



Research Article

**MATHEMATICS**

## On Solving Fully Rough Multi-Objective Integer Linear Programming Problems

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*Affiliations:* Department of mathematics, Faculty of science. Tanta University.**KEY WORDS**

Integer linear programming, weightings,  $\varepsilon$ -Constraint, Upper approximation, Lower approximation, Multi Objective, Crisp coefficients.

**ABSTRACT**

In this paper a suggested algorithm to solve fully rough multi-objective integer linear programming problem [FRMOILP] is described. In order to solve this problem and find rough value efficient solutions and decision rough integer variables by the slice-sum method with the branch and bound technique, we will use two methods, the first one is the method of weights and the second is  $\varepsilon$ -Constraint method. The basic idea of the computational phase of the algorithm is based on constructing two LP problems with interval coefficients, and then to four crisp LPs. In addition to determining the weights and the values of  $\varepsilon$ -constraint. Also, we reviewed some of the advantages and disadvantages for them. We used integer programming because many linear programming problems require that the decision variables are integers. Also, rough intervals (RIs) are very important to tackle the uncertainty and imprecise data in decision making problems. In addition, the proposed algorithm enables us to search for the efficient solution in the largest range of possible solutions range. Also, we obtain N suggested solutions and which enables the decision maker to choose the best decisions. Finally, two numerical examples are given to clarify the obtained results in the paper.

**1. Introduction**

Linear programming (LP) is one of the most popular models used in decision making and optimization problems. Many researches, studies and applications of LP models have

been reported in numerous books, monographs, articles and chapters in books, for instance see [3,5]. Taha. H. T, (1997) Integer programming (IP) problems are optimization problems that

min or max the objective function taking into consideration the limits of constraints and integer variables. More widely application of integer programming can be used to appropriately describe the decision problems on the management and effective use of resources in engineering technology, business management and other numerous fields [15]. Rough set theory (RST) was initiated by Pawlak in (1982) as a method for ambiguity management [10]. RST approach has fundamental importance in the fields of pattern recognition, data mining, artificial intelligence, machine learning and medical applications [8]. For a vague concept  $R$ , a lower approximation is contained of all objects which surely belong to the concept  $R$  and an upper approximation is contained of all objects which possibly belong to the concept  $R$ . In other words, the lower approximation of the concept is the union of all elementary concepts which are included in it, whereas the upper approximation is the union of all elementary concepts which have nonempty intersection with the concept [11]. Ammar and Khalifa in (2014) applied a new method named, separation method for solving Rough Interval Multi Objective Transportation Problems (RIMOTP), where transportation cost, supply and demand are rough intervals [1] Also, they discussed the separation method as an important tool for the decision makers when they are handling various types of logistic problems having rough interval parameters of transportation problems. Osman et al, in (2016) presented a solution approach for RIMOTP. The concept of solving conventional interval programming combined with fuzzy programming is used to build the solution approach for RIMOTP [9]. G. Mavrotas in (2009) Effective implementation of the  $\varepsilon$ -constraint method in Multi-Objective Mathematic programming problems see [20]. In this paper, the focus of our study is improvement a method to solve fully rough multi objective integer linear programming

(FRMOILP) problems. For determine rough value efficient solutions and rough decision integer optimal value. Also, we will obtain on solutions such as completely satisfactory solutions (surely solutions) and rather satisfactory solutions (possibly solutions) by lower approximation interval and upper approximations interval respectively. In our problem we assume that all parameters and decision variables in the both of constraints and the objective functions are rough intervals (RIs). We used integer programming because many linear programming (LP) problems require that the decision variables are integers. Also, rough intervals are very important to tackle the uncertainty and imprecise data in decision making problems. In addition, the proposed algorithm enables us to search for the optimal solution in the largest range of possible solutions range. The rest of the paper is organized as follows. In Section 2, some basic about the preliminaries of RIs are presented and problem formulation and solution concept. In section 3, the weighting problem and procedures for the solution FRMOILP problems were considered. In section 4 the  $\varepsilon$ -constraint method was described. In addition, we will use slice-sum method [12] with the branch and bound technique for solving FRMOILP problems. Numerical examples for demonstrating the solution procedure of the proposed method and the conclusion are given.

## 2. Problem Formulation and solution concept

### 2.1. Fully Rough Multi objective Integer Linear Programming Problems

Let  $A^R, B^R$  represent the two sets of rough intervals. A rough interval multi-objective linear programming (FRMOILP) problems, see [17] with "k" linear objective functions

$$\text{Max } f_r^R(x) = C^R x^R, \quad r = 1, 2, \dots, k, \text{ as}$$

$$\left. \begin{array}{l} \text{Max } f_r^R(x) = \\ (f_1^R(c^R, x^R), f_2^R(c^R, x^R), \dots, f_k^R(c^R, x^R))^T \\ \text{Subject to} \\ x^R \in X^R = \{x^R \in \mathcal{R}^n \mid g_j^R(A^R, x^R) \leq B^R, x^R \geq 0\} \end{array} \right\} (1)$$

where

$$\begin{aligned} A^R &= [a_{ji}^R]_{m \times n}, \quad B^R = (b_1^R, b_2^R, \dots, b_m^R)^T, \quad x^R = \\ &(x_1^R, x_2^R, \dots, x_n^R)^T \quad \text{and} \quad C_r^R = (c_1^r, c_2^r, \dots, c_k^r), \quad r = \\ &1, \dots, \text{set of } k, \quad i \in I, \quad j \in J, \quad I = 1, \dots, n, \quad J = \\ &1, \dots, m \quad \text{are rough interval integer parameters} \\ x^R &\text{denote a set of decision rough variables, } f^R \\ &\text{denote rough objective function.} \end{aligned}$$

