DJS Vol. 38 (2017) 33- 42

Delta Journal of Science

Available online at

https://djs.journals.ekb.eg/

Research Article

MATHEMATICS

COORDINATED SEARCH FOR A RANDOMLY LOCATED TARGET

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Abstract: The coordinated search problem faced by two searchers who start together from some point on the line in order to search for a lost target which randomly located on the line. In this paper the distribution of the lost target is symmetric around the point of the intersection of two lines, where the intersected point is a starting point of the motion of the searchers. There are four searchers start together the search for the lost target at the intersection point under this condition we defined the search plan and computed the expected value of the first meeting time between one of the searchers and the target. The search plan which minimized this first meeting time is studied. Finally we obtained some special cases for a search problems. An illustrative examples is given to demonstrate the applicability of this model.

Key words: *symmetric distribution, Located target, coordinated search*

Introduction:

The study of search plans for any lost target either located or moved and having symmetric or unsymmetric distribution is important and has recently various applications, such as searching for a faulty unit in large linear system, such as electrical power lines, this kind of search is called linear search problem,(see [2], [4], [5] and [6]).

The coordinated search technique is one of a set of techniques, which studied on the line when the target has symmetric or unsymmetric distribution, (see [7], [8] ,[9] and [10]). Stone studied the located target on a known region, like petrol or gas supply underground, (see [12]). Also, the search for a located target in the plane and has symmetric or unsymmetric distribution has been studied see [1]and [3]. The search for a moving target like missing boats, submarines and missing system, a Bayesian approach would formulate for a target whose prior distribution and probabilistic motion model are known and generalized the approach for multi-vehicle search, (see [13] and [14]).

Let X be a random variable which represented the position of the target if it on the first line L_1, and Y be a random variable which represented the position of the target if it on the second line L_2, we have two searchers S_1 and S_2 on the first line L_1. Also, we have another two searchers S_3 and S_4 on the second line L_2. The four searchers S_1 ,S_2, S_3 and S_4 start together looking for the target from the point of the intersection of the two lines, on the first line L_1 the searcher S_1 searches to the right of the intersection point and the searcher S_2 searches to the left of the intersection point. Also, on the second line L_2 the searcher S_3 searches to the right of the intersection point and the searcher S_4 searches to the left of the intersection point, either the two searchers S_1 and S_2 detect the target on the first line L_1or the other two searchers S_3 and S_4 detect the target on the second line L_2 .

The four searchers return to the intersection point after searching successively common distances until the target is found. We assume that the speeds of the searchers are V_1,V_2,V_3 andV_4. Our aim is to calculate the first meeting time between one of the 4 searchers and the lost target. Also, we wish to find the optimal search plan which minimize the expected value.

2. Search plan

The generalized coordinated search problem asks how the four searchers who start together at the point of the intersection of two lines in a search region can minimize the expected time to meet back at the starting point after finding the lost target. The target is located according to a known symmetric distribution on one of two intersected lines L_1 and L_2, where the symmetric distribution of the target on L_1 is differs from the symmetric distribution of the target on L_2. The point of the intersection of the two lines is some point $a_0 = b_0$, where a_0 refers to the distance which the first searcher S_1 far from the origin of L_1, and b_0 refers to the distance which the second searcherS_2 far from the origin of L_1. Also, a_0 is the position of S_1on L_1 at the beginning of search and b_0 is the position of S_2, on L_1 at the beginning of search. Let $k_0 = z_0$, where k_0 refer to the distance which the third searcher S_3 far from the origin of L_2, and z_0 refer to the distance which the fourth searcher S_4 far from the origin of L_2. Also, k_0 is the position of S_3 on L_2 at the beginning of search and z_0 is the position S_4 on L_2 at the beginning of search .

The point of the intersection of the two lines is some point e_1 at which $a_0 = b_0$ on L_1, and e_2 at which $k_0 = z_0$ on L_2.The probable paths for the searchers S_1, S_2, S_3 and S_4 take the following cases:

Case(1),
in this case we have $e_1 > 0$ and $e_2 > 0$

Case(2),
in this case we have $e_1 > 0$ and $e_2 < 0$

Case(3),
in this case we have $e_1 < 0$ and $e_2 > 0$

Case(4),
in this case we have $e_1 < 0$ and $e_2 < 0$

Case(5),
in this case we have $e_1 = 0$ and $e_2 = 0$

Case(6),
in this case we have $e_1 > 0$ and $e_2 = 0$

Case(7),
in this case we have $e_1 < 0$ and $e_2 = 0$

Case(8),
in this case we have $e_1 = 0$ and $e_2 < 0$

Case(9),
in this case we have $e_1 = 0$ and $e_2 > 0$

On L_1 the two searchers S_1 and S_2 follow the search paths a and b, respectively to detect the target. The search path a of S_1 is completely defined by a sequence $\{ a_i ,i \geq 0\}$ and the search path b of S_2 is completely defined by a sequence $\{ b_i ,i \geq 0\}$ where i is a nonnegative integer. let the search plan be represented by $\phi = \{ a_i , b_i \} \in \Phi_1$, where Φ_1 is the set of all search plans. Also, we consider H be a set of positive integer numbers, where $H = \{1, 2, 3,\ldots\}$ and $L = \{1, 2, 3,\ldots,j \}$ be a finite subset of H. Also, we consider, $j \in L$, $i \in H$, where

$$i = \begin{cases} 1 & \text{, if L is empty set} \\ j+1 & \text{, if L is non-empty set} \end{cases}$$

