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# Modeling of Photovoltaic Systems Using Matlab

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## نمذجة المنظومة الفوتوفولتية باستخدام Matlab

يقدم هذا البحث نمذجة لعناصر منظومة القوى الفوتوفولتية. و قد تم نمذجة الخلية الشمسية رياضيا لاستنتاج الاداء الكهربى لها وهى خواص التيار و الجهد و القدرة و الجهد لكل خلية شمسية حيث يمكن استنتاجها بمساعدة هذا النموذج عند مستويات اشعاع مختلفة. هذا و يمكن ان يتم عمل امتداد لنموذج الخلية للحصول على نموذج لالواح الخلايا الشمسية وكذلك ايضا لعدة الواح من الخلايا الموصلة ببعضها و قد اوضحت المقارنة بين انتائج النظرية و النتائج العملية تطابقا جيدا بينهم.

تلعب البطارية دورا هام فى منظومة القوى الفوتوفولتية لذلك فان البحث يوضح كيفية نمذجة خلية واحدة من خلايا بطارية الرصاص الحامضية. و ينشأ نموذجان ليوضحا اداء البطارية اثناء عمليتي الشحن و التفريغ. و ينشأ هذان النموذجان كدالة فى حالة الشحن اثناء عمليتى الشحن او التفريغ. و يعطى النموذجين جهد الخلية عند اى قيمة لحالة الشحن فى البطارية الذى بدورة يكون دالة فى زمن التشغيل. هذا و يمكن عمل امتداد لهذان النموذجان ليشملا البطارية كلها حيث ان نموذج البطارية الكامل يعتمد على عدد الخلايا المكونة لها.و يعتبر النموذج الرياضى للبطارية مهم جدا لايجاد ادائها الكهربى حيث ان عملية الحصول على الاداء الكهربى للبطارية من العمليات الصعبة التى تحتاج لفترات زمنيه طويله. و قد اوضح النموذج الرياضى تطابقا جيدا مع الاداء المعملى.

يعطى النموذج الرياضى للعاكس علاقة بين الخرج و الدخل حيث ان الخرج يعتمد على الزمن و على العكس من ذلك فان دخل العاكس لا يتوقف على الزمن و يعتمد النموذج الرياضى للعاكس على كيفة التعديل السعوى للنبضة حيث ان هذا التعديل السعوى يمكن الحصول علية اما مستخدما نبضة معدله من النوع المثلثى المتساوى الساقين او نبضة معدلة من المثلث القائم. ان تكنولوجيا النبضة المعدلة يتم استخدامها من اجل تحسين شكل موجة خرج العاكس و كذلك ايضا من اجل تقليل التوافقيات التى تحتويها نبضة الخرج. و تكمن اهمية نموذج العاكس فى انة يعطى استراتيجية جيدة للاشارات التى يتم استخدامها من اجل عمل اشعال لمفاتيح العاكس و قد اظهر النموذج تطابقا جيد بين الاستراتجيه النبضة النظرية و الاستراتجية المعملية التى تم الحصول عليها من دائرة نبضة تم انشائها بالمعمل.

## Abstract

This paper presents the modeling of photovoltaic power system elements. The solar cell is modeled mathematically for determining its electrical behavior. Current voltage and power voltage characteristics of each solar cell are determined by aiding of the model at different insolation levels. The solar cell model is then extended to obtain the module along with the array model. Experimental verification of the model indicates a good fitting of the model.

The battery plays an important role in photovoltaic power system. Hence, the paper illustrates the modeling of one cell of a lead acid battery. The cell model is built up for discharging and charging processes. The two models are a function of the state of charge of the cell during charging and discharging conditions. These models give the cell voltage at any desired value of state of charge and hence against the operating time. The cell model can be extended for obtaining the whole battery model depending upon the number of battery cells. The battery models are very important for determining the battery electrical performance. For obtaining the battery performance experimentally, this needs a very large period of time for the experiments need. The models exhibit a good fitting with the battery experimental performance.

The inverter model gives the relationship between its output and input. The inverter output is time dependence. Conversely its input is independent of time. The inverter model depends upon the pulse width modulation used. The pulse width modulation can be obtained by using rather triangular carrier or ramp carrier wave. The technique of pulse width modulation is used for improving the inverter output waveform and for reducing the harmonic contents. The importance of inverter model is for giving good strategy of pulse signals used for triggering the inverter switches. The model exhibits good conforms between the theoretical pulse strategy and the experimental strategy obtained by the inverter triggering circuit constructed in the laboratory.

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## Introduction

There is an increasing interest in the analysis of photovoltaic power systems. Therefore, there is a detailed model that can be used for dynamic simulation of photovoltaic systems. A general scheme of a Stand Alone Power System based on solar energy technology is shown in Fig. 1. The main components of the system include a PV-generator, a DC/AC inverter (at the user end), and a storage battery. Starting from the analysis of the equivalent model of the system components, a complete simulation model of the system is realized in the Matlab environment. The models have been designed to be as general as possible and all the elements take the design parameters.
The research presents the models and simulation of the stand-alone photovoltaic (PV) system with a battery and inverter. The work has been supported by the experimental results. The main purpose of the research is to establish a library of simple mathematical models for each individual element of a stand-alone PV system, namely solar cells, battery and inverter. The models for PV module and battery are based on the model descriptions found in the literature. The other component models in the PV system are based on simple electrical knowledge. The implementation of the programs for obtaining the system performance is carried out using Matlab. Validation of each part of the photovoltaic system based on the implemented mathematical model is performed by an interactive analysis and comparison between simulation results and measurements, which are experimentally acquired from the stand-alone PV system.

