DJS Vol. 38 (2017) 19-26

Delta Journal of Science

Available online at

https://djs.journals.ekb.eg/

Research Article

MATHEMATICS

Rough Convexity of A Set and A Function

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Abstract: In this paper a new concept of convexity approximation for non convex set and nonconvex function with respect to family of convex sets and to family of convex function respectively is presented. This approximation of convexity is called rough convexity. Some properties of these kind of sets and functions are discussed.

Key words: *Rough convexity with respect to family of convex sets , rough supporting hyperplane , rough convex function with respect to family of convex functions.*

1 -Introduction

The concept of convexity of sets and functions plays an important role in the field of mathematical programming. The convexity ensure the globality of solutions. In real life problems the convexity maybe not satisfied, so the objective function or the constraints or both may not be convex that pushes the researchers to relax non convexity of sets and functions to what is called generalized convexity such as, quasi, pseudo, E-convexity.

Recently E.A.Youness [3] discussed an optimization problem that involves roughness notion in the constraint set. Also Fang D., [6] presented rough approximation of non convex set.

In this paper the rough approximation non convex set and non convex function in terms of family of convex sets and family of convex functions , are presented and some their properties are discussed.

2 - Rough convex set

Definition 2.1 : Let X be an universal set , F be a family of non-disjoint convex sets $F=\{A_1,A_2,\ldots,A_n\}$, $B\subset X$ and $\sigma(x,y)$ is a space of segments between x and y in B. B is called rough convex with respect to family F if for each $x,y\in B$, $\sigma(x,y)$ either contained in $\bigcap_{i=1}^{n}A_i$ or $\sigma\bigcap A_i\neq\varphi$ for at least one i .

The Lower convexity of B is:

$$L(Conv)=\left\{\sigma(x,y):\sigma(x,y)\subset\bigcap_{i=1}^{n}A_i\ ,\sigma(x,y)\subset B\right\}$$

and

The upper convexity of B is:

$U(Conv) = \{\sigma(x,y) : \sigma(x,y) \bigcap A_i \neq \phi \text{ for at least one } i\}$

The following figures show the rough convexity of a set B with respect to a family of a convex sets. In figure (2 - 1) and figure(2 - 2) the sets B and B' are rough convex with respect to A_1, A_2 and A_3, but in figure (2 - 3) and figure (2 - 4) the sets B'' and B''' are non rough convex with respect to them.

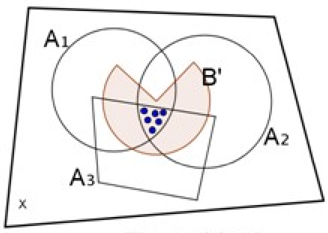

Figure (2-1)

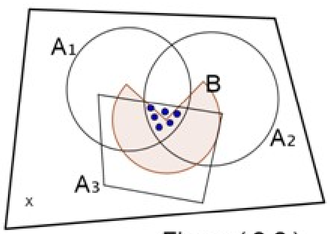

Figure (2-2)

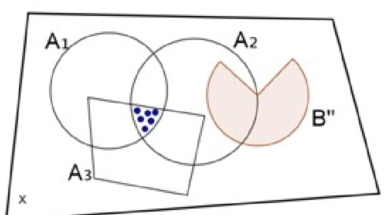

Figure (2-3)

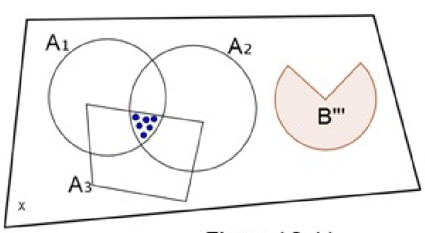

Figure (2-4)

Example 2.1 : Consider a universal

$$X = \{ (x,y) \in R^2 : -3 \leq x \leq 3 \quad , \quad -3 \leq y \leq 3 \}$$

and family F :

$$F = \begin{Bmatrix} A_1 = \{(x,y) \in R^2 : (x)^2 + (y+1)^2 \leq 2\} \\ A_2 = \{(x,y) \in R^2 : (x-1)^2 + (y-1)^2 \leq 3\} \\ A_3 = \{(x,y) \in R^2 : (x+1)^2 + (y-1)^2 \leq 3\} \end{Bmatrix}$$