On L_2 the two searchers S_3 and S_4 follow the search paths k and z, respectively to detect the target. The search path k of S_3 is completely defined by a sequence $\{k_{\bar{\imath}}, \bar{\imath} \geq 0\}$ and the search path z of S_4 is completely defined by a sequence $\{z_{\bar{\imath}}, \bar{\imath} \geq 0\}$, where $\bar{\imath}$ is a nonnegative integer .let the search plan be represented by $\bar{\phi} = \{k_{\bar{\imath}} ,z_{\bar{\imath}}\} \in \Phi_2$, where Φ_2 is the set of all search plans. Also we consider H be a set of positive integer numbers , where and $\bar{L} =.\{1, 2, 3,\ldots, \bar{\jmath}\}$ be a finite subset of H, $\bar{\imath} \in H$. Also, we consider $\bar{\jmath} \in \bar{L}$, where :

$$\bar{\imath} = \begin{cases} 1 & \text{, if } \bar{L} \text{ is empty set} \\ \bar{\jmath} + 1 & \text{, if } \bar{L} \text{ is non -empty set} \end{cases}$$

The probable search plans for the searchers S_1, S_2,S_3 and S_4 take the following cases according to the previous cases of the probable paths for the searchers S_1, S_2, S_3 and S_4 respectively.

Case(1), in this case we have the following search plan for S_1 and S_2 on L_1

$\phi = \{(a_i ,b_i)_{i \geq 0} : c < \ldots b_i < 0 < b_j < \ldots < b_3 < b_2 < b_1 < b_0 = a_0 < a_1 < a_2 < \ldots < d\}$

,where $d = \lim_{i \to \infty} a_i$, $c = \lim_{i \to \infty} b_i$ and $|a_{i+1} - a_i| = |b_{i+1} - b_i|$, $i \geq 0$

and the following search plan for S_3 and S_4 on L_2

$\bar{\phi} = \{(k_{\bar{\imath}}, z_{\bar{\imath}})_{\bar{\imath} \geq 0} : e < \ldots < z_{\bar{\imath}} < 0 < z_j < \ldots < z_3 < z_2 < z_1 < z_0 = k_0 < k_1 < k_2 < \ldots < g\}$

,where $g = \lim_{\bar{\imath} \to \infty} k_{\bar{\imath}}$, $e = \lim_{\bar{\imath} \to \infty} z_{\bar{\imath}}$ and $|k_{\bar{\imath}+1} - k_{\bar{\imath}}| = |z_{\bar{\imath}+1} - z_{\bar{\imath}}|$, $\bar{\imath} \geq 0$

Case(2), in this case we have the following search plan for S_1 and S_2 on L_1

$\phi = \{(a_i, b_i)_{i \geq 0} : c < \ldots b_i < 0 < b_j < \ldots < b_3 < b_2 < b_1 < b_0 = a_0 < a_1 < a_2 < \ldots < d\}$

and the following search plan for S_3 and S_4 on L_2

$\bar{\phi} = \{(k_{\bar{\imath}}, z_{\bar{\imath}})_{\bar{\imath} \geq 0} : e < \ldots < z_3 < z_2 < z_1 < z_0 = k_0 < k_1 < k_2 < \ldots < k_j < 0 < k_{\bar{\imath}} < \ldots < g\}$

Case(3), in this case we have the following search plan for S_1 and S_2 on L_1

$\phi = \{(a_i, b_i)_{i \geq 0} : c < \ldots < b_3 < b_2 < b_1 < b_0 = a_0 < a_1 < a_2 < \ldots < a_j < 0 < a_i < \ldots < d\}$

and the following search plan for S_3 and S_4 on L_2

$\bar{\phi} = \{(k_{\bar{\imath}}, z_{\bar{\imath}})_{\bar{\imath} \geq 0} : e < \ldots < z_i < 0 < z_j < \ldots < z_3 < z_2 < z_1 < z_0 = k_0 < k_1 < k_2 < \ldots < g\}$

Case(4), in this case we have the following search plan for S_1 and S_2 on L_1

$\phi = \{(a_i, b_i)_{i \geq 0} : c < \ldots < b_3 < b_2 < b_1 < b_0 = a_0 < a_1 < a_2 < \ldots < a_j < 0 < a_i < \ldots < d\}$

and the following search plan for S_3 and S_4 on L_2

$\bar{\phi} = \{(k_{\bar{\imath}}, z_{\bar{\imath}})_{\bar{\imath} \geq 0} : e < \ldots < z_3 < z_2 < z_1 < z_0 = k_0 < k_1 < k_2 < \ldots < k_j < 0 < k_{\bar{\imath}} < \ldots < g\}$

Case(5), in this case we have the following search plan for S_1 and S_2 on L_1

$\phi = \{(a_i, b_i)_{i \geq 0} : c < \ldots < b_3 < b_2 < b_1 < b_0 = a_0 = 0 < a_1 < a_2 < a_3 < \ldots < d\}$

and the following search plan for S_3 and S_4 on L_2

$\bar{\phi} = \{(k_{\bar{\imath}}, z_{\bar{\imath}})_{\bar{\imath} \geq 0} : e < \ldots < z_3 < z_2 < z_1 < z_0 = k_0 = 0 < k_1 < k_2 < k_3 < \ldots < g\}$