## Description of the stand-alone PV system under modeling

The PV stand-alone system consists of PV arrays, storage battery and inverter. The first part is PV panel consists of 36 single-crystalline solar cells deliver, 75 W at excellent performance [1]. The terminals of the solar panel are connected to the batteries, second part of the system. The solar cell panel and the battery are connected to AC load via an inverter. The battery type lead-acid 12 V/ 35 Ah. Figure 1 represents the system components of the photovoltaic system.

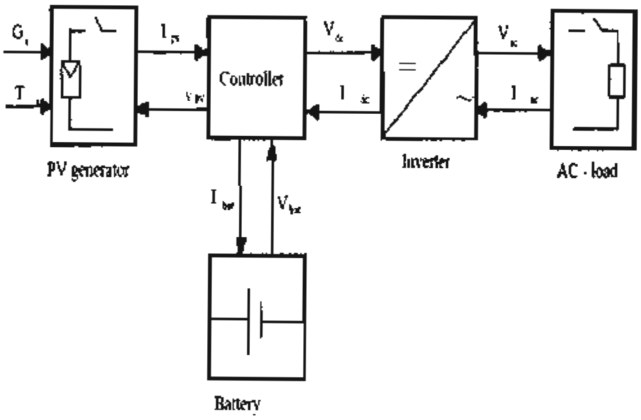


Figure 1: Block diagram for stand-alone PV system

## PV generator (cell, module, and array)

Solar cells are made of semiconductor materials (silicon), which are specially treated to form an electric field, positive on one side (backside) and negative on the other (towards the sun). When solar energy (photons) hits the solar cell, electrons are knocked loose from the atoms in the semiconductor material, creating electron-hole pairs [2]. If electrical conductors are then attached to the positive and negative sides, forming an electrical circuit, the electrons are captured in the form of electric current $I_{ph}$ (photocurrent).

## Solar cell model

The simplest equivalent circuit of a solar cell is a current source in parallel with a diode. The output of the current source is directly proportional to the light falling on the cell. The diode determines the I-V characteristics of the cell. Increasing sophistication, accuracy and complexity can be introduced to the model by adding in turn

- Temperature dependence of the diode saturation current Io

- Temperature dependence of the photo current $I_L$.
- Series resistance $R_S$, which gives a more accurate shape between the maximum power point and the open circuit voltage.
- Shunt resistance $R_P$ in parallel with the diode.
- Either allowing the diode quality factor n to become a variable parameter (instead of being fixed at either 1 or 2) or introducing two parallel diodes (one with A = 1, one with A = 2) with independently set saturation currents [3].

The equivalent circuit is shown in figure 2.

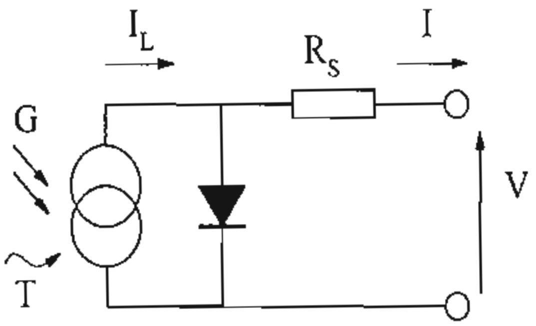


Figure 2: The circuit diagram of the PV model.

The equations which describe the I-V characteristics of the cell are according to Figure 2 are:

$$I = I_L - I_o(e^{(\frac{q}{nkT}(V+IR_s))} - 1)$$

$$I_L = I_L(T_o)(1 + K_o(T - T_o))$$

$K_o = (I_{SC}(T_{n+1}) - I_{SC}(T_n)) / (T_{n+1} - T_n)$

$G = G * I_{SC(To,nom)} / G_{o,(nom)}$ (1)

Where:

$I_L(T)$ is the photogenerated current,

$I_o$ is dark current of the diode,

$T_o$ is the standard temperature =25$^0$C,

$K_o$ is the short circuit current temperature coefficient,

T is the temperature,

G is the insolation at temperature T and,

$G_{o,nom}$ is nominal radiation (1000W/m$^2$)

$$I_o = I_{o(T_o)} * (\frac{T}{T_o})^{\frac{3}{n}} e^{(\frac{-qV_g}{nk}(\frac{1}{T} - \frac{1}{T_o}))} \quad (2)$$

Where:

N is diode quality factor and equal 2 for crystalline solar cell and <2 for amorphous

$$I_{o(T_o)} = I_{SC(T_o)} / (e^{(\frac{qV_{OC(T_o)}}{nkT_o})} - 1) \quad (3)$$

The series resistance of the panel has a large impact on the slope of the I-V curve at V = $V_{OC}$. The cell resistance is determined as following;

Differentiating equation (1) with respect to the cell voltage;

$$\frac{dI}{dv} = 0 - I_o(e^{\frac{q}{nKT_1}(V+IR_s)} * \frac{q}{nKT_1}(1 + \frac{dI}{dV}R_s) \quad (4)$$

Rearrange the equation

$$\frac{dI}{dV}(1 + I_o\frac{q}{nKT_d}R_s e^{\frac{q}{nKT_d}(V+IR_s)}) = -I_o\frac{q}{nKT_d}e^{\frac{q}{nKT_d}(V+IR_s)} \quad (5)$$

