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Electric Monopole Transitions in Nd Nuclei within IBM-2

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THE monopole transitions of $^{144-154}$Nd isotopes have been investigated in this work within the interacting boson model-2 (IBM-2) framework. The low-lying energy levels, electric quadrupole transition probability rates B(E2), electric quadrupole moments for first excited states $Q(2_1^+)$ monopole transition matrix elements $\rho(E0)$ and the intensity ratio $X(E0/E2)$ were calculated in this work. The results of IBM-2 have been compared with the available experimental data; we have obtained a reasonable agreement. Unfortunately, the experimental data available on E0 transitions are very rare.

Keywords: Nuclear structure, Energy levels, Transition Probability, Monopole Transition, Interacting Boson model.

Introduction

The monopole transitions (E0) are known to be pure penetration effect where the transition is caused by an electromagnetic interaction between the nuclear charge and atomic electron penetrating [1]. The nucleus E0 transition could be pure for $\Delta I = I_i - I_f = 0$, $I_i = I_f = 0$, where I is the total angular momentum of the nuclear state . This transition competes with E2 and M1 transitions [1].

The excited states up on 5 MeV excitation energies in even-even $^{140-146}$Nd isotopes, been investigated in [2]. The even-even $^{142-150}$N isotopes is characterized by a fast transition from spherical (vibrational shape) to axially rotor deformed shapes as many properties are changing rapidly at the deformation onset and provides, a sensitive testing ground for nuclear models [3]. Because this reasons, these isotopes have special attention during the past years. The low-lying collective states have been investigated in different models and methods [4-6] .

Gupta [7], has been investigated the $^{144-150}$Nd nuclear structure in IBM-1 framework. In this study analyzed the energy levels and electric transition probability of these nuclei. Devi and Gupta [8] analyzed the nuclear structure of even-even Neodymium isotopes using Interacting Boson Model (IBM-1), in this study showed the ground state energy levels, quasi beta band and quasi gamma band in $^{142-148}$Nd isotopes. These properties of IBM-1 can be examined experimentally by the staggering of energy in the gamma-band as a signature of $\gamma -$unstable.

Dieperinger and Iachello [9] have been suggested that the IBM could be used in the electron scattering data analyzing and discussed the properties of the expected behaviour of the inelastic excitation of some 2$^+$ states in the transitional samarium-neodymium region. The structures of the neutron-deficient Nd isotopes of $A = 128 - 140$ are studied in a schematic Hamiltonian in the interacting boson model-1, investigated by Long Guilu [10]. The level structure and *E2* transitions can be well described in the scheme and the particular, the back-bending in the ground state band is well reproduced in this study.

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Holden et al., [11] pointed to a single particle degrees of freedom in the transition from spherical to deformed Nd nuclei in IBM. Turkan and Inci [12] applied the IBM-2 on some even-even neodymium nuclei. In this study used the best-fitted values of parameters in the IBM-2 Hamiltonian and have been calculated energy levels and electric transition probability rates B(E2) in ^{144,146,148,150,152,154}Nd isotopes. The results were compared with the available experimental data. Abdul-Kader [13], applied the IBM-2 on $^{140-160}$Nd isotopes, in this study have been calculated the energy levels, electromagnetic transition rates B(E2), B(M1), $\delta(E2/M1)$ and monopole transition matrix element and the intensity ratio $X(E0/E2)$. Hummadi [14], studied the nuclear structures of even-even isotopes $^{148-152}$Nd are studied by using IBM-1. The energy levels of ground state, beta and gamma bands, energy ratios are calculated. The results showed dynamical symmetry of these isotopes SU(3)-SU(6), SU(5)-SU(6). The spectra, B(E2), branching ratios and potential energy surface are studied in the IBM-1 [15]. It is found that ^{146,148}Nd isotopes are in the transition region U(5)-O(6).

The $^{144-154}$Nd isotopes under consideration have $Z = 60$ and $84 \le N \le 94$, which means that we have 10 proton particles outside the major shell at 50. The neutron numbers are 84, means that we have 2 neutrons out side the major shell 82 in ^{144}Nd isotope to 12 neutrons outside the major closed shell at 82 in ^{154}Nd isotope. The nucleon numbers out side the major shell make the nucleus closed to Sm, Gd, Dy and Er nuclei [16-18].

The purpose of this work is to the connection between strong E0 transitions and coexistence of shape in $^{144-154}$Nd isotopes within IBM-2 framework. Unfortunately, the theoretical and experimental data on monopole (E0) transitions are very rare and also the approximate nature of theory does not make it possible to settle the question of nuclear non-axiality. We do attempt an exhaustive all aspects review of E0 transitions and intensity ratio between E0 and E2 transitions in $^{144-154}$Nd isotopes.

The Interacting Boson Model (IBM)

The properties of low-lying collective states in nuclei are dominated by the pairing and quadrupole degrees of freedom. In the IBM-1 model these are incorporated by introducing six bosonic degrees of freedom, divided into a scalar boson with angular momentum $L = 0$ (called an s-boson) and a quadrupole boson with angular momentum $L = 2$ ($d_\mu - boson$ $\mu = -2,-1,0,1,2$). The creation (s^+, d_μ^+) and annihilation $(s^\sim, d_\mu^\sim)$ operator which obey the standard boson commutation relations, span a six-dimensional space and thus provide a basis for the representations of the group $U(6)$. The basis states for an *N*-boson system span the totally symmetric representations $[N]$ of $U(6)$ and can be expressed as $\left| s^{N-n_d} d^{n_d} \; [\alpha L] \right\rangle$, where n_d is the number of d-bosons that are coupled to angular momentum *L*. One then assumes that the properties of low-lying collective states in even-even nuclei can be described by a Hamiltonian that conserves the boson number, is rotationally invariant, and contains at most two-body interactions.