Case(6), in this case we have the following search plan for S_1 and S_2 on L_1

$\phi = \{(a_i, b_i)_{\bar{\imath} \geq 0} : c < \ldots b_i < 0 < b_j < \ldots < b_3 < b_2 < b_1 < b_0 = a_0 < a_1 < a_2 < \ldots < d\}$

and the following search plan for S_3 and S_4 on L_2

$\bar{\phi} = \{(k_{\bar{\imath}}, z_{\bar{\imath}})_{\bar{\imath} \geq 0} : e < \ldots < z_3 < z_2 < z_1 < z_0 = k_0 = 0 < k_1 < k_2 < k_3 < \ldots < g\}$

Case(7), in this case we have the following search plan for S_1 and S_2 on L_1

$\phi = \{(a_i, b_i)_{i \geq 0} : c < \ldots < b_3 < b_2 < b_1 < b_0 = a_0 < a_1 < a_2 < \ldots < a_j < 0 < a_i < \ldots < d\}$

and the following search plan for S_3 and S_4 on L_2

$\bar{\phi} = \{(k_{\bar{\imath}}, z_{\bar{\imath}})_{\bar{\imath} \geq 0} : e < \ldots < z_3 < z_2 < z_1 < z_0 = k_0 = 0 < k_1 < k_2 < k_3 < \ldots < g\}$

Case(8), in this case we have the following search plan for S_1 and S_2 on L_1

$\phi = \{(a_i, b_i)_{i \geq 0} : c < \ldots < b_3 < b_2 < b_1 < b_0 = a_0 = 0 < a_1 < a_2 < a_3 < \ldots < d\}$

and the following search plan for S_3 and S_4 on L_2

$\bar{\phi} = \{(k_{\bar{\imath}}, z_{\bar{\imath}})_{\bar{\imath} \geq 0} : e < \ldots < z_3 < z_2 < z_1 < z_0 = k_0 < k_1 < k_2 < \ldots < k_j < 0 < k_{\bar{\imath}} < \ldots < g\}$

Case(9), in this case we have the following search plan for S_1 and S_2 on L_1

$\phi = \{(a_i, b_i)_{i \geq 0} : c < \ldots < b_3 < b_2 < b_1 < b_0 = a_0 = 0 < a_1 < a_2 < a_3 < \ldots < d\}$

and the following search plan for S_3 and S_4 on L_2

$\bar{\phi} = \{(k_{\bar{\imath}}, z_{\bar{\imath}})_{\bar{\imath} \geq 0} : e < \ldots < z_{\bar{\imath}} < 0 < z_J < \ldots < z_3 < z_2 < z_1 < z_0 = k_0 < k_1 < k_2 < \ldots < g\}$

The four searchers S_i, $i = 1, 2, 3$ and 4 start together looking for the target from the intersection point of the two lines L_1 and L_2, where $a_0 = b_0$, $k_0 = z_0$.On L_1 the searcher S_1 goes from the intersection point to a_1 and the searcher S_2 goes from the intersection point to b_1 then they return to the intersection point. Also, on L_2 the searcher S_3 goes from the intersection point to k_1 and the searcher S_4 goes from the intersection point to z_1 then they return to the intersection point. The two searchers S_1 and S_2 have the same distances on L_1 and the other two searchers S_3 and S_4 have the same distances on L_2 but differ from the distances of S_1 and S_2 on L_1 because the lost target has the two difference symmetric distributions on L_1 and L_2. The four searchers meet back at the intersection point but may be the first two searchers S_1 and S_2 meet back at the intersection point before the second two searchers S_3 and S_4 in this case the searchers S_1 and S_2 will be wait the other two searchers S_3 and S_4 until they arrived to the intersection point on the other hand the two searchers S_3 and S_4 may be meet back at the intersection point before the second two searchers S_1 and S_2 in this case the searchers S_3 and S_4 will be wait the other two searchers S_1 and S_2 until they arrived to the intersection point. If they did not detect the target they start the search again from the intersection point to the right and the left and again they return to the intersection point and so on until one of them detect the target.

Let $V_i = 1$, $i = 1, 2, 3, 4$ and the search plan of the searchers represented by $\Phi = (\phi, \bar{\phi}) \in \tilde{\Phi}$, where $\tilde{\Phi}$ is the set of all search plans . On L_1 we assume that the probability of the position of the target at each point in $\{c,d\}$ can be calculated from a given distribution function $F_1(x)$ with a density function $f_1(x)$ which is symmetric about the intersection point. Also, on L_2 the probability of the position of the target at each point in $\{e,g\}$ can be calculated from another distribution function $F_2(y)$ with a density function $f_2(y)$ which is symmetric about the intersection point. Let $|a_i - a_{i-1}| = |b_i - b_{i-1}|$ on L_1 are differ from $|k_i - k_{i-1}| = |z_i - z_{i-1}|$ on L_2, $i \geq 0$.

There is a known probability measure $\nu = \nu_1 + \nu_2$ on $\{c,d\} \cup \{e,g\}$ which describes the position of the target ,where ν_1 is the probability measure induced by the position of the target on $\{c,d\}$ .Also , ν_2 is the probability measure induced by the position of the target on $\{e,g\}$, and $\nu_1\{c,d\} + \nu_2\{e,g\} = 1$, where.

$\nu_1 (x_1,x_2) = F_1 (x_2) - F_1 (x_1)$.

and

$\nu_2 (y_1,y_2) = F_2 (y_2) - F_2 (y_1)$

Let D_i, $i = 1, 2$ be the time for the searcher S_i , $i =1,2$ to return to the starting point if the other searcher has found the target .Also, let $\overline{D}_i$, $i = 3, 4$ be the time for the searcher S_i , $i = 3, 4$ to return to the starting point if the other searcher has found the target and $D(\Phi)$ be the time for the searchers to return to the starting point if one of them has found the target .

