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DEPENDENCE OF THE BOND WORK INDEX ON THE FILLING RATIO AND TEST SIEVE APERTURE

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ABSTRACT:

Knowledge of the grindability behavior of solid materials can be considered as an important factor for the design of their grindability systems. Bond work index (Wi) can be considered as one of the most important affecting factors in the design of grinding systems which indicates the value of energy (kwh) consumed per ton of the ground ore. In this research the Bond work index (Wi) was appointed using the standard Bond test and Karra's and Kapur's abbreviated algorithms for some Egyptian ores which can be used in some strategic industries such as ceramics and cement. These ores include quartz, white marble, Nephline syenite, cement clinker, kaolin, ilmenite and barite. Experiments were carried out at different ore filling ratio percentages and at different test sieve apertures. From the obtained results it was clear that, values of the Bond work index (Wi) resulted from the three mentioned methods were quite similar. It was shown also that the filling ratio affects greatly on the value of the work index. The index value increases by decreasing the test screen aperture (degree of fineness).

It was recommended to identify the filling ratio and degree of fineness for appointing the work index as a prerequisite for the design of grinding systems.

Keywords: Comminution – Bond work index – Design of grinding systems – Methods of appointing work index – Energy consumed in comminution.

1. Introduction

The Bond grindability test is widely used for the prediction of the power requirement of an industrial grinding mill. Procedures for conducting this test are available in many references [1-2]. In case of a ball mill, batch grindabilities have been correlated to Bond indices to help monitor day-to-day variations in ore grindabilities for the purpose of plant control [3]. Of special interest grindability is the algorithms presented by Magdalinovic [3], Karra [4] and Kapur [5] for calculating the work index numerical values. Bond type simulated closed-circuit grindability and batch grindability tests were tried. The results of these trials were compared [6-9]. It is indicated that often it should be possible to estimate Bond grindabilities from batch grindabilities if care is taken in making the estimation. In the field of coal, the interrelationships between Bond work index and the Impact strength index, Point load index, and Friability index were found in a good correlation within the mean relative values [10-11].

Recently, numerous attempts have been made to install faster and easier methods to determine the work index, a complete description of these methods is presented by Yap al [12] .It is unusual to find Bond standard mills in mineral processing laboratories. Because of this it is important to find alternative ways for determining the work index. An analysis of the results obtained by Menendez et. al [13] shows a direct relationship between the Bond work index and the Denver work index when the Bond test is defined adequately to the Denver lab mill. A new size laboratory ball mill is presented [14] that is smaller in size than the standard Bond mill. Therefore, it requires smaller amount of sample and the test is faster. The test is conducted just as the standard Bond test but with a smaller amount of sample. A global operational work index is defined for a conventional grinding plant consisting of a rod mill followed by three inverse hydrocyclone ball mill grinding circuit and its value was estimated online by Gonzalez et. al [15].

Other alternative modified Bond test procedures were tried to solve the problem of fine limiting sizes of feed [16-19], the size of sample [20-21] and grinding crystalline grains [22]. Attempts were made to predict the Bond's work index using samples rock properties such as density, Protodyakonov's strength index, and rebound hardness number (Viker's hardness) [23-24].

These properties can be determined at the site as there is no need to carry the blocks to the laboratory to prepare the suitable samples. The main objective of this study was to confirm that the Bond work index (Wi) - as an indicator of grindability- is not a material constant but depends on the filling ratio of the Bond ball mill charge as well as the aperture of the test sieve. These considerations should be considered when energy consumption calculated according to the Bond formula and its abbreviated algorithms.

2. Experimental Work

2.1 Materials Preparation:

Six Egyptian minerals in addition to cement clinker were used to identify their work indices at different filling ratios and different meshes of grind. Standard Bond ball mill grindability test was used to identify the work index (Wi) for the studied materials. To prepare the Bond grindability test feeds, the samples were stage crushed to pass 6 mesh sieves. A sample weighing about 10 kgs of each material was prepared for the test. For convenience, each sample was splitted into 8 lots by riffle sampler to facilitate handling. A representative sample of each crushed sample was size analyzed, then the 80% passing size (F) was identified graphically.