A set

$$B = \{(x,y) \in R^2 : y \geq \frac{1}{2}x^2 , \quad y \leq |0.5x| + 1 \quad , \quad -2 \leq x \leq 2 \quad \}$$

is rough convex with respect to F with:

Lower convexity of

$$L = \left\{ \sigma : \sigma = \lambda x + (1-\lambda) y \, , \, 0 \leq \lambda \leq 1 \; x, y \in \bigcap_{i=1}^{3} A_i \right\}$$

And

upper convexity

$U = \{\sigma : \sigma = \lambda x + (1-\lambda) y \, , 0 \leq \lambda \leq 1 \; x, y \in B\}$,

see Figure (2-5).

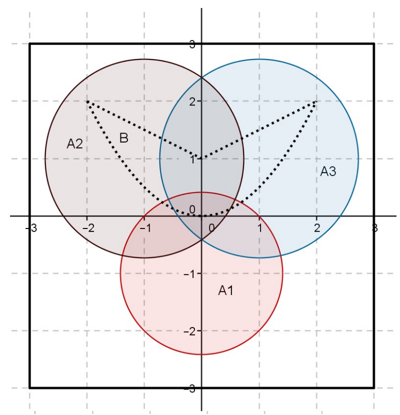

Figure (2 - 5)

Remark 2.1 :.If **L(conv)** = **U(conv)** of the set **B** ,with respect to the family $F = \{A_1, A_2, \ldots, A_n\}$,then the set B is completely convex and hence $B \subset \bigcap_{i=1}^{n} A_i$.

A convex set is rough convex with respect to itself and its **L(conv)** = its **U(conv)**

Theorem 2.1 : If **B** is a roughly convex with respect to a family of convex sets $A_i \, , i = 1,2,3,\ldots,n$, then $B' = f(B)$ is a roughly convex with respect to family $C_i \, , i = 1,2,3,\ldots,n$,such that $f(x_{Ci}) = x_{Ai} \mp \alpha \, , x_{Ci} \in C_i$ where

$x_{ci} \in C_i$, $x_{Ai} \in A_i$,and **B'** is the image of **B (i.e :** $f(B) = B'$**).**

Proof: **B** is roughly convex with respect to A_i , so

$\sigma(x,y) \subset \bigcap_{i=1}^{n} A_i$ or $\sigma(x,y) \cap A_i \neq \varphi$, for all $x, y \in B$.

uppose $x, y \in B' = f(B)$, $x, y \in \cup C_i$

thus, $f(x_i) = x_{A_i} + \alpha$, $f(y_{C_i}) = y_{A_i} + \alpha$,

and

$$\sigma(x_{ci}, y_{ci}) = \lambda x_{Ci} + (1-\lambda) y_{Ci}$$
$$= \lambda [x_{Ai} + \alpha] + (1-\lambda)[y_{Ai} + \alpha]$$
$$= \sigma(x_{Ai}, y_{Ai}) + \alpha$$

So,

$\sigma(x_{C_i}, y_{C_i}) \subset \bigcap_{i=1}^{n} C_i$ *or* $\sigma(x_{C_i}, y_{C_i}) \cap C_i \neq \varphi$. Hence $B' = f(B)$ is roughly convex with respect to C_i

Theorem 2.2 : If $B \subset X$ is a roughly convex with respect to a family of convex sets A_i , $i = 1,2,3,...,n$ .Let $f : X \to X$ be a map such that $f(A_i) = A_j, i, j \in \{1,2,3,......,n\}$ then , $B' = f(B)$ is a roughly convex with respect to

family A_i .

Proof : The proof is clear, since for $x', y' \in B'$, then there exist $x, y \in B$ such that

$$\lambda x' + (1-\lambda) y' = \lambda f(x) + (1-\lambda) f(y) \subset \bigcap_{i=1}^{n} A_i$$

or $[\lambda f(x) + (1-\lambda) f(y)] \cap A_i \neq \phi$, for some i .