<u>Remark 1:</u> The waiting time between the searchers at the intersection point is out of the cost.

In the following theorem we assume that $D_1 = D_2$ on the first line L_1, and $\overline{D}_3 = \overline{D}_4$ on the second line L_2 according to the probability of the position of the lost target on the two lines.

We consider from now case (1).

<u>Theorem 1</u> The expected value of the time for the searchers to return to the intersection point $a_0 = b_0$, $k_0 = z_0$ if one of them has found the target is given by

$$\begin{aligned} E(D(\Phi)) = {} & 2\sum_{w=1}^{j}(a_w - a_0)[v_1(c,d) - v_1(b_{w-1},a_{w-1})] \\ & +2\sum_{w=i}^{\infty}(a_w - a_0)[v_1(c,d) - v_1(b_{w-1},a_{w-1})] \\ & +2\sum_{\bar{w}=1}^{\bar{j}}(k_{\bar{w}} - k_0)[v_2(e,g) - v_2(z_{\bar{w}-1},k_{\bar{w}-1})] \\ & +2\sum_{\bar{w}=\bar{i}}^{\infty}(k_{\bar{w}} - k_0)[v_2(e,g) - v_2(z_{\bar{w}\ 1},k_{\bar{w}\ 1})] \end{aligned}$$

<u>Proof</u>

1)If the target is detected on L_1 we find :

If the target lies in $]a_0,a_1]$, then

$D_2 = -2(b_1-b_0)$,

If the target lies in $]a_1,a_2]$, then

$D_2 = -2(b_1-b_0) + (b_2 - b_0)]$,

If the target lies in $]a_2,a_3]$, then

$D_2 = -2(b_1-b_0) + (b_2 - b_0)] + (b_3 - b_0)]$,

If the target lies in $]a_{j-1},a_j]$, then

$D_2 = -2(b_1-b_0) + (b_2 - b_0)] + (b_3 - b_0)] + \ldots + (b_{j-1} - b_0) + (b_j - b_0)]$,

If the target lies in $]a_i,a_j]$, then

$D_2 = -2(b_1-b_0) + (b_2 - b_0)] + (b_3 - b_0)] + \ldots + (b_{j-1} - b_0) + (b_j - b_0)+ (b_i - b_0)]$,

If the target lies in $]a_j,a_{j+1}]$, then

$D_2 = -2(b_1-b_0) + (b_2 - b_0)] + (b_3 - b_0)] + \ldots + (b_{j-1} - b_0) + (b_j - b_0)+ (b_i - b_0)+ (b_{i+1} - b_0)]$.

and so on.

If the target lies in $[b_1,b_0[$, then

$D_1 = 2(a_1-a_0)$,

If the target lies in $[b_2,b_1[$, then

$D_1 = 2(a_1-a_0) + (a_2 - a_0)]$,

If the target lies in $[b_3,b_2[$, then

$D_1 = 2(a_1-a_0) + (a_2 - a_0)] + (a_3 - a_0)]$,

If the target lies in $[b_j,b_{j-1}[$, then

$D_1 = 2(a_1-a_0) + (a_2 - a_0)] + (a_3 - a_0)] + \ldots + (a_{j-1} - a_0) + (a_j - a_0)]$,

If the target lies in $[b_i,b_j[$, then

$D_1 = 2(a_1-a_0) + (a_2 - a_0)] + (a_3 - a_0)] + \ldots + (a_{j-1} - a_0) + (a_j - a_0) + (a_i - a_0)]$,

If the target lies in $[b_{i+1},b_i[$, then

$D_1 = 2(a_1-a_0) + (a_2 - a_0)] + (a_3 - a_0)] + \ldots + (a_{j-1} - a_0) + (a_j - a_0) + (a_i - a_0) + (a_{i+1} - a_0)]$,

and so on.

2) If the target is detected on the second line L_2 we find:

If the target lies in $]k_0,k_1]$, then

$\overline{D}_4 = -2(z_1-z_0)$,

If the target lies in $]k_1,k_2]$, then

$\overline{D}_4 = -2[(z_1-z_0) + (z_2 - z_0)]$,

If the target lies in $]k_2,k_3]$, then

$\overline{D}_4 = -2[(z_1-z_0) + (z_2 - z_0) + (z_3 - z_0)]$,

If the target lies in $]k_{\bar{j}-1}, k_{\bar{j}}]$, then

$\overline{D}_4 = -2[(z_1-z_0) + (z_2 - z_0) + (z_3 - z_0) + \ldots + (z_{\bar{j}-1}-z_0) + (z_{\bar{j}} - z_0)]$,

If the target lies in $]k_{\bar{j}}, k_{\bar{i}}]$, then

$\overline{D}_4 = -2[(z_1-z_0) + (z_2 - z_0) + (z_3 -z) + \ldots + (z_{\bar{j}-1}-z_0) + (z_{\bar{j}} - z_0)+ (z_{\bar{i}} - z_0)]$,

If the target lies in $]k_{\bar{i}}, k_{\bar{i}+1}]$, then

$\overline{D}_4 = -2[(z_1-z_0) + (z_2 - z_0) + (z_3 - z_0)+ \ldots + (z_{\bar{j}-1}-z_0) + (z_{\bar{j}} - z_0)+ (z_{\bar{i}} - z_0)+ (z_{\bar{i}+1}- z_0)]$,

and so on.