**Definition 1** (Rough efficient solution) [1]: The rough vector  $x^{*R}(A^{*R}, B^{*R})$  which satisfies the condition in problem (1), is called a rough efficient solution of problem (1), if and only if there does not exist another  $x^R(A^R, B^R) \in X^R$  such that

$$f_r^R(c^R, x^R) \leq f_r^{*R}(c^{*R}, x^{*R})$$

For all  $r$  and  $f_r(x^R) \neq f_r(x^{*R})$  for at least one  $r = 1, 2, \dots, K$  and  $i \in I, j \in J$  where from (1) we have:

$$C_{ri}^{*R} = [(c_{ri}^{*LL}, c_{ri}^{*UL}), (c_{ri}^{*LU}, c_{ri}^{*UU})]$$

$$a_{ji}^{*R} = [(a_{ji}^{*LL}, a_{ji}^{*UL}), (a_{ji}^{*LU}, a_{ji}^{*UU})]$$

$$b_j^{*R} = [(b_j^{*LL}, b_j^{*UL}), (b_j^{*LU}, b_j^{*UU})]$$

$$x_j^{*R} = [(x_j^{*LL}, x_j^{*UL}), (x_j^{*LU}, x_j^{*UU})]$$

$$r = 1, 2, \dots, k, \quad I = 1, 2, \dots, n, \quad J = 1, 2, \dots, m$$

### 3. The weighting problem

The idea is to associate each objective function with a weighting coefficient and minimize the weight sum of the objectives. In this way multiple objective function are transformed into a single objective function. We suppose that the weighting coefficient  $w_r$  are real numbers such that  $w_r \geq 0$  for all  $r = 1, 2, \dots, k$ . It is also usually supposed that the weight is normalized that is  $\sum_{r=1}^k w_r = 1$ .

To be more exact, the multi objective optimized problem is modified into the following problem to be called a weighting that problem (2)  $LP^R(w)$ :

$$\left. \begin{array}{l} \text{Maximize} \quad \sum_{r=1}^k w_r f_r^R(c^R, x^R) \\ \text{Subject to} \\ x \in X = \{x \in \mathcal{R}^n \mid A^R x^R \leq B^R, x^R \geq 0 \text{ and integers}\} \\ \text{Where } w_r \in W = \{w \in \mathcal{R}^n, w_r \geq 0 \text{ and } \sum_{r=1}^k w_r = 1\} \end{array} \right\} (2)$$

The relationship between the optimal solution  $x^*$  of the weighting problem (2) and the efficient solution of problem (1) can be characterized by the following theorems [16, 18].

**Theorem 1** If  $x^*(w^*) \in X$  is optimal solution of the weighting problem (2) for some  $w^* > 0$ , then  $x^*$  is optimal solution of problem (1).

**Theorem 2** If  $x^* \in X$  is an optimal solution of problem (1) then there exists  $w^* \in W$  such that  $x^*$  solves  $LP^R(w^*)$  and if either one of the following two conditions holds:

- (i)  $w_r^* > 0$ , for all  $r = 1, 2, \dots, k$ , or
- (ii)  $x^*$  is the unique optimal solution for a given  $(w^*)$ .

The weighting problem determines the complete set of optimal solution of problem (2) if the problem is convex.

#### 3.1. Procedures for the solution FRMOILP problems:

**Step (1):** The idea is to associate each objective function with a weighting coefficient and maximize the weight sum of the objectives:

$$\left. \begin{array}{l} \text{Max } w_1(c_1^R x_1^R) + w_2(c_2^R x_2^R) + \dots + w_k(c_k^R x_k^R) \\ \text{S. t } \quad \sum_{j=1}^m a_{ij}^R x_j^R \leq b_i^R \quad j \in J \\ \text{where } x_j^R \geq 0 \text{ rough integers } j \in J = \{1, \dots, m\} \\ w_r \geq 0 \text{ for } r = 1, 2, \dots, k \text{ and } \sum_{r=1}^k w_r = 1 \end{array} \right\} (3)$$

and then will deal with a single objective function.

**Step (2):** The general FRMOILP problem after the first step to become as a single problem (4).

$$\left. \begin{aligned} & ([ILP^{LAI}], [ILP^{UAI}]) = \text{Max} \\ & \sum_{r=1}^k w_r ([c_r^{LL}, c_r^{UL}]: [c_r^{LU}, c_r^{UU}]) \otimes ([x_r^{LL}, x_r^{UL}]: [x_r^{LU}, x_r^{UU}]) \\ & \text{s.t.} \\ & \sum_{j=1}^m ([a_{ij}^{LL}, a_{ij}^{UL}]: [a_{ij}^{LU}, a_{ij}^{UU}]) \otimes ([x_j^{LL}, x_j^{UL}]: [x_j^{LU}, x_j^{UU}]) \\ & \leq ([b_i^{LL}, b_i^{UL}]: [b_i^{LU}, b_i^{UU}]) \\ & x_j^{LL}, x_j^{LU}, x_j^{UL}, x_j^{UU} \geq 0, \text{ where } j \in J \\ & \text{and rough integer variables} \end{aligned} \right\} (4)$$

Where  $([c_r^{LL}, c_r^{UL}], [c_r^{LU}, c_r^{UU}])$ ,

$([a_{ij}^{LL}, a_{ij}^{UL}], [a_{ij}^{LU}, a_{ij}^{UU}])$ ,  $([b_i^{LL}, b_i^{UL}], [b_i^{LU}, b_i^{UU}])$

And  $([x_j^{LL}, x_j^{UL}], [x_j^{LU}, x_j^{UU}])$  ( $I = 1, \dots, n; J = 1, \dots, m$ )

are rough intervals coefficient and variables of the objective function and the constraints.

**Step (3):** Find the possibly optimal range or the upper approximation interval [UAI] as  $[ILP^{LU}, ILP^{UU}]$  By solving integer interval linear programming as following:

$$\left. \begin{aligned} & ILP^{UAI} = \text{Max} \sum_{r=1}^k w_r [c_r^{LU}, c_r^{UU}] \otimes [x_r^{LU}, x_r^{UU}] \\ & \text{s.t.} \sum_{j=1}^m [a_{ij}^{LU}, a_{ij}^{UU}] \otimes [x_j^{LU}, x_j^{UU}] \leq [b_i^{LU}, b_i^{UU}] \\ & x_i^{LU}, x_i^{UU} \geq 0, \text{ where } i \in I, j \in J \\ & \text{and integer variables, } (I = 1, \dots, n; J = 1, \dots, m). \end{aligned} \right\} (5)$$

**Step (4):** Find the surly optimal range or the lower approximation interval [LAI] as  $[ILP^{LL}, ILP^{UL}]$  by solving the following:

$$\left. \begin{aligned} & ILP^{LAI} = \text{Max} \sum_{j=1}^m w_j [c_j^{LL}, c_j^{UL}] \otimes [x_j^{LL}, x_j^{UL}] \\ & \text{s.t.} \sum_{j=1}^m [a_{ij}^{LL}, a_{ij}^{UL}] \otimes [x_j^{LL}, x_j^{UL}] \leq [b_i^{LL}, b_i^{UL}] \\ & x_j^{LL}, x_j^{UL} \geq 0, \text{ where } i \in I, j \in J \\ & \text{and integer variables } (I = 1, \dots, n; J = 1, \dots, m). \end{aligned} \right\} (6)$$