2.2 Equipment:

The test mill used was the standard one (Bond mill), 12 inch inside diameter and length, without lifters, operated at 65 rev/min. which represent 82% of its critical speed. The grinding charge consisted of 296 steel balls weighing about 20.125 kg as was used in the standard Bond grindability test [25-26]. It consisted of about 45×1.45-inch balls, 68×1.17-inch balls, 10×1-inch balls, 71×0.75-inch balls, and 102×0.61-inch balls. The ball voids as measured by displacement of water are equal to 1920 cm^3. It represents 41 percent of the volume of the ball charge with voids.

2.3 Methods of Calculation:

Two abbreviated methods are used to calculate the numerical values of the work index (Wi) of the tested materials. The first one was that proposed by Magdalinovic [3]. It simplified the determination of the work index by just two grinding tests. The procedure of this method was explained in details by the author. The data obtained from the two grinding tests were mainly used to calculate the ball mill grindability (G). with respect to this method the Bond work index is calculated by the following well known Bond formula [25]. Eq. (1)

$$Wi = 44.5/ [P\_c\text{^}(0.23) \times (G)0.82 \times \square(10/\sqrt{P}) - \square(10/\sqrt{F})] \qquad (1)$$

Where:

Wi = Bond work index, (kwh/t),
Pc = Selected test sieve, microns,
G = Ball mill grindability, grams per revolution,
P = 80% passing size of product in the test cycle, microns,
F = 80% passing size of the feed materials, microns.

Because the above mentioned needs a special feed preparation as mentioned by the author, it was only carried out at 36.4% ore filling ratio with the test sieve of 200 µm to check its validity. In the second method, suggested by Karra [4] the information available from batch grinding of one sample in two grinding cycles are used to calculate the work index (Wi) by Karra's algorithm [4] and modified Kapur's algorithm [5]. It should be known that this method does not need any special feed preparation, therefore, the effect of ore filling ratio on the work index was studied at filling ratios of 36.4%, 60% and 90% at different apertures of the test sieve. The algorithm of Karra was expressed mathematically by Eq. (2).

$$Wi = 9.934\ (Pc)0.308 \times (Gk)\text{-}0.696 \times (F)\text{-}0.128 \qquad (2)$$

And the modified algorithm of Kapur was expressed by Eq. (3)

$$Wi = 3.987\ (Pc)0.3 \times (\text{-}R)\text{-}0.708 \times (X\text{-}XY)\text{-}0.745 \times (Y)\text{—}0.141 \qquad (3)$$

Where:

Wi = Work index; (kwh/t),
Pc = Test sieve; microns,
Gk = Karra simulated Bond grindability; grams per revolution,
F = 80% passing size of the original feed sample; microns,
R = Kapur batch grindability; rev.-1,
X = Original feed weight of the sample; grams,
Y = Fraction of product grade material in the original feed.

The work index values were calculated by solving Equations (2) and (3) mathematically. The abbreviated procedure of Karra [4], and Kapur [5] for finding the work index of the tested materials can be summarized as follows:

- Dry grinding of 700 cm3 bulk volume (weighing X grams) of material, crushed previously to -6 mesh, in Bond mill for an arbitrary chosen number of revolution (N1).

- After grinding, the entire sample of the first cycle was screened on the test sieve of 500 microns for example. The undersize was weighed (Z1). and representative sample was screened on a set of screens having apertures of 400, 315, 250, 200, 160, 125, 100 and 71 microns.
- The whole sample was collected again and ground to another number of revolutions (N2). The entire sample of the second cycle was screened on the test sieve of 500 microns. The undersize was weighed (Z2) and a representative sample was screened using the same set of screens that was used in the first cycle.

The information from the two grinding cycles were Pc, X, N1, Z1, N2 and Z2 in addition to F and Y obtained from the size distribution of the original feed. These data were used to compute G and R of Equations (2) and (3) respectively, then the work indices of Karra and Kapur were computed. It should be noted that these calculations have been carried out at different filling ratios and different apertures of the test-screen for each studied material.