If the target lies in $[z_0,z_1[$, then

$\overline{D}_3 = 2(k_1-k_0)$,

If the target lies in $[z_2,z_1[$, then

$\overline{D}_3 = 2(k_1-k_0) + (k_2 - k_0)]$,

If the target lies in $[z_3,z_2[$, then

$\overline{D}_3 = 2(k_1-k_0) + (k_2 - k_0)] + (k_3 - k_0)]$,
If the target lies in $[z_{\bar{j}-1}, z_{\bar{j}}[$, then
$\overline{D}_3 = 2(k_1-k_0) + (k_2 - k_0)] + (k_3 - k_0)] + \ldots + (k_{\bar{j}-1} - k_0) + (k_{\bar{j}} - k_0)]$,
If the target lies in $[z_{\bar{i}}, z_{\bar{j}}[$, then
$\overline{D}_3 = 2(k_1-k_0) + (k_2 - k_0)] + (k_3 - k_0)] + \ldots + (k_{\bar{j}-1} - k_0) + (k_{\bar{j}} - k_0) + (k_{\bar{i}} - k_0)]$,
If the target lies in $[z_{\bar{i}}, z_{\bar{i}+1}[$, then
$\overline{D}_3 = 2(k_1-k_0) + (k_2 - k_0)] + (k_3 - k_0)] + \ldots + (k_{\bar{j}-1} - k_0) + (k_{\bar{j}} - k_0) + (k_{\bar{i}} - k_0) + + (k_{\bar{i}+1} - k_0)]$,

And so on, then we can calculate $E[D(\Phi)]$ as follows:

$E[D(\Phi)] = 2(a_1-a_0[v_1(a_0, a_1) + v_1(a_1, a_2) + v_1(a_2, a_3) + \ldots + v_1(a_j, a_i) + v_1(a_i, a_{i+1}) + \ldots + v_1(b_1, b_0) + v_1(b_2, b_1) + v_1(b_3, b_2) + \ldots + v_1(b_i, b_j) + v_1(b_{i+1}, b_i) + \ldots]$

$+2(a_2-a_0)[v_1(a_1, a_2) + v_1(a_2, a_3) + \ldots + v_1(a_j, a_i) + v_1(a_i, a_{i+1}) + \ldots + v_1(b_2, b_1) + v_1(b_3, b_2) + \ldots + v_1(b_i, b_j) + v_1(b_{i+1}, b_i) + v_1(b_{i+2}, b_{i+1})]$
$+2(a_3-a_0))[v_1(a_2, a_3) + \ldots + v_1(a_j, a_i) + v_1(a_i, a_{i+1}) + \ldots + v_1(b_3, b_2) + \ldots + v_1(b_i, b_j) + v_1(b_{i+1}, b_i) + v_1(b_{i+2}, b_{i+1})] + \ldots$

$+2(k_1-a_0)[v_2(k_0, k_1) + v_2(k_1, k_2) + v_2(k_2, k_3) + \ldots + v_2(k_{\bar{j}}, k_{\bar{i}}) + v_2(k_{\bar{i}}, k_{\bar{i}+1}) + \ldots + v_2(z_1, z_0) + v_2(z_2, z_1) + v_2(z_3, z_2) + \ldots + v_2(z_{\bar{i}+1}, z_j) + v_2(z_{\bar{i}+2}, z_{\bar{i}+1}) + \ldots]$

$+2(k_2-a_0)[v_2(k_1, k_2) + v_2(k_2, k_3) + \ldots + v_2(k_{\bar{j}-1}, k_{\bar{j}}) + v_2(k_{\bar{j}}, k_{\bar{i}}) + v_2(k_{\bar{i}}, k_{\bar{i}+1}) + \ldots + v_2(z_2, z_1) + v_2(z_3, z_2) + \ldots + v_2(z_{\bar{j}}, z_{\bar{j}-1}) + v_2(z_{\bar{i}}, z_{\bar{j}}) + v_2(z_{\bar{i}+1}, z_{\bar{i}}) + \ldots]$

$+2(k_3-a_0)[v_2(k_2,k_3) + \ldots + v_2(k_{\bar{j}-1},k_{\bar{j}}) + v_2(k_{\bar{j}},k_{\bar{i}}) + v_2(k_{\bar{i}},k_{\bar{i}+1}) + \ldots + v_2(z_3,z_2) + \ldots + v_2(z_{\bar{j}},z_{\bar{j}-1}) + v_2(z_{\bar{i}},z_{\bar{j}}) + v_2(z_{i+1},z_{\bar{i}}) + \ldots] + \ldots$

$= 2(a_1 - a_0)[v_1(c,d)] + 2(a_2 - a_0)[v_1(c,d) - v_1(b_1,a_1)] + 2(a_3 - a_0)[v_1(c,d) - v_1(b_2,a_2)] + \ldots$

$+ 2(k_1 - a_0)[v_2(e,g)] + 2(k_2 - a_0)[v_2(e,g) - v_2(z_1,k_1)] + 2(k_3 - a_0)[v_2(e,g) - v_2(z_2,k_2)] + \ldots$

$$= 2\sum_{w=1}^{j}(a_w - a_0)[v_1(c,d) - v_1(b_{w-1}, a_{w-1})]$$

$$+2\sum_{w=i}^{\infty}(a_w - a_0)[v_1(c,d) - v_1(b_{w-1}, a_{w-1})]$$

$$+2\sum_{\bar{w}=1}^{\bar{j}}(k_{\bar{w}} - k_0)[v_2(e,g) - v_2(z_{\bar{w}-1}, k_{\bar{w}-1})]$$

$$+2\sum_{\bar{w}=\bar{i}}^{\infty}(k_{\bar{w}} - k_0)[v_2(e,g) - v_2(z_{\bar{w}-1}, k_{\bar{w}-1})]$$

$$, 1 \le \bar{j} < \bar{i}, \bar{i} = \bar{j} + 1 \qquad [2.1]$$

Definition 1 : let $\Phi^* \in \Phi$ be a search plan, then Φ^* is an optimal search plan if :
$E[D(\Phi^*)] = \inf\{E[D(\Phi)], \Phi \in \Phi\}$,
Where Φ is the set of all search plans.

