge of the shape of the lactation curve (the graphical representation of the time-milk yield relationship) and its related traits; initial milk yield, peak yield, peak week, persistency, total milk yield and lactation period are important for dairy farmers in setting proper production, feeding and selection strategies; developing more accurate correction factors for different non-genetic factors affecting milk production; extending incomplete lactation records and predicting total milk yield from the ongoing lactations.

Many attempts had been made for mathematical modeling the lactation curve, composing a mathematical function that usually consisted of a scalar (initial point), a positive monotonically increasing function, and a monotonically decreasing function. Some of these functions were: the widely used Incomplete Gamma (Wood, 1967), Mitscherlich Exponential and Michaelis-Menten Exponential functions (Rook *et al.*, 1993), the Compartmental and Modified Compartmental functions (Leon-Velard, 1995) and the Multiphase Logistic function (Grossman *et al.*, 1988) that had proposed a new conception of lactation curve modeling considering milk yield to result from an accumulation of more than one phase of lactation.

For Egyptian buffalo, some useful attempts had been made mainly using the Incomplete Gamma (Ali, 1972; Samak *et al.*, 1988; Soliman, 1989; Mansour *et al.*, 1993; Ibrahim, 1995 and Sadek *et al.*, 1998), whereas, Abdel-Aziz *et al.* (1973b), after correcting the data for some non-genetic effects, used a least squares analysis of variance to calculate regression factors for predicting 12-month lactation from both single-month and accumulative monthly lactations.

The objective of this study was to investigate how best to mathematically describe the lactation curve of the Egyptian buffalo and to develop a rule for predicting lactation period from the ongoing lactation at an early stage of lactation by applying a discriminant analysis technique.

MATERIALS AND METHODS

Data

Lactation data of (normally terminated at ≥8 wks of lactation) 2226 lactation records of buffalo cows that calved from January 1,1987 through June 30, 1995 were used. These data were from six farms that belong to the Animal Production Research Institute, Ministry of Agriculture. Data were grouped into 46 groups according to the normal lactation length in weeks, i.e., 8, 9, 10, ..., 53-or-more wks of lactation.

First milk yield of more than three days was corrected to a week yield, otherwise, it was excluded from the analysis and second week data were considered to be the first.

Managerial conditions and feeding system followed in these herds were described by Abdel-Raof (1995).

Modeling the lactation curve

Seven mathematical functions were evaluated for fitting the buffalo lactation. These functions were:

The Incomplete Gamma : $y_t = a\, t^b e^{-ct}$,

The Compartmental : $y_t = m\,(1 - e^{-p(t-q)})\, e^{-nt}$,

the Modified Compartmental : $y_t = g\,(1/(1 + e^{-s(t-r)}))\, e^{-ht}$,

the Multiphasic function: $y_t = \sum_{i=1}^{n} \{a_i b_i [1 - \tanh^2(b_i(t - c_i))]\}$,

in its both diphasic and triphasic forms,

Mitscherlich Exponential: $y_t = a\,(1 - b_o\, e^{-b_1 t})\, e^{-ct}$,

Michaelis-Menten Exponential: $y_t = a\,(1/(1+b_o/(b_1+t)))\, e^{-ct}$, and

the proposed Linear (EMM) function: $y_t = á + â\, t^2 \ln t + ã\, e^t + ä\, e^{-t}$,

which was adopted from TC (1994), where, y_t is the weekly milk yield at the t^{th} wk of lactation, e is the base of natural logarithms, and á, â, ã, and ä are parameters that determine the curve shape.

All the seven functions were individually fitted to the 2226 complete lactation records. The first six models were fitted to the lactation data using nonlinear regression, the marquardt iterative method (SAS, 1988). The Multiphasic Logistic function was fitted for its both diphasic (n=2) and triphasic (n=3) forms. Number of parameters to be estimated depended on the number of phases; diphasic function had six parameters, while the triphasic had nine parameters. The EMM function was fitted to the data as a multiple regression model using SAS (1988).

