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OPTIMAL DESIGN FOR COMPOSITE GIRDER UNDER BIAXIAL BENDING

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This paper presents an efficient computer-based method for optimal criteria design of composite girder under biaxial bending. The width, depth for concrete slab and steel section are taken as the design variables. The strength constraints for the design are formulated using the finite element method. The method solves composite girders taking into consideration the material non-linearity due to the change in stress-strain curves of steel and concrete, and geometric non-linearity due to the change of the path of the composite girder during deformation. The formulation depends on the principle of Virtual Work. An optimality criteria method is applied to minimize the cost of concrete slab, steel, and form subject to constraints on strength and stiffness. Four full composite girder examples are presented to illustrate the features of the design optimization method.

It is shown that the design method provides an effective iterative optimization strategy that converges in relatively few cycles to a least-cost design of reinforced concrete element satisfying all relevant requirements of the governing design code. The iterative process is insensitive to the selected initial design and converges smoothly to a final design involving concrete slab dimensions and steel section consistent with usual design practice. A complete computer program has been developed to solve the problem of full composite-beams under biaxial bending.

KEYWORDS: *Composite girders, Concrete-slab, Finite element, Material and Geometric non Linearities, Incremental loading, Virtual work, Optimization.*

INTRODUCTION

Considerable research can be found in the structural optimization literature that has focused on reinforced concrete structures. Many studies have been concerned with the optimization of cross-section dimensions because of the repeated use of standard reinforced concrete members in prefabricated construction (e.g. Chou 1977, and Friel 1974). Similar studies have considered individual construction elements such as shear walls, retaining walls, plates, and slabs (e.g. Hajek and Frangopol 1991; Rhomberg and Street 1981). Still other optimization studies have accounted for plastic behavior in reinforced concrete frameworks (e.g. Cohn and Mac Rae 1984). In their work, the objective is to achieve minimum structure cost through redistribution of member forces while satisfying all equilibrium, serviceability, and compatibility conditions for the structure. Optimum member capacities are determined rather than optimum cross-sectional dimensions of individual members. Another type of optimization problem is concerned with the optimal design of the cross sections of reinforced concrete members within the context of the assembled structure. Elastic behavior of the structure is generally assumed, and the width, depth, and steel reinforcement for member's cross sections are taken as the design variables (e.g. Kanagasundaram and Karihaloo 1990). To this point studies concerned with this design problem have used various types of formal mathematical programming (MP) algorithms to conduct the optimization with varying degrees of success.

The present paper is concerned with the latter design optimization problem noted in the foregoing discussion. Specifically, the optimal determination of section dimensions and reinforcement within the context of an assembled reinforced concrete framework under gravity and lateral loads. Such a design problem involves numerous design variables and constraints, even for modest-size structures, which is perhaps the main reason why formal MP optimization techniques have had limited success in achieving a solution for practical frameworks (i.e. because the basis matrix generally reaches a prohibitive size for the numbers of variables and constrains involved for such structures). On the other hand, the optimality criteria method (Venkayya 1989) is readily applied for the solution of large-scale optimization problems involving many design variables and constrains (primarily because the variable values are established one at a time through a recursive procedure).

Moharrami and Grierson 1993 suggested the optimal criteria (O.C.) which were adopted herein as it has the advantage of converging rapidly compared to other methods and achieving good results. Due to the efficiency of the method, it was adopted in several researches, Chun- Man Chan 2001, used the O.C. method for optimum lateral stiffness design of tall steel and concrete building. The method was applied to an 88-storey building in Hong-Kong. Also Chun-Man and Qian Wang 2006 applied the optimal criteria method and presented a formwork example.

Yasir I. Musa, and Manuel A. Diaz, M. 2007 are studys the composite girders consisting of concrete deck on built-up girders are frequently used in bridge construction for their economic advantages. The use of composite girders results in a very economical design. Additional savings can be obtained in design and material costs for some members by automating design approaches based on optimization techniques. The other describes the use of EXCEL Solver to find the minimum weight for a composite trapezoidal box cross section for a two lane bridge. Design aid tables were generated for structural steel Grades 250, 345, 485, and 690 MPa, and different spans varying from 3.0 — 100 m. The search for the minimum cross section used in this research satisfies the 17th Edition of the American Association of State Highway and Transportation Officials Specifications Load Factor Design method.

Multi Science Publishing 2009, are study the structural optimization seeks the selection of design variables to achieve within the limit (constraints) placed on the structural behaviour, geometry or other factors; its goal of optimality defined by the objective function for specified loading conditions. The three basic features design variables, objective function and constraints contrive to form the design problem. There are several mathematical techniques to solve such problems. The polynomial optimization technique is a recently evolved procedure which is concerned with finding the minimum of a polynomial objective function subjected to constraints. A structural design problem has been formulated in this manner which enables minimum cost design to be derived rapidly and simply. It deals with the application of Polynomial optimization technique to Reinforced Concrete (R.C.) beam-member design problem. In the present study this technique is used to determine the minimum cost of reinforced concrete members by considering several design variables such as breadth, depth, area of reinforcing steel etc. Since it is difficult for the designer in the office to become familiar with the mathematical computation required, further attempt is made to represent the resulting optimum design expressions in the form of "Nomograms" which will facilitate the work in the design office.

Shan Suo Zheng, Huan Juan Lou, Lei Li, Zhi Qiang Li, Wei Wang 2011are studes the optimization methodology of the steel-concrete composite beam. The objective function is the cost of the composite beams, and the design variables are the geometry parameters, including height and width of the concrete deck, as well as thickness of the steel flange and web. The constraint conditions are main requirements stated in Chinese code for the design of composite beam, reasonable calculating theories and indispensable constructions, as well as some mature and consistent conclusions confirmed by experimental studies. Stiffness reduction coefficient is used to consider the effect of bond-slip between concrete and steel when calculating the beam deformation. The optimization for composite beam under uniform loads is given as a demonstration example finally. The methodology proposed should be useful for obtaining the solution of this kind of optimization problem.

Therefore, this paper gives the details of the method and presents a computer-based program achieving the minimum cost of full composite girders under biaxial bending. The optimum width, depth, and steel section of girder sections are sought, while ensuring that stresses for girder are within acceptable limits. The explicit design optimization problem is first formulated including the corresponding design sensitivity analysis and then the details of the OC method and design optimization procedure are given. Finally, four full composite girders examples are presented to illustrate the features of the design method. Moreover a design formula expressing the minimum cost was deduced by the writer.

CHARACTERISTICS OF COMPOSITE GIRDER SECTION

The basic assumptions for the analysis of composite girders in the present analysis are there exists a full composite action or (complete bond) between steel and concrete slab, the strain distribution across the section is assumed to be linear (the plane section before bending remains plane after bending), neglected the effect of shear deformations, torsion deformations, shrinkage and creep of concrete.

The stress strain relationships used in the present work for concrete slab and steel are given by El-Shaer 1997.