3. Results and Discussions:

Firstly, the work indices of the tested minerals were estimated using Equations (1), (2) and (3) at 200 µm test sieve aperture and 36.4% ore filling ratio as an example to check the validity of the results. These results are given in Table (1). From Table (1) it is noticed that the values of the work index using the three algorithms were quite similar.

Table (1) Values of the work index of the studied materials by means of the three algorithms (at 36.4% filling ratio and 200 microns test sieve)

Material Type	**80% passing size**		**Work index (kwh/t)**			
	Feed (µm)	**Product (µm)**	**Eq. (1) Bond**	**Eq. (2) Karra**	**Eq. (3) Kapur**	**av. W_i**
Quartz	2555	157	15.97	14.79	16.12	15.63
White marble	2472	144	13.10	12.05	12.20	12.45
Nephline Syenite	2507	145	14.03	13.12	13.72	3.62
Cement clinker	1959	128	9.78	9.60	8.97	9.45
Kaolin	2592	160	8.92	8.45	8.12	8.50
Ilmenite	2567	147	9.14	8.84	8.59	8.86
Barite	2518	129	4.68	4.89	4.36	4.64

Secondly, the values of the work index of the seven tested materials were calculated at different filling ratios and test sieves using Karra's and Kapur's algorithms. The results obtained are given in Table (2) and represented graphically in Figs. (1-3). This table and figures reveal the following interesting features.

The work indices of the tested materials continually increase with finer mesh of grind at the same filling ratio. This increase is high with respect to quartz, Nephline syenite and marble, medium for ilmenite cement clinker and kaolin and low for barite. This behavior may be attributed to the breakage behavior of each group which needs more extensive studies in the future. The behavior of each group may be attributed to the increase of the fine particle's strength which needs more energy to break [27].

On the other hand, work indices decrease with increasing the ore filling ratio at the same mesh of grind as shown in Figs (4-6). This decrease may be due to the change of the load trajectories to include cataracting beside cascading mechanism which increases the efficiency of comminution due to abrasion and chipping and decreases balls collision waste energy [28]. The obtained results agree to a great extent with those obtained by Menedez et al [22]. These results can be considered as a convenient reply for the criticisms of Austin et al [29] and Austin and Brame [30] as they mentioned that there is a lack in studying the effect of the two studied parameters on the Bond work index value.

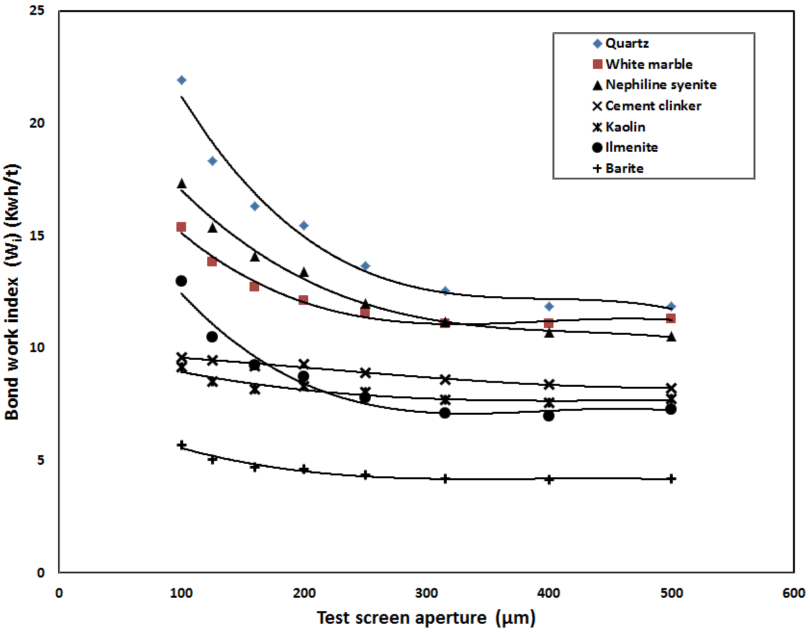

Fig. (1): Effect of test screen aperture on the Bond index (W$_i$) value at 36.4 % filling ratio.