Let V=$V_{OC}$ and I=0 (intersection of I-V characteristics with V-axis)

$$\frac{dI_{V_{OC}}}{dV}(1 + I_o\frac{q}{nKT_o}R_s e^{\frac{q}{nKT_o}(V_{OC})}) = -I_o\frac{q}{nKT_o}e^{\frac{q}{nKT_o}(V_{OC})} \quad (6)$$

Let

$$X_V = I_{o(T_o)}\frac{q}{nkT_o}e^{(\frac{qV_{OC(T_o)}}{nkT_o})}$$

$$\therefore 1 + X_V R_s = \frac{-X_V}{\frac{dI_{V_{OC}}}{dV}} = -X_V\frac{dV}{dI_{V_{OC}}} \quad (7)$$

$$\therefore R_s = -\frac{dV}{dI_{V_{OC}}} - \frac{1}{X_V} \quad (8)$$

**Model accuracy**

The accuracy of the previous model (cell model) is obtained experimentally by measuring I-V characteristics of solar cells module (one panel) has the following specifications:

| At Temperature | T | 25° C |
|---|---|---|
| Open Circuit Voltage | $V_{OC}$ | 21.7 V |
| Short Circuit Current | $I_{SC}$ | 4.8 A |
| Voltage, max power | $V_m$ | 17.0 V |
| Current, max power | $I_m$ | 4.4 A |
| Maximum Power | $P_m$ | 75 W |

The mathematical models describe the solar cell equations is modified to represent the characteristics of one module constitute one array contains m cells connected in series as follows [4];

$$I = I_L - I_o\left(e^{\left(\frac{mq}{nkT}(V + IR_s)\right)} - 1\right) \quad (9)$$

$$I_L = I_L(T_o)(1 + K_o(T - T_o))$$

$$I_o = I_{o(T_o)} * \left(\frac{T}{T_o}\right)^{\frac{3}{n}} e^{\left(\frac{-mqV_g}{nk}\left(\frac{1}{T} - \frac{1}{T_o}\right)\right)} \quad (10)$$

$$I_{o(T_o)} = I_{SC(T_o)} / \left(e^{\left(\frac{mqV_{OVC(T_o)}}{nkT_o}\right)} - 1\right) \quad (11)$$

The model series resistance is evaluated by differentiating equation (9) with respect to V;

$$\frac{dI}{dv} = 0 - I_o\left(e^{\frac{mq}{nKT_1}(V + IR_s)}\right) * \frac{mq}{nKT_1}\left(1 + \frac{dI}{dV}R_s\right) \quad (12)$$

Rearrange equation (12);

$$\frac{dI}{dV}\left(1 + I_o\frac{mq}{nKT_o}R_s e^{\frac{mq}{nKT_o}(V+IR_s)}\right) = -I_o\frac{mq}{nKT_o}e^{\frac{mq}{nKT_o}(V+IR)} \quad (13)$$

let $V=V_{OC}$ and $I=0$

$$\frac{dI_{V_{OC}}}{dV}\left(1 + I_o\frac{mq}{nKT_o}R_s e^{\frac{mq}{nKT_o}(V_{OC})}\right) = -I_o\frac{mq}{nKT_o}e^{\frac{mq}{nKT_o}(V_{OC})} \quad (14)$$

$$\text{let } X_V = I_{o(T_o)}\frac{mq}{nkT_o}e^{\left(\frac{mqV_{oc(T_o)}}{nkT_o}\right)}$$

$$\therefore 1 + X_V R_s = \frac{-X_V}{\frac{dI_{V_{oc}}}{dV}} = -X_V\frac{dV}{dI_{V_{oc}}} \quad (15)$$

$$\therefore R_s = -\frac{dV}{dI_{V_{oc}}} - \frac{1}{X_V} \quad (16)$$

The last model is programmed using Matlab to obtain the theoretical I-V and power characteristics of one module at specific temperature and insolation level as shown in Figure:

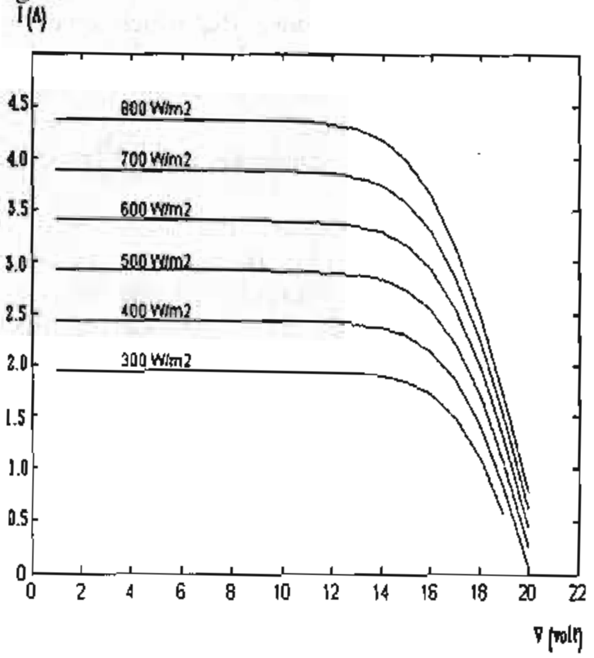


Figure 3 I-V characteristics at different insolation levels

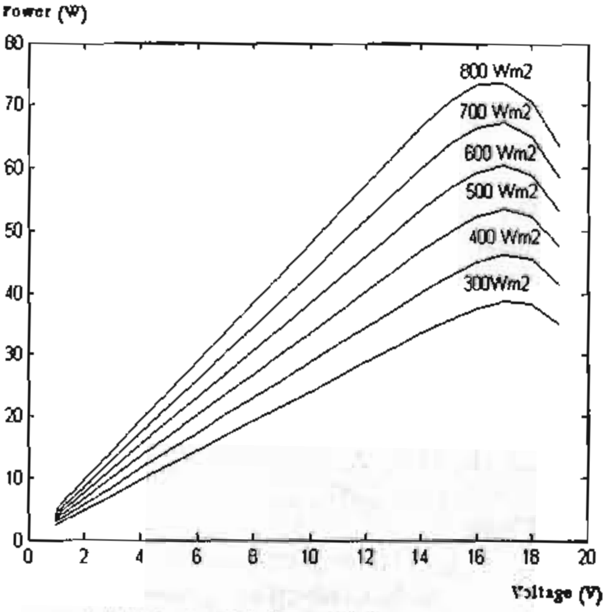


Figure 4 I-V Power-Voltage characteristics at different levels of insolation

Figure 3 illustrates the theoretical characteristics as obtained from the program representing the module model. On the other hand Figure5 shows the experimental I-V characteristics of the module at different insolation levels.

**Model validation**

The experimental and theoretical results show a good conform between the results and the maximum error is 11.3 %. Consequently the model illustrates a good fitting of the I-V characteristics of the solar cells module.

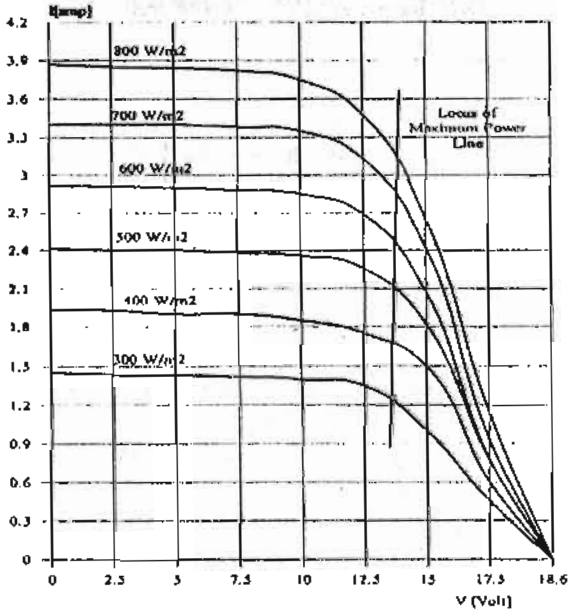


Figure 5: I-V Experimental characteristics

**Modeling of photovoltaic array**

The photovoltaic power system contains series parallel modules connected with each other for obtaining the required current and voltage suitable for specific load as shown in Figure 6. Hence, the model which describes these types of connections between the modules is derived as follows:

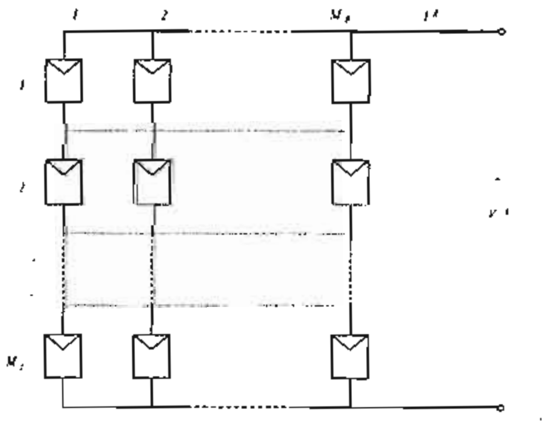

Figure 6: Solar cells array consists of $M_p$ parallel branches, each with $M_s$ Modules in series

Now suppose that the number of series and parallel modules are $M_s$ and $M_p$ respectively. Hence, the mathematical model represents the electrical behavior of these combinations of module may be written as;

$$I^A = M_p * I$$

Where:

$I^A$ is array current

The validation of the previous model is represented by the aid of the theoretical and experimental measurements of the electrical behavior as shown in figure 5.

**Modeling of the storage battery**

The lead acid battery typically used in a photovoltaic or hybrid system is a complex non-linear device. A mathematical model of this device has been developed for the purposes of photovoltaic system modeling. It is an experimental model of the current-voltage characteristics. The model has been fit to measure data from one lead acid battery, flooded-vented cell. It accounts for many aspects of the battery's behavior, including temperature dependence, self-discharge, gassing, water loss, aging, and heat generation.

The battery is the central component in the photovoltaic system. The operating point of the PV array is determined by the voltage of the battery depending on the state the battery. Simulation can be used to quickly compare and identify the control strategies even for discharging or charging. It is evident that the battery model will be at the heart of the simulation.

**Modeling the Ampere-hour Relation during discharge**

The simplest battery models treat the battery as store of energy with certain efficiency. The model will predict the battery voltage on the basis of charge or discharge current and the state of charge (SOC) of the battery. The voltage-current/SOC relations are based upon [5].