DESCRIPTION OF THE FULL COMPOSITE GIRDER

The full composite cross-section studied is shown in Fig. 1 where a force F_z is considered to act at eccentricities e_y and e_x.

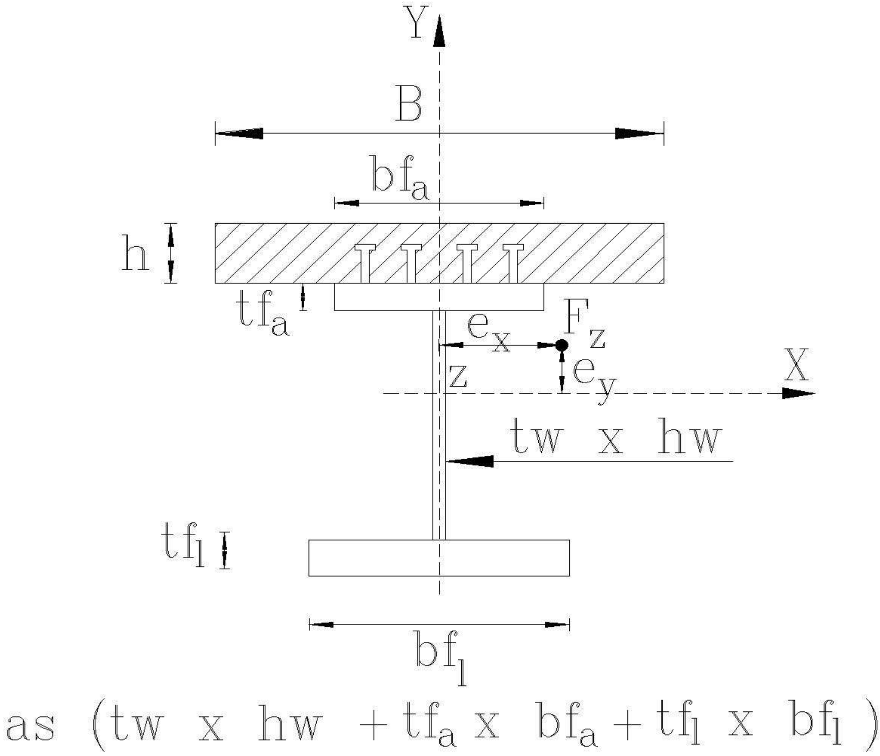


Fig. 1: Geometric Configuration of full composite girder

EXPLICIT DESIGN PROBLEM

Consider a composite girder whose section for concrete slab is of width b, height h and area of steel beam a_s the following is the optimization problem.
Minimize:

$$Z= C_c [bh + a_s (C_s - 1) + C_f (2 b + 2h)]L \quad (1)$$

Subject to:-

$$F_z – F_{zn} \leq 0 \quad (2)$$

$$M_x- M_{xn} \leq 0 \quad (3)$$

$$My- Myn \leq 0 \quad (4)$$

$$b_l \leq b \leq b_u \,;\, h_l \leq h \leq h_u \,;\, a_{sl} \leq a_s \leq a_{su} \quad (5)$$

where
Z= the cost; C_c= Cost of unit volume of concrete; C_s= ratio of cost of unit volume of steel to the cost of unit volume of concrete; C_f= ratio of cost of unit area of formwork to cost of unit volume of concrete; F_z, M_x and M_y= internal forces acting on the section concerned; the forces are the axial force, moment about x-axis and moment about y-axis respectively; F_{zn} , M_{xn} and M_{yn}= the corresponding nominal forces.
b_l , b_u , h_l , h_u , a_{sl} and a_{su} the lower and upper bounds of b, h and a_s .

Equations (2 to 4) can be generalized as:

$F - S \leq 0$ (6)

Where

F= the internal forces (F_z , M_x , M_y); S= the strength of the section (F_{zn}, M_{xn}, M_{yn}).

FORCE AND STRENGTH SENSITIVITIES

For the purpose of this study, adopt the variable notation:

x_1=b, x_2=h, x_3= a_s (7)

Also, adopt a first - order Taylor series expansion to Eqn. (6) to obtain:

$$F^0 - S^0 + \sum_{K=1}^{3} \left(\frac{\partial F^0}{\partial X_K} - \frac{\partial S^0}{\partial X_K} \right) \left(X_K - X_K^0 \right) \leq 0 \quad (8)$$

Where

superscript zero (0)= known or calculated quantities for the current design (eg. initial trial design)

X_k = the design variables ; k= 1, 2, 3.

The derivative $\frac{\partial F}{\partial X_K}$ is the internal force sensitivity to the design variables X_k.

The derivative $\frac{\partial S}{\partial X_K}$ is the strength sensitivity to the design variables X_k.

The sensitivities may be evaluated using the finite-difference technique as follows:

Consider the composite girder axial force capacity, F_{zn} , for the current design variables {b, h and a_s} and the six neighboring designs {b+ δb, h+δh, a_s+ δa_s} and {b-δb, h-δh, a_s-δa_s } where δb, δh and δa_s are small specified increments in the design variable. The sensitivities of the composite girder axial force capacity are then found as:

$$\frac{\partial F_{zn}}{\partial b} = \frac{F_{zn}(b + \delta b) - F_{zn}(b - \delta b)}{2\delta b} \quad (9)$$

$$\frac{\partial F_{zn}}{\partial h} = \frac{F_{zn}(h + \delta h) - F_{zn}(h - \delta h)}{2\delta h} \quad (10)$$

$$\frac{\partial F_{zn}}{\partial a_s} = \frac{F_{zn}(a_s + \delta a_s) - F_{zn}(a_s - \delta a_s)}{2\delta a_s} \quad (11)$$

The other force and strength sensitivities are determined using the same procedure.

OPTIMALITY CRITERIA METHOD

The optimization problem can be expressed as minimize:

$\mathbf{Z} = Z(X_k)$ (12)

Subject to:

$g_j(X_k) \leq 0$ (j=1,……..m) (13)

$\mathbf{X}_k^{\mathbf{L}} < \mathbf{X}_k < \mathbf{X}_k^{u}$ (14)

Where equations (12, 13 and 14) correspond to equations (1, 6 and 5) respectively.