**Step (5):** According to step (3), the possibly optimal range of problem (5) by solving two classical LPs as follows

$$\left. \begin{aligned} & ILP^{UU} := \text{Max} \sum_{j=1}^m w_j c_j^{UU} x_j^{UU} \\ & \text{s.t.} \sum_{j=1}^m a_{ij}^{UU} x_j^{UU} \leq b_i^{UU} \\ & x_j^{UU} \geq 0 \text{ and integers } i \in I, j \in J \\ & J = 1, \dots, m, I = 1, \dots, n \end{aligned} \right\} (7)$$

And

$$\left. \begin{aligned} & ILP^{LU} := \text{Max} \sum_{j=1}^m w_j c_j^{LU} x_j^{LU} \\ & \text{s.t.} \sum_{j=1}^m a_{ij}^{LU} x_j^{LU} \leq b_i^{LU} \\ & x_j^{LU} \geq 0 \text{ and integers } i \in I, j \in J \\ & J = 1, \dots, m, I = 1, \dots, n \end{aligned} \right\} (8)$$

**Step (6)** The surly optimal range of problem (6) follows by solving the two classical LPs:

$$\left. \begin{aligned} & ILP^{LL} := \text{Max} \sum_{j=1}^m w_j c_j^{LL} x_j^{LL} \\ & \text{s.t.} \sum_{j=1}^m a_{ij}^{LL} x_j^{LL} \leq b_i^{LL} \\ & x_j^{LL} \geq 0 \text{ and integers } i \in I, j \in J \\ & J = 1, \dots, m, I = 1, \dots, n \end{aligned} \right\} (9)$$

And

$$\left. \begin{aligned} & ILP^{UL} := \text{Max} \sum_{j=1}^n w_j c_j^{UL} x_j^{UL} \\ & \text{s.t.} \sum_{j=1}^n a_{ij}^{UL} x_j^{UL} \leq b_i^{UL} \\ & x_j^{UL} \geq 0 \text{ and integers } i \in I, j \in J \\ & J = 1, \dots, m, I = 1, \dots, n \end{aligned} \right\} (10)$$

where the rough optimal values ( $Z^{*R}$ ) and rough integer efficient solutions ( $x_j^{*R}$ ) will be as:

$$Z^{*R} = ([z^{*LL}, z^{*UL}], [z^{*LU}, z^{*UU}])$$

$$x_j^{*R} = ([x_j^{*LL}, x_j^{*UL}], [x_j^{*LU}, x_j^{*UU}]) (j = 1, \dots, n)$$

In addition, the possible optimal values range for problem (5) are  $[z^{*LU}, z^{*UU}]$  and the surely optimal values range for problem (6) are  $[z^{*LL}, z^{*UL}]$ . Also, the intervals  $[x_j^{*LL}, x_j^{*UL}] j \in J$  are the integer completely satisfactory solutions. Furthermore, the intervals  $[x_j^{*LU}, x_j^{*UU}] j \in J$  are integer rather satisfactory solutions.

**Definition 2** Consider all of the corresponding FRMOILP problems and LP of problem (4).

- (a) The interval  $[ILP^{LL}, ILP^{UL}]([ILP^{LU}, ILP^{UU}])$  is called the surely (possibly) optimal range symbolized  $[ILP^{LAI}] ([ILP^{UAI}])$  of problem (4), if the optimal range of each (ILPFRI) is a superset (subset) of  $[ILP^{LL}, ILP^{UL}]([ILP^{LU}, ILP^{UU}])$ .
- (b) Let  $[ILP^{LL}, ILP^{UL}]([ILP^{LU}, ILP^{UU}])$  be surely optimal (possibly) optimal range of the problem (4). Then the rough interval  $([ILP^{LL}, ILP^{UL}], [ILP^{LU}, ILP^{UU}])$  is called the rough optimal range of problem (4), also any point, optimal value belongs to  $[ILP^{LL}, ILP^{UL}], ([ILP^{LU}, ILP^{UU}])$  is called a completely (rather) satisfactory solution of the problem (4).
- (c) A solution  $x^*$  is surely-feasible, iff it belongs to the lower approximation of the feasible set.
- (d) A solution  $x^*$  is possibly -feasible, iff it belongs to the upper approximation of the feasible set.
- (e) A solution  $x^*$  is surely-not feasible, iff it does not belong to the upper approximation of the feasible set [2,7].

Now, the establish of the relation between optimal solutions of the integer linear programming problem with fully rough intervals for problem (4) and four problems  $ILP^{UU}, ILP^{LU}, ILP^{LL}$  and  $ILP^{UL}$  the problems (7), (8), (9) and (10) respectively. The established relation is used in the proposed method, namely, slice-sum method.

**Theorem 3** [14]

If the set  $\{x_j^{UU}, \text{ for all } j \in J\}$  is an optimal solution for the  $(ILP^{UU})$  or (7) problem of the problem (4) with the maximum optimal value for  $(Z^{UU})$ , the set  $\{x_j^{UL}, \text{ for all } j \in J\}$  is an optimal solution for the  $(ILP^{UL})$  or (10) problem of the problem (4) with the maximum optimal value for  $(Z^{UL})$ , the set  $\{x_j^{LL}, \text{ for all } j \in J\}$  is an optimal solution for the  $(ILP^{LL})$  or (9)

problem of the problem (4) with the maximum optimal value for  $(Z^{LL})$ , the set  $\{x_j^{LU}, \text{ for all } j \in J\}$  is an optimal solution for the  $(ILP^{LU})$  or (8) problem of the problem (4) with the maximum optimal value for  $(Z^{LU})$ , then the set of rough integer intervals  $\{([x_j^{LL}, x_j^{UL}], [x_j^{LU}, x_j^{UU}]), \text{ for all } j \in J\}$  is an optimal solution for the problem (4) with maximum optimal values  $([z^{LL}, z^{UL}], [z^{LU}, z^{UU}])$  provided  $x_j^{LU} \leq x_j^{LL} \leq x_j^{UL} \leq x_j^{UU}$ , for all  $j \in J$ .