Residual mean squares (MSE), coefficient of determination (R^2) and Durbin Watson statistics, a measure of residuals autocorrelation, (Durbin and Watson, 1951), were all used to compare goodness of fit for all the used functions except for the multiphasic ones. Then, the standardized residual distribution of the EMM function was checked for being randomly distributed.

Some actual lactation traits; accumulative milk yield up to wk 'n' of lactation (Q_n); initial milk yield (y_i), the milk yield of the first week of lactation; week of peak (t_{max}), the week at which the peak yield had been obtained; yield at week of peak (y_{max}), the maximum weekly yield through the first eight wks of lactation; and persistency (S), measured as [(total milk yield - yield up to peak)/yield up to peak] according to Anakawiang (1963), were all individually measured for each record. Then the stepwise regression was performed on those traits for the EMM function estimated parameters to obtain the statistical relationships between those actual traits and the estimated parameters. Actual lactation traits were estimated as:

The accumulative yield to week 'n' of lactation (kg): $Q_n = \sum_{t=1}^{n} y_t$,

First week yield (kg): $y_i = b_0 á + b_1 ä$,

Week of peak (wk): $t_{max} = b_0 á + b_1 â$,

Yield at week of peak (kg): $y_{max} = b_0 á$ and

Persistency: $S = b_0 á + b_1 â + b_2 ã + b_3 ä$,

where, b_0, b_1, b_2 and b_3 are regression coefficients and other terms are the estimated parameters of the EMM function.

To check the resulting mathematical relationship between week of peak and the estimated parameters of the EMM function, the first derivative of the function was equated to zero to determine the peak point of the curve (time of peak in wks) for different lactation period groups. The resulting values were then compared to those obtained by the stepwise regression.

First derivative of the EMM function was in the form:

$dy/dt = ât (1 + 2 \ln t) + e^{t} (ã - ä e^{-2t})$,

where,

t is wk of lactation,

e is the base of natural logarithms, and

â, ã and ä are the estimated parameters for the EMM function.

Discriminant analysis and prediction of lactation period

It was found in this data that the percentages of records in each lactation group were almost equal, ranging from 0.7 to 3.7% with an average of 2.17%. Less than 70% of the records reached 23 wks of lactation and only 14.2% reached 44 wks. It is therefore quite difficult to standardize a lactation length for Egyptian buffaloes under this situation.

In an attempt to accurately extend the lactation period to correct the milk production of the obligatory dried buffalo cows and to develop a means for predicting the lactation performance of lactating buffalo cows at an early stage of lactation (8 wks and forward), the data were classified into four classes according to the lactation period. These classes were: short (8-16 wk), medium (17-27 wk), long (28-47 wk) and very long (48-53 wk) of lactation. The EMM function was fitted for the first 8 wks for all the 2226 records and the function parameters were estimated. The discriminant analysis (Johnson and Wichern, 1982) was applied on the resulting parameters to allocate each record into one of the four previously described classes according to the EMM function parameters values obtained from the fitting. This process was performed for the data of the first 16, 20, 24, 28, 32, 36, 40, 44, 48 and 52 wks of lactation for only the records that achieved these lactation periods. Range of the EMM function parameters (á, â, and ä) values were used to develop a prediction rule of the lactation period of any lactating animal at an early stage of lactation according to those three parameters range of values.

RESULTS AND DISCUSSION

Comparison between studied functions

For the Multiphasic Logistic functions, the triphasic form only fitted lactation data of more than 47 wks in milk while the diphasic one only fitted lactation data of more than 37 wks in milk, otherwise, the C's parameters (time of peak of each phase) had negative values. In addition, R^2 values were obviously lower than the EMM function as it ranged between 0.37 to 0.61; 0.53 to 0.71 for the diphasic and triphasic forms, respectively. Therefore, the Multiphasic function was excluded from the comparison.

Table (1) depicts the significant effects of lactation period (wk), the studied function and the interaction between them on the R^2 value of the studied six functions. Other criteria of comparison showed similar trend when analyzed for variances.