The design optimization problem can be reformulated as the minimization of the Lagrangian function

$$L(x_k, \lambda_j) = Z(x_k) + \sum_{j=1}^{m} \lambda_j g_j(\mathbf{X}_k) \quad (15)$$

Where the Lagrange multipliers are such that $\lambda_j > 0$ if constraint j is active or $\lambda_j = 0$ if constraint j is inactive. Differentiate (15) *w.r.t.* the design variables ($\mathbf{X}_k$) and rearrange the terms to obtain

$$1 = -\sum_{j=1}^{m} \lambda_j \left[\left(\frac{\partial g_j}{\partial X_k} \right) \Big/ \left(\frac{\partial z}{\partial X_k} \right) \right] \quad (16)$$

Multiply both sides of Eq. (16) by X_k and take the η_{th} root and then, apply a first order binomial expansion to obtain

$$X_K^{\nu+1} = X_K^{\nu} \left[1 - \frac{1}{\eta} \left[1 + \sum_{J=1}^{m} \lambda_J \left[\left(\frac{\partial g_j}{\partial X_k} \right) \Big/ \left(\frac{\partial z}{\partial X_k} \right) \right] \right] \right] \quad (17)$$

Where

η= step-size parameter that controls convergence. $\upsilon+1$ and υ indicate successive iterations. Consider the change Δg_l in the 1th constraint due to changes ΔX_k in the design variables ie,

$$\Delta g_l = g_l\left(X_k^{\nu} + \Delta X_k\right) - g_l\left(X_k^{\nu}\right) = \sum_{k=1}^{3} \frac{\partial g_l}{\partial X_k} \Delta X_k \quad (18)$$

from Eqns. (17 and 18) we deduce that

$$\Delta X_k = X_k^{\nu+1} - X_k^{\nu} = \frac{-X_k^{\nu}}{\eta} \left[1 + \sum_{j=1}^{m} \lambda_j \left(\frac{\partial g_l}{\partial X_k} \right) \Big/ \left(\frac{\partial Z}{\partial X_k} \right) \right] \quad (19)$$

We have from Eqns. (18 and 19) that

$$\sum_{j=1}^{m} \lambda_j \sum_{k=1}^{3} x_k^{\nu} \left[\left(\frac{\partial g_l}{\partial X_k} \right) \left(\frac{\partial g_j}{\partial X_k} \right) / \left(\frac{\partial Z}{\partial X_x} \right) \right] = \eta g_l\left(X_k^{\nu}\right) - \sum_{k=1}^{3} X_k^{\nu} \frac{\partial g_l}{\partial X_k} \; (1=1, \; \ldots m) \quad (20)$$

The optimization problem is solved using Eq. (17) and Eq.(20) in an iterative procedure. However the components of the gradient vector $\partial Z/\partial X_k$, ∂g_j, ∂X_k are - replaced by the normalized forms.

$$\frac{\partial Z}{\partial X_k} = \|\nabla z\| \frac{\partial \overline{Z}}{\partial X_k} \quad (21)$$

$$\frac{\partial g_j}{\partial X_k} = \|\nabla g_j\| \frac{\partial \overline{g_j}}{\partial X_k} \quad (22)$$

where:

$$\|\nabla Z\| = \sqrt{(\partial Z / \partial b)^2 + (\partial Z / \partial h)^2 + (\partial Z / \partial a_s)^2}$$

and

$\|\nabla_{gj}\|$ is computed in the same sense eg.

$$\left\|\nabla F_x\right\| = \sqrt{(\partial F_x / \partial b)^2 + (\partial F_x / \partial h)^2 + (\partial F_x / \partial a_s)^2}$$

Therefore, equations (17 and 20) respectively yield to the two following equations.

$$X_k^{v+1} = X_k^v \left\{ 1 - \frac{1}{\eta} \left[1 + \sum_{j=1}^{m} \Lambda_j \left(\frac{\partial \overline{g}_j}{\partial X_k} / \frac{\partial \overline{Z}}{\partial X_k} \right) \right] - \right\} \tag{23}$$

and substituting from Eqns. (21 and 22) into Eq. (20), the norma1ized system of linear equations in terms of Lagrange variables is

$$\sum_{j=1}^{m} \Lambda_j \sum_{k=1}^{3} X_k^v \left[\left(\frac{\partial \overline{g}_l}{\partial X_k} \right) \left(\frac{\partial \overline{g}_l}{\partial X_k} \right) / \left(\frac{\partial \overline{Z}}{\partial X_x} \right) \right] = \frac{\eta g_l \left(X_k^v \right)}{\left\| \nabla g_l \right\|} - \sum_{k=1}^{3} X_k^v \frac{\partial \overline{g}_l}{\partial X_k} \tag{24}$$

where the normalized Lagrange Variables are

$$\Lambda_j = \lambda_j \frac{\left\| \nabla g_j \right\|}{\left\| \nabla Z \right\|} \tag{25}$$

The Gauss-Seidel technique is applied to solve Eq. (24) for the Lagrange variables Λ_j. The Gauss-Seide1 technique involves an iterative procedure given by:

$$\Lambda_j^{\pm 1} = \frac{1}{e_u} \left(b_l \sum_{j=1}^{l-1} e_{lj} \Lambda_j^{=+1} - \sum_{j=l+1}^{l-1} e_{lj} \Lambda_j^{-} \right) \tag{26}$$

noting that Λ^-_j and Λ^{-+1}_j in the R.H.S. of Eq. (26) are the old and new Lagrange variables respectively where from Eq. (24)

$$e_{ll} = \sum_{k=1}^{3} X_k^v \left[\left(\frac{\partial \overline{g}_l}{\partial X_k} \right) \left(\frac{\partial \overline{g}_l}{\partial X_k} \right) / \left(\frac{\partial \overline{Z}}{\partial X_k} \right) \right] \tag{27}$$

$$e_{lj} = \sum_{k=1}^{3} X_k^v \left[\left(\frac{\partial \overline{g}_l}{\partial X_k} \right) \left(\frac{\partial \overline{g}_j}{\partial X_k} \right) / \left(\frac{\partial \overline{Z}}{\partial X_k} \right) \right] \tag{28}$$

$$b_l = \eta \frac{g_l \left(X_k^v \right)}{\left\| \nabla g_l \right\|} - \sum_{k=1}^{3} X_k^v \left(\frac{\partial \overline{g}_l}{\partial X_k} \right) \tag{29}$$