The IBM-2 [19] is the natural extension of the IBM-1 considering explicitly the neutron-proton degree of freedom. In contrast to the IBM-1 which is purely phenomenological, the IBM-2 has at least qualitatively a microscopic justification and in principle it is possible to derive the parameters of the IBM-2 from microscopic considerations. However, until today this connection is not quantitative, *i.e.* the derived parameter using the OAI-mapping [20] differ from the one required to fit the data. This is a serious caveat of the IBM-2. The microscopic counterparts of *s*- and *d*-bosons are correlated nucleon pairs of the same type. The main problem of shell model calculations is the drastically increasing size of the model space when going from magic nuclei to open-shell systems. Typically, it would be necessary to diagonalize matrices of the dimension of ≈ 10^{20}, a number where one could not even think of diagonalizing it. The IBM-2 can be seen essentially as a very vast and rough truncation of this huge shell model space reducing the problem even at mid-shell to matrices of $\approx 10^2$ which can be handled by conventional diagonalization techniques easily.

The most general IBM-2 Hamiltonian has the form [19, 21, 22]:

$$H = H_\pi + H_\nu + V_{\pi\nu} \qquad \text{..............} \quad (1)$$

$$H = \varepsilon(n_{d\pi} + n_{d\nu}) + \kappa(Q_\pi . Q_\nu) + V_{\pi\pi} + V_{\nu\nu} + M_{\pi\nu} \quad \ldots\ldots (2)$$

The fermionic analog on of the d-boson energies ε_π and ε_ν is the monopole pairing part, while the analogon of $\kappa Q_\pi . Q_\nu$ is the proton-neutron quadrupole interaction. $M_{\pi\nu}$ is the so called Majorano-Operator which has no direct microscopic counterpart. It's most general form is **[23]:**

$$M_{\pi\nu} = \left(s_\pi^+ \times d_\nu^+ - d_\pi^+ \times s_\nu^+\right)^{(2)} \left(s_\pi \times d_\nu^{\sim} - d_\nu^{\sim} \times s_\pi\right)^{(2)}$$

$$-2\sum_{K=1}^{3} [d_\nu^+ \times d_\pi^+]^{(K)} [\, d_\nu^{\sim} \times d_\pi^{\sim}]^{(K)} \quad \ldots\ldots. (3)$$

The underlying algebra of the IBM-2 is $U_\pi(6) \times U_\nu(6)$. The three dynamical symmetries SU(3), O(6) and U(5) are still contained and can be used for interpreting nuclear structure phenomena.

The operator of quadrupole moment in the IBM-2 is written as [23]:

$$Q_\pi^{\chi_\pi} = (d_\pi^+ d_\pi^{\sim})^{(2)} + \chi_\pi (s_\pi^+ d_\pi^{\sim} + d_\pi^+ s_\pi)^{(2)}$$

and $Q_\nu^{\chi_\nu} = (d_\nu^+ d_\nu^{\sim})^{(2)} + \chi_\nu (s_\nu^+ d_\nu^{\sim} + d_\nu^+ s_\nu)^{(2)}$ (4)

the terms $V_{\pi\pi}$ is the interaction of proton-proton bosons and $V_{\nu\nu}$ is the interaction of neutron-neutron bosons only and given by **[23]:**

$$V_{\pi\pi} = \sum_{J=0.2.4} c_{L\rho} \left[\left(d^+ d^+\right)_\pi^{(L)} \left(\tilde{d}\,\tilde{d}\right)_\pi^{(L)} \right]^{(0)} \text{ and}$$

$$V_{\nu\nu} = \sum_{J=0.2.4} c_{L\rho} \left[\left(d^+ d^+\right)_\nu^{(L)} \left(\tilde{d}\,\tilde{d}\right)_\nu^{(L)} \right]^{(0)} . (5)$$

Results and Discussion

Energy Spectra

To present the monopole (E0) matrix elements, we have to obtain the best fit for energy levels and the reproduced the reduced electric transition probability. So, fit to experimental energy levels of the $^{144-154}$Nd isotopes. The required boson numbers are $N_\pi = 5$ (number of proton bosons) and the neutron bosons vary from $N_\nu = 1$ for ^{144}Nd isotope to $N_\nu = 6$ for ^{154}Nd isotope. After several iterations it is found that the following Hamiltonian parameter values in Eq. (2) gave the best fit to experimental energy levels for the ground state band, $\beta-$band and $\gamma-$ band.

The Hamiltonian parameters for $^{144-154}$Nd isotopes are given in Table 1, from the parameter values, one can observe the ε , κ , χ_ν and ξ_K treated as free parameters, varies from isotope to another, where change one parameter, and other parameters remains constants until to get a best fit result with experimental value. These parameters are used to calculate the nuclear properties, such as, energy levels, electric transition probability and monopole transition matrix elements using the NPBOS and NPBTRN computer code programs **[24]** to evaluate these nuclear properties.

The free parameters ε , κ and ξ_K are functions of neutron and proton boson number, while $\chi_\nu(N_\nu)$ as a function of neutron bosons number and $\chi_\pi(N_\nu)$ as a function of proton bosons number, this parameter χ_π is constant for all $^{144-154}$Nd isotopes because the number of proton bosons constant in these isotopes. The $C_{0\pi}$ and $C_{2\pi}$ are two important terms in $V_{\pi\pi}$ parameter, the parameter $V_{\nu\nu}$ play minor role but not ignored, this due to $N_\nu < N_\pi$. The Majorana parameter $\xi_1 = \xi_2 = \xi_3$, these terms are taken equal for whole isotopes, this parameter is important to push the mixed proton-neutron bosons symmetry states.

The IBM-2 calculations for energy levels and experimental values of $^{144-154}$Nd isotopes are given in Fig. 1 - 6, we can observe the agreement with three lower energy bands is quit well, especially the low-lying levels. The discrepancies between IBM-2 results and experimental data appear in high spin states, this is due to, these states don't have collective nature and outside the IBM-2 space.

