

139-156

مقاله  
MORKOV CHAIN FINANCIAL FAILURE MODEL

BY USING ACCOUNTING NUMBERS

الدراسه  
→

Dr. ZAKARIA M. E. ISMAIL

Assistant Professor of Accountancy

Tanta University, Egypt

## MORCOV CHAIN FINANCIAL FAILURE MODEL BY USING ACCOUNTING NUMBERS

### 1. Introduction :

In recent years, the financial Accounting Standards Board (FASB) emphasized the usefulness of Accounting numbers to decision making. For example, paragraph 52 of the Statement of Financial Accounting concepts No. 1 (FASB 1978, p. 25) Concluded that : «Financial reporting should provide information that is useful to managers and directors in making decisions in the interests of owners». Also, the accounting Profession focused recently on the importance of reporting cash flow, liquidity, and financial flexibility. Such information will help investors, creditors and managers in their decisions.

In light of these objectives, questions can be raised as to the usefulness of accounting information for assessing corporate management decisions. This study examines the usefulness of this information for financial failure prediction by using Finite Absorbing Markov Chain with stationary transition matrix probabilities.

### Organization of this study :

This study is divided into four sections. In the next section, the existing financial failure prediction models are reviewed. The prior models are : (1) Models based on univariate analysis, (2) Models based Multiple Discriminant Analysis (MDA) and (3) Models based on Gamblers ruin process. Section three describes the finite absorbing Markov Chain Financial Failure model, which a contribution by this author. A study case will be established in section four.

## 2. Financial Failure Prediction Models :

The ability to predict corporate financial failure is important from a social and private point of view, since corporate bankruptcy, is obviously an indication of resource misallocation (Lev, 1974). There are many Financial Failure prediction models in the literature. They are include (1) Financial Failure models based on univariate analysis, (2) Financial Failure models based on Multiple Discriminant Analysis (MDA) and (3) Financial Failure Models based on Gambler's ruin process.

### 2.1 Univariate Analysis Approach :

A univariate approach to predict financial failure attempts to identify a single financial ratio that could classify failed and non-failed firms. There are two key assumptions in this approach. (Foster, 1978, p. 463) :

- (1) The distribution of the variables for distressed firms differs systematically from the distribution of the variables for nondistressed firms, and
- (2) These systematic differences can be capitalized on for prediction purposes.

Beaver (1966) built his model based in this approach. His sample included 79 failed firms and 79 nonfailed firms. A firm was designated as failed when only one of the following events occurred in the 1954-1964 period : bankruptcy, bond default, an overdrawn bank account, or nonpayment of a preferred stock dividend. Beaver (1966) computed the mean of 30 financial ratios for each of the failed and nonfailed groups in each of the five annual financial statements issued prior to failure. His best ratio was cash flow to total debt. The overall accuracy was 87 percent for one year before the bankruptcy.

## 2.2 Multiple Discriminant Analysis Approach :

One basic limitation of the univariate approach is that it can consider only one aspect of the firm at a time. Since the firm has multidimensional characteristics, this constitutes a severe restriction in the use of information. In an attempt to improve the efficiency of information used, a Multiple Discriminant Analysis (MDA) was developed by Altman (1968) and later modified by many other authors.

Altman (1968) used the MDA to predict financial failure by using accounting information. The initial sample was composed from 66 firms. The sample of firms included 33 firms that filed a bankruptcy petition under American Bankruptcy Act during the 1946-1965. The firms were stratified by industry and by size. Firms in group two were still in existence in 1966. For the initial sample test, the data was derived from financial statements reported the period prior to bankruptcy. A list of 22 variables was compiled for evaluation. The variables were classified into five standard ratio categories, including liquidity, profitability, leverage, solvency, and activity ratios. From the original list of variables, five independent variables were selected as doing the best overall job together in the prediction of corporate bankruptcy. The Altman Model (Z-Model) as follows :

$$Z = 1.2 x_1 + 1.4 x_2 + 3.3 x_3 + 0.6 x_4 + x_5$$

where,

$Z$  = Overall index.

$X_1$  = Working capital/total assets.

$X_2$  = Retained earnings/total assets.