**A - Modeling state-of-charge**

The model of the battery uses SOC as a state variable. SOC is considered as a discontinuity quantity because every time the discharge current changes. Every time step, the Ah discharged variable is updated according to:

$$AhD = -\int (I - \max(I_{gas}, I_{SD}))dt + AhD_o \qquad (17)$$

During the discharging condition, both gassing and self discharging currents are neglected [6], hence:

$$AhD = -\int (Idt + AhD_o$$

Where:

AhD is the number of Ah that would have to be discharged from a fully charged battery to bring it to its current state;

I is the normalized discharge current;

$I_{gas}$ is the gassing current (zero on discharging);

$I_{SD}$ is self-discharge current (always positive);

$AhD_o$ is AhD at the beginning of the simulation or discharge.

The $AhD_o$ variable is calculated from

$$AhD_o = C_{ref} - C_{ref} SOC_o \qquad (18)$$

Where:

$C_{ref}$ is the reference cell capacity, in Ah;

$SOC_o$ is the initial state-of-charge specified by during the experimental work in term capacity per time at 25°C.

The normalized discharge current, I,(Internal generated current) is calculated by:

$$I = I_{sim} \frac{C_{ref}}{C_{sim}} \qquad (19)$$

Where:

$I_{sim}$ is the discharge current output from the model, (actual discharge current)in amps;

$C_{sim}$ is the capacity of the battery specified for the simulation (in Ah), battery capacity corresponding to $I_{sim}$ which is less than I.

**B - Modeling Battery Voltage during discharge**

When the charge current, I, is negative, the battery is discharging and the cell voltage, V, is calculated from [7]:

$$V = DV_{oc} + \frac{\partial V_{oc}}{\partial T}(T - 25) - DV_{slope}(1 - AhR) + \frac{1}{DK_T}\left(\frac{DP_1}{1 + (-I)DP_2} + \frac{DP_3}{(AhR + 0.0001)^{DP_4}}\right)(1 - DaTr(T - 25)) \qquad (20)$$

Where:

$DV_{oc}$ is a fit parameter roughly corresponding to the open-circuit voltage of the battery when it is fully charged;

T is the battery temperature (°C);

$DV_{slope}$ is a fit parameter roughly corresponding to the partial derivative of the open circuit voltage with respect to state-of-charge;

AhR is the unusable amount of charge remaining in the battery (Ah);

$DK_T$ is the discharge capacity of the reference cell at the temperature T, in Ah;

$DP_{1,2,3,4}$ are fit parameters;

DaTr is a parameter describing the change in internal battery resistance with temperature.

The variable $DK_T$ is calculated by:

$$DK = DK_T(1 + DaTc(T - 25)) \qquad (21)$$

Where:

DK is a fit parameter corresponding to the discharge capacity of the reference cell at an infinitesimal discharge current in Ah;

DaTc is a parameter describing the change in the useable battery capacity with temperature.

The term AhR is calculated by:

$$AhR = 1 - \frac{AhD}{DK_T} \qquad (22)$$

The partial derivative of the open circuit voltage with respect to temperature is calculated based on a curve fitting:

$$\frac{\partial V_{oc}}{\partial T} = \frac{\left(\begin{array}{l} -44.500655 + 120.18547\,SG^2 \\ -120.79444\,SG^4 \\ +53.966821\,SG^6 - 9.0628981\,SG^8 \end{array}\right)}{1000} \qquad (23)$$

Where:

SG is the specific gravity of the battery (assumed fully mixed, no stratification) measured at 25°C and is calculated from:

$$SG = SG_{full} - \frac{AhD(SG_{full} - SG_{empty})}{DK} \qquad (24)$$

Where:

The subscripts full and empty refer to the battery being fully charged and fully discharged.

When the charge current, I is zero, the battery is open circuited and the cell voltage, V, equals to $V_{OC}$ and is calculated as:

$$V_{OC} = -168.22968 + 174.13606SG - 1.4836919e^{SG}$$
$$-169.0027\sqrt{SG}.\ln SG - \frac{0.000077765039}{\ln(SG)} \qquad (25)$$

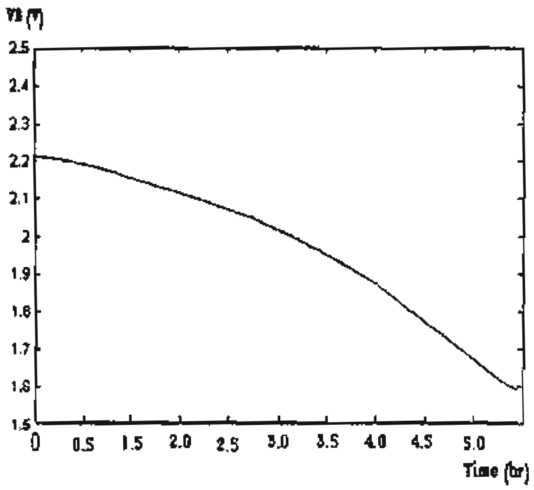

Figure 7-a: Discharge characteristics of one cell model using Matlab

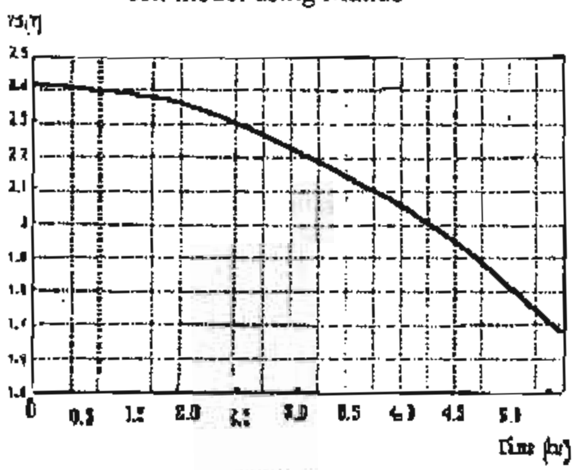

Figure 7-b: Experimental discharge characteristics of one cell

**Modeling the Ampere-hour Relation during charge**

When the charge current, I, is positive, the battery is charging and cell voltage, V is calculated from:

$$V = \min \begin{pmatrix} V_{max}, \\ CV_{OC} + \frac{\partial V_{OC}}{\partial T}(T-25) + CV_{slope} AhC \\ + \frac{I}{CK}\left(\frac{CP_1}{1+I^{CP_2}} + \frac{CP_3}{(1-AhC)^{CP_4}}\right)(1 - CaTr(T-25)) \end{pmatrix} \qquad (26)$$