TABLE 1. The Hamiltonian parameters for [144-154]Nd isotopes in IBM-2, all parameters in MeV units except χ_π and χ_ν are dimensionless

Isotopes	N_π	N_ν	ε	κ	χ_ν	χ_π	$C_{0\pi}$	$C_{2\pi}$	$\xi_1 = \xi_2 = \xi_3$
Nd-144	5	1	0.95	-0.18	0.00	-1.20	0.40	0.20	0.06
Nd-146	5	2	0.90	-0.15	0.00	-1.20	0.40	0.20	0.08
Nd-148	5	3	0.70	-0.10	-0.80	-1.20	0.40	0.20	0.22
Nd-150	5	4	0.47	-0.07	-1.00	-1.20	0.40	0.20	0.37
Nd-152	5	5	0.34	-0.089	-1.10	-1.20	0.40	0.20	0.22
Nd-154	5	6	0.30	-0.085	-1.20	-1.20	0.40	0.20	0.20

$C_{0\nu} = C_{2\nu} = C_{4\nu} = 0.0,\ \ C_{4\pi} = 0.0$

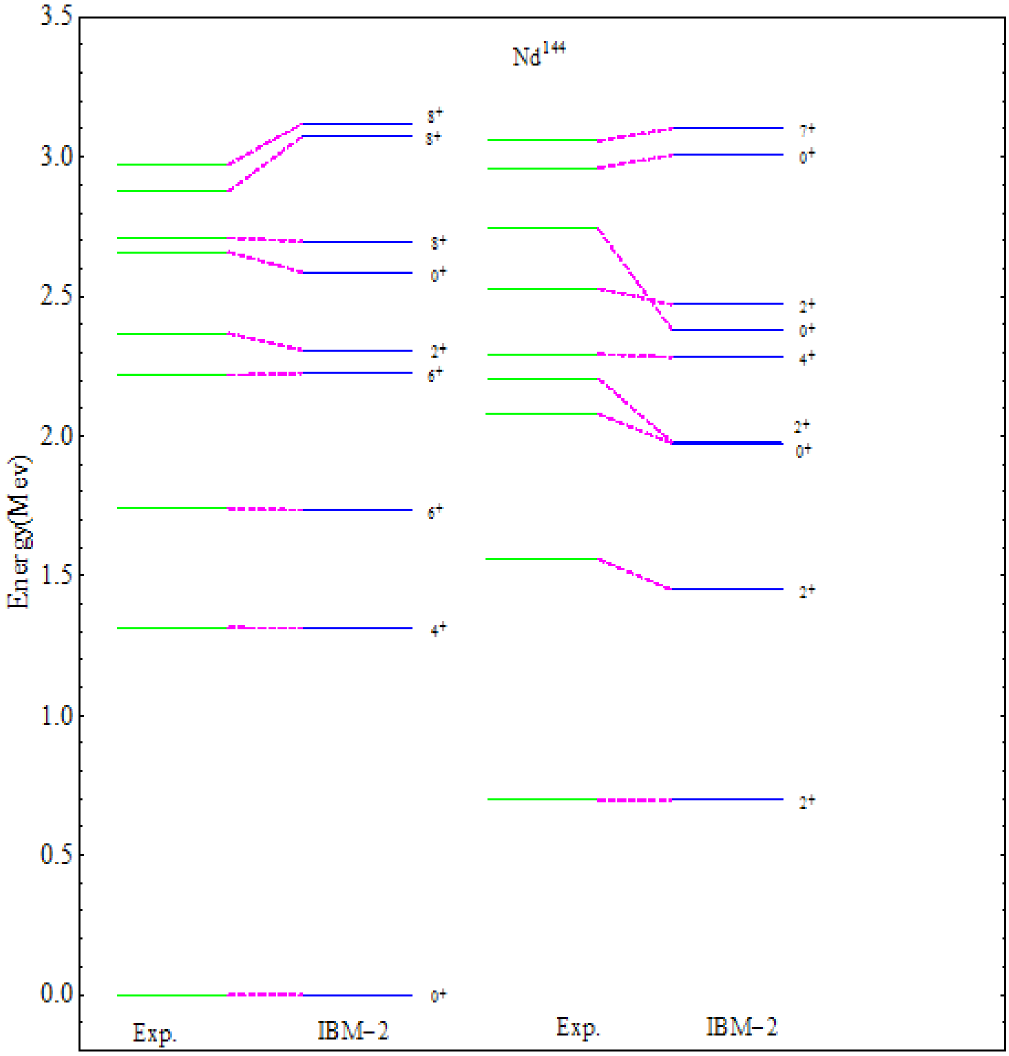

Fig. 1. Comparison between experimental data [25] and IBM-2 calculated energy levels for [144]Nd isotope.