Table 1. Analysis of variance for the effects of the studied functions, the lactation period and the interaction between them on the coefficient of determination (R^2) value

Source of variance	D.F.	Mean squares
Lactation period (LP)	45	0.4604**
Function	5	41.9470**
LP * Function	225	0.1091**
Remainder	13080	0.0090

** = P<0.01.

The EMM function was the best function for all lactation period groups according to the proposed criteria of comparison, high R^2 and low MSE values, ranged between 0.9225 to 0.995; and 0.643 to 15.898 kg with averages of 0.978 and 2.723 kg, respectively (Table 2). Durbin Watson Statistics (d) values indicated that residuals were only autocorrelated for the long lactation period groups (LP>41 wks). In general, residuals of all the groups of lactation period showed much less autocorrelation with the EMM function (with d average equals 1.24) than with other functions that had averages of 0.62, 0.55, 0.34, 0.34, and 0.35 for the Gamma, Mitscherlich Exponential, Michaelis- Menten Exponential, Compartmental and Modified

Table 2. Averages of parameter estimates, their standard errors (SE), mean squares of error (MSE), coefficient of determination (R^2) and Durbin Watson statistics (d) for the EMM function fitted to lactation data of different lactation period groups

L.P group	â ±	SE	â ±	SE	â ±	SE	â ±	SE	MSE	R^2	d
8	46.599	3.767	0.008521	0.099126	-0.011029	0.004037	-37.190	16.586	15.898	0.922	1.895
9	44.984	1.132	-0.074701	0.021186	-0.002592	0.000439	-35.646	5.334	1.926	0.989	2.819
10	48.910	1.142	-0.105440	0.015897	-0.000505	0.000162	-36.327	5.724	2.498	0.985	2.430
11	46.637	0.723	-0.057687	0.007745	-0.000314	3.79E-05	-33.936	3.833	1.228	0.992	2.176
12	48.261	0.590	-0.059193	0.004998	-9.57E-05	1.15E-05	-39.682	3.292	0.974	0.993	2.094
13	42.381	0.759	-0.038858	0.005194	-3.43E-05	5.50E-06	-38.856	4.435	1.873	0.983	1.077
14	47.848	0.810	-0.032721	0.004562	-1.71E-05	2.20E-06	-40.017	4.938	2.436	0.982	2.684
15	48.593	0.609	-0.039337	0.002865	-3.72E-06	6.00E-07	-39.288	3.859	1.548	0.988	1.279
16	48.064	0.505	-0.036786	0.002013	-1.34E-06	2.00E-07	-32.992	3.321	1.186	0.991	0.895
17	48.193	0.714	-0.024169	0.002439	-7.77E-07	1.00E-07	-35.288	4.862	2.613	0.979	1.022
18	48.837	0.530	-0.024343	0.001566	-2.47E-07	0	-38.499	3.726	1.573	0.987	1.191
19	50.390	0.423	-0.021132	0.001091	-9.96E-08	0	-35.520	3.063	1.086	0.991	1.695
20	49.424	0.508	-0.017918	0.001153	-3.49E-08	0	-34.882	3.786	1.691	0.985	1.477
21	50.871	0.596	-0.021335	0.001197	-8.71E-09	0	-37.401	4.559	2.492	0.981	1.084
22	54.701	0.378	-0.019948	0.000677	-3.91E-09	0	-46.574	2.966	1.071	0.993	1.409
23	52.079	0.573	-0.014525	0.000919	-1.52E-09	0	-36.625	4.605	2.615	0.976	0.737
24	52.945	0.387	-0.014255	0.000560	-6.27E-10	0	-36.393	3.185	1.266	0.990	1.458
25	55.102	0.545	-0.014599	0.000712	-1.83E-10	0	-52.083	4.580	2.645	0.980	0.996
26	56.254	0.389	-0.011221	0.000463	-1.19E-10	0	-44.519	3.344	1.424	0.990	1.543
27	58.744	0.633	-0.011637	0.000687	-3.94E-11	0	-46.890	5.548	3.955	0.974	0.487
28	58.617	0.438	-0.010628	0.000435	-1.47E-11	0	-51.412	3.915	1.986	0.987	1.473
29	57.800	0.244	-0.009337	0.000223	-5.17E-12	0	-45.050	2.220	0.643	0.995	1.811
30	56.476	0.294	-0.008091	0.000248	-1.90E-12	0	-41.967	2.730	0.980	0.992	1.329
31	54.686	0.285	-0.007865	0.000222	-6.29E-13	0	-44.103	2.690	0.957	0.992	1.051
32	57.776	0.333	-0.007827	0.000240	-2.68E-13	0	-46.821	3.191	1.355	0.990	1.045
33	58.082	0.434	-0.007103	0.000291	-1.00E-13	0	-54.431	4.236	2.401	0.982	1.137
34	57.823	0.614	-0.007079	0.000383	-3.32E-14	0	-53.123	6.079	4.972	0.964	0.896
35	56.081	0.393	-0.006289	0.000229	-1.30E-14	0	-42.195	3.951	2.111	0.983	0.877
36	54.436	0.388	-0.005989	0.000212	-4.22E-15	0	-49.881	3.963	2.133	0.982	0.875
37	55.695	0.308	-0.006301	0.000157	-1.35E-15	0	-50.922	3.189	1.387	0.990	1.193
38	57.463	0.493	-0.005585	0.000237	-5.88E-16	0	-48.626	5.178	3.672	0.972	0.552
39	55.076	0.524	-0.005389	0.000237	-1.74E-16	0	-56.900	5.581	4.282	0.966	2.285
40	55.714	0.347	-0.004785	0.000148	-8.59E-17	0	-57.864	3.748	1.938	0.985	1.317