$X_3$  = Earnings before interest and income tax/total assets.

$X_4$  = Market value of equity/book value of total debt.

$X_5$  = Sales/total assets.

The Altman's cutoff point is 2.675. If a firm's Z-Score is greater than 2.675, this indication to the healthy firm. On the other hand, if a firm's Z-score is less than 2.675 that indication to sick firm. Altman's Model was an accurate forecaster of bankruptcy up to two years prior bankruptcy and the prediction ability was a decreasing function for the year before bankruptcy. In the second year through the fifth years prior to bankruptcy, the Altman model led to more misclassification than did Beaver's model, using only cash flow to total debt ratio.

### 2.3 Gambler's Ruin Process Approach :

Tinsley (1970) and Wilcox (1971, 1973 and 1976) are examples of this approach. Wilcox (1971) built a probabilistic model of financial ruin. The problem is depicted as a Markov Process in the form of a one-dimensional random walk with an absorbing barrier at the lower boundary and no upper boundary. This is similar to Tinsley's (1970) random walk model with the expectation that the latter included an upper boundary. Both are modeled after the gambler's ruin process. Wilcox selected as a unit of measurement the standard deviation ( $\sigma$ ) of gains and losses. He stated that, suppose there exists a firm of wealth  $C$ , which every year plays a game which nets it a gain or loss of constant size  $\pm\sigma$  where the probability of a gain equals  $P$ , and of a loss,  $q$ . Suppose  $P > q$ , then the probability of this firm's ultimate failure is :

$$p(\text{ultimate failure}) = (q/p)^{c/\sigma}$$

where  $c/\sigma$  = number of losses the firm can take in a row being ruined.

In his model, the random walk has a drift rate which is an average tendency to gain or loss during sequential trials. The drift of the random walk is represented as the average rate of return on total capital invested.

Wilcox emphasized developing a model which will enable calculation of the probability of ruin rather than the effect of failure risk on the value of the firm. Wilcox used the following accounting information in his model : (1) net income, (2) dividends, (3) stock issued, (4) cash, (5) current assets, (6) total assets, (7) total liabilities.

### **Finite Absorbing Markov Chain of Corporate Financial Failure:**

#### **3.1 Introduction :**

The basic concept of the Markov Chain was introduced by a Russian mathematician, A.A. Markov, in 1907. Since that time, the Markov chain theory has been developed by a number of leading mathematicians. It is only in very recent times that the importance of the Markov Chain theory to the social and biological sciences has been recognized. One of the earliest practical applications was to predict changes in the occupational status of workers by Blumen, Kogan, and McCarthy (1955).

The application of the Markov Chain to accounting is suggested by some authors. Cyert, Davidson, and Thompson in their paper (1962) suggested this theory for predicting doubtful accounts. Corcran (1978) applied the Markov Chain also for accounts receivable with a different approach. He updated the transition matrix via exponential smoothing and ignored steady-state results. Shank (1971) used this analysis to income determination under uncertainty. The Markov chain theory is widely used in Human Resource Accounting (HRA) literature. Gillespie

(1978) utilizes this analysis in the manpower planning and human resource valuation.

### 3.2 A General Description of the Markov Chain :

A finite Markov Chain is defined as a process which starts in an initial state determined by a probability vector and moves from State  $S_i$  to State  $S_j$  in discrete steps. The probabilities of state changes are specified in a transition matrix, the element of which specifies the probability of movement from state ( $S_i$ ) to state ( $S_j$ ). The initial probability vector together with the transition matrix specifies a true measure completely determining the transition process (Kemeny and snell 1976, pp. 24-26).

A Markov Chain is one particular type of stochastic process. In general terms, a stochastic model is one which provides only the probability of likelihood associated with a set of possible future outcomes. Thus, whereas in deterministic law state  $S_i$  is always followed by  $S_j$  with probability  $P$ , and by state  $S_i$  with probability  $q = 1 - p$ . Stochastic models may be classified on the basis of being «discrete» or «continuous», depending on whether the time variable is treated in these terms or depending on whether or not they possess the Markov property. Markov Chain Models possess this property and can be regarded as generalizations of Markov chains; in a Markov chain model transition from one state to another can take place at any point in time, i.e., time is continuous, but in a Markov chain the state varies only at discrete time intervals.