Where:

$V_{max}$ is the maximum attainable voltage or the voltage that would occur with all charge current being used in the gassing reaction;

$CV_{OC}$ is a fit parameter roughly corresponding to the open-circuit voltage of the battery when it is fully discharged; $CV_{slope}$ is a fit parameter roughly corresponding to the partial derivative of the open circuit voltage with respect to state-of-charge;

AhC is the useable amount of charge remaining in the battery;

CK is a fit parameter corresponding to the capacity of the reference cell, in Ah;

$CP_{1,2,3,4}$ are fit parameters;

CaTr is a parameter describing the change in internal battery resistance with temperature.

The useable amount of charge remaining in the battery is calculated from:

$$AhC = 1 + \frac{AhD}{CK} \qquad (27)$$

The AhD is calculated from equation 18 based on positive value of the normalized current.

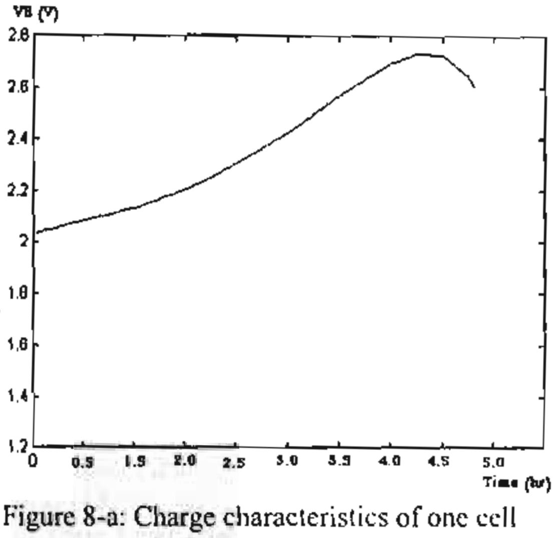


Figure 8-a: Charge characteristics of one cell model using Matlab

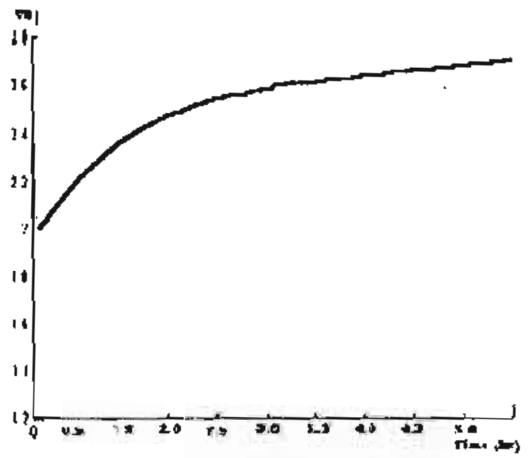

Figure 8-b: Charge characteristics of one cell model using Matlab

**Modeling of Inverter**

**Sinusoidal Pulse Width Modulation Model**

The famous technique used for obtaining pure sine wave output of the inverter is the pulse width modulation technique. The output voltage waveform for sinusoidal-pulse width modulation (pulse duration modulation) is illustrated in Figure 9. In this Figure, the duration of each pulse changes sinusoidally according to its position in the sinusoidal waveform [8]. The Figure represents how to determine the duration of each pulse. For obtaining pure sine form output of the inverter two waves must be employed. The base wave is a sinusoidal wave with frequency equal to the inverter frequency (called as a modulated function). On the other hand, the carrier waveform called the modulating function is employed as a ramp or triangular waveform. The last waveform is the best one because it gives many pulses greater than that obtained by the ramp during each period. The control function is the result of intersection between the modulating function (triangular) and the modulated function (sine wave). This function consists of a train of signals sinusoidally-variable pulse width generated for the triggering and the operation of the inverter.

The duration of each pulse of the control function is represented by:

$$d(t)=m_o\,|\sin 2\pi ft| \qquad (28)$$

Where:

$d(t)$ is the instantaneous value of the duty ratios of the generated pulse corresponding to time t which is related to the ratio of (2 pulse duration)/T, T=1/f. this duty ratio of the instantaneous pulse is written as in (28) ;

$m_o$ is the modulation index which is selected in the range of $0<m_o<1$;

and f is the desired frequency from the inverter.

The voltage output of the inverter is a time varying function is each half wave. This voltage depends upon the duty ratio of each pulse corresponding to instantaneous time t. This voltage is given by:

$$V_o = d(t)V_B \qquad (29)$$

Where:

$V_o$ is the output voltage from the inverter;

and $V_b$ is input DC (batteries) voltage.