Table 2. cont.

41	56.986	0.228	-0.004824	0.000091	-2.77E-17	0	-58.328	2.487	0.856	0.993	1.370
42	53.542	0.353	-0.004942	0.000134	-6.80E-18	0	-48.367	3.911	2.124	0.984	0.539
43	53.926	0.486	-0.004836	0.000174	-2.07E-18	0	-47.691	5.444	4.129	0.969	0.425
44	50.878	0.467	-0.004086	0.000159	-6.69E-19	0	-50.416	5.293	3.915	0.963	0.467
45	51.713	0.439	-0.003830	0.000142	-3.09E-19	0	-54.825	5.034	3.550	0.967	0.766
46	53.119	0.631	-0.003384	0.000193	-1.31E-19	0	-38.341	7.317	7.520	0.925	0.648
47	51.737	0.301	-0.003340	0.000088	-6.19E-20	0	-40.863	3.530	1.755	0.984	1.422
48	47.441	0.294	-0.003069	0.000082	-2.02E-20	0	-34.742	3.489	1.719	0.982	0.796
49	47.378	0.421	-0.002898	0.000111	-4.87E-21	0	-29.933	5.042	3.596	0.959	0.658
50	50.373	0.438	-0.002727	0.000111	-3.29E-21	0	-33.601	5.308	3.995	0.959	1.087
51	50.243	0.397	-0.002499	0.000096	-1.30E-21	0	-26.624	4.853	3.346	0.964	0.858
52	51.118	0.448	-0.002888	0.000103	-2.67E-22	0	-25.107	5.539	4.368	0.961	1.352
53	46.773	0.400	-0.002316	0.000088	-6.69E-24	0	-27.252	4.991	3.553	0.947	0.273
Maximum	58.744	3.767	0.008521	0.099126	-6.7E-24	0.004037	-9.933	16.586	15.898	0.995	2.819
Minimum	42.381	0.228	-0.105440	0.000082	-0.01103	0	-58.328	2.220	0.643	0.922	0.273
Average	52.190	0.568	-0.017	0.004	-3.17E-04	0.000	-41.261	4.532	2.723	0.978	1.238

Compartmental functions, respectively. In addition, standardized residuals showed random distribution with values ranged between 2 and -2 for all the groups of lactation period. For the rest five functions, the Gamma was the best followed by the Mitscherlich Exponential having R^2 values in the range between 0.67 to 0.98; 0.32 to 0.91 with averages of 0.85 and 0.68 and MSE values ranged between 1.54 to 54.1 and 6.24 to 110.78 with averages of 18.16 and 38.69, respectively. The remaining three functions had no significant differences with almost the same average values for the MSE and R^2 being 52 and 0.56, respectively. Tukey's studentized range test (HSD) gave similar results for the criteria of comparison.