DESIGN OPTIMIZATION PROCEDURE

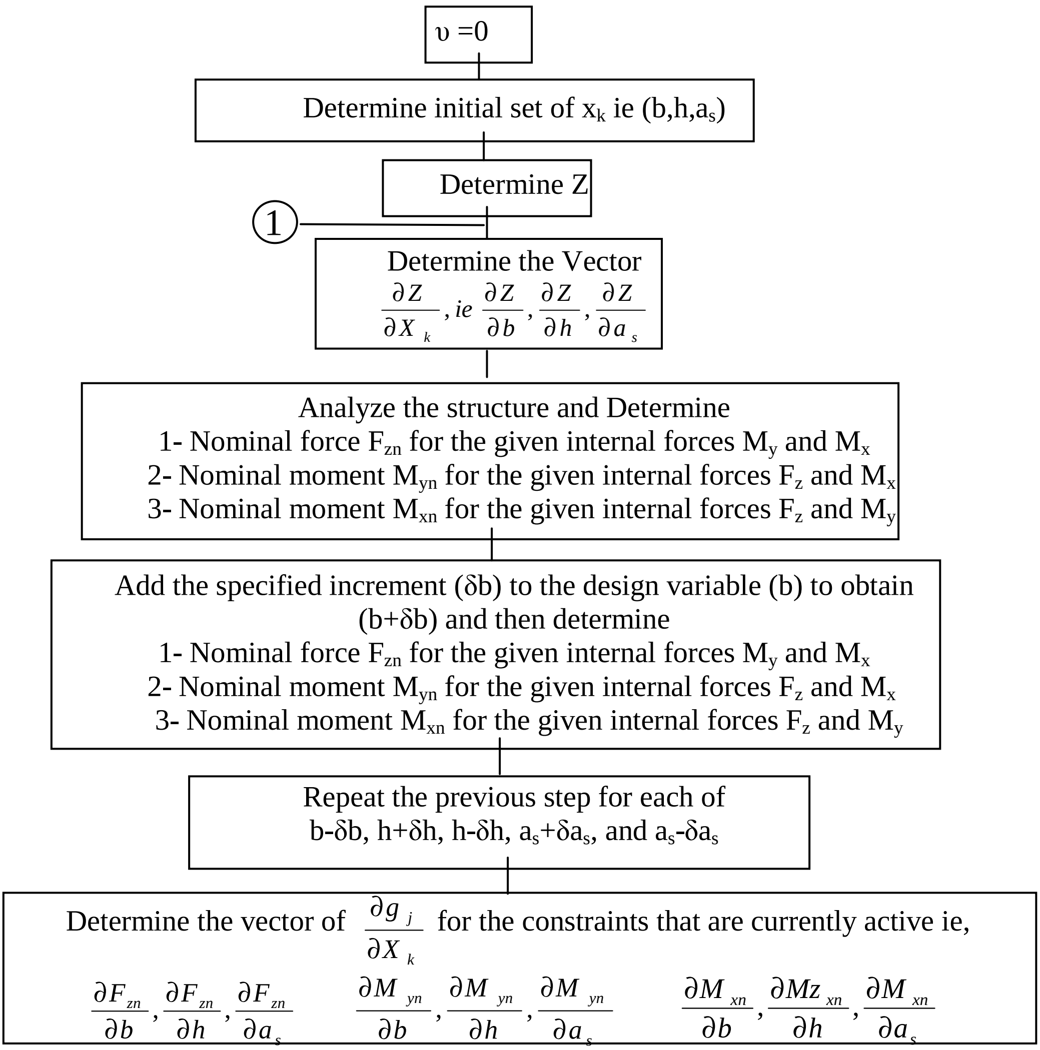
The following arc the steps of design:

1. Set $\upsilon = 0$ and adopt on initial set of design variables X_k
2. For the current X^{υ}_k ,establish the gradient vector $\partial Z / \partial X_k$
3. For the current X_k , analyses the structure and establish the gradient vectors $\partial g_j / \partial X_k$ (j= 1, …..m) for the m constraints that are currently active.
4. For the current active X_k^v ; use Gauss-Seidel technique Eq.(29) to solve Eq.(24) for the set of Lagrange multipliers Λ^v_j . When convergence of the Gauss- Seidel technique has occurred such that $\Lambda^-_j = \Lambda^{-+1}_j$ the solution of Eq. (24) has been found as $\Lambda^v_j = \Lambda^{-+1}_j$

5. For the current active X_k^v and current Λ^v_j , find the new set of active design variables x_k^{v+1} from Eq (23).
6. If all $x_k^{\upsilon+1} = x_k^{\upsilon}$ and $\Lambda^{\upsilon}_j = \Lambda^{\upsilon -1}_j$, go to step 7; otherwise set $\upsilon = \upsilon +1$ and update Eq (24). For the current xk υ values and return to step 4.
7. If the cost is the same for two successive design cycles, terminate with the minimum cost, otherwise set $\upsilon = 0$ and return to step2.

A computer program was developed by the writer to solve the optimization problem the flow-chart of the program is given in Fig. 2.

The optimal criteria, (O.C.), is adopted herein to solve several composite girders under biaxial bending. It is shown that the O.C. provides an effective iterative optimization strategy that converges in relatively few cycles to the least cost. The convergence is achieved whether the start point is feasible or infeasible. Also, a comparison between the O.C. and the penalty function method is held to show the difference of the rate of convergence of the two methods.



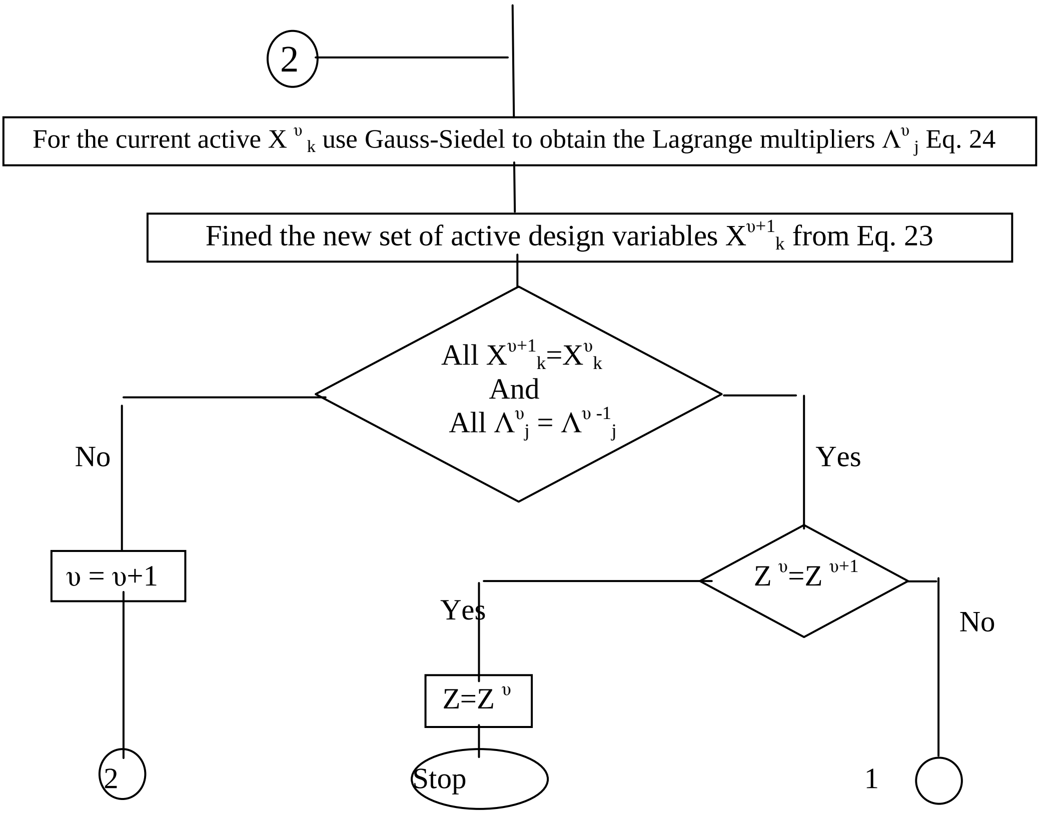
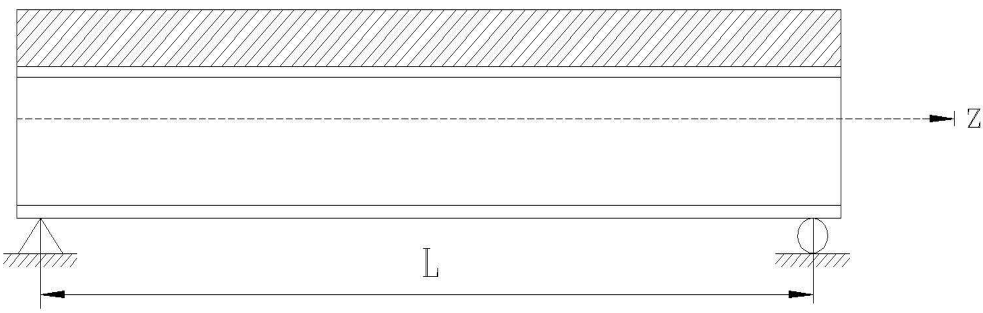