Table (2) Values of the work index Wi (kwh/t) of the materials tested evaluated by Karra's and Kapur's algorithm's

Materials	Mesh of grind P_o (µm)	500			400			315			250			200			160			125			100		
	Ore filling ratio%	1*	2*	3*	1*	2*	3*	1*	2*	3*	1*	2*	3*	1*	2*	3*	1*	2*	3*	1*	2*	3*	1*	2*	3*
Quartz	36.4	12.67	11.03	11.85	12.45	11.29	11.87	12.8	12.31	12.56	13.56	13.78	13.67	14.79	16.12	15.46	15.36	17.29	16.33	16.82	19.84	18.33	19.45	24.43	21.94
	60.0	9.85	8.76	9.31	10.18	9.51	9.85	10.33	10.04	10.19	11.25	11.53	11.39	12.34	13.37	12.86	13.10	14.68	13.89	14.38	16.79	15.59	16.29	20.26	18.28
	90.0	9.77	8.06	8.42	8.90	8.39	8.65	9.39	9.20	9.30	10.30	10.54	10.42	11.44	12.28	11.86	12.04	13.24	12.64	13.05	14.81	13.93	14.20	18.61	16.41
White marble	36.4	11.61	10.98	11.30	11.32	10.81	11.07	11.24	10.89	11.07	11.70	11.52	11.61	12.05	12.20	12.13	12.58	12.82	12.70	13.62	14.09	13.85	14.50	16.27	15.38
	60.00	8.88	8.35	8.62	8.80	8.36	8.58	8.91	8.59	8.75	9.21	9.06	9.14	9.47	9.55	9.51	9.52	9.71	9.62	10.06	10.62	10.34	11.13	12.65	11.89
	90.0	7.42	6.94	7.18	7.25	6.82	7.04	7.34	6.97	7.16	7.54	7.26	7.40	8.07	8.01	8.04	8.21	8.24	8.23	8.68	8.67	8.67	10.88	10.05	10.47
Nephline syenite	36.4	10.93	10.12	10.52	10.99	10.41	10.70	11.31	11.01	11.16	11.98	12.03	12.00	13.12	13.72	13.42	13.61	14.56	14.09	14.53	16.19	15.36	16.21	18.50	17.36
	60.0	8.10	7.42	7.76	8.19	7.66	7.93	8.48	8.15	8.32	9.08	8.98	9.03	9.94	10.16	10.05	10.16	10.66	10.41	10.90	11.85	11.37	10.94	12.05	11.50
	90.0	6.71	6.17	6.44	6.87	6.43	6.65	7.15	6.82	6.99	7.74	7.63	7.69	8.46	8.55	8.51	8.49	8.73	8.61	9.31	9.99	9.65	10.45	11.63	11.04
Cement clinker	36.4	8.64	7.84	8.24	8.94	7.86	8.40	9.04	8.13	8.59	9.28	8.52	8.90	9.60	8.97	9.29	9.37	9.02	9.20	9.61	9.32	9.47	9.70	9.50	9.60
	60.0	6.80	5.91	6.36	6.75	5.92	6.34	6.83	6.10	6.47	6.93	6.29	6.61	7.17	6.62	6.90	7.12	6.65	6.89	7.22	6.85	7.04	7.40	6.97	7.19
	90.0	5.64	5.06	5.35	5.68	5.09	5.39	5.71	5.15	5.43	5.72	5.18	5.45	5.99	5.50	5.75	5.96	5.52	5.74	6.10	5.71	5.91	6.64	6.28	6.47
Kaolin	36.4	8.48	6.97	7.73	8.23	6.93	7.58	8.20	7.22	7.71	8.37	7.74	8.06	8.45	8.12	8.29	8.24	8.09	8.17	8.43	8.60	8.51	8.61	9.68	9.15
	60.00	6.72	5.76	6.24	6.57	5.77	6.17	6.53	5.91	6.22	6.62	6.19	6.41	6.63	6.38	6.50	6.58	6.45	6.52	6.84	6.90	6.87	6.97	7.47	7.22
	90.0	5.97	5.34	5.66	5.80	5.26	5.53	5.79	5.37	5.58	5.87	5.58	5.73	5.88	5.71	5.80	5.67	5.58	5.63	5.98	5.90	5.94	5.90	6.55	6.23
Ilmenite	36.4	7.61	6.79	7.29	7.39	6.61	7.00	7.44	6.79	7.12	8.03	7.57	7.80	8.84	8.59	8.72	9.30	9.20	9.25	10.39	10.56	10.48	12.53	13.38	12.96
	60.00	5.58	4.91	5.25	5.46	4.83	5.15	5.60	5.08	5.34	5.94	5.50	5.72	6.69	6.40	6.55	7.04	6.90	6.97	7.75	7.79	7.77	9.47	9.87	9.67
	90.0	4.71	4.30	4.50	4.69	4.28	4.49	4.85	4.44	4.65	5.19	4.78	4.99	5.87	5.52	5.70	6.26	5.97	6.12	6.76	6.58	6.67	7.97	7.92	7.94
Barite	36.4	5.00	3.35	4.18	4.82	3.45	4.14	4.72	3.66	4.19	4.76	3.98	4.37	4.88	4.36	4.62	4.89	4.53	4.71	5.12	5.02	5.07	5.58	5.82	5.70
	60.0	3.85	2.79	3.32	3.73	2.85	3.29	3.67	2.98	3.33	3.81	3.31	3.56	3.95	3.60	3.78	3.91	3.66	3.79	4.00	3.91	3.95	4.36	4.50	4.43
	90.0	3.32	2.65	2.99	3.27	2.70	2.99	3.28	2.81	3.05	3.41	3.07	3.24	3.55	3.32	3.44	3.56	3.42	3.49	3.72	3.69	3.70	3.88	4.03	3.96