Theorem 2 : let F_1*(x) be a continuous distribution with a density function* f_1*(x) and* f_2*(y) be a continuous distribution with a density function* f_2*(y).if* $\Phi = (\phi, \bar{\phi}) \in \Phi^*$ is an optimal search plan, then :

$$a_{i+1} = \frac{[v_1(c,d) - v_1(b_j, a_j)]}{f_1(a_j)} + a_0$$

$$, 1 \le j < i, i = j + 1 \qquad [2.2]$$

$$k_{\bar{i}+1} = \frac{[v_2(e,g) - v_2(z_{\bar{j}}, k_{\bar{j}})]}{f_2(k_{\bar{j}})} + k_0$$

$$, 1 \le \bar{j} < \bar{i}, \bar{i} = \bar{j} + 1 \qquad [2.3]$$

Proof: we have from [2.1]

$$E[D(\Phi)] = 2\sum_{w=1}^{j}(a_w - a_0)[v_1(c,d) - v_1(b_{w-1}, a_{w-1})] + 2\sum_{w=i}^{\infty}(a_w - a_0)[v_1(c,d) - v_1(b_{w-1}, a_{w-1})]$$

$$+2\sum_{\bar{w}=1}^{\bar{j}}(k_{\bar{w}} - a_0)[v_2(e,g) - v_2(z_{\bar{w}-1}, k_{\bar{w}-1})] + 2\sum_{\bar{w}=\bar{i}}^{\infty}(k_{\bar{w}} - a_0)[v_1(e,g) - v_2(z_{\bar{w}-1}, k_{\bar{w}-1})]$$

$$= 2\begin{bmatrix}(a_1-a_0)[v_1(c,d)]+(a_2-a_0)[v_1(c,d)-v_1(b_1,a_1)]\\ +(a_3-a_0)[v_1(c,d)-v_1(b_2,a_2)]+\cdots\\ +(a_j-a_0)[v_1(c,d)-v_1(b_{j-1},a_{j-1})]\\ +(a_j-a_0)[v_1(c,d)-v_1(b_j,a_j)]\\ +(a_{j+1}-a_0)[v_1(c,d)-v_1(b_i,a_i)]+\cdots\end{bmatrix}$$

$$+ 2\begin{bmatrix}(k_1-a_0)[v_2(e,g)]+(k_2-a_0)[v_2(e,g)-v_2(z_1,k_1)]\\ +(k_3-a_0)[v_2(e,g)-v_2(z_2,k_2)]+\cdots\\ +(k_j-a_0)[v_2(e,g)-v_2(z_{j-1},k_{j-1})]\\ +(k_j-a_0)[v_2(e,g)-v_2(z_j,k_j)]\\ +(k_{i+1}-a_0)[v_2(e,g)-v_2(z_i,k_i)]+\cdots\end{bmatrix}$$

Hence,

$$\frac{\partial E[D(\Phi)]}{\partial a_1} = 2[v_1(c,d)-(a_2-a_0)f_1(a_1)] = 0$$

Thus,

$$a_2 = \frac{v_1(c,d)}{f_1(a_1)} + a_0$$

Also,

$$\frac{\partial E[D(\Phi)]}{\partial a_2} = 2[v_1(c,d)-v_1(b_1,a_1)-(a_3-a_0)f_1(a_2)] = 0$$

Thus,

$$a_3 = \frac{[v_1(c,d)-v_1(b_1,a_1)]}{f_1(a_2)} + a_0$$

And so on, then:

$$\frac{\partial E[D(\Phi)]}{\partial a_{j-1}} = 2[v_1(c,d)-v_1(b_{j-2},a_{j-2})-(a_j-a_0)f_1(a_{j-1})] = 0$$

Thus,

$$a_j = \frac{[v_1(c,d)-v_1(b_{j-2},a_{j-2})]}{f_1(a_{j-1})} + a_0, 1\le j<i, i=j+1$$

Also,

$$\frac{\partial E[D(\Phi)]}{\partial a_j} = 2[v_1(c,d)-v_1(b_{j-1},a_{j-1})-(a_i-a_0)f_1(a_j)] = 0$$

Thus,

$$a_i = \frac{[v_1(c,d)-v_1(b_{j-1},a_{j-1})]}{f_1(a_j)} + a_0, 1\le j<i, i=j+1$$

Hence,

$$a_{i+1} = \frac{[v_1(c,d)-v_1(b_j,a_j)]}{f_1(a_i)} + a_0, 1\le j<i, i=j+1$$

By a similar way we can prove (2.3).