Fig. 2 Flow chart of optimization program

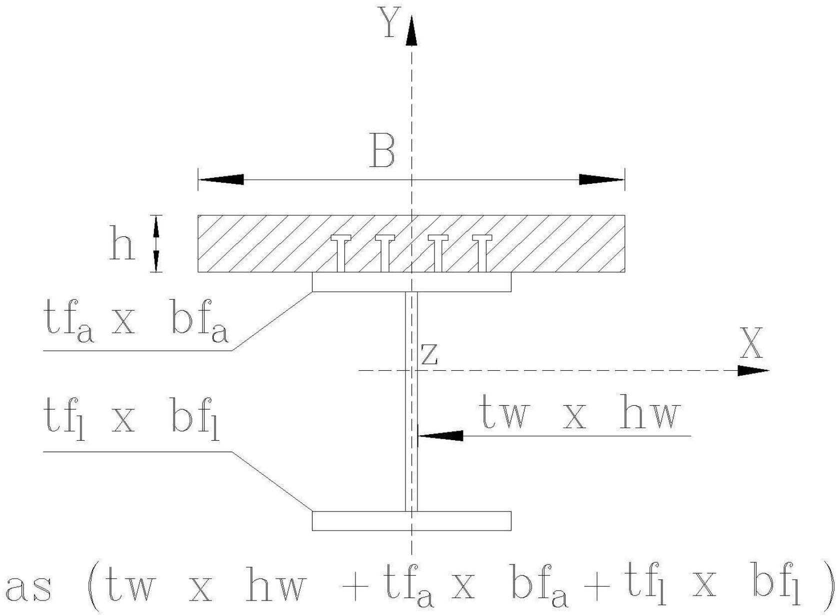
EXAMPLES FOR COMPOSITE GIRDER SOLVED BY O.C

Composite girder *1(CG1):*

The first problem solved, herein, is a full composite girder for length and cross-section is shown in Fig. 3. The cross-section has the following properties:



a-Elevation of Composite girder



b- Girder Cross-section

Fig. 3 Composite Girder (CG1& CG2& CG3& CG4& CG5)

f_{sy} = 3.6 E 4 t/ m^2, $f_{c'}$= 2550 t /m^2 ,E_s = 2.06E7 t/m^2.

The composite beam is subjected to the forces Fz= 400 t, M_x=50 mt. and M_y=20 mt. The design variables are the width, height of the concrete slab b, h and area of steel a_s. The design optimization problem is to find the values of the design variables such as to minimize the cost of the composite girder, accounting for the costs of concrete slab, steel and formwork while satisfying constraints given in Eqns. (2 to 5).

The ratio of the unit volume cost of steel to that of concrete is taken as 60, while the ratio of unit area cost of shattering to the unit volume cost of concrete is 0.6. The design optimization problem has the following objective function, strength and sizing constraints:

Minimize Z= [b h + (60-1) a_s +2*0.6 (b *h)] L (30)

Subject to

$F_z \leq F_{zn}$ (31)

$M_x \leq M_{xn}$ (32)

$M_y \leq M_{yn}$ (33)

0.80m < b < 3.00m (34)

0.05m < h < 0.50m (35)

20 cm^2 < as < 500 cm^2 (36)

Eq.(30) is the objective function, Eqns (31 to 33) are constraints on the axial force, moments about x-axis and y-axis respectively. Eqns (34 to 36) are sizing constraints on concrete slab section dimensions and steel area. The steps presented hereafter are followed to solve the problem.

1- Set $\upsilon = 0$, where υ is the counter of iterations and start with the design variables b = 1.20 m, h= 0.10 m and a_s = **(0.8x22.0+2x1.0x15.0)=47.60** cm^2 .

2- For the current X_k, where $X_k = \{b,h,a_s\}$,establish $\partial Z/\partial X_k$

where

$$\partial Z/\partial b = [\, h + 2*0.6(h)]L \tag{37}$$

$$\partial Z/\partial h = [b + 2*0.6(b)]L \tag{38}$$

$$\partial Z/\partial a_s = [60 - 1]L \tag{39}$$

3- The strength gradient $\partial S/\partial X_k$ is found using the interaction diagram presented in details as follows:

The axial force capacity, F_{zn}, is computed for the current design variables (b =1.20 m, h = 0.10 m, and a_s = 47.60 cm^2} by fixing M_y=20mt and M_x = 50mt and running the computer program to give a point on the interaction diagram of the composite girder solved. Each of the other force capacities M_{yn} and M_{xn} are computed in the same sense. Each of F_{zn} , M_{yn} and M_{xn} are then computed in the six designs $\{b+\delta b, b-\delta b, h+\delta h, h-\delta h, a_s+\delta a_s$ and $a_s-\delta a_s\}$

The gradient vector $\partial g/\partial X_k$ is then computed where

$$\partial g/\partial X_k = \partial F/\partial X_k - \partial S/\partial X_k$$

The strength sensitivities $\partial S/\partial Xk$ is given as $\{\partial F_{xn}/\partial b$, $\partial F_{xn}/\partial h$, $\partial F_x/\partial a_s$, $\partial M_{yn}/\partial b$, $\partial M_{yn}/\partial h$, $\partial M_{yn}/\partial a_s$, $\partial M_{zn}/\partial b$, $\partial M_{zn}/\partial h$ and $\partial M_{zn}/\partial a_s\}$

where

$$\frac{\partial F_{xn}}{\partial b} = \frac{F_{zn}(b+\delta b) - F_{zn}(b-\delta b)}{2\delta b} \tag{40}$$

and the other components are computed in the same sense.

The components of the gradient vectors $\partial Z/\partial X_k$ and $\partial g/\partial X_k$ are replaced by the normalized forms given in Eqns. (21 and 22) in which the increments of change δb and δh are taken as 0.05m, 0.01 respectively and the increment of change $\delta\, a_s$ is taken as the average between the differences of a_s of the preceding and proceeding steel profiles to the steel profile specified in the iteration considered.