Generally, all functions showed better fitting for lactation records with longer lactation period than with shorter ones except the EMM which showed good fitting for all the lactation period groups (Table 2).

Actual and predicted mean weekly milk yields for the 44-wks lactation period group are shown in Figure (1). The plot shows that the EMM function tended to slightly underpredict the peak yield (4-8 wk) and overpredict yield of mid-lactation (25-38 wk). The increase of milk production in the last few weeks of actual lactation might be due to the autumn hump as Ali (1972) indicated in a previous study. Autumn hump is usually resulted from the availability of green fodder and the climatic changes in the autumn season (about the end of the present lactation season) which positively affect the milk yield.

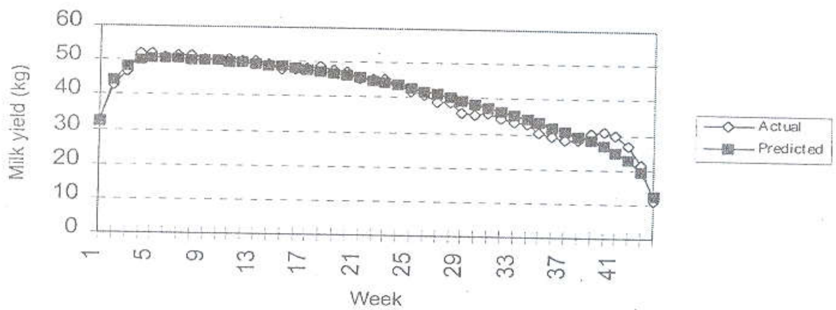

Figure 1. Actual and the predicted mean weekly milk production (kg) for the 44-wks

Interpretation of the EMM function estimated parameters

Parameter 'á' was found to be related to the peak yield. It had an effect of changing the whole scale of the curve. Parameter 'â' was related to the persistency. It changed the slope of the curvature after peak till the end of lactation. Parameter 'ã' was related to the last part (death curvature) of the lactation curve, whereas parameter 'ä' was found to be related to the initial milk yield.

Values of parameter á, which are related to peak yield, tended to increase regularly with increasing the lactation period till the 33 wks lactation period when no significant increase was observed afterwards.

Values of both parameters â (except for the first group of lactation period) and ã consistently increased with increasing the lactation period which was associated with increased persistency, i.e., reducing the slope of the curve after peak.

The only positive value of parameter â was observed for the first group of lactation period (8 wks), because of the limited number of points in such curve accompanied with much less slope of curvature after peak than longer ones.

Parameter ä value tended to decrease with longer lactation periods till the 41 wks lactation period group and then increased again till the 53 wks group. It is indicated that, although initial milk yield increased with longer lactation periods, it decreased again for the very long lactation periods.

It is therefore, concluded that the highest milk production can be achieved, with the highest initial milk yield, highest peak yield and highest persistency, with normal length of lactation period (i.e. not too long, about 40 wks in milk).

Statistical relationships between estimated parameters and some lactation traits

The following mathematical relationships were found between the EMM function estimated parameters and some actual lactation traits:

First week yield = 0.985 á + 0.337 ä, R^2 0.995,

Week of peak = 0.090 á + 5.988 â, R^2 0.909,

Yield of week of peak = 1.1 á, R^2 0.991 and

Persistency = 0.043 á + 11.678 â + 287 ã - 0.0055 ä, R^2 0.827.

Actual week of peak trait, stepwise regression estimated, and the predicted one, as the first derivative of the EMM function, were found to be 4.24, 4.34, 5.03, 5.0 and 4.52; 4.17, 4.64, 5.39, 6.07 and 6.08 wk for the 8, 20, 30, 40 and 50 weeks in milk lactation period groups, respectively. Results indicated that the stepwise regression method accurately determined the week of peak for lactation period groups up to 36 wks in milk, where the divergence between the two estimates (actual and predicted) got wider for lactation records of longer lactation period. Stepwise regression was therefore recommended to be done in two separate steps, one for the records up to 36 wks in milk, and the other for the longer ones. Therefore two sets of more accurate statistical relationships could be developed for estimating the lactation criteria.