The technique which is used in this study is a finite absorbing Markov chain with stationary transition matrix probabilities. This is a Markov process because it has the single stage property. It is called a chain because the stages are discrete. It is an absorbing

ing process because it contains at least one absorbing stage. It is called stationary because after a specific number of years, it will reach to the equilibrium.

### 3.3 Wilcox's variable as a Measure of Long-Run Profitability:

Profitability is one of the economic characteristics of the firm. There are many methods to measure the profitability of a firm. It is hard for anyone to decide which is the best measure. It is, to large extent, subject to the researcher's judgment.

This study will use Wilcox's variable (X) as a measure of profitability. However, this variable will be used with a completely different technique (Absorbing Markov Chain Model). where,

(average net income) (1-dividends payout ratio)  
 (1- average proportion of net cash less dividends reinvested  
 in illiquid assets).

$$X = \frac{\text{(average net income) (1-dividends payout ratio) (1- average proportion of net cash less dividends reinvested in illiquid assets)}}{\text{Standard deviation of (net cash flow less capital expenditures for illiquid assets and less dividends)}}$$

X represents a measure of long-run profitability compared to the variability of the income stream. Firms with negative X's are losing cash and thus liquidity as claimed by Katz, Lilion, and Bert in their paper (1983).

### 3.4 Financial Failure Model :

- a) The first step is to define the states of the transition matrix. In our model, there are two states, i.e.,

$$X > 0 \text{ and } X < 0$$

b) Next step is to built a transition matrix (T) as follows

	Absorbing states	Transient states
Absorbing states	I	O
Transient states	R	Q

T =

Key :

I = identity Square matrix ( $2 \times 2$ ) for the absorbing state  
 once in an absorbing state, the probability of remaining there is 1.

O = null matrix that reflects the impossibility of leaving absorbing states. This matrix consists entirely of

R = Settlement matrix showing how the nonabsorbing states are absorbed. Also, its showing the relationships between the transient states and absorbing states.

Q = Matrix of the non-absorbing states.

The elements of matrix T denote the probability of moving from state  $S_i$  to  $S_j$  in the next step. Since the elements of this matrix must be non-negative and the sum of the elements in any row is one, each row is called a probability vector and T is a stochastic matrix such that :

$$\sum_{j=1}^m T_{ij} = 1 \quad (\text{for } i = 1, 2, \dots, m)$$

- c) One important procedure in this kind of model is determining the transition period. The optimal choice of the period depends on the financial and operating characteristics of the firms. However, the year is chosen as the transition period.
- d) The fundamental matrix (N), described in the stochastic matrix, allows the calculation of the average number of steps which are required to pass from state  $S_i$  to state  $S_j$ . The fundamental matrix allows the calculation of the mean and variance of the first passage time for any state, which is the mean and variance of the number of steps which are expected to pass before reentering any state for the first time. For any absorbing Markov chain (1-Q) has an inverse. The fundamental matrix can be calculated by the following formula :

$$N = (I - Q)^{-1}$$

- e) The expected length of time before Financial Failure can be calculated by the following formula :

$$E(t) = NS$$

where,

$$S = \text{Sum vector} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ \cdot \\ \cdot \\ 1 \end{pmatrix}$$

This formula also can tell us whether there is a relationship between the value of the variable (X) and the expected length of time before the firms go to Financial Failure.

- f) The probability that the process of Financial Failure is

absorbed in state  $S_j$  is given at the beginning of state  $S_i$  and can be expressed by the following formula :

$$B = NR$$

- g) To calculate the variance of number of steps which are expected to be passed before entering any state for the first time, Kemeny and Snell (1976) used the following formula :

$$V = N (2 N_{dg} - I) - N_{sq}$$

where,

$V$  = variance Matrix

$N_{dg}$  = Diagonal matrix  $N$

$N_{sq}$  = Square matrix  $N$

- h) To calculate the distribution of firms who are expected to remain in business after one accounting period, this study is using the following formula :

$$\begin{bmatrix} \text{Firms} \\ \text{remain in} \\ \text{business} \end{bmatrix} = \begin{bmatrix} \text{Nonfailed} & \text{failed} \\ \text{Firms} & \text{Firms} \end{bmatrix} \begin{bmatrix} Q \end{bmatrix}$$

Accuracy can be considered in two dimensions :

Type I error, the accuracy of correctly classifying the failed firms, and Type II error, the accuracy of classifying nonfailed firms. An error of Type I would be predicting a nonfailed firms to fail.