4- Apply Gauss-Seidel technique, Eqns. (26 to 29) to solve Eq. (24). for the normalized Lagrange variables Λ_j .

The steps are as follows:

Knowing that, each of *l* and j is the counter for the constraints corresponding to F_z, M_x and M_y respectively, then e_{11}in eq. (7.27) is computed as

$$e_{11} = b(\frac{\partial \overline{F_z}}{\partial b} * \frac{\partial \overline{F_z}}{\partial b}) / \frac{\partial \overline{z}}{\partial b} + h(\frac{\partial \overline{F_z}}{\partial h} * \frac{\partial \overline{F_z}}{\partial h}) / \frac{\partial \overline{z}}{\partial h} + a_s(\frac{\partial \overline{F_z}}{\partial a_s} \frac{\partial \overline{F_z}}{\partial a_s}) / \frac{\partial \overline{z}}{\partial a_s} \tag{41}$$

e_{22} and e_{33} are computed in the same sense as e_{11} by replacing F_z by M_x for e_{22} and F_z by M_y for e_{33}.

e_{12} in Eq. (28) is computed as:

$$e_{12} = b(\frac{\partial \overline{F_z}}{\partial b} * \frac{\partial \overline{M_x}}{\partial b}) / \frac{\partial \overline{Z}}{\partial b} + h(\frac{\partial \overline{F_z}}{\partial h} * \frac{\partial \overline{M_x}}{\partial h}) / \frac{\partial \overline{Z}}{\partial h} + a_s(\frac{\partial \overline{F_z}}{\partial a_s} * \frac{\partial \overline{M_x}}{\partial a_s}) / \frac{\partial \overline{Z}}{\partial a_s} \quad (42)$$

e_{13} and e_{23} are computed in the same sense as e_{12} but by taking the forces corresponding to l and j in return. It is thus obvious that ee_j=e_{jl} .
b_1 in Eq. (29) is computed as

$$b_1 = \eta * \frac{(F_z - F_{zn})}{\nabla F_z} - b(\frac{\partial \overline{F_z}}{\partial b}) - h(\frac{\partial \overline{F_z}}{\partial h}) - a_s(\frac{\partial \overline{F_z}}{\partial a_s}) \quad (43)$$

and b_2 and b_3 are computed in the same sence as b_1 , but by replacing F_z by M_x and M_y respectively.
Eq . (26) computes the normalized lagrange variables.
Set Λ_j=(Λ_1, Λ_2, Λ_3)=(0,0,0)
And compute

$$\Lambda_1 = \frac{1}{e_{11}}(b_1 - e_{12}\Lambda_2^{old} - e_{13}\Lambda_3^{old}) \quad (44)$$

$$\Lambda_2 = \frac{1}{e_{22}}(b_2 - e_{11}\Lambda_2^{new} - e_{23}\Lambda_3^{old}) \quad (45)$$

$$\Lambda_3 = \frac{1}{e_{33}}(b_3 - e_3\Lambda_1^{new} - e_{32}\Lambda_2^{new}) \quad (46)$$

Replace the old values of Λ_j by the new set (Λ_1, Λ_2, Λ_3) and repeat the three previous Eqns. until convergence is achieved 5.

5- Apply Eq. (23) to find the new set of design variables (b, h, a_s). As an example the variable b is computed as:

$$b^{new} = b^{old}\{1 - \frac{1}{\eta}\left[1 + \Lambda_1 * \frac{\partial \overline{F_z}}{\partial b} / \frac{\partial \overline{Z}}{\partial b} + \Lambda_2 * \frac{\partial \overline{M_x}}{\partial b} / \frac{\partial \overline{Z}}{\partial b} + \Lambda_3 * \frac{\partial \overline{M_y}}{\partial b} / \frac{\partial \overline{Z}}{\partial b}\right]\} \quad (47)$$

h and a_s are computed in the same sense as b but by replacing b by h and a_s respectively.
The new set of variables obtained are b=1.10 m , h=10.0 cm and a_s=47.60 cm^2.
Set υ= υ+1 and go to step 2. Proceed with the steps to achieve a new section. Repeat several times till convergence is achieved.

6- For the last cross-section check that the deflection is within the limits of the code. Table (1) shows the steps of convergence.

Composite girders 2 to 4 (CG2 TO CG4)

The three full composite girders, the length and cross-section are presented in tables 2 to 4, the design parameters, and end cost given bellow.
The results of the previous examples are plotted on the Fig. 4 to 7.

From Figs. 4 to 7 , we observe that the final cost of the composite girders (CG) is less than the initial cost by a percentage ranging from 18.7% to 22.4%. The equation of the cost as deduced from Figs. 4 to 7 is:

$$Cost=-A \ln(x)+B \tag{48}$$

Where
A = the constant from range (3.1 to 4.6);
B = the constant from range (11.8 to 23.9).
The constants A and B depend on the first iteration which depends on F_z, M_x, M_y, L, f_{sy}, $f_c^{'}$.

Table 1 Convergence of Composite Girder 1 (CG1)

No. of Iterations	b (cm)	h (cm)	as= (tw x hw + tf_a x bf_a+tf_l x bf_l) cm^2	Stress percentage	Z The cost
1	110.0	10.0	(0.8x22.0+1.0x15.0 +1.0x15) =47.60	32.80	10.99
2	105	9.0	(0.7x21.0+0.9x14.0 +0.9x14) =39.90	66.90	10.19
3	100.0	8.0	(0.6x20.0+0.84x14.0 +0.84x14) =35.52	85.20	9.51
4	100.0	7.0	(0.58x19.12+0.78x13.38+0.78 x13.38) =31.96	97.00	9.26

F_z=200t, M_x=25.0 mt, M_y=5.0 mt, L=6.0 m,
f_{sy} = 36000 t/ m^2, $f_c^{'}$= 2550 t/m^2

Table 2 Convergence of Composite Girder 2 (CG2)

No. of Iterations	b (cm)	h (cm)	as= (tw x hw + tf_a x bf_a+tf_l x bf_l) cm^2	Stress percentage	Z The cost
1	190.0	18.0	(1.2x65.0+1.8x28.0 +1.8x28.0 =178.80	34.30	46.72
2	185.0	16.0	(1.1x63.0+1.7x26.0 +1.7x26.0 =157.70	49.10	43.66
3	180.0	15.0	(1.0x60.0+1.6x24.0 +1.6x24.0 =136.80	67.10	41.01
4	175.0	14.0	(0.96x57.0+1.5x23.44+1 .5x23.44 =125.04	94.20	39.01

F_z=500t, M_x=50.0 mt, M_y=12.5 mt, L=12.0 m,
f_{sy} = 24000 t/ m^2, $f_c^{'}$= 0.85*4000 t/m^2

Table 3 Convergence of Composite Girder 3 (CG3)