Example 2.2 : Consider the following two families of convex sets, $A = \{A_1, A_2, A_3\} \subset X \subset R$ and $C = \{ C_1, C_2, C_3 \}$,and the relation between the two families is given by the following

$f : A_i \to C_i$ such that $f(x,y) = (x-6, y-2)$

$$X = \{(x,y) \in R^2 : -4 \le x \le 9, -4 \le y \le 5\}$$

$$A = \begin{Bmatrix} A_1 = \{(x,y) : (x+1)^2 + (y+1)^2 \le 4 \\ A_2 = \{(x,y) : (x)^2 + (y-1)^2 \le 2 \\ A_3 = \{(x,y) : (x-1)^2 + (y)^2 \le 3 \end{Bmatrix}$$

The set

$B = \{x, y \in R^2 : 5(x-0.3)^2 - 1,\ |0.5(x-0.3)| + 0.7 \ ,\ -0.34 \le x \le 0.94 \ \}$

is rough convex with respect to ***A***

$$C = \begin{Bmatrix} C_1 = \{(x,y) : (x-5)^2 + (y-1)^2 \le 4\} \\ C_2 = \{(x,y) : (x-6)^2 + (y-3)^2 \le 2\} \\ C_3 = \{(x,y) : (x-7)^2 + (y-2)^2 \le 3\} \end{Bmatrix}$$

and the set

$B' = \{x, y \in R^2 : 5(x-6.3)^2 + 1,\ |0.5(x-6.3)| + 2.7 \ ,\ 5.66 \le x \le 6.94 \ \}$

is rough convex with respect to ***C*** ,See Figure(2-6).

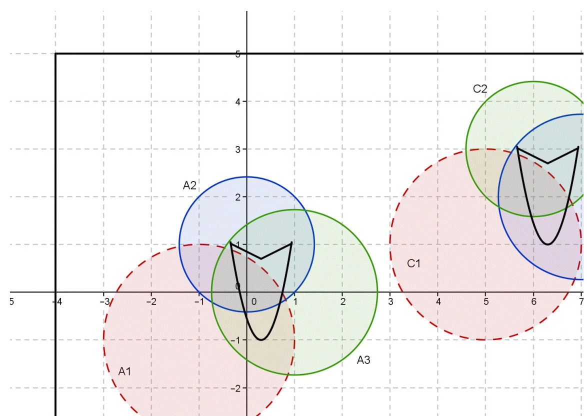

Figure (2-6).

Theorem 2.3 : Let X be an universal set ,

$F = \{A_1, A_2, \ldots, A_n\}$ is a family of convex subsets

of X .If B and B' are roughly convex with respect to $F = \{A_1, A_2, \ldots, A_n\}$, then

1. $B \cap B'$ is a rough convex with respect to a family F.
2. $B \cup B'$ is not necessarily to be a rough convex with respect to a family F .

Proof :

1. Let $x, y \in B \cap B' \Rightarrow x, y \in B$ and $x, y \in B'$.

Assume $\sigma(x,y)$ the segment whose ends x and y.

From the roughly convexity of B and B' with respect to $F=\{A_1,A_2,\ldots,A_n\}$.

we have

$\sigma(x,y)\subset\bigcap A_i$ or $\sigma(x,y)\bigcap A_i\neq\varphi$, for at least one i.

So, $B\bigcap B'$ is rough convexity with respect to a family of convex sets.

2. Let $x,y\in B\bigcup B'$,

So may

$x,y\in B$, $x,y\in B'$ or $x,y\in B\bigcap B'$

Thus

$\sigma(x,y)\subset\bigcap A_i$ or $\sigma(x,y)\bigcap A_i\neq\varphi$, for at least one i

$\Rightarrow B\bigcup B'$ is rough convexity but , If $x\in B$ and $y\in B'$ implies that $B\bigcup B'$ does not necessarily contain σ , see Figure (2-7)

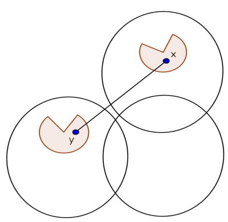

Figure (2-7)

Remark 2.2 :

The lower and upper approximation of convexity for intersection and union of two roughly convex sets B and B' given as:

$$\left\langle \begin{array}{l} L_{\cap}(Con)=\left\{\sigma(x,y):\sigma(x,y)\subset\bigcap_{i=1}^{n}A_i\ ,\sigma(x,y)\in B\cap B'\right\} \\ U_{\cap}(Con)=\left\{\sigma(x,y):\sigma(x,y)\cap\left(\bigcap_{i=1}^{n}A_i\right)\neq\varphi\ ,\text{for at least one } i\right\} \end{array}\right\rangle$$

see Figure (2-8)