1*- W_i Karra **2*- W_i Kapur** **3*- Average W_i**

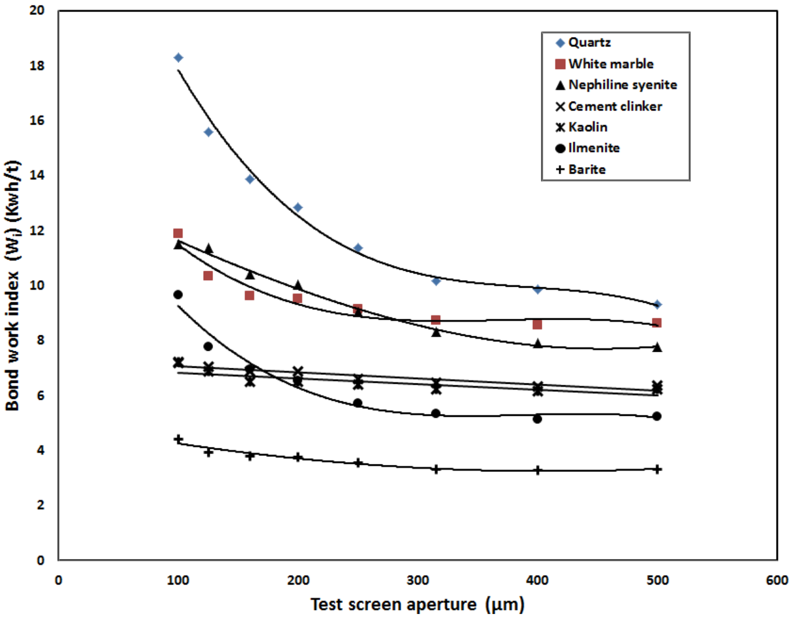

Fig. (2): Effect of test screen aperture on the Bond index (W_i) value at 60 % filling ratio.