As mentioned previously, the overall accuracy of the models will be measured in terms of Type I error and Type II error. The sum of the diagonal elements in accuracy matrix equals the total correct, and when divided into the total number of firms classified, yields the measure of success of the Markov chain corporate Financial Failure in classifying firms, that is, the percent of firms correctly classified.

#### 4. A Case Study :

The general procedures of sample selection for this case study should be on the basis of identifying a group of firms having negative X values and then choosing positive X values firms. The matching variables are industry, size, and age of the firm.

Suppose the research sample is 110 firms, including 50 firms having negative X value and 60 firms having positive X value on January 1, 1980. The following table shows the movement of the firms from and to the two financial ratio levels during 1980.

	Firms Having Negative X Values	Firms Having Positive X Values
Beginning of the period	50	60
To negative X values		(15)
To positive X values	(20)	
Failed firms	(15)	(5)
Ending of the period	15	50

The following are some of the empirical steps which will follow from this model :

a. The transition matrix can build based on the above table. On January 1, 1980, there were 50 firms having negative X values. Any change comes from those 50 firms. Hence, 20 firms having negative X value improve their profitability to positive X value. This accounts for .40 of those firms. Also, 15 firms failed during the year or .30 of the firms having negative X. Finally, (50-20-15 = 15) or .30 remained at negative level though the year.

Another row in the matrix can be found in the same manner as the above analysis. The transition matrix for the first year is given below :

**Transition Matrix — Year One**  
**Jan. 1, 1980 to Dec. 31, 1980**

From	To	Transient states		
		Absorbing state Failed Firms	Negative	Positive
Failed Firms		1	0	0
Negative level		.30	.30	.40
Positive level		.08	.25	.67

A transition matrix can be developed for each year and also for multiple years (e.g., five years, ten years...).

b. Two tests must be implemented after a transition matrix had ben built :

i. Stability Test :

The objective of the stability test is to know whether the probabilities are stationary or not. Fluctuations in the transition matrix during different time periods decrease the predictive ability of the model and thus invalidate its use. For the stability test, it is possible to use a chi-square test.

ii. Testing the Markov Property :

The markov property means that the process is a first-order Markov chain. Testing for the presence of the Markov property is the same as testing to see whether the process is a first-order chain. Again, it is possible to use the chi-square test.

c. Develop a Fundamental Matrix (N) which allows the calculation of the average number of steps which are required to pass from state  $S_i$  to state  $S_j$ . The fundamental matrix can be calculated by the following formula :

$$N = (I - Q)^{-1}$$

For any absorbing Markov Chain  $(I-Q)$  has an inverse.

$$N = \begin{bmatrix} \boxed{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}} & \boxed{\begin{matrix} .30 & .40 \\ .25 & .67 \end{matrix}} \\ \boxed{\begin{matrix} .7 & .4 \\ .25 & .33 \end{matrix}} & \boxed{1} \end{bmatrix}^{-1}$$

		Negative	Positive	
	Neg.	1.91	5.34	]
N =	Pos.	2.52	3.05	

From N above, the firms having negative X values would pass, on the average, through the first state by 2.52 transition periods and the second state 3.05 by transition periods. For firms having positive X values would pass, on the average, through the first state 1.91 transition periods and the second state 5.34 transition periods.

d. The expected length of time before failure is calculated as follows :

$$E(t) = N S$$

$$= \begin{bmatrix} 2.52 & 3.05 \\ 1.91 & 5.34 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$E(t) = \begin{bmatrix} 5.57 \\ 7.25 \end{bmatrix}$$

e. As expected, there is a positive relationship between X value and expected length of time before the firms go to financial failure. In the example, expected time for the firms having negative X value is, on the average, 5.57 transient period and for firms having positive X value it is, on the average, 7.25 transient period.