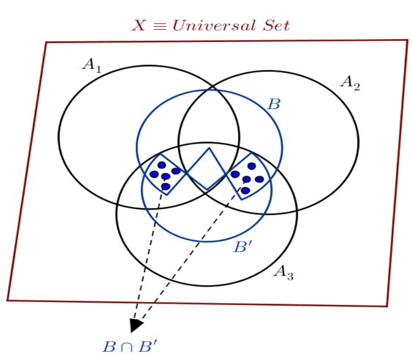

Figure (2-8)

$$\left\langle \begin{array}{l} L_{\cup}(Con)=\left\{\sigma(x,y):\sigma(x,y)\subset\bigcap_{i=1}^{n}A_i\ ,\sigma(x,y)\in B\cup B'\right\} \\ U_{\cup}(Con)=\left\{\sigma(x,y):\sigma(x,y)\cap\left(\bigcap_{i=1}^{n}A_i\right)\neq\varphi\ ,\text{for at least one } i\right\} \end{array}\right\rangle$$

see Figure (2-9)

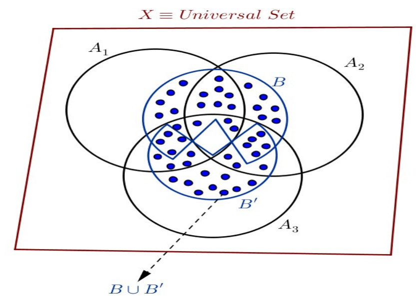

Figure (2-9)

Preposition 2.1 : Let X be an universal set ,and B and B' are roughly convex with respect to a family of convex sets $F=\{A_1,A_2,\ldots,A_n\}$, then the following properties are satisfying

1. Intersection properties

$$L(conv)_{B\bigcap B'}\subset L(conv)_B\cup L(conv)_{B'}$$

(always true) .

$$U(conv)_{B\bigcap B'}\subset U(conv)_B\cup U(conv)_{B'}$$ 3

(always true) .

2. Union properties

$$L(conv)_{B\cup B'}\supset L(conv)_B\quad ,\quad L(conv)_{B'}$$

(always true) .

$$U(conv)_{B\bigcup B'}\supset U(conv)_B\ ,U(conv)_{B'}$$

(not always true).

Proof :

1. Intuitively $B\cap B'\subset B$ and $B\cap B'\subset B'$

If $\sigma(x,y)\subset B\bigcap B'\Rightarrow\sigma(x,y)\subset B$ and $\sigma(x,y)\subset B'$

So, $L(conv)_{B\bigcap B'}\subset L(conv)_B\cup L(conv)_{B'}$ and

$$U(conv)_{B\bigcap B'}\subset U(conv)_B\cup U(conv)_{B'}$$

2. Since $B\bigcup B'\supset B$ and $B\bigcup B'\supset B'$

If $\sigma(x,y)\subset B$, $\sigma(x,y)\subset B'$ or $\sigma(x,y)\subset B\cap B'$ *then* $\sigma(x,y)\subset B\cup B'$

So,

$L(conv)_{B\cup B'}\supset L(conv)_B$ *and* $L(conv)_{B'}$

$U(conv)_{B\cup B'}\supset U(conv)_B$ *and* $U(conv)_{B'}$

But

$U(conv)_{B\cup B'}\supset U(conv)_B$ *and* $U(conv)_{B'}$

is not necessary to be true when $x\in B$ and $y\in B'$.

3 - Rough Supporting Hyperplane

Definition 3.1 : Let X be an universal set in R^n ,B is a rough convex set with respect to family of convex sets $\Im=\{A_1,A_2,\dots,A_n\}$,

$H=\{\tilde{H}_1,\tilde{H}_2,\tilde{H}_3\}\subset$ Ƕ is the set of all supporting hyperplanes of the family .Consider the following three classes :

$$\tilde{H}_1=\left\{H:\begin{matrix}H \text{ is supporting for at least one}\\ \text{of the family and } H\cap B=\phi\end{matrix}\right\}$$

$$\tilde{H}_2=\left\{H:\begin{matrix}H \text{ is supporting for at least one of the}\\ \text{family and } H\cap B\neq\phi \text{ at one point}\end{matrix}\right\}$$

$$\tilde{H}_3=\left\{H:\begin{matrix}H \text{ is supporting for at least one of the family}\\ \text{and } H\cap B\neq\phi \text{ at least one point}\end{matrix}\right\}$$

Each element $h\in H\subset$ Ƕ is called rough supporting of B , if H is a rough set in Ƕ .