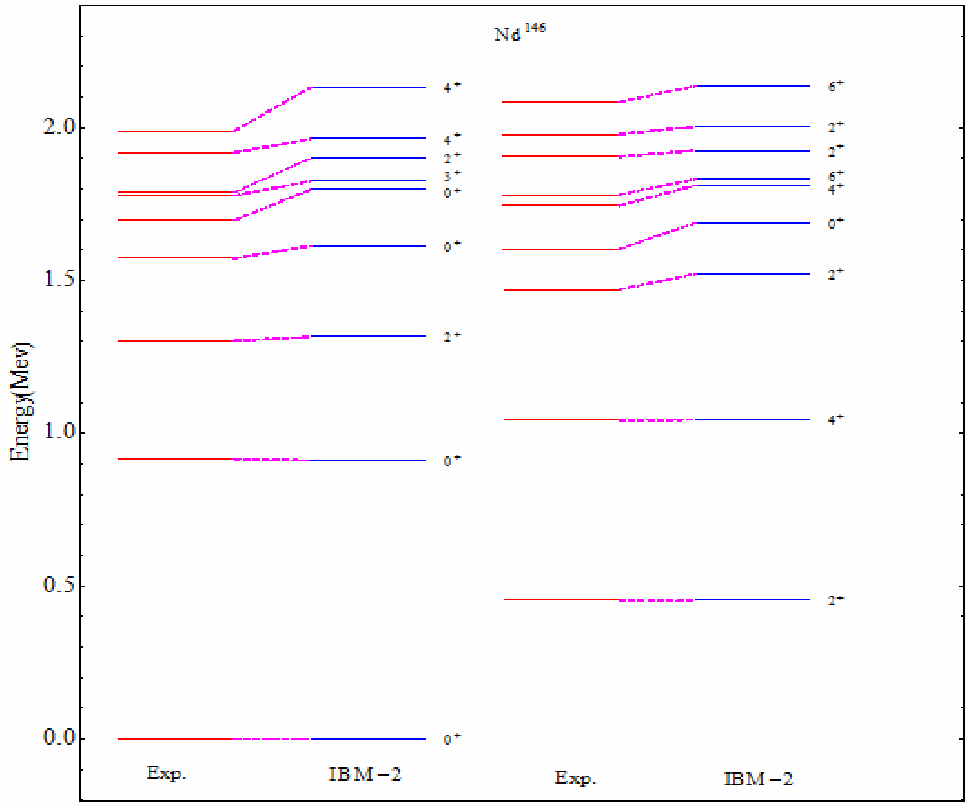

Fig. 2. Comparison between experimental data [26] and IBM-2 calculated energy levels for ^{146}Nd.

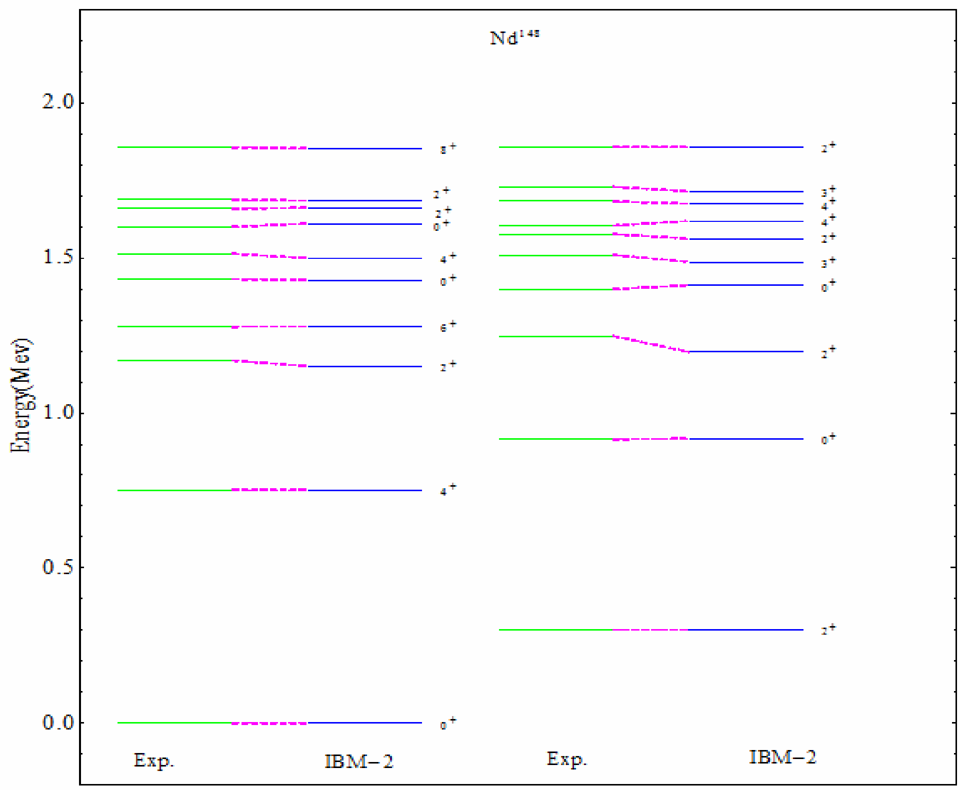

Fig. 3. Comparison between experimental data [27] and IBM-2 calculated energy levels for ^{148}Nd.