**Proof:** Since  $\{x_j^{LU}, \text{ for all } j \in J\}, \{x_j^{LL}, \text{ for all } j \in J\}, \{x_j^{UL}, \text{ for all } j \in J\}, \{x_j^{UU}, \text{ for all } j \in J\}$  are optimal solutions for the problems  $ILP^{LU}, ILP^{LL}, ILP^{UL}$  and  $ILP^{UU}$ , respectively and  $x_j^{LU} \leq x_j^{LL} \leq x_j^{UL} \leq x_j^{UU}$ , for all  $j \in J$ , then we can conclude that the set of rough integer intervals  $\{([x_j^{LL}, x_j^{UL}], [x_j^{LU}, x_j^{UU}]), \text{ for all } j \in J\}$  is a feasible solution to the problem (4).

Let  $\{([x_j^{*LL}, x_j^{*UL}], [x_j^{*LU}, x_j^{*UU}]), \text{ for all } j \in J\}$  be a feasible solution to the problem (4).

Therefore  $\{x_j^{*LU}, \text{ for all } j \in J\}, \{x_j^{*LL}, \text{ for all } j \in J\}, \{x_j^{*UL}, \text{ for all } j \in J\}, \{x_j^{*UU}, \text{ for all } j \in J\}$  are feasible solutions to the problems  $ILP^{LU}, ILP^{LL}, ILP^{UL}$  and  $ILP^{UU}$ , respectively.

Since  $\{x_j^{LU}, \text{ for all } j \in J\}, \{x_j^{LL}, \text{ for all } j \in J\}, \{x_j^{UL}, \text{ for all } j \in J\}, \{x_j^{UU}, \text{ for all } j \in J\}$  are feasible solutions to the problems  $ILP^{LU}, ILP^{LL}, ILP^{UL}$  and  $ILP^{UU}$ , respectively.

We have:

$$Z^{LU} = \sum_{j=1}^m c_j^{LU} x_j^{*LU} \geq \sum_{j=1}^m c_j^{LU} x_j^{LU},$$

$$Z^{LL} = \sum_{j=1}^m c_j^{LL} x_j^{*LL} \geq \sum_{j=1}^m c_j^{LL} x_j^{LL},$$

$$Z^{UL} = \sum_{j=1}^m c_j^{UL} x_j^{*UL} \geq \sum_{j=1}^m c_j^{UL} x_j^{UL},$$

$$\text{and } Z^{UU} = \sum_{j=1}^m c_j^{UU} x_j^{*UU} \geq \sum_{j=1}^m c_j^{UU} x_j^{UU}.$$

This implies that  $([Z^{LL}, Z^{UL}]: [Z^{LU}, Z^{UU}]) =$

$$\sum_{j=1}^m ([c_j^{LL}, c_j^{UL}]: [c_j^{LU}, c_j^{UU}]) \otimes ([x_j^{LL}, x_j^{UL}]: [x_j^{LU}, x_j^{UU}])$$

And

$$\sum_{j=1}^m ([c_j^{LL}, c_j^{UL}]: [c_j^{LU}, c_j^{UU}]) \otimes ([x_j^{*LL}, x_j^{*UL}]: [x_j^{*LU}, x_j^{*UU}])$$

$$\geq \sum_{j=1}^m ([c_j^{LL}, c_j^{UL}]: [c_j^{LU}, c_j^{UU}]) \otimes ([x_j^{LL}, x_j^{UL}]: [x_j^{LU}, x_j^{UU}])$$

Therefore, the set of rough integer intervals  $\{([x_j^{*LL}, x_j^{*UL}]: [x_j^{*LU}, x_j^{*UU}]), \text{ for all } j \in J\}$  is optimal solution for the problem (4) with maximum optimal values  $([Z^{LL}, Z^{UL}], [Z^{LU}, Z^{UU}])$ . Hence, the theorem is proved [14].

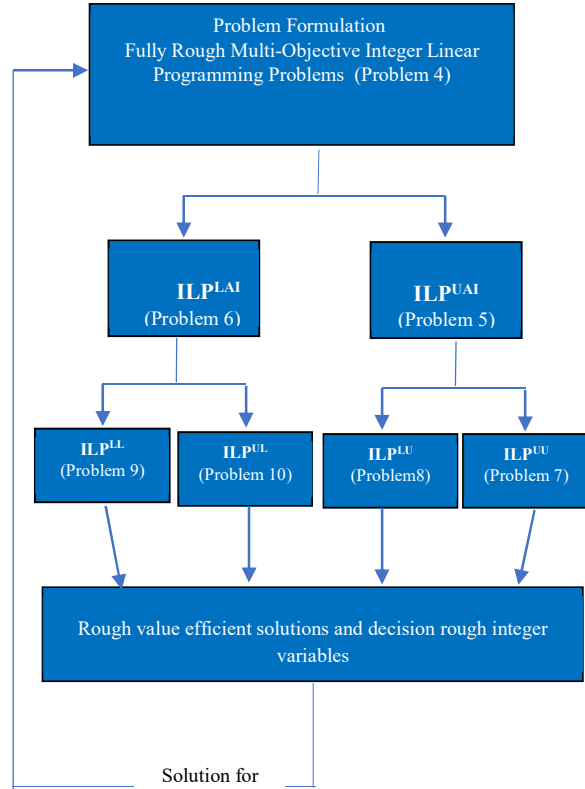


Figure (1): Flowchart of the proposed approach for solving Fully rough multi objective integer linear programming problems.

To demonstrate the solution method let us consider a numerical example of the following fully rough Multi Objective integer linear

programming problem, where we will use "WinQSB" program to solve it.