Upper Supporting approximation:

$U_H=\{H:H \text{ is supporting for at least one, of family and } H\cap B\neq\varphi\}$

, see Figure (3-1).

Lower Supporting approximation:

$L_H=\{H:H \text{ is supporting for at least one, of family and } H\cap B=\varphi\}$

, see Figure (3-2).

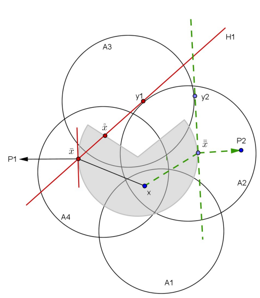

Figure (3-1)

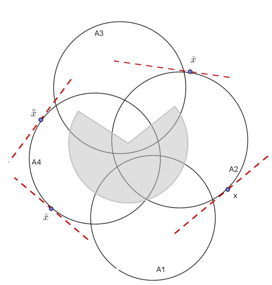

Figure (3- 2)

4 - Rough Convexity of a Function

Definition 4.1 : Let

$\Im=\{f_i:B\to R\ ,i=1,2,..,k\ ,f_i(x) \text{ convex}\}$;

be a family of convex functions , $f(x)$ is called rough convex on a convex set $B\subset R^n$ with respect to $\Im$, if for each $x,y\in B$, there exist $f_l(x),f_t(x)\in\Im$ **,** such that

$f(\lambda x+(1-\lambda)y)\ \leq\ \lambda f_l(x)+(1-\lambda)f_t(y)$, $0\leq\lambda\leq1$

The upper convexity of f is denoted by $U(conv\ f)$ **and is defined as**

$U(conv\ f)=\{f_i\in\Im:f_i(z)>f(z)\ ,\ z\in\lambda x+(1-\lambda)y\ ,i\in\{1,.....,k\}\}$

The lower convexity of f is denoted by $L(conv\ f)$

and is defined as

$L(conv\ f)=\{f_i\in\Im:f_i(z)\leq f(z)\ ,\ z\in\lambda x+(1-\lambda)y\ ,i\in\{1,.....,k\}\}$

It is clear that for each $f_h\in L(conv\ f)$, implies $f_h(x)<f_i(x)\in U(conv\ f)$.

Rough concavity it is defined in similar way by reversing the sign $\leq$ to $\geq$ **,i.e:** $f(\lambda x+(1-\lambda)y)\ \geq\ \lambda f_l(x)+(1-\lambda)f_t(y)$, $0\leq\lambda\leq1$

Examples 4.1 :

Let

$$\Im=\left\{\begin{matrix}f_1(x)=e^{-x}+2\ ,\ f_2(x)=2x^2-6x+4\ ,\\ f_3(x)=x^2-2\ ,\ f_4(x)=e^{-x}+20\ ,\ f_5(x)=x^2+16\end{matrix}\right\}$$

be a family of convex function defined on $B=[0,5]$. The function

$f(x)=\frac{1}{3}x^3-x^2-x+15$ be a rough convex function on B with respect to a family of convex functions $\Im$. and its upper and lower convexity are

$$\left\langle\begin{matrix}U(conv\ f)=\{f_4(x),f_5(x)\}\\ L(conv\ f)=\{f_1(x),f_3(x)\}\end{matrix}\right\rangle$$

, see Figure (4-1)

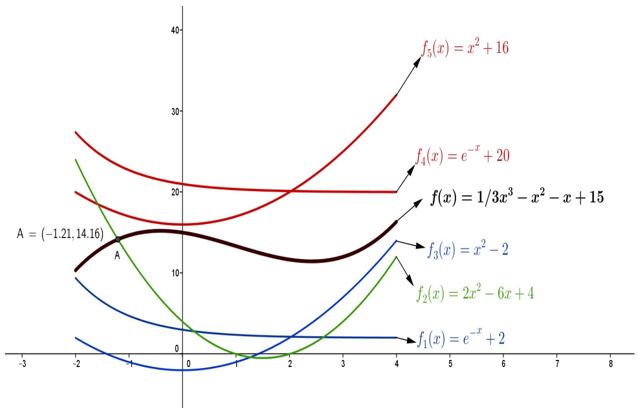

Figure (4-1)

In the following, the characterization of rough convexity of a function , in terms of rough convexity of its level set and its epigragh , is discussed .