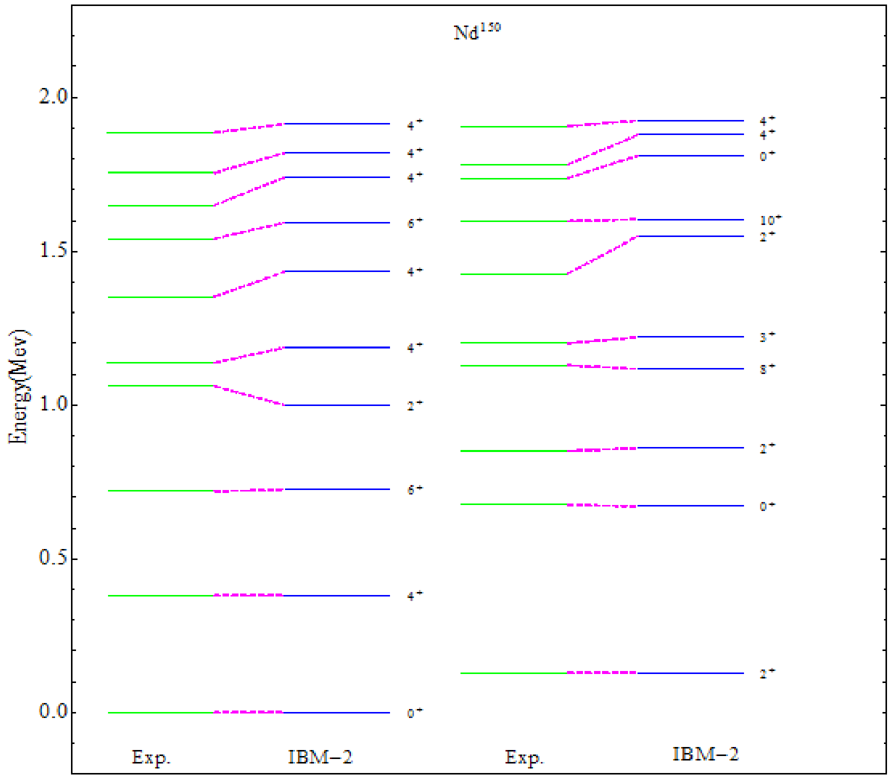

Fig. 4. Comparison between experimental data [28] and IBM-2 calculated energy levels for ^{150}Nd.

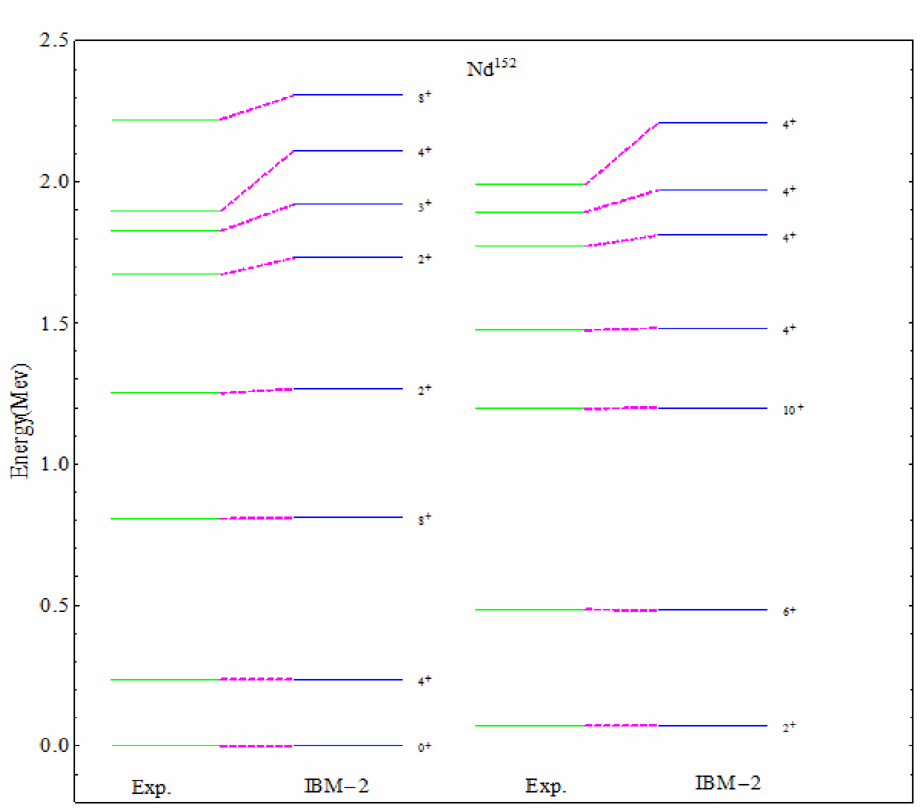

Fig. 5. Comparison between experimental data [29] and IBM-2 calculated energy levels for ^{152}Nd.

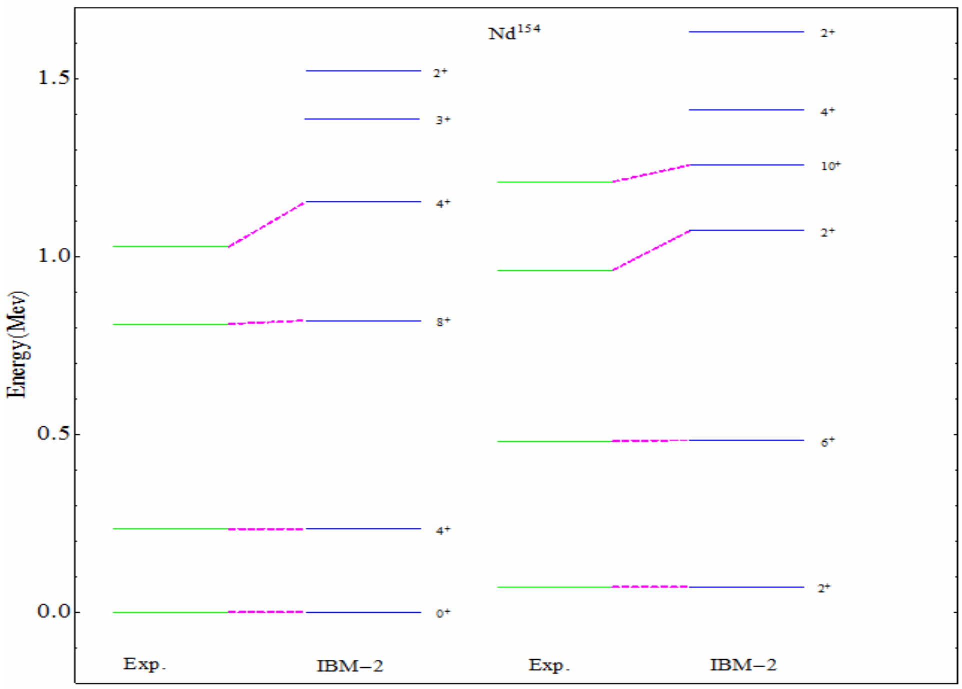

Fig. 6. Comparison between experimental data [30] and IBM-2 calculated energy levels for ^{154}Nd.