### Example 1

$$Max \left( \begin{array}{l} \{([2,3]: [1,4]) \otimes ([x_1^{LL}, x_1^{UL}]: [x_1^{LU}, x_1^{UU}]) \oplus \\ ([3,4]: [2,5]) \otimes ([x_2^{LL}, x_2^{UL}]: [x_2^{LU}, x_2^{UU}]) \oplus \} \\ \{([5,6]: [4,8]) \otimes ([x_1^{LL}, x_1^{UL}]: [x_1^{LU}, x_1^{UU}]) \oplus \\ ([6,8]: [4,10]) \otimes ([x_2^{LL}, x_2^{UL}]: [x_2^{LU}, x_2^{UU}]) \oplus \} \end{array} \right)$$

Subject to

$$\left\{ \begin{array}{l} ([3,3.5]: [2.5,5]) \otimes ([x_1^{LL}, x_1^{UL}]: [x_1^{LU}, x_1^{UU}]) \oplus ([2.5,3]: [2,4]) \\ \otimes ([x_2^{LL}, x_2^{UL}]: [x_2^{LU}, x_2^{UU}]) \leq ([600,700], [500,800]) \end{array} \right\}$$

$$\left\{ \begin{array}{l} ([2.5,3]: [2,4]) \otimes ([x_1^{LL}, x_1^{UL}]: [x_1^{LU}, x_1^{UU}]) \oplus ([3,3]: [1.5,4]) \\ \otimes ([x_2^{LL}, x_2^{UL}]: [x_2^{LU}, x_2^{UU}]) \leq ([450,550]: [400,600]) \end{array} \right\}$$

$$\left\{ \begin{array}{l} ([1,1]: [0.5,1.5]) \otimes ([x_1^{LL}, x_1^{UL}]: [x_1^{LU}, x_1^{UU}]) \oplus ([3.5,4]: [3,4]) \\ \otimes ([x_2^{LL}, x_2^{UL}]: [x_2^{LU}, x_2^{UU}]) \leq ([250,350], [100,400]) \end{array} \right\}$$

$$\left\{ \begin{array}{l} x_i^{LL}, x_i^{LU}, x_i^{UL}, x_i^{UU} \geq 0, \text{ where } i = 1, 2 \\ \text{and rough integer variables} \end{array} \right\}$$

Convert the FRMOILP problem to single objective by weighting method  $f(x) = w_1 f_1 + w_2 f_2$  where  $w_1 = w_2 = 0.5$  as follows:

$$Max \left\{ \begin{array}{l} 0.5 \left( ([2,3]: [1,4]) \otimes ([x_1^{LL}, x_1^{UL}]: [x_1^{LU}, x_1^{UU}]) \oplus \right) \oplus \\ ([3,4]: [2,5]) \otimes ([x_2^{LL}, x_2^{UL}]: [x_2^{LU}, x_2^{UU}]) \oplus \\ 0.5 \left( ([5,6]: [4,8]) \otimes ([x_1^{LL}, x_1^{UL}]: [x_1^{LU}, x_1^{UU}]) \oplus \right) \oplus \\ ([6,8]: [4,10]) \otimes ([x_2^{LL}, x_2^{UL}]: [x_2^{LU}, x_2^{UU}]) \oplus \end{array} \right\}$$

where the single objective function is

$$Max \left\{ \begin{array}{l} ([3.5,4.5], [2.5,6]) \otimes ([x_1^{LL}, x_1^{UL}], [x_1^{LU}, x_1^{UU}]) \\ \oplus ([4.5,6], [3,7.5]) \otimes ([x_2^{LL}, x_2^{UL}], [x_2^{LU}, x_2^{UU}]) \end{array} \right\}$$

To solve Example 1 we have to solve two integer ILP problems with upper approximation interval (UAI) and lower approximation interval (LAI) [ILP<sup>LAI</sup>, ILP<sup>UAI</sup>] as follows:

$$ILP^{UAI} = [LP^{LU}, LP^{UU}] =$$

$$Max [2.5,6] \otimes [x_1^{LU}, x_1^{UU}] \oplus [3,7.5] \otimes [x_2^{LU}, x_2^{UU}]$$

Subject to

$$[2.5,5] \otimes [x_1^{LU}, x_1^{UU}] \oplus [2,4] \otimes [x_2^{LU}, x_2^{UU}] \leq [500,800]$$

$$[2,4] \otimes [x_1^{LU}, x_1^{UU}] \oplus [1.5,4] \otimes [x_2^{LU}, x_2^{UU}]$$

$$\leq [400,600]$$

$$[0.5,1.5] \otimes [x_1^{LU}, x_1^{UU}] \oplus [3,4] \otimes [x_2^{LU}, x_2^{UU}]$$

$$\leq [100,400]$$

$$x_i^{LU}, x_i^{UU} \geq 0, \text{ where } j = 1, 2,$$

$$i = 1, 2 \text{ and integer variables}$$

$$[LP^{LL}, LP^{UL}] = ILP^{LAI} =$$

$$\text{Max } [3.5,4.5] \otimes [x_1^{LL}, x_1^{UL}] \oplus [4.5,6] \otimes [x_2^{LL}, x_2^{UL}]$$

Subject to

$$[3,3.5] \otimes [x_1^{LL}, x_1^{UL}] \oplus [2.5,3] \otimes [x_2^{LL}, x_2^{UL}]$$

$$\leq [600,700]$$

$$[2.5,3] \otimes [x_1^{LL}, x_1^{UL}] \oplus [3,3] \otimes [x_2^{LL}, x_2^{UL}]$$

$$\leq [450,550]$$

$$[1,1] \otimes [x_1^{LL}, x_1^{UL}] \oplus [3.5,4] \otimes [x_2^{LL}, x_2^{UL}]$$

$$\leq [250,350]$$

$$x_i^{LL}, x_i^{UL} \geq 0, \text{ where } j = 1, 2 ;$$

$$i = 1, 2 \text{ and integer variables}$$

In the ILPFI Problem ( $ILP^{UAI}$ ) is transformed to LP problems  $ILP^{LU}$  and  $ILP^{UU}$ , and in the ILPFI Problem ( $ILP^{LAI}$ ) is transformed to LP problems  $ILP^{LL}$  and  $ILP^{UL}$  as following:

$$ILP^{UU} := \max 6x_1^{UU} + 7.5x_2^{UU}$$

$$s.t \quad 5x_1^{UU} + 4x_2^{UU} \leq 800$$

$$4x_1^{UU} + 4x_2^{UU} \leq 600$$

$$1.5x_1^{UU} + 4x_2^{UU} \leq 400$$

$$x_j^{UU} \geq 0, \quad j = 1, 2$$

and rough integer variables,

$$ILP^{UL} := \max 4.5x_1^{UL} + 6x_2^{UL}$$

$$s.t \quad 3.5x_1^{UL} + 3x_2^{UL} \leq 700$$

$$3x_1^{UL} + 3x_2^{UL} \leq 550$$

$$1x_1^{UL} + 4x_2^{UL} \leq 350$$

$$x_j^{UL} \leq x_j^{UU}, \quad x_j^{UL} \geq 0, \quad j = 1, 2$$

rough integer variables. And

$$ILP^{LL} := \max 3.5x_1^{LL} + 4.5x_2^{LL}$$

$$s.t \quad 3x_1^{LL} + 2.5x_2^{LL} \leq 600$$

$$2.5x_1^{LL} + 3x_2^{LL} \leq 450$$

$$x_1^{LL} + 3.5x_2^{LL} \leq 250$$

$$x_j^{LL} \leq x_j^{UL}, \quad x_j^{LL} \geq 0, \quad j = 1, 2$$

rough integer variables. And

$$ILP^{LU} := \max 2.5x_1^{LU} + 3x_2^{LU}$$

$$s.t \quad 2.5x_1^{LU} + 2x_2^{LU} \leq 500$$

$$2x_1^{LU} + 1.5x_2^{LU} \leq 400$$

$$0.5x_1^{LU} + 3x_2^{LU} \leq 100$$

$$x_j^{LU} \leq x_j^{LL}, \quad x_j^{LU} \geq 0, \quad j = 1, 2$$

and rough integer variables

We used "WinQSB" program to find efficient values and efficient integer solutions for the UAI and the LAI for example (1), also, we will use apply branch and bound algorithm for integer programming, as following results:

$$ILP^{UU} = 1005, \text{ where } x_1^{UU} = 80, \quad x_2^{UU} = 70$$

$$ILP^{UL} = 762, \text{ where } x_1^{UL} = 80, \quad x_2^{UL} = 67$$

$$ILP^{LL} = 550, \text{ where } x_1^{LL} = 80, \quad x_2^{LL} = 48$$

$$ILP^{LU} = 260, \text{ where } x_1^{LU} = 80, \quad x_2^{LU} = 20$$

here the rough efficient values range solutions for  $Z^{*R} = ([ILP^{LAI}], [ILP^{UAI}]) = [550, 762] ([260, 1005])$

The integer rough optimal solutions are

$$x_1^{*R} = ([80, 80], [80, 80]), x_2^{*R} = ([48, 67], [20, 70]).$$

And the possibly optimal values range solutions for

$$ILP^{UAI} \text{ are } [ILP^{LU}, ILP^{UU}] = [260, 1006].$$

Moreover, the surely optimal values range solutions for

$$ILP^{LAI} \text{ are } [ILP^{LL}, ILP^{UL}] = [550, 760].$$

In addition, the integer completely satisfactory for

$$[x_1^{*LL}, x_1^{*UL}] = [80, 80], [x_2^{*LL}, x_2^{*UL}] = [48, 67]$$

and the integer rather satisfactory solution for

$$[x_1^{*LU}, x_1^{*UU}] = [80, 80], [x_2^{*LU}, x_2^{*UU}] = [20, 70].$$

#### 4. The $s^{th}$ Objective $\varepsilon$ -constraint problem $LP^R(\varepsilon)$

The  $\varepsilon$ -constraint method for characterizing Pareto optimal solution is to solve the following constrained problem by taking one objective function as the objective function and letting all the other objective functions be inequality constraints  $LP^R(\varepsilon)$  [16, 18].

$$\left. \begin{array}{l} \text{Max } f_s^R(c^R, x^R) \\ \text{Subject to} \\ f_s^R(c^R, x^R) \leq \varepsilon_r, \quad r = 1, \dots, k, s = 1, \dots, k \quad r \neq s \\ X = \{x \in \mathbb{R}^n \mid g_j^R(c^R, x^R) \leq b_j, x_j^R \geq 0, j = 1, \dots, m\} \end{array} \right\} (11)$$

where  $\varepsilon = (\varepsilon_1, \dots, \varepsilon_{s-1}, \varepsilon_{s+1}, \dots, \varepsilon_k)$  For a given point  $x^*$ , we shall use the symbol  $LP^R(\varepsilon^*)$  to present the problem  $LP^R(\varepsilon)$ , where  $\varepsilon_r = \varepsilon_r^* = x^{*R}(c^{*R}, a^{*R}, b^{*R})$ ,  $r \neq s$  and  $x = (x_1, x_2, \dots, x_n)^T$  are integers and an dimensional vector of decision variables.

**Theorem 4** [16, 18] If  $x^* \in X$  is unique optimal solution to the  $\varepsilon$ -constraint problem  $IL^R(\varepsilon)$  for some  $\varepsilon_r, r = 1, \dots, k, r \neq s$ , then  $x^*$  is a pareto optimal solution to problem (1).

**Theorem 5** [16, 18] If  $x^*$  is a Pareto optimal solution of problem (1), then  $x^*$  is an optimal solution of the constrained problem  $LP^R(\varepsilon)$  for some  $\varepsilon_i, r = 1, \dots, k, r \neq S$ .

To demonstrate the solution method  $LP^R(\varepsilon)$  let us consider a numerical example of the following fully rough interval multi objective integer linear programming problem.

**Example (2):** Consider the following fully rough multi objective linear programming problem:

$$\text{Max } \left( \begin{array}{l} \{ ([0.5, 1.5]: [0, 2]) \otimes ([x_1^{LL}, x_1^{UL}]: [x_1^{LU}, x_1^{UU}]) \oplus \} \\ \{ ([1.5, 2.5]: [1, 3]) \otimes ([x_2^{LL}, x_2^{UL}]: [x_2^{LU}, x_2^{UU}]) \} \\ \{ ([2.5, 3.5]: [2, 4]) \otimes ([x_1^{LL}, x_1^{UL}]: [x_1^{LU}, x_1^{UU}]) \oplus \} \\ \{ ([1.5, 2.5]: [1, 3]) \otimes ([x_2^{LL}, x_2^{UL}]: [x_2^{LU}, x_2^{UU}]) \} \end{array} \right)$$