<u>Theorem 3:</u>

If $\Phi = (\phi, \bar{\phi}) \in \boldsymbol{\Phi}$ is an optimal search plan, then

$f_1(a_i) < f_1(a_j)$ and $f_1(b_i) < f_1(b_j)$ on the first line L_1

$\forall\ 1 \le j < i,\ i = j+1$ (2.4)

Also,

$f_2(k_i) < f_2(k_j)$ and $f_2(z_{\bar{i}}) < f_2(z_j)$. on the second line

$L_2\ \forall\ 1 \le \bar{j} < \bar{i}, \bar{i} = \bar{j}+1$ (2.5)

Proof *: We have from* (2.2)and further from the definition of the search plan

$$f_1(a_i) = \frac{v_1(c,d)-v_1(b_j,a_j)}{a_{j+1}-a_0}, 1\le j<i, i=j+1$$

$$f_1(a_2) = \frac{v_1(c,d)-v_1(b_1,a_1)}{a_3-a_0}$$

And

$$f_1(a_1) = \frac{v_1(c,d)}{a_2-a_0}$$

But

$v_1(c,d) > v_1(c,d) - v_1(b_1,a_1) > v_1(c,d) - v_1(b_2,a_2)$

$> v_1(c,d) - v_1(b_j,a_j),\ 1 \le j < i, = j+1$

And

$a_2 - a_0 < a_3 - a_0 < a_{j+1} - a_0$

Hence,

$$\frac{v_1(c,d)}{a_2 - a_0} > \frac{v_1(c,d) - v_1(b_1,a_1)}{a_3 - a_0} > \frac{v_1(c,d) - v_1(b_j,a_j)}{a_{i+1} - a_0}$$

Thus,

$f_1(a_1) > f_1(a_2) > f_1(a_j),\ j \geq 1,$

Then

$f_1(a_i) < f_1(a_j)\ \forall\ 1 \leq j < I,\ I = j + 1$

by similar way we can prove $f_1(b_i) < f_1(b_j)$ and (2.5).

Special cases:

Case(i)

In case (1) if the target is located according to a known symmetric distribution on one of two intersected lines L_1 and L_2, where the distribution of the lost target is the same on the first line L_1 and on the second line L_2, then the expected value of the time of finding the lost target is :

$$E[D(\Phi)] = 2\sum_{w=1}^{j}(a_w - a_0)[1 - 2v_1(b_{w-1},a_{w-1})]$$

$$+2\sum_{w=i}^{\infty}(a_w - a_0)[1 - 2v_1(b_{w-1},a_{w-1})] \qquad (2.6)$$

$$a_{i+1} = \frac{[1 - 2v_1(b_{j-1},a_{j-1})]}{f(a_j)} + a_0,\ 1 \leq j < i,\ i = j + 1 \qquad (2.7)$$

$$k_{\bar{i}+1} = \frac{[1 - 2v_2(z_{\bar{j}-1},k_{\bar{j}-1})]}{f(k_{\bar{j}})} + k_0,\ 1 \leq \bar{j} < \bar{i},\ \bar{i} = \bar{j} + 1 \qquad (2.8)$$

Case (ii)

In case (*1*) if the point of intersection of the two lines is the origin , where $a_0 = b_0 = k_0 = z_0 = 0$, then the expected value of the time of finding the lost target is given by :

$$E[D(\Phi)] = 2\sum_{w=1}^{\infty} a_w[v_1(c,d) - v_1(b_{w-1},a_{w-1})]$$

$$+2\sum_{\bar{w}=1}^{\infty} k_{\bar{w}}[v_2(e,g) - v_2(z_{\bar{w}-1},k_{\bar{w}-1})] \qquad (2.9)$$

Also,

$$a_{i+1} = \frac{[v_1(c,d) - v_1(b_{i-1},a_{i-1})]}{f_1(a_i)} \quad ,i \geq 1 \qquad (2.10)$$

$$k_{\bar{i}+1} = \frac{[v_2(e,g) - (z_{\bar{i}},k_{\bar{i}})]}{f_2(k_{\bar{i}})} \quad ,\bar{i} \geq 1 \qquad (2.11)$$

Case (iii)

In case (*i*) if the target is located according to a known symmetric distribution on one of two intersected lines L_1 and L_2, where the distribution of the lost target is the same on the first line L_1 and on the second line L_2, and the point of intersection of the two lines is the origin, then the expected value of the time of finding the lost target is given by the following :

$$E[D(\Phi)] = 2\sum_{w=1}^{j} a_w[1 - 2v_2(b_{w-1},a_{w-1})]$$

$$+2\sum_{w=1}^{\infty} a_w[1 - 2v_1(b_{w-1},a_{w-1})] \qquad (2.12)$$

Also,

$$a_{i+1} = \frac{[1 - 2v_1(b_{j-1},a_{j-1})]}{f(a_j)},\ 1 \leq j < i,\ i = j + 1 \qquad (2.13)$$

$$k_{\bar{i}+1} = \frac{[1 - 2v_2(z_{\bar{j}-1},k_{\bar{j}-1})]}{f(k_{\bar{j}})},\ 1 \leq \bar{j} < \bar{i},\ \bar{i} = \bar{j} + 1 \qquad (2.14)$$

Case (iv)

In case (i) if the target is located according to a known symmetric distribution on one of two intersected lines L_1 and L_2, where the distribution of the lost target is the same on the first line L_1 and on the second line L_2, then $v_1 = v_2$ and $f_1(x) = f_2(y)$ when c=e and d=g we find :

$$\int_c^d f_1(x)dx = \int_e^g f_2(y)dy = \frac{1}{2}$$

But if the target is located according to a known symmetric distribution on one of two intersected lines L_1 and L_2 ,where the distribution of the lost target is the same on the first line L_1 and on the second line L_2 but $c \neq e$ and $d \neq g$ we find :

$$\int_c^d f_1(x)dx \neq \int_e^g f_2(y)dy$$

Case (v)

In case (i) if $v_2(e,g) = 0$, then we can get the same result of [10].