Then Wilcox's variable, at least in this example, is acceptable to predict financial failure because it is distinct between nonfailed firms and failed firms.

f. The probability that the process is absorbed in state  $S_i$  given entrance in state  $S_i$  for this example is :

$$B = NR$$

$$B = \begin{bmatrix} 2.52 & 3.05 \\ 1.91 & 5.34 \end{bmatrix} \begin{bmatrix} .30 \\ .08 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ .60 \end{bmatrix}$$

Therefore, the firms having negative X value would have a probability of one to go to financial failure. The firms having positive X value would have a probability of .60 to go to financial failure.

g. It is important to measure the accuracy of classification of the models. The sample research represents a vector. When this vector is multiplied by the given Q matrix, the resulting vector shows the distribution of firms who are expected to remain in business after one accounting period.

$$\begin{bmatrix} 60 & 50 \end{bmatrix} \begin{bmatrix} .30 & .40 \\ .25 & .67 \end{bmatrix} = \begin{bmatrix} 50 & 53 \end{bmatrix}$$

With respect to this example, the model predicts 50 out of 60 firms having positive X value will remain in business. The model classified 53 firms (50 of them having negative X values and three firms having positive X values) as failed.

h. Also from the above we can measure the overall accuracy of the model. Accuracy can be considered in two dimensions : Type I error, the accuracy of correctly classifying the nonfailed firms. An error of Type I (Alpha) would be predicting a failed firms not to fail a Type II error (Beta) would be predicting a non-

failed firm to fail. The results of the empirical research for the model should show in «Accuracy Matrix» the following :

Actual Group	Percent Correct	Predicted	
		$X > 0$	$X < 0$
$X > 0$	$C_p$	$\alpha$	
$X < 0$	$B$		$C_n$

Key :

$C_p$  = correct classification for firms having positive value.

$C_n$  = correct classification for firms having negative value.

## REFERENCES

- Altman, Edward, «Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy», **The Journal of Finance**, Sep. 1968, pp. 587 - 609.
- Corcoran, Wayne, «Forecasting of Cash flows from Accounts Receivable», **Management Science**, March 1978, pp. 732-739.
- Cyert, R.M., Davidson, H.J., and Thompson, G.L., «Estimating of the Allowance for Doubtful Accounts by Markov Chains», **Management Science**, April, 1962, pp. 287-303.
- Beaver, William, «Financial Ratios as Predictors of Failure», **Empirical Research in Accounting : Selected Studies, 1966**, Supplement to the **Journal of Accounting Research**, pp. 71-111.
- Financial Accounting Standards Board, «**Objectives of Financial Reporting by Business Enterprises**», Statement of Financial Accounting Concepts No. 2, FASB, May, 1978.
- Foster, George, «**Financial Statement Analysis**», Prentice-Hall, Inc., 1978.
- Gilospite, Jackson, «A Markovian Manpower Planning, Human Resource Valuation Model», **Unpublished Dissertation**, Virginia Polytechnic Institute and State University, Ph. D., 1978.
- Ketz, Steven, Lilien, Steven, and Nelson, Bert, «Stock Market Behavior around Distress and Recovery Predictions from Bankruptcy Models», **Unpublished Paper**, 1983.
- Kemeny, John and Snell, Laurie, «**Finite Markov Chains**», D. Van Nostrand Co. Inc., Princeton, New Jersey, 1960.

Lev, Baruch, «Financial Statement Analysis : A New Approach»,  
Prentice-Hall, Englewood Cliffs, N.J., 1974.

Shank, John, «Income Determination under Uncertainty : An Appli-  
cation of Markov Chains», **The Accounting Review**, Jan. 1971,  
pp. 57-74.

Wilcox, Jarrod, «The Gamblers Ruin Approach to Business Risk»,  
**Sloan Management Review**, Massachusetts Institute of  
Technology, Fall, 1976, pp. 33-46.

Wilcox, Jarrod, «A Prediction of Business Failure using Account-  
ing Data», **The Journal of Accounting Research**, Empirical  
studies in Accounting, Supplement 1973, pp. 163-190.