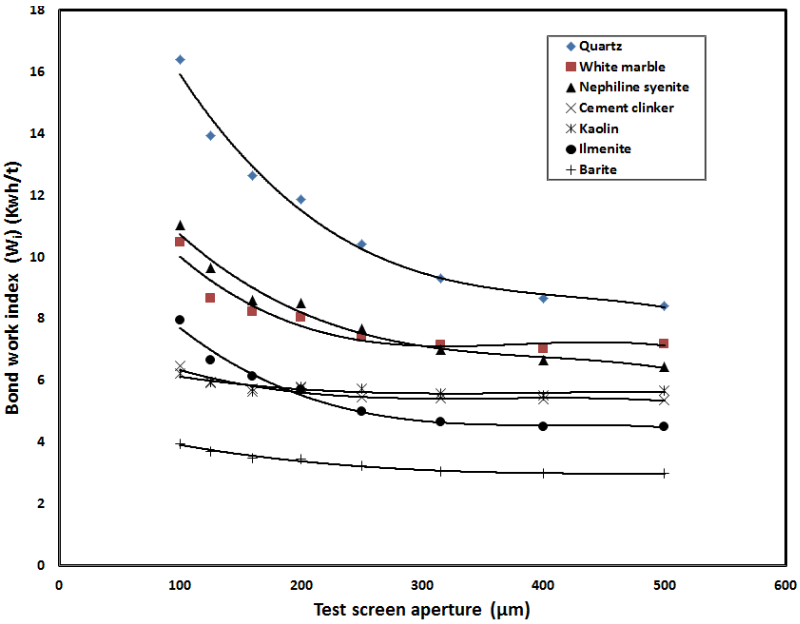

Fig. (3): Effect of test screen aperture on the Bond index (W_i) value at 90 % filling ratio.

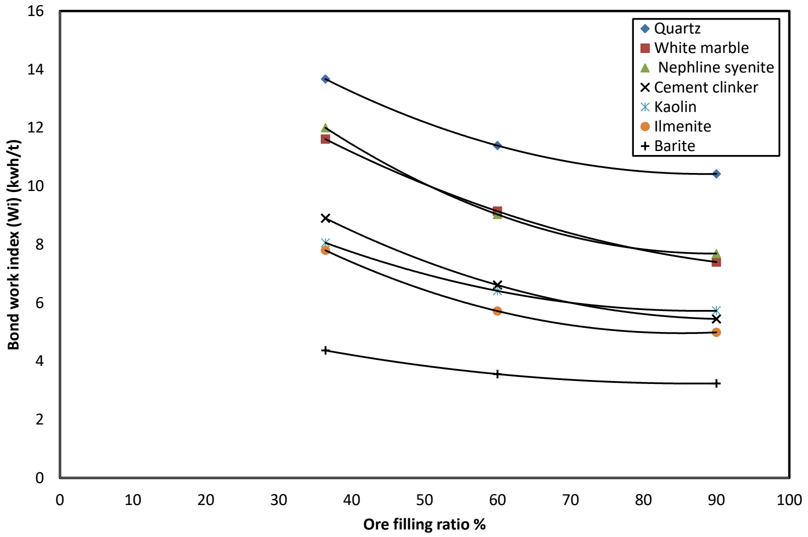

Fig. (4): Effect of the ore filling ratio on the Bond work index (W_i) value at a test screen aperture of 5oo µm.

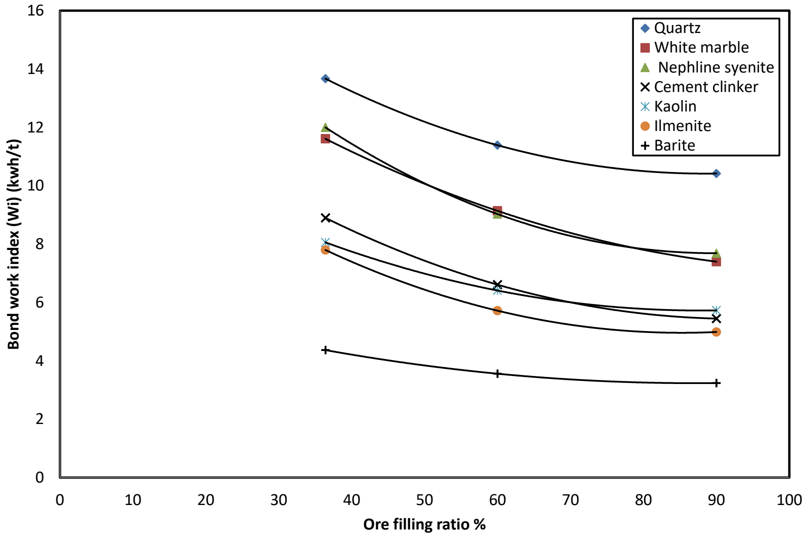

Fig. (5): Effect of the ore filling ratio on the Bond work index (W_i) value at a test screen aperture of 250 µm.