The energy level ratios for IBM-2 results and experimental data are presented in Table 2, from these ratios, we can be observed, the $^{144+146}$Nd isotopes are corresponding to a anharmonic vibrator character (near spherical shape, U(5) symmetry), while the $^{148-150}$Nd isotopes appears transitional nuclei (γ-unstable), finally, the $^{152-154}$Nd isotopes appears a deformed nuclei (rotor nuclei, lie in SU(3) limit).

Electric Transition Probability

The E2-transition operator is given by [**209]:**

$$T(E2) = e_\pi Q_\pi^{\chi_\pi} + Q_\nu^{\chi_\nu} \quad \text{..............} (6)$$

where the quadrupole operators $Q_\pi^{\chi_\pi}$, $Q_\nu^{\chi_\nu}$ can be found in Eq. (4) and e_π , e_ν are boson effective charges accounting for states which contribute to the transition of interest but are not included in the small IBM-2 model space. The reduced electric transition probability is written as [31]:

$$B(E2; J_i^+ \rightarrow J_f^+) = \frac{\left| < J_i \| T(E2) \| J_f > \right|^2}{2J_i + 1} \text{..................} (7)$$

Then we have to choose the parameter for the calculations of electric transition probability which is a sensitive test for our procedure. The method of evaluate the perfect fitting parameters is discussed in ref.**[32].** The proton bosons effective charge $e_\pi = 0.353$ *b* which is a constant value for all $^{144-145}$Nd isotopes, and the neutron bosons effective charges are tabulated in Table 3.

The reduced electric transition probabilities are presented in Table 4 together with the experimental values. One can see that the B(E2) transitions in intraband have values are large than the transition in interband, this is due to, the selection rules. Our agreement with the experimental values is quit well. It should be noted that there is no attempt is made to fitting to any electric transition probabilities value will determining the parameters in the collective Hamiltonian.

To estimate the quadrupole moments for fist excited states, we depend on the following equation **[31]**:

$$Q_{2_1^+} = \sqrt{\frac{6\pi}{175}} < 2_1^+ \| T(E2) \| 2_1^+ > \text{........} (8)$$

TABLE 2. The Energy level ratios for $^{144-154}$Nd isotopes.

Isotopes	$R_1 = E(4_1^+)/E(2_1^+)$ Exp.	R_1 IBM-2	$R_2 = E(6_1^+)/E(2_1^+)$ Exp.	R_2 IBM-2	$R_3 = E(8_1^+)/E(2_1^+)$ Exp.	R_3 IBM-2
^{144}Nd	1.890	1.887	2.576	2.499	3.897	3.880
^{146}Nd	2.302	2.302	3.929	4.028	5.4610	5.684
^{148}Nd	2.498	2.497	4.249	4.250	6.166	6.168
^{150}Nd	2.930	2.930	5.538	5.541	8.684	8.750
^{152}Nd	3.277	3.277	6.722	6.722	11.195	11.21
^{154}Nd	3.290	3.290	6.871	6.871	11.571	11.223
$SU(5)$	2		3		4	
$O(6)$	2.5		4.5		7	
$SU(3)$	3.3		7		12	

TABLE 3. The neutron bosons effective charges in eb units.

	^{144}Nd	^{146}Nd	^{148}Nd	^{150}Nd	^{152}Nd	^{154}Nd
$e_\nu(eb)$	0.0848	0.0851	0.0863	0.0872	0.0881	0.0912

TABLE 4. B(E2) values for *$^{144-154}$Nd* isotopes in e^2b^2 Units.

Isotope	$2_1^+ \to 0_1^+$ Exp.	IBM-2	$4_1^+ \to 2_1^+$ Exp.	IBM-2	$6_1^+ \to 4_1^+$ Exp.	IBM-2	$8_1^+ \to 6_1^+$ Exp.	IBM-2
^{144}Nd	0.188(4) [a]	0.180	0.143(6)[a]	0.137	-	0.256	-	0.247
^{146}Nd	0.233(3)[b]	0.231	0.348[b]	0.337	-	0.412	-	0.4077
^{148}Nd	0.480[c]	0.411	0.765[c]	0.731	-	0.821	-	0.839
^{150}Nd	0.978[d]	0.998	1.486[d]	1.272	0.92(9)[e]	1.072	-	1.577
^{152}Nd	-	0.872	-	1.212	1.039(213)	1.131		2.383
^{154}Nd	0.47(13)[f]	0.471	-	0.621	-	1.430		1.332

Isotope	$0_2^+ \to 2_1^+$ Exp.	IBM-2	$4_2^+ \to 2_2^+$ Exp.	IBM-2	$2_2^+ \to 0_1^+$ Exp.	IBM-2	$2_2^+ \to 0_2^+$ Exp.	IBM-2
^{144}Nd	-	0.184	-	0.130	0.0051(2)[a]	0.005	-	0.0134
^{146}Nd	-	0.261	-	0.210	0.128[b]	0.130	-	0.0504
^{148}Nd	-	0.410	-	0.410	0.0345[c]	0.0367	-	0.0606
^{150}Nd	-	0.251	-	0.669	0.0218[d]	0.022	-	0.218
^{152}Nd	-	0.149		0.918	-	0.125		0.372
^{154}Nd	-	0.177		1.357	-	0.0452		0.560

Isotope	$2_2^+ \to 2_1^+$ Exp.	IBM-2	$3_1^+ \to 2_1^+$ Exp.	IBM-2	$3_1^+ \to 2_2^+$ Exp.	IBM-2	$3_1^+ \to 4_1^+$ Exp.	IBM-2
^{144}Nd	0.1619[a]	0.162	-	0.343	-	0.32	-	0.032
^{146}Nd	0.1557[b]	0.163	-	0.291	-	0.423	-	0.0345
^{148}Nd	0.214[c]	0.221	-	0.285	-	0.450	-	0.0412
^{150}Nd	0.0665[d]	0.077	-	0.200	-	0.140	-	0.0431
^{152}Nd	-	0.923	-	0.199	-	0.265	-	0.0451
^{154}Nd	-	1.313	-	0.144	-	0.251	-	0.0113

a-[25] b-[26] c- [27] d-[28] e-[33] f-[34]

The IBM-2 values for quadrupole moment for first excited states $Q(2_1^+)$ are tabulated in Table (5) with the experimental data. Our agreement with the available experimental values is a good. The values of $Q(2_1^+)$ increased in negative with increasing neutron numbers.