5 - Rough Epigraph

Definition 5.1 : Let E_i be an epigraph of a convex function f_i , let f be a rough convex with respect to the family f_i .An epigraph of f is denoted by

E_f and is defined as : $E_f = \{(x,\alpha): f(x) \le \alpha\}$.

Theorem 5.1 : If f is a rough convex with respect to f_i ,then E_f is rough convex set with espect to E_i .

Proof: Assume (x_1, y_1) and $(x_2, y_2) \in E_f$, i.e.,

$f(x_1) \le y_1$, $f(x_2) \le y_2$.

Since f is rough convex with respect to family $\Im$, so, there exist $f_l, f_t \in \Im$ such that

$$f(\lambda x_1 + (1-\lambda)x_2) \le \lambda f_l(x_1) + (1-\lambda) f_t(x_2), \lambda \in [0,1]$$

If f_l and f_t are in $L(conv\ f)$, then

$f_l(x_1) \le f(x_1) \le y_1$ and

$f_t(x_2) \le f(x_2) \le y_2$. Therefore, $f(\lambda x_1 + (1-\lambda)x_2) \le \lambda y_1 + (1-\lambda)y_2$.

Since f_l and f_t are convex, so if

$f_t(x_1) \le f_t(x_2) \le y_2$

then $(\lambda x_1 + (1-\lambda)x_2), (\lambda y_1 + (1-\lambda)y_2) \in E_l(f_l)$.

Similarly

$(\lambda x_1 + (1-\lambda)x_2), (\lambda y_1 + (1-\lambda)y_2) \in E_t(f_t)$

which implies

$\{(\lambda x_1 + (1-\lambda)x_2), (\lambda y_1 + (1-\lambda)y_2)\} \cap E_i \ne \varphi$

for at least one i.

Thus the rough convexity of $E(f)$ on the other hand, if f_l or $f_t \in U(conv\ f)$, then for (x_1, y_1) ,$(x_2, y_2) \in E_i(f_i)$, we get $(x_1, y_1), (x_2, y_2) \in E(f)$.

Since f_l is convex

, $(\lambda x_1 + (1-\lambda)x_2), (\lambda y_1 + (1-\lambda)y_2) \in E_l(f_l)$,

i.e., $f_l(\lambda x_1 + (1-\lambda)x_2) \le \lambda y_1 + (1-\lambda)y_2$.

Thus

$f_l \in U(conv\ f)$ implies to

$$f(\lambda x_1 + (1-\lambda)x_2) \le f_l(\lambda x_1 + (1-\lambda)x_2) \\ \le \lambda y_1 + (1-\lambda)y_2 .$$

Hence the result .

Lemma 5.1 : Let S be a nonempty convex set subset in R^n , let f_1 ,f_2 be rough convex with respect to family of convex functions

$\Im = \{f_i(x): R^n \to R \ , i = 1,2,..,n\}$ .If $\Im$ is closed under addition then $f_1(x) + f_2(x)$ is rough convex.

Proof: Let $g(x) = f_1(x) + f_2(x)$, $f_1(x)$,$f_2(x)$ are rough convex that means, for each $x_1, x_2 \in S$, $0 \le \lambda \le 1$, there exist $f_t(x), f_l(x), f_n(x), f_m(x) \in \Im$ such that :

$z = \lambda x_1 + (1-\lambda)x_2 \in S$, $0 \le \lambda \le 1$ and from rough convexity of $f_1(x)$ and $f_2(x)$,We have