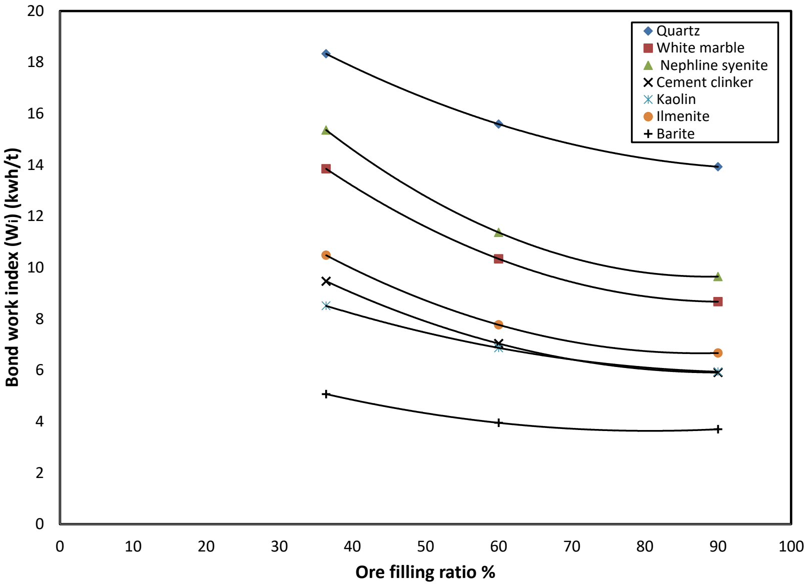

Fig. (6): Effect of the ore filling ratio on the Bond work index (W_i) value at test screen aperture of 125 µm.

6. Conclusions

From this study the following conclusions can be drawn.

- The Bond work index depends on the properties of the studied material.
- Bond work index (W_i) is not a material constant but changes with the change of the filling ratio within ball mill charge, and the aperture of the test sieve.
- The simplified algorithms for the determination of the work index are very useful monitor to show day to day variation of the ore grindability.
- The work index (W_i) -as an indicator of grindability- decreases with increasing the mesh of grind.
- Filling ratio has a significant influence on the value of the work index. This value decreases as the filling ratio increases.

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اعتماد دليل بوند للطاقة على نسبة ملئ فراغات الوسط الطاحن وفتحة منخل الاختبار

الملخص العربي:

تعتبر معرفة سلوك طحن المواد الصلبة عاملا هاما في تصميم نظم طحنها. ويعتبر دليل بوند للطاقة أحد أهم العوامل المؤثرة في تصميم نظم الطحن، والذي يعبر عن مقدار الطاقة المستهلكة (كيلووات/ساعة) لكل طن من الخام المطحون.

وفى هذا البحث تم تعيين دليل بوند للطاقة بالطريقة القياسية وكذا بطريقتي كارا وكابور المختصرتين لبعض الخامات المصرية والتي تدخل في بعض الصناعات الاستراتيجية مثل السيراميكات والاسمنت. وهذه الخامات هي الكوارتز والرخام الأبيض والنيفيلين سيانيت والكلنكر والكاولين والألمنيت والباريت وذلك عند قيم مختلفة لنسبة ملئ الفراغات البينية بين كرات الوسط الطاحن، وكذا عند قيم مختلفة لفتحة منخل الاختبار، والتي تعبر عن درجة نعومة المنتج المطلوب من نظام الطحن.

وقد أوضحت الدراسة أن قيم دليل بوند للطاقة الناتجة باستخدام الطرق الثلاث سالفة الذكر متطابقة إلى حد كبير، وأن نسبة ملئ الفراغات البينية تؤثر على قيمة دليل بوند تأثيرا ملحوظا. وأن قيم هذا الدليل تزداد بزيادة درجة النعومة المطلوبة للمنتج.

وتوصى الدراسة أنه عند تعيين الدليل كمتطلب لتصميم نظم الطحن بتحديد نسبة ملئ الفراغات البينية وكذا درجة نعومة المنتج المطلوب، حيث اتضح أن قيمة هذا الدليل تعتمد بدرجة كبيرة عليهما.