Electric Monopole Matrix Elements

The B(E0) reduced transition probability of monopole transition is given as **[35]:**

$$B(E0;J_i^+ \rightarrow J_f^+) = e^2 R_0^4 \rho^2 (E0) \quad \text{........ (9)}$$

Where $J_i^+ = J_f^+$, $R_0 = 1.25A^{1/3}$ *fm*, *e* is the electronic effective charge. The operator of electric monopole transition is **[36]:**

$$T(E0) = \beta_{0\pi}^{\sim} d_\pi^+ . d_\pi^{\sim} + \beta_{0\nu}^{\sim} \gamma_\rho \, d_\nu^+ . d_\nu^{\sim} + \gamma_{0\pi} N_\pi + \gamma_{0\nu} N_\nu \text{.............(10)}$$

Where $\beta_{0\pi}^{\sim} = (\beta_{0\pi} / \sqrt{5}) - \gamma_{0\pi}$ and

$$\beta_{0\nu}^{\sim} = (\beta_{0\nu} / \sqrt{5}) - \gamma_{0\nu} \quad \text{............. (11)}$$

The terms $\gamma_{0\nu} N_\nu$ and $\gamma_{0\pi} N_\pi$ are constants. The monopole transition matrix is written by **[36]:**

$$\rho_{if}(E0; i \rightarrow f) = \frac{Z}{R_o^2} \sum (\beta_{0\pi}^{\sim} < J_f^+ | d_\pi^+ d_\pi^{\sim} | J_i^+ > + \beta_{0\nu}^{\sim} < J_f^+ | d_\nu^+ d_\nu^{\sim} | J_i^+ >) \quad \text{.......... (12)}$$

To estimated the matrix element of monopole transition, must be evaluated the parameters $\beta_{0\pi}$, $\beta_{0\nu}$ and $\gamma_{0\nu}$, the values of these parameters may be estimated by fitting the isotopic shifts or isomer shifts.

The intensity ratio $X(E0/E2)$ of monopole transition E0 to competing electric quadrupole transition E2 is given as **[32]:**

$$X(E0/E2)_{iff'} = \frac{B(E0; J_i^+ \rightarrow J_f^+)}{B(E2; J_i^+ \rightarrow J_{f'}^+)} \text{.................(13)}$$

Where $I_i = I_f = 0$, $I_{f'} = 2$ or $I_i = I_f \neq 0$, $I_i = I_{f'}$

The Eq. (13) rewritten as:

$$X(E0/E2)_{iff'} = \frac{e^2 R_0^4 \rho^2 (E0; J_i^+ \rightarrow J_f^+)}{B(E2; J_i^+ \rightarrow J_f^+)} \text{................(14)}$$

The isotope shift $\Delta < r^2 >$ is defined as a measure of the different in $< r^2 >$ between two neighboring isotopes in their ground state. The value of isotopic shifts is given by **[36]:**

$$\Delta < r^2 > = < 0_1 | r^2 | 0_1 >_A - < 0_1 | r^2 | 0_1 >_{A-2} = \beta_{0\pi} \Delta n_{d\pi} + \beta_{0\nu} \Delta n_{d\nu} - \gamma_{0\nu}$$

$$\Delta < r^2 > = \beta_{0\pi}^{\sim} [< 0_1 | d_\pi^+ . d_\pi^{\sim} | 0_1 >_{N_\nu} - < 0_1 | d_\pi^+ . d_\pi^{\sim} | 0_1 >_{N_\nu + 1}] + \beta_{0\nu}^{\sim} [< 0_1 | d_\nu^+ . d_\nu^{\sim} | 0_1 >_{N_\nu} - < 0_1 | d_\nu^+ . d_\nu^{\sim} | 0_1 >_{N_\nu + 1}] - \gamma_{0\nu} \quad \text{.......... (15)}$$

The isomer shift is defined as the difference between the mean square radius $< r^2 >$ of an excited state and the ground state in a given nucleus [36]:

$$\delta < r^2 > = < r^2 >_{e.s} - < r^2 >_{g.s}$$

TABLE 5. Quadrupole moments for first excited states $Q(2_1^+)$ in b units.

Isotopes	Exp.	IBM-2
Nd-144	-	-0.723
Nd-146	-0.78(9) [26]	-0.76
Nd-148	-1.46(24) [27]	-1.33
Nd-150	-2.0(5) [28]	-2.10
Nd-152	-	-2.246
Nd-154	-	-2.31

$$\delta < r^2 > = \beta_{0\pi}^{\sim}[<2_1|d_\pi^+ \tilde{d}_\pi|2_1> - <0_1|d_\pi^+ \tilde{d}_\pi|0_1>] +$$

$$\beta_{0\nu}^{\sim}[<2_1|d_\nu^+ \tilde{d}_\nu|2_1> - <0_1|d_\nu^+ \tilde{d}_\nu|0_1>] \quad (16)$$

The monopole transition matrix is given in Eq.(12), to estimate the parameters $\beta_{0\pi}$, $\beta_{0\nu}$ and $\gamma_{0\nu}$ in monopole matrix element $\rho(E0)$, are calculated from fitting the experimental value of $\rho(E0)$ for ^{144}Nd isotope ($\rho(E0) = 1.82\ (6)$ b) **[37]** and isomer shift for the same isotope $\delta < r^2 > = 0.162\,fm^2$ **[37]**, we get the best fit values of these parameters are ($\beta_{0\pi} = 0.0428\,fm^2, \beta_{0\nu} = 0.0204\,fm^2$ and $\gamma_{0\nu} = -0.044\,fm^2$). The results of IBM-2 for $\rho(E0)$ values are given in Table 6.