$f_1(z) \le \lambda f_t(x_1) + (1-\lambda) f_l(x_2)$

and $f_2(z) \le \lambda f_n(x_1) + (1-\lambda) f_m(x_2)$

which implies

$$g(z)=(f_1+f_2)(z)=f_1(z)+f_2(z)$$
$$\le \lambda f_t(x_1)+(1-\lambda)f_l(x_2)+\lambda f_n(x_1)+(1-\lambda)f_m(x_2)$$

$$=\lambda[f_t(x_1)+f_n(x_1)]+(1-\lambda)[f_l(x_2)+f_m(x_2)]$$
$$=\lambda f_k(x_1)+(1-\lambda)f_r(x_2)$$, where

$$f_k=f_t+f_n \quad , \quad f_r=f_l+f_m$$

From closeness of $\Im$ under addition, $f_k, f_r \in \Im$. Hence the result.

corollary 5.1 : Let S be a nonempty convex set subset in R^n , let $f_1(x)$ be rough convex and $f_2(x)$ be rough concave with respect to family of convex functions $\Im=\{f_i : R^n \to R \,, i=1,2,..,n\}$. Then $f_1(x)-f_2(x)$ is rough convex.

Proof: The proof immediately yields from lemma 5.1 .

6 - Level set

Definition 6.1 : Let S be a nonempty convex set subset in R^n . Let $f : S \to R$ be a rough convex with respect to family of convex functions $\Im=\{f_i : R^n \to R \,, i=1,2,..,n\}$. A level set of $f(x)$ is defined as $S_\alpha=\{x \in S : f(x)\le \alpha\}$, it is clear that $S_\alpha \subset \bigcup S_{\alpha_i}$, where $S_{\alpha_i}=\{x \in S : f_i(x)\le \alpha_i\}$. see Figure (6-1)

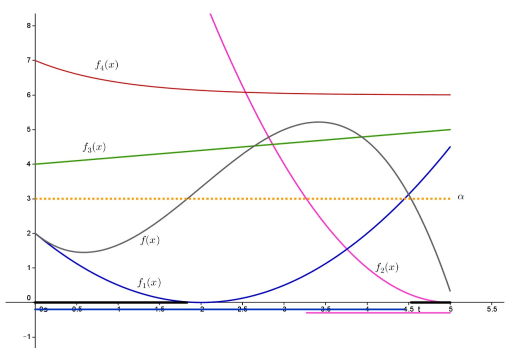

Figure (6-1)

Theorem 6.2 : Let f be a rough convex with respect to family convex functions

$\Im=\{f_i : R^n \to R \,, i=1,2,..,n\}$. Assume $S_{\alpha i}$ is the level set of f_i . A level set S_α of f is rough convex with respect to S_{α_i} , $\alpha=\max_i \alpha_i$.

Proof: Since $S_\alpha \subset \bigcup S_{\alpha_i}$

$x, y \in \bigcup S_{\alpha_i}$ then x, y may be in $\bigcap S_{\alpha_i}$ which implies $\lambda x+(1-\lambda)y \subset \bigcap_i S_{\alpha_i}$

Thus $f_i(\lambda x+(1-\lambda)y)\le \alpha_i \le \alpha$

since f is rough convex , there exist f_l and f_t ,such that

$$f(\lambda x+(1-\lambda)y)\le \lambda f_l(x)+(1-\lambda)f_t(y)\le \lambda\alpha_l+(1-\lambda)\alpha_t \le \alpha$$

On the other hand , $x, y \notin \bigcap_i S_{\alpha_i}$, $x, y \in \bigcup S_{\alpha_i}$,

$\{\lambda x+(1-\lambda)y\}\bigcap S_{\alpha_i}\ne \phi$ for at least one i .

Conclusion :

In this paper the definition of rough convexity of set with respect to a family of convex sets ,and rough convexity of function with respect to a family of convex functions . Therefore we discussed the relationship between roughness with respect to a family of functions and their epigraphs and level sets.

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التحدب الأستقرابي للمجموعة والدالة

في هذا البحث تم تقديم مفهوم جديد لتقريب التحدب لكل من المجموعات غير المحدبة (non-convex sets) بالنسبة لعائلة من المجموعات المحدبة وكذلك الدوال غير المحدبة (non-convex factions) بالنسبة لعائله من الدوال المحدبة . سمي هذا التقريب للتحدب بالتحدب ألاستقرابي , وتم مناقشة بعض الخصائص لكل من المجموعات والدوال من هذا النوع .