Subject to

$$\left\{ \begin{array}{l} ([1.5, 2.5]: [1, 3]) \otimes ([x_1^{LL}, x_1^{UL}], [x_1^{LU}, x_1^{UU}]) \oplus ([5.5, 6.5], [5, 7]) \otimes ([x_2^{LL}, x_2^{UL}], [x_2^{LU}, x_2^{UU}]) \leq ([200, 250][150, 300]) \end{array} \right\}$$

$$\left\{ \begin{array}{l} ([7.5, 8.5]: [7, 9]) \otimes ([x_1^{LL}, x_1^{UL}]: [x_1^{LU}, x_1^{UU}]) \oplus ([5.5, 6.5], [5, 7]) \otimes ([x_2^{LL}, x_2^{UL}]: [x_2^{LU}, x_2^{UU}]) \leq ([350, 450]: [300, 500]) \end{array} \right\}$$

$$\left\{ \begin{array}{l} ([2.5, 3.5]: [2, 4]) \otimes ([x_1^{LL}, x_1^{UL}]: [x_1^{LU}, x_1^{UU}]) \oplus ([0.5, 1.5], [0, 2]) \otimes ([x_2^{LL}, x_2^{UL}]: [x_2^{LU}, x_2^{UU}]) \leq [240, 280]: [200, 300] \end{array} \right\}$$

$$\left\{ \begin{array}{l} ([2.5, 3.5], [2, 4]) \otimes ([x_1^{LL}, x_1^{UL}], [x_1^{LU}, x_1^{UU}]) \oplus ([1.5, 2.5]) \\ \{ [1, 3] \} \otimes ([x_2^{LL}, x_2^{UL}], [x_2^{LU}, x_2^{UU}]) \leq ([\varepsilon_2^{LL}, \varepsilon_2^{UL}], [\varepsilon_2^{LU}, \varepsilon_2^{UU}]) \end{array} \right\}$$

$$\left\{ \begin{array}{l} x_i^{LL}, x_i^{LU}, x_i^{UL}, x_i^{UU} \geq 0, \text{ where } i = 1, 2 \\ \text{and rough integer variables} \end{array} \right\}$$

To solve it we have to solve four multi-objective integer crisp linear programming problems as with an example (2). Moreover, in order to properly apply the  $\varepsilon$ -constraint method we must have the range of every objective function, at least for the  $k-1$  objective function that will be used as constraints [19, 20]. The calculation of the range of  $\varepsilon$  illustrative in table (1) and (2) as follows

$$ILP_1^{UU} := \text{Max } 2x_1^{UU} + 3x_2^{UU}$$

$$ILP_2^{UU} := \text{Max } 4x_1^{UU} + 3x_2^{UU}$$

$$S.t. \quad 3x_1^{UU} + 7x_2^{UU} \leq 300$$

$$9x_1^{UU} + 7x_2^{UU} \leq 500$$

$$4x_1^{UU} + 2x_2^{UU} \leq 300$$

$$x_j^{UU} \geq 0, \quad j = 1, 2$$

rough integer variables. And

$$ILP_1^{LU} := \text{Max } x_2^{LU}$$

$$ILP_2^{LU} := \text{Max } 2x_1^{LU} + x_2^{LU}$$

$$S.t. \quad x_1^{LU} + 5x_2^{LU} \leq 150$$

$$7x_1^{LU} + 5x_2^{LU} \leq 300$$

$$2x_1^{LU} \leq 200$$

$$x_j^{LU} \leq x_j^{LL}, \quad x_j^{LU} \geq 0, \quad j = 1, 2$$

and rough integer variables.

Table (1): To determine the parameters  $\varepsilon_2^{LU}$  and  $\varepsilon_2^{UU}$ .

$(x_1, x_2)$	$x_1^{LU} + x_2^{LU}$	$(x_1, x_2)$	$4x_1^{UU} + 3x_2^{UU}$
$ILP_1^{LU}$	30	$ILP_1^{UU}$	215
$(0, 30)$		$(32, 29)$	



ILP <sub>2</sub> <sup>LU</sup>	85	ILP <sub>1</sub> <sup>UU</sup>	222
(42, 1)		(54, 2)	
Max	85	Max	222
Min	30	Min	215
ε	30 ≤ ε <sub>2</sub> <sup>LU</sup> ≤ 85	ε	215 ≤ ε <sub>2</sub> <sup>UU</sup> ≤ 222

$$ILP_1^{UL} := \text{Max } 1.5x_1^{UL} + 2.5x_2^{UL}$$

$$ILP_2^{UL} := \text{Max } 3.5x_1^{UL} + 2.5x_2^{UL}$$

$$s.t \quad 2.5x_1^{UL} + 6.5x_2^{UL} \leq 250$$

$$8.5x_1^{UL} + 6.5x_2^{UL} \leq 450$$

$$3.5x_1^{UL} + 1.5x_2^{UL} \leq 280$$

$$x_j^{UL} \leq x_j^{UU}, x_j^{UL} \geq 0, j = 1, 2$$

rough integer variables. And

$$ILP_1^{LL} := \text{Max } 0.5x_1^{LL} + 1.5x_2^{LL}$$

$$ILP_2^{LL} := \text{Max } 2.5x_1^{LL} + 1.5x_2^{LL}$$

$$s.t \quad 1.5x_1^{LL} + 5.5x_2^{LL} \leq 200$$

$$7.5x_1^{LL} + 5.5x_2^{LL} \leq 350$$

$$2.5x_1^{LL} + 0.5x_2^{LL} \leq 240$$

$$x_j^{LL} \leq x_j^{UL}, x_j^{LL} \geq 0, j = 1, 2$$

and rough integer variables

Table (2): To determine the parameters ε<sub>2</sub><sup>LU</sup> and ε<sub>2</sub><sup>UL</sup>.

(x <sub>1</sub> , x <sub>2</sub> )	2.5x <sub>1</sub> <sup>LU</sup> + 1.5x <sub>2</sub> <sup>LU</sup>	(x <sub>1</sub> , x <sub>2</sub> )	3.5x <sub>1</sub> <sup>UL</sup> + 2.5x <sub>2</sub> <sup>UL</sup>
ILP <sub>1</sub> <sup>LU</sup>	102.5	ILP <sub>1</sub> <sup>UL</sup>	177
(3, 3)		(32, 26)	
ILP <sub>2</sub> <sup>LU</sup>	115.5	ILP <sub>2</sub> <sup>UL</sup>	184.5
(3, 3)		(52, 1)	
Max	115.5	Max	184.5
Min	102.5	Min	177
ε	102.5 ≤ ε <sub>2</sub> <sup>LU</sup> ≤ 115.5	ε	177 ≤ ε <sub>2</sub> <sup>UL</sup> ≤ 184.5