If the duty ratio d(t) is made to vary slowly with respect to the switching frequency, the local average value of the voltage is given by:

$$\bar{V}_o = \bar{d}(t)V_B \qquad (30)$$

The chopped voltage is according to the time varying modulation pulses d(t). The output of the pulse width modulator is shown in Figure 9 must be similar to the triggering pulses required for triggering the power switches of the inverter. The instantaneous value of the output voltage of the inverter is given by [9];

$$V_o = \begin{cases} +d(t)Vb & +ve \\ -d(t)Vb & -ve \end{cases} \quad (31)$$

The control function can be modulated as follows:

1. Create a sine wave with the frequency equal to the desired inverter frequency (f=50) and variable amplitude A.

$$c(t) = A \sin(2\pi ft) \quad (32)$$

2. Set the amplitude ($A_p$) of the modulating signal (triangular wave) such that:

$$2A_p > A \quad (33)$$

3. Set the frequency ($f_c$=fcarrier) of the modulating signal equal to the switching frequency.

4. Define the period of the modulating signal as;

$$T_c = \frac{1}{f_c} \quad (34)$$

Where:

$T_c$ is the period of the modulating signal.

5. The instantaneous value of the modulating signal is given by;

$$m(t) = 2A_c\left[\left(\frac{t}{T_c}\right) - floor\left(\frac{t}{T_c}\right) - A_c\right] \quad (35)$$

Where:

m(t) is the function representing the modulating triangular signal.

The generation of the PWM waveform will depend on the triangular and instantaneous amplitude of the sine wave.

6. The comparison between the instantaneous value of the triangular and sine waves gives the sequence of the triggering pulses of the inverter switches, hence;

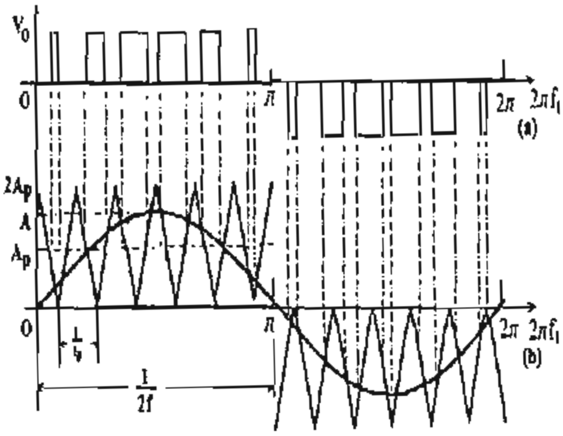


Figure 9: Output waveform with sinusoidal-pulse modulation

$$S(t) = \begin{cases} E \ at \ m(t) < c(t) \\ 0 \ at \ m(t) > c(t) \end{cases} \quad (36)$$

Where E is the voltage of the triggering signal (approximately 5V)

The number of pulses output of the inverter which are generated during a half cycle of the output waveform is;

$$N = \frac{f_c}{2f} = \text{Integer} \quad (37)$$

This number is also equal to the number or pulses of the carrier wave during half cycle of the output wave. The output voltage is controlled by varying the amplitude A over a range $0 \le A \le Amax$, where $Amax < 2A_c$. if Amax is made very large, within the limit, output voltage $V_o$ approaches to the rectangular waveform.

The determination of the harmonic amplitudes is relatively complicated, however, it is found that for $0 \le A/A_c \le 2$ all harmonics of order n<2N are eliminated. For $A/A_c > 2$, low-order harmonics appear. Since pulse width is no longer a sinusoidal function of the angular position of the pulse. For the PWM technique, the frequency harmonics are in the area of the switching frequency (equal to carrier frequency). The content of the harmonic is determined according to the frequency of the triangular carrier wave m(t). The harmonic frequency $f_n$ is given by:

$$f_n = \left(j\frac{f_c}{f_1} \pm k\right)f_c \quad (38)$$

j, k are always odd.

The triggering pulses of the inverter are represented in figure 12 which is similar to pulses switching triggering obtained by using ramp signal as a carrier waveform. the ramp signal in figure 10 has an amplitude such that $A_r=2A$, where A is the amplitude of the modulated signal shown in figure 11. Figure 12 conforms to figure 9. Hence, the modulating signal used in the pulse width modulation technique may be a ramp or triangular wave form.