No. of Iterations	b (cm)	h (cm)	as= (tw x hw + tf_a x bf_a+tf_l x bf_l) cm^2	Stress percentage	Z The cost
1	245	24	(1.8x170.0+2.5x35.0 +3.5x50.0 =568.50	31.70	179.25
2	235	23	(1.7x165.0+2.3x32.0 +3.2x48.0 =507.70	52.34	165.80
3	230	22	(1.6x160.0+2.1x32.0 +3.0x46.0 =461.2	69.74	156.28
4	225	20	(1.5x150.0+2.0x30.0 +3.0x45.0 =420.0	98.96	146.70

F_z=400t, M_x=105.0 mt, M_y=15mt, L=25.0 m,
f_{sy} = 24000 t/ m^2, f_c'= 0.85*3000 t/m^2

Table 4 Convergence of Composite Girder 4 (CG4)

No. of Iterations	b (cm)	h (cm)	as= (tw x hw + tf_a x (bf_a+tf_l x bf_l) (cm^2)	Stress percentage	Z The cost
1	230.0	28.0	(1.4x225.0+2.0x30.0 +5.5x45.0 =622.50	44.1	266.86
2	220.0	26.0	(1.3x220.0+1.8x28.0 +5.3x42.0 =559.00	73.4	245.60
3	210.0	24.0	(1.2x215.0+1.6x26.0 +5.1x40.0 =503.60	88.0	226.20
4	205.0	23.0	(1.2x212.0+1.5x27.5 +5.0x37.50 =483.15	98.1	218.09

F_z=600t, M_x=320.0 mt, M_y=70mt, L=36.0 m,
f_{sy} = 36000 t/ m^2, f_c'= 0.85*3000 t/m^2

CONCLUSION

There is a reliable analytical solution for the problem of optimization for biaxial full composite girders.

A computer program is now available to give a quick and accurate solution of the optimization for biaxial full composite girders cross-sections.

The stress percentage in concrete slab, and steel girder increase when increase the iteration. At iteration number four the stress percentage reach to more than 95%.

The O.C. is applied to achieve the composite girder reduces the cost by 18.7% to 22.4%.

We recommend by much research in these fields, taking into account the effect of slipping and uplift between the concrete slab and steel girder.