Discrimination and classification

Figure 2 shows that classifying the records according to the values of the parameters á, â and ä into the four proposed groups of lactation period (short, medium, long and very long) was quite successful. The upper and lower values on the diagonal represent the successful numbers and percentages (between parenthesis) of records, respectively, the off diagonal represent the misclassified (error rates).

Only few error rates (misclassifications) were observed ranging from 0 to 23.79%. The highest error rate was misclassify the long lactation period records for the very long ones. The percentages of successful classification were 81.53, 85.17, 74.88 and 99.48% with an average of 85.27% for the short, medium, long and very long lactation period classes, respectively.

This result indicated that a lactation record can be successfully assigned into one of the proposed four classes with accuracy exceeding 85%, starting from the 8th wk of lactation (less than two months). Parameter ã was excluded from the discriminant analysis since it is related to the last part of the lactation curve (death curvature) that had no stable trend within the same group of lactation period.

Discriminant analysis that was applied on the EMM function parameters; á, â and ä resulting from fitting the first 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, and 52 wks of lactation, for the only records that achieved the fitted period, showed similar trends.

		Predicted classification				
		Short	Medium	Long	Very long	Total
Actual classification (according to lactation period in wks)	Short (8-16 wk)	**287** **(81.53)**	48 (13.64)	17 (4.83)	0 0	352
	Medium (17-27 wk)	0 0	**534** **(85.17)**	25 (3.99)	68 (10.84)	627
	Long (28-47 wk)	0 0	14 (1.33)	**790** **(74.88)**	251 (23.79)	1055
	Very long (48-53 wk)	0 0	0 0	1 (0.52)	**191** **(99.48)**	192
						2226
	Error rates	0.1847	0.1483	0.2512	0.0052	0.1473

Figure 2. The confusion matrix for short, medium, long and very long lactation period records

Range of the values of the EMM function parameters (á, â and ä) obtained from the previous fitting were used to develop a prediction rule for lactation period from the ongoing lactations (Table 3). For instance, if a lactating animal was obligatory dried off after the first eight weeks of lactation, the animal could be assigned into one of the four proposed lactation classes with a specific error rate. Firstly, the EMM function should be fitted to the lactation data of the first eight weeks, which the animal had performed. Secondly, the obtained parameter estimates for á, â and ä should be compared to their corresponding expected range of values, for the same fitted period, from Table 3. For example, If the estimated parameters for this animal were: á<48, â<0.022 and -35>ä>-37, this animal is expected to perform a maximum lactation period of 16 wks with 14% error rate.

Table 3. EMM function parameters (á, â and ä) range of values when fitted to different lactation periods (wk), with four wks intervals, for the proposed four lactation classes of lactation period