The $\rho(E0)$ values of IBM-2 in Table 7, there is no experimental data to compare these values. One can see these values are increased with increasing of neutron number in some isotopes; this is due to the $\rho(E0)$ proportional with the nuclear radius, (see Eq.(12)) and this because the isotopes they possess excess amount energy and that they are trying to get rid of this by lessen the E0 transitions to the state stability, this implies that these isotopes are deformed. Unfortunately, the experimental data available on E0 transitions are very rare.

To evaluate the intensity ratio $X(E0/E2)$ we depend on Eq.(14), the IBM-2 values are given in Table 7. We can observe that the values of the intensity ratios are small, for some transitions, because the small contributions of E0 in the life time of 0^+ states. The $X(E0/E2)$ are high values for the transitions $0_2^+ \rightarrow 0_1^+$ in $^{144-154}$Nd isotopes, that's implies that the decay of the 0_2^+ state by the (E0) monopole transition $0_2 \rightarrow 0_1$ is greater than B(E2) for $0_2^+ \rightarrow 2_1^+$, for this property, we could say that the this state study give information about the nucleus shape.

The IBM-2 values and available experimental data for $\delta < r^2 >$ are presented in Table 8, were the $\delta < r^2 >$ calculated in IBM-2 in satisfactory agreement have obtained to the available experimental values.

TABLE 6. Monopole transition matrix element $\rho(E0)$ for $^{144-154}$Nd isotopes.

Isotopes	$0_2 \rightarrow 0_1$		$0_3 \rightarrow 0_1$		$0_3 \rightarrow 0_2$		$2_2 \rightarrow 2_1$	
	Exp. [24]	IBM-2	Exp.	IBM-2	Exp.	IBM-2	Exp.	IBM-2
Nd-144	1.82(19)	1.99	-	0.00033	-	0.073	-	0.033
Nd-146	-	0.073	-	0.0075	-	0.00035	-	0.0054
Nd-148	-	0.079	-	0.0078	-	0.00091	-	0.0075
Nd-150	-	0.084	-	0.0081	-	0.00095	-	0.0082
Nd-152	-	0.087	-	0.0085	-	0.00098	-	0.0086
Nd-154	-	0.095	-	0.0094	-	0.0011	-	0.0095

TABLE 7. The IBM-2 *X*(*E*0/*E*2) intensity ratio for $^{144-154}$Nd Isotopes.

Isotopes	$0_2 \rightarrow 0_1$	$0_3 \rightarrow 0_1$	$0_3 \rightarrow 0_2$	$2_2 \rightarrow 2_1$
Nd-144	3.22	0.33	3.8	4.22
Nd-146	6.90	0.056	5.6	8
Nd-148	11	0.0055	10.6	9.5
Nd-150	13.7	0.0078	12	11
Nd-152	18	0.0088	16	24
Nd-154	2.5	0.0091	1.6	28

TABLE 8. Isomer shifts $\delta < r^2 >$ for 144-154Nd isotopes in fm^2 Units.

Isotope	Nd-144	Nd-146	Nd-148	Nd-150	Nd-152	Nd-154
$\delta < r^2 >$ **Exp.**	0.162 [37]	0.164 [37]	-	0.167	-	-
$\delta < r^2 >$ **IBM-2**	0.170	0.172	0.177	0.182	0.192	0.22

Conclusion

The monopole transition matrix elements $\rho(E0)$ for $^{144-154}$Nd isotopes have been investigated in details in this work within IBM-2 framework. For this study we see the following notes:

1- The IBM-2 values for energy levels for $^{144-154}$Nd isotopes were calculated by IBM-2, the agreement between the calculated and experimental data are very good for the low–lying collective states and poor for high spin states which may be due to band crossing (band mixing).

2- The energy ratios are given in Table 3, the energy ratio R_1 is increased smoothly from ^{144}Nd isotope to ^{154}Nd isotope, because far-off than the major shell. The value of this ratio is equal $R_1 = E(4_1^+)/E(2_1^+)=$ 1.890 in ^{144}Nd isotope and increased smoothly with increasing neutron number, for ^{154}Nd isotope which equal $R_1 = E(4_1^+)/E(2_1^+)=$ 3.290. From the values of energy ratios, the ^{144}Nd isotopes shows intermediate a nuclear structure in the shape transition from the spherical shape (SU(5) symmetry). The energy level ratios in $^{144-146}$Nd isotopes correspond to a spherical anharmonic vibrator, and those in $^{148-150}$Nd isotopes being a transitional nuclei lie in O(6) symmetry or $\gamma-$unstable. While the isotopes $^{152-154}$Nd characterizes a strong deformation tendency lies in SU(3) symmetry.

3- The electric transition probability rates of beta and gamma bands to ground state band were calculated are fairly good agreement with available experimental values. Concerning the electric transition rates properties in IBM-2, we find that all calculations trends is reproduced well reasonably. The effective charges for neutron bosons and proton bosons calculated within IBM-2 are depending on the IBM-2 symmetries, we get suitable values for e_π which is a constant for all $^{144-154}$Nd isotopes because the number of proton bosons is constant. The effective charge for neutron bosons varies from isotope to another.

4- The electric monopole transition strength is calculated in IBM-2. Unfortunately, the experimental data on monopole (E0) transitions are very rare (little) and also the approximate nature of theory does not make it possible to settle the question of nuclear nonaxiality. They show satisfactory agreement to the available of isomer shifts experimental values.

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