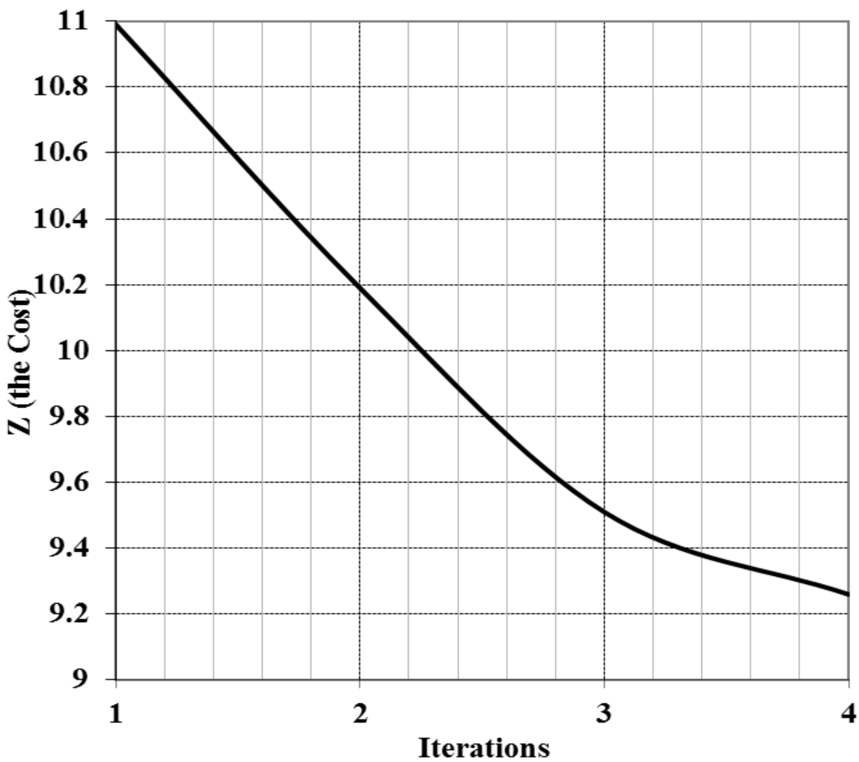


Fig. 4 The Cost Iterations for CG 1

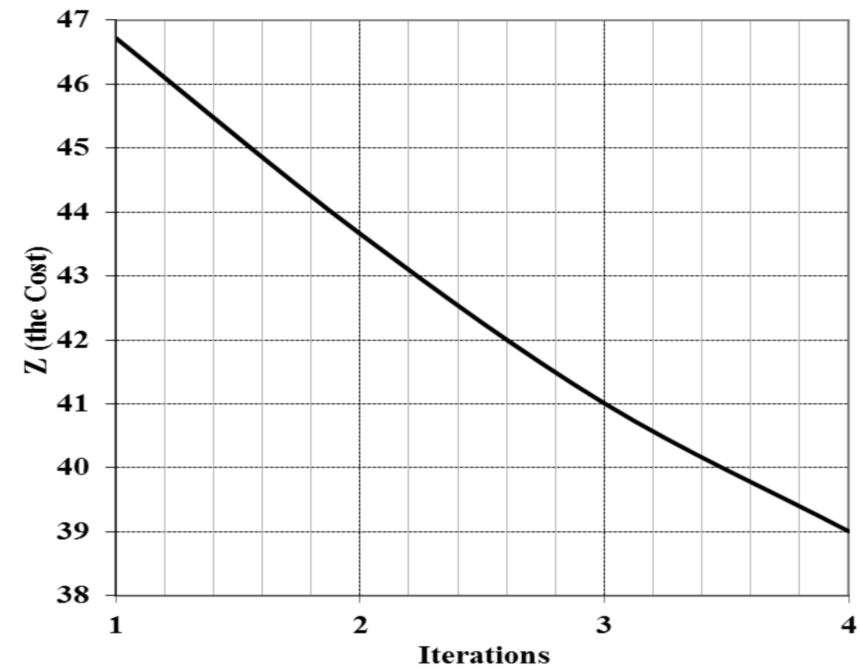


Fig. 5 The Cost Iterations for CG 2

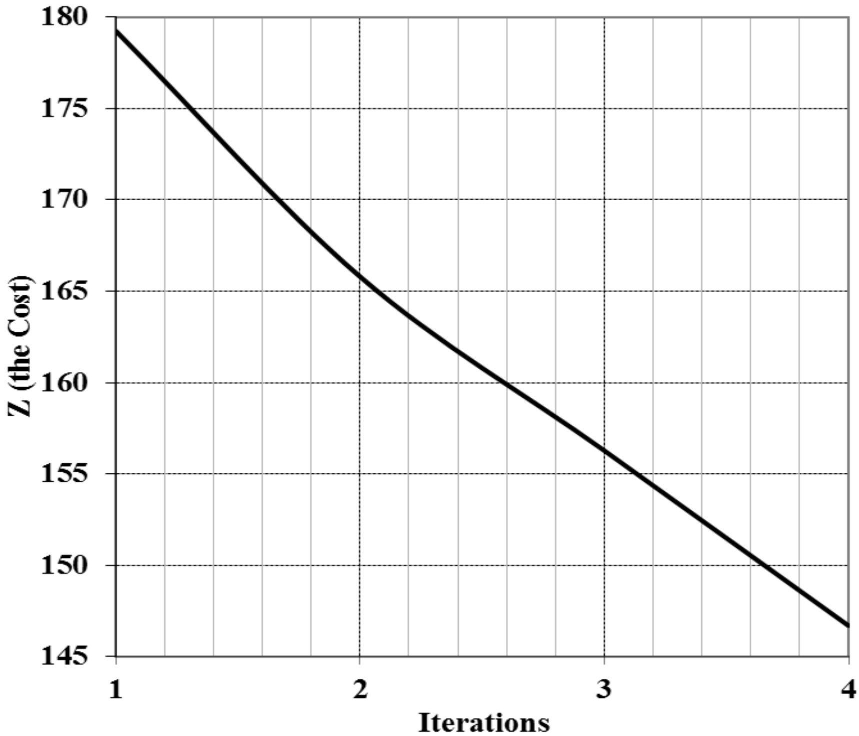


Fig. 6 The Cost Iterations for CG 3

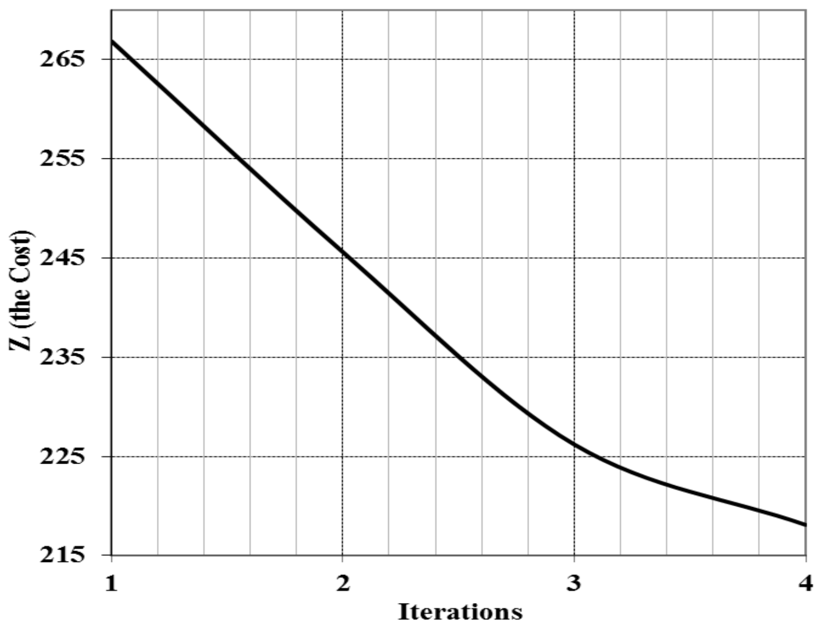


Fig. 7 The Cost Iterations for CG4

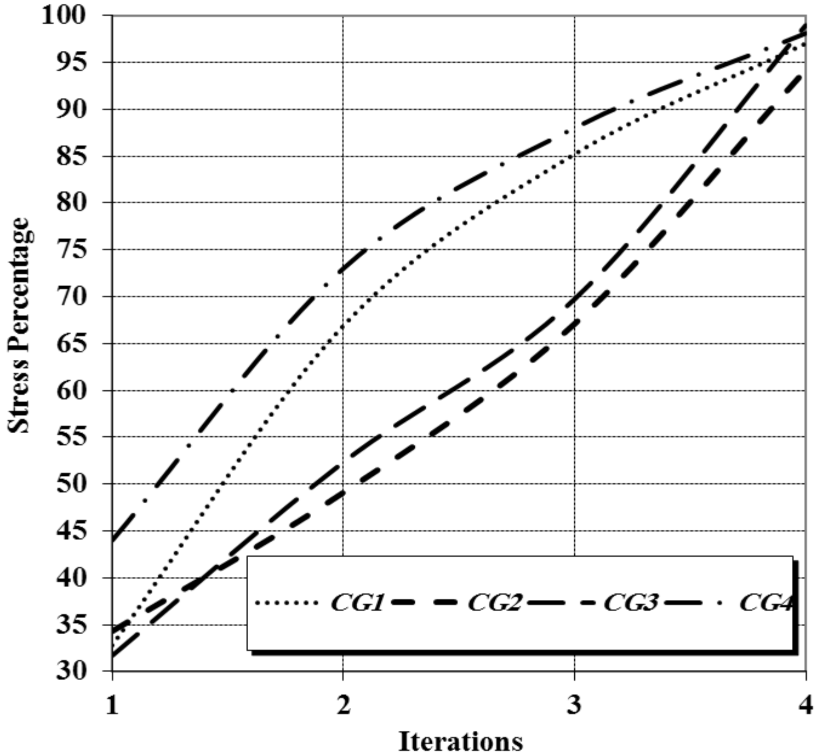


Fig. 8 Stress Percentage Iteration
(CG1& CG2& CG3& CG4)

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التصميم الاقتصادي للكمرات المركبة المعرضة لعزوم مزدوجة

تقدم هذة المقالة طريقة فعالة مبنية علي الحاسب الالي لتصممبم الكمرات المركبة تحت تأثير الانحاء الثنائي باستخدام طريقة المعيار الأمثل (Optimal Criteria) وقد اخذ عرض وسمك القطاع الخرسانى للبلاطة الخرسانية المسلحة وقطاع الحديد كمتغيرات التصميم.

وقد تم استنتاج حدود المقاومة اللازمة للتصميم باستخدام طريقة العناصر المحددة. والطريقة المذكورة تقوم بحساب الكمرات المركبة آخذة في الاعتبار السلوك اللاخطي للمادة نتيجة التغير في منحنيات الإجهاد–الانفعال لكل من الخرسانة والحديد، وكذلك السلوك اللاخطي هندسيا نتيجة التغير في مسار الكمرة المركبة أثناء الانبعاج. ويعتمد الاستنتاج علي استخدام طريقة الشغل التخيلي. وقد طبقت طريقة المعيار الأمثل للوصول لاقل تكلفة للخرسانة والحديد والشدات الي الحد الادني وقد تم تطبيق الطريقة المذكورة علي اربعة كمرات مركبة.

وقد أثبتت طريقة المعيار الأمثل انها تمنح تكرارا فعالا يؤول الي التكلفة الأقل للكمرات المركبة بعد عدد دورات قليلة نسبيا. والطريقة غير مرتبطة بالتصميم الإفتراضي الأول للكمرة المركبة بل تؤول إنسيابيا للتصميم النهائي للكمرة بتوافق مع إشتراطات التصميم.