$$\text{Let } \epsilon_2^{LU} = 85, \quad \epsilon_2^{UU} = 220$$

$$ILP_1^{UU} := \text{Max } 2x_1^{UU} + 3x_2^{UU}$$

$$s.t \quad 3x_1^{UU} + 7x_2^{UU} \leq 300$$

$$9x_1^{UU} + 7x_2^{UU} \leq 500$$

$$4x_1^{UU} + 2x_2^{UU} \leq 300$$

$$4x_1^{UU} + 3x_2^{UU} \leq 220$$

$$x_j^{UU} \geq 0, j = 1, 2$$

rough integer variables. And

$$ILP_1^{LU} := \text{Max } x_2^{LU}$$

$$s.t \quad x_1^{LU} + 5x_2^{LU} \leq 150$$

$$7x_1^{LU} + 5x_2^{LU} \leq 300$$

$$2x_1^{LU} + \quad \leq 200$$

$$2x_1^{LU} + x_2^{LU} \leq 85$$

$$x_j^{LU} \leq x_j^{LL}, x_j^{LU} \geq 0, j = 1, 2$$

rough integer variables. And

$$\text{Let } \epsilon_2^{LL} = 102, \quad \epsilon_2^{UL} = 180$$

$$ILP_1^{UL} := \text{Max } 1.5x_1^{UL} + 2.5x_2^{UL}$$

$$s.t. \quad 2.5x_1^{UL} + 6.5x_2^{UL} \leq 250$$

$$8.5x_1^{UL} + 6.5x_2^{UL} \leq 450$$

$$3.5x_1^{UL} + 1.5x_2^{UL} \leq 280$$

$$3.5x_1^{UL} + 2.5x_2^{UL} \leq 180$$

$$x_j^{UL} \leq x_j^{UU}, x_j^{UL} \geq 0, j = 1, 2$$

rough integer variables. And

$$ILP_1^{LL} := \text{Max } 0.5x_1^{LL} + 1.5x_2^{LL}$$

$$s.t \quad 1.5x_1^{LL} + 5.5x_2^{LL} \leq 200$$

$$7.5x_1^{LL} + 5.5x_2^{LL} \leq 350$$

$$2.5x_1^{LL} + 0.5x_2^{LL} \leq 240$$

$$2.5x_1^{LL} + 1.5x_2^{LL} \leq 102$$

$$x_j^{LL} \leq x_j^{UL}, x_j^{LL} \geq 0, j = 1, 2$$

and rough integer variables.

We used "WinQSB" program to find efficient value solutions and efficient integer solutions for the UAI and the LAI for example (2). Also, we used to apply branch and bound algorithm for integer programming, as following results:

$$ILP^{UU} = 151, \text{ where } x_1^{UU} = 32, x_2^{UU} = 29$$

$$ILP^{UL} = 113, \text{ where } x_1^{UL} = 32, x_2^{UL} = 26$$

$$ILP^{LL} = 51.5, \text{ where } x_1^{LL} = 25, x_2^{LL} = 26$$

$$ILP^{LU} = 26, \text{ where } x_1^{LU} = 0, x_2^{LU} = 26$$

Where the rough efficient values range solutions for  $Z^{*R} = ([ILP^{LAI}], [ILP^{UAI}]) = ([51.5, 113], [26, 151])$ .

The integer rough optimal solutions are

$$x_1^{*R} = ([25, 32], [0, 32]), x_2^{*R} = ([26, 26], [26, 29])$$

And the possibly optimal values range solutions for  $ILP^{UAI}$  are  $[ILP^{LU}, ILP^{UU}] = [26, 151]$ .

Moreover, the surely optimal values range solutions for  $ILP^{LAI}$  are  $[ILP^{LL}, ILP^{UL}] = [51.5, 113]$ .

In addition, the integer completely satisfactory for  $[x_1^{*LL}, x_1^{*UL}] = [25, 32], [x_2^{*LL}, x_2^{*UL}] = [26, 26]$  and the integer rather satisfactory solution for  $[x_1^{*LU}, x_1^{*UU}] = [0, 32], [x_2^{*LU}, x_2^{*UU}] = [26, 29]$ .

### Advantages and differences

1. For linear programming problems, the weighting strategy is applied to the pristine feasible region and results in a corner solution (extreme points), hence producing only efficient extreme solutions. On the opposite, the  $\varepsilon$ -Constraint method alters the pristine feasible region and is able to engender non-extreme efficient solutions. As a conclusion, with the weighting strategy we may spend a plethora of runs that are redundant in the sense that there can be an abundance of amalgamations of weights that result in the same efficient extreme solution. On the other hand, with the  $\varepsilon$ -constraint we can exploit virtually every run to engender a different efficient solution, thus obtaining a richer representation of the efficient set.
2. The weighting method cannot engender unsupported efficient solutions in multi-objective integer and commixed integer programming quandaries, whereas the  $\varepsilon$

-constraint method does not endure from this trap [19, 20].

3. In the weighting strategy, the scaling of the target capacities impacts the acquired outcomes. Subsequently, we have to scale the target capacities to a typical scale before shaping the weighted entirety. In the  $\varepsilon$ -constraint technique this is not essential.

With the additional favorable position of the  $\varepsilon$ -imperative strategy is that we can control the quantity of the created proficient arrangements by legitimately altering the quantity of matrix focuses in every last one of the target work ranges. This is not so natural with the weighting technique.

### Conclusion

In the presented paper a solution algorithm has been proposed to solve fully rough multi-objective integer linear programming problems by two methods and found rough value efficient solutions and decision rough integer variables. In the first phase of the solution approach and to avoid the complexity of this problem we began by converting the rough nature of this problem into an equivalent crisp problem. In the second phase we used some of the concepts to determine and choose values of the weights and  $\varepsilon$ -constraint technique. We obtained rough efficient values and integer rough efficient solutions. Also, the used rough intervals are very important to tackle the uncertainty in decision making problems. In addition, we obtained on N suggested solutions and enabling the decision maker from making the best decision. Furthermore, we got on approximations interval respectively. Also, we posted the advantages and solutions such as completely satisfactory solutions (surely solutions) and rather satisfactory solutions (possibly solutions) by lower approximation interval and upper differences between the weighting method and the  $\varepsilon$ -constraint technique. We believe that this paper is an

attempt to establish underlying results which hopefully help others to answer some of these questions.

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