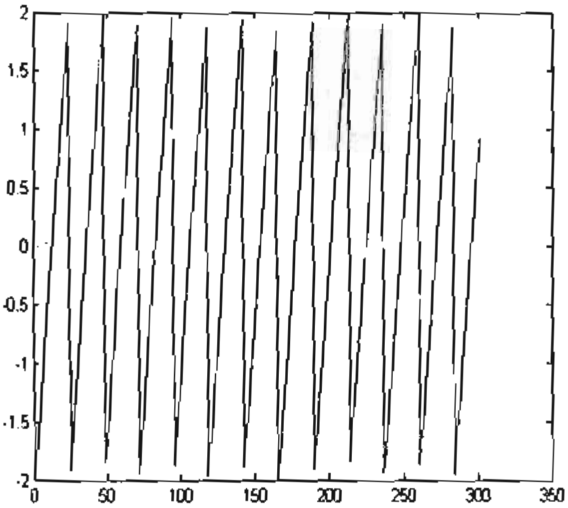

Figure10: Sawtooth modulating signal

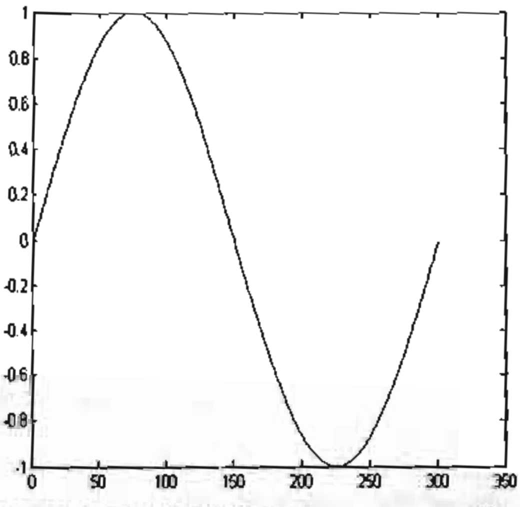

Figure11: modulated sine wave corresponding to inverter output frequency

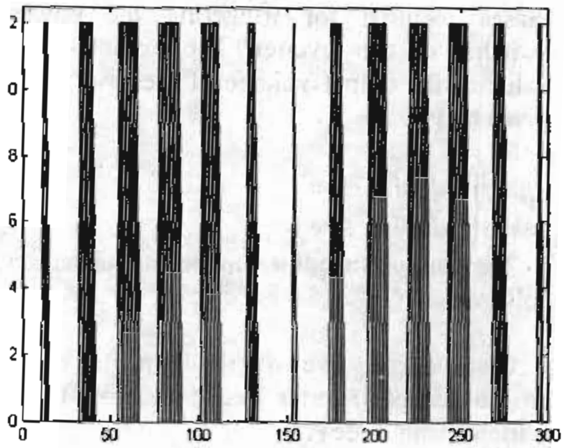

Figure 12: triggering pulses train for inverter switches

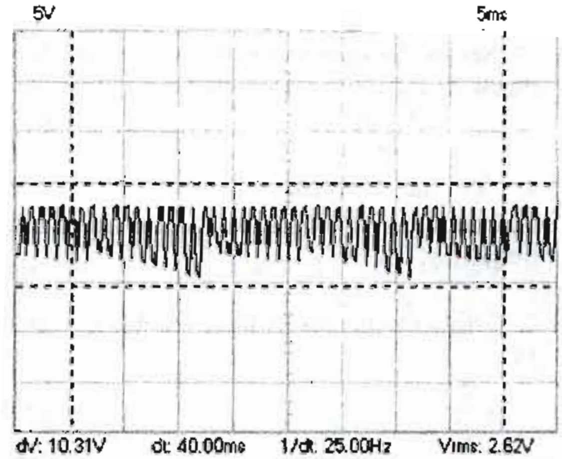


Figure 13: experimental triggering pulses train for inverter switches

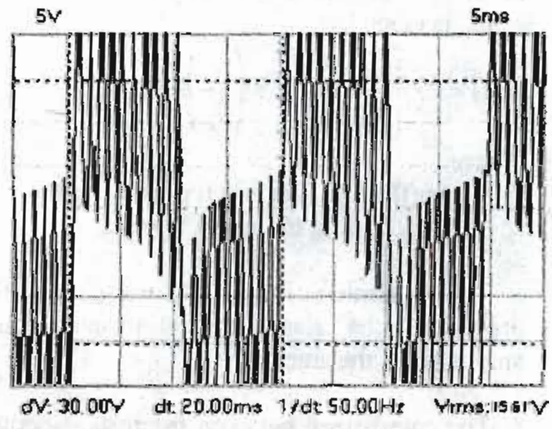


Figure 14: Output Voltage of PWM inverter

The figure shows that, the voltage output of the two half of the wave is a sinusoidally time varying. Consequently, by using suitable filter (capacitor), pure sine wave is delivered from the inverter.

**Model validation**

The device circuit of inverter switches is built up in the laboratory for giving suitable pulses to the inverter switches. Train of pulse is recorded experimentally as shown in figure 13. The comparison between figure 13(experimentally train of pulses) and figure 12 shows the correspondence between them. Figure 12 illustrates that the pulse duration time varying sinusoidally. Figure 13 also shows that, the duration of the pulses in the train has a sinusoidally time varying also. The output voltage of the inverter is recorded as shown in figure 14.

**Conclusions**

This paper presents the mathematical models of photovoltaic power system elements. The solar cell model is illustrated mathematically. The model gives the I-V characteristics of a solar cell as well as its output power-voltage characteristics at different insolation levels. The model can be extended for giving the mathematical model of any connected arrays. The array model is validated by measuring array characteristics experimentally and comparing them with those obtained by using the mathematical model. The comparison shows a good fitting between them.

The paper presents the modeling of a storage battery which is the important element of the photovoltaic power system. The battery has two distinct models during discharging and charging conditions. The two models illustrate that, the battery terminal voltage during discharging and charging conditions are a function of the battery state of charge. Hence the battery voltage can be derived as a function of state of charge which is in turn a function of time. The validation of the two models is obtained experimentally by recording the discharge, charging characteristics of the battery experimentally against time. The comparison between the results obtained from the battery models and that obtained experimentally show a good fitting between each other.

For obtaining pure sinusoidal wave from the inverter, the triggering pulses of the inverter switches must be modulated sinusoid ally. Consequently, the mathematical model of pulse width modulation inverter is illustrated in this paper. The output voltage of the inverter is modeled sinusoidally. The triggering pulses of the inverter power switches are also modeled such that their durations are changed sinusoidally. The paper presents the triggering pulses obtained by using the model and that obtained experimentally. The results show that the correspondence between the model and the experimentally work. The inverter output obtained experimentally is illustrated in this paper. This voltage as illustrated is a sinusoidally time-varying.

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