L.P class	Fitted period (wk)	Range of 'á'	Range of 'â'	Range of 'ä'	Classification
8 - 16	8	á < 48	â<0.022	-35> ä > -37	SHORT
17 - 27	8	55 > á > 50	0.033> â>0.022	-37 > ä > -40	MEDIUM
28 - 47	8	55 > á > 50	0.080> â>0.033	-40 > ä > -48	LONG
48 - 53	8	á < 50	â>0.080	ä > -40	VERY LONG
8 - 16	12	á < 50	â<-0.030	-36 > ä > -40	SHORT
17 - 27	12	55 > á > 50	-0.020>â>-0.030	-39 > ä > -40	MEDIUM
28 - 47	12	60 > á > 50	-0.005>â>-0.010	-45 > ä > -54	LONG
48 - 53	12	á < 50	â>-0.005	ä > -40	VERY LONG
8 - 16	16	á < 50	â<-0.020	ä > -35	SHORT
17 - 27	16	55 > á > 50	-0.010>â>-0.020	-35 > ä > -43	MEDIUM
28 - 47	16	60 > á > 50	-0.004>â>-0.010	-43 > ä > -53	LONG
48 - 53	16	á < 50	â>-0.004	ä > -40	VERY LONG
17 - 27	20	á < 55	-0.010>â>-0.020	ä > -40	MEDIUM
28 - 47	20	60 > á > 55	-0.004>â>-0.010	-40 > ä > -55	LONG
48 - 53	20	á < 50	â>-0.004	ä> -40	VERY LONG
17 - 27	24	á < 53	â<-0.015	ä> -40	MEDIUM
28 - 47	24	60 > á > 50	-0.004>â>-0.015	-40 > ä > -55	LONG
48 - 53	24	á < 50	â>-0.004	ä > -40	VERY LONG
28 - 47	28	60 > á > 50	-0.004>â>-0.015	-40 > ä > -55	LONG
48 - 53	28	á < 50	â>-0.004	ä > -40	VERY LONG
28 - 47	32	á > 50	-0.004>â>-0.008	-45 > ä > -55	LONG
48 - 53	32	á < 50	â>-0.004	ä > -40	VERY LONG
28 - 47	36	55 > á > 50	â<-0.006	-50 > ä > -55	LONG
48 - 53	36	á < 50	â>-0.004	ä > -40	VERY LONG
28 - 47	40	55 > á > 50	â<-0.004	-50 > ä > -60	LONG
48 - 53	40	á < 50	â>-0.004	ä > -40	VERY LONG
28 - 47	44	á > 50	â<-0.004	ä < -50	LONG
48 - 53	44	á < 50	â>-0.004	ä > -40	VERY LONG
48 - 53	48	á < 50	â>-0.004	ä> -40	VERY LONG
48 - 53	52	á < 50	â>-0.004	ä> -30	VERY LONG

REFERENCES

Abdel-Aziz, A. S., M. T. Ragab and A. Kamal, 1973. Regression factors for extending part lactation milk records in buffaloes. Egypt. J. Anim. Prod. 13(2):109-114.

Abdel-Raof, Hoda M., 1995. Factors affecting the shape of lactation curve in buffaloes. M. Sc. Thesis, Cairo Univ., Egypt, pp. 31-33.

Ali, A. K. A., 1972. Some environmental factors affecting the lactation curve in buffaloes. M. Sc. Thesis, Cairo Univ., Egypt, pp. 66-76.

Anakawiang, T., 1963. Persistency of lactation. Indian J. Dairy Sci. 16:267-268.

Durbin, J. and G. S. Watson, 1951. Testing for serial correlation in least squares regression. Biometrika. 37:409-428.

Grossman, M., and W. J. Koops, 1988. Multiphasic analysis of lactation curves in dairy cattle. J. Dairy Sci. 71:1598-1608.

Ibrahim, M. A. M., 1995. The use of gamma-type function in describing the lactation curve of Egyptian buffaloes. Egypt. J. Anim. Prod. 32(2):113-123.

Johnson, R. A. and D.W. Wichern, 1982. Applied multivariate statistical analysis (1st Ed.). Prentic-Hall Inc., Englewood Cliffs, N. J., USA., pp. 461-531.

Leon-Velard, C. U., I. McMillan, R. D. Gentry, and J. W. Wilton, 1995. Models for estimating typical lactation curves in dairy cattle. J. Anim. Breed. Genet. 112:333-340.

Mansour, H., I. A. Soliman, and G. A. Abd El-Hafiz, 1993. Factors affecting lactation curve of buffaloes in upper Egypt. Proceedings of the International Symposium on Prospects of Buffalo Production in the Mediterranean and Middle East. Cairo, Egypt 9-12 Nov. 1992. 62:234-237.

Rook, A. J., J. France, and M. S. Dhanoa, 1993. On the mathematical description of lactation curves. J. Agric. Sci. 121:97-102.

Sadek, R. R., Mahasin, M. Mohamed, M. A. M. Ibrahim and Hoda, M. A. Abdel-Lattef, 1998. Estimation of lactation curve parameters in Egyptian buffaloes. Egypt. J. Anim. Prod., 35(1):1-27.

Samak, M. A., G. A. Hassan, A. Hassan, and N. Yassen, 1988. The lactation curve and performance of the Egyptian buffalo. Alex. Sci. Exch. 9(1):83-97.

SAS, 1988. SAS/STAT user's guid, Release 6.03 Edition. SAS institute Inc. Cary, North Carolina, USA., pp. 675-710, 773- 875.

Soliman, I. A., 1989. Studies on lactation curves of water buffaloes in upper Egypt. M. Sc., Thesis, Assiut Univ., Assiut, Egypt, pp. 29-31.

TC, 1994. Table curve 2D windows V 2.01 by Jandel Scientific, Eastern Valley Way, Middle Cove MSW 2068, Australia.

Wood, P. D., 1967. Algebraic model of the lactation curve in cattle. Nature, (Lond). 216:164-165.

دراسة التوصيف الرياضى لمُنحنى الحليب فى الجاموس المصرى

أحمد البلتاجى[١] – حسين منصور[٢] – محاسن محمد[١]

١– معهد بحوث الإنتاج الحيوانى، الدقى، الجيزة ٢– قسم الإنتاج الحيوانى، كلية الزراعة، جامعة عين شمس

تم إستخدام عدد ٢٢٢٦ سجل حليب لمُقارنة سبعة دوال رياضية مُختلفة من حيث ملاءمتها لتوصيف مُنحنى الحليب للجاموس المصرى والدوال المُستخدمة هى:دالة جاما $y_t=a\ t^be^{-ct}$، الدالة التقسيمية (الكومبارتمنتال) $y_t=m\ (1-e^{-p(t-q)})e^{-nt}$ ، الدالة التقسيمية المُعدلة (موديفايد كومبارتمنتال) $y_t=g[1/(1+e^{-s(t-r)})]e^{-ht}$، الدالة اللوغارتمية مُتعددة الأطوار (ملتيفيزيك لوجيستيك) فى كل من صورتيها الثنائية والثلاثية الأطوار ، دالة ميتشرليتس الأسية ، دالة ميكاليس منتن الأسية ، بالإضافة إلى دالة خطية مُقترحة

إتخذ كل من متوسط مُربعات الخطأ، مُعامل التحديد ومقياس دوربين واطسون الإحصائى كمقاييس لمقارنة الدوال المُستخدمة. وقد أظهرت الدالة الخطية أفضل مُلائمة لبيانات الحليب المُستخدمة حيث تراوحت قيم كل من متوسط مربعات الخطأ، معامل التحديد ومقياس دوربين واطسون الإحصائى بين (٠,٦٤– ١٥,٩٠)، (٩٢,٢– ٩٩,٥٪)، (٠,٢٧ – ٢,٨٢) بمتوسطات تساوى ٢,٧٢، ٩٧٪ و ١,٢٤ على الترتيب بينما تراوحت قيم تلك المقاييس بالنسبة لباقى الدوال بين (١,٥٤ – ١١٥,٥٥)، (١٥–٩٧٪) و (٠,١٧– ١,٣٦) لمتوسط مُربعات الخطأ، مُعامل التحديد ومقياس دوربين واطسون على الترتيب.

كما تم إشتاق العلاقات الرياضية بين الدالة الخطية المُقترحة وبعض الصفات المُتعلقة بإنتاج اللبن مثل كمية اللبن التجميعية لعدد ن من أسابيع الحليب (كجم) وحليب بداية الموسم (كجم) ووقت (أسبوع) أقصى حليب ومقدار هذا الحليب الأقصى (كجم)، بالإضافة إلى المُثابرة.

وقد تم وضع قاعدة للتنبؤ بطول موسم الحليب للحيوانات التى تجبر إضطرارياً على الجفاف المبكر (٨ أسابيع فأكثر) بإستخدام تقنية التحليل الإحصائى التمييزى Discriminant analysis. وقد أظهرت النتائج أن دقة هذا التقسيم قد تراوحت بين ٧٥٪ للسجلات قصيرة موسم الحليب إلى ٩٩,٥٪ للسجلات شديدة طول موسم الحليب.