Example 1

If X has the density function

$$f_1(x) = |x| , -1 < -\sqrt{\frac{1}{2}} \leq x \leq \sqrt{\frac{1}{2}} < 1$$

,where x representing the position of the located target on the first line L_1,

$$c = -\sqrt{\frac{1}{2}} \text{ and } d = \sqrt{\frac{1}{2}},$$

and y has the density function

$$f_2(y) = |y| , -1 < -\sqrt{\frac{1}{2}} \leq y \leq \sqrt{\frac{1}{2}} < 1$$

,where y representing the position of the located target on the second line L_2,

$$g = -\sqrt{\frac{1}{2}} \text{ and } e = \sqrt{\frac{1}{2}},$$

and the two lines L_1 and L_2 are intrested in the origin.

Then the optimal search plan $\Phi^* = (\emptyset, \bar{\emptyset}^*)$ consists of $\emptyset = \left\{-\sqrt{\frac{1}{2}}, 0, \sqrt{\frac{1}{2}}\right\}$ and $\bar{\emptyset}^* = \left\{-\sqrt{\frac{1}{2}}, 0, \sqrt{\frac{1}{2}}\right\}$.

Proof

Let the optimal search plan Φ contains of

$$\emptyset = \left\{-\sqrt{\frac{1}{2}}, -a_1, 0, a_1, \sqrt{\frac{1}{2}}\right\} \text{ and}$$

$$\bar{\emptyset}^* = \left\{-\sqrt{\frac{1}{2}}, -k_1, 0, k_1, \sqrt{\frac{1}{2}}\right\}$$

But

$$f_1(a_1) < f_1\left(\sqrt{\frac{1}{2}}\right) \text{ and } f_2(k_1) < f_2\left(\sqrt{\frac{1}{2}}\right)$$

which contracted relation (2.4) and (2.5). Then $a_1 = k_1 = \sqrt{\frac{1}{2}}$.

Example 2

If X is a continuous random variable that has a uniform distribution with a probability density function

$$f_1(x) = \frac{1}{b-a} , -12 < -3 \leq x \leq 3 < 12$$

,where x representing the position of the located target on the first line L_1, c = -3 and d = 3,

And Y be a continuous random variable has a triangular distribution with a probability density function

$$f_2(y) = \frac{1}{4} - \frac{1}{16}|y| , -4 < -2 \leq y \leq 2 < 4$$

,where y representing the position of the located target on the second line L_2, g = 2 and e = -2, and the two lines L_1 and L_2 interested in the origin.

Let $\Phi_i = (\emptyset_i, \emptyset_i) \in \Phi_0$ be a search plans and $E[D(\Phi_i)]$, I =1,2,3

(i) suppose that $\emptyset_1$ consists of the points {-3,0,3} and $\emptyset_1$ consists of the points {-2,0,2}

if $0 < x \leq 3$	,then	$D_2 = 6$
if $-3 \leq x < 0$	,then	$D_1 = 6$
if $0 < y \leq 2$	,then	$D_4 = 4$
if $-2 \leq y < 0$	,then	$D_3 = 4$

Hence:

$$E[D(\Phi_1)] = 4.5$$

0(ii) suppose that $\emptyset_2$ consists of the points {-3, -2, 0, 2, 3} and $\emptyset_2$ consists of the points {-2, -1, 0, 1, 2}.

if $0 < x \leq 2$	,then	$D_2 = 4$
if $2 < x \leq 3$	,then	$D_2 = 10$
if $-2 \leq x < 0$	,then	$D_1 = 4$
if $-3 \leq x < -2$	,then	$D_1 = 10$
if $0 < y \leq 1$	,then	$D_4 = 2$
if $1 < y \leq 2$	,then	$D_4 = 6$
if $-1 \leq y < 0$	,then	$D_3 = 2$
if $-2 \leq y < -1$	,then	$D_3 = 6$

,Hence the expectation value can be obtained as follows :

$$E[D(\Phi_2)] = 4.25$$

(iii) suppose that $\emptyset_3$ consists of the points {-3, -2, -1, 0, 1, 2, 3}, and $\emptyset_3$ consists of the points {-2, -1, -0.5, 0, 0.5, 1, 2}.

if $0 < x \leq 1$	,then	$D_2 = 2$
if $1 < x \leq 2$	,then	$D_2 = 6$

if $2 < x \le 3$,then $D_2 = 12$
if $-1 \le x < 0$,then $D_1 = 2$
if $-2 \le x < -1$,then $D_1 = 6$
if $-3 \le x < -2$,then $D_1 = 12$
if $0 < y \le 0.5$,then $D4 = 1$
if $0.5 < y \le 1$,then $D4 = 3$
if $1 < y \le 2$,then $D4 = 7$
if $-0.5 \le y < 0$,then $D3 = 1$
if $-1 \le y < -0.5$,then $D3 = 3$
if $-2 \le y < -1$,then $D1 = 7$
,Hence the expectation value can be obtained as follows :

$$E[D(\Phi_3)] = 4.69791666666,$$

Hence ø2 is the optimal search plan.

Example 3

If X is a continuous random variable that has a triangular distribution with a probability density function.

$$f_1(x) = \frac{1}{4} - \frac{1}{16}|x|, -4 < -2 \le x \le 2 < 4$$

where x representing the position of the located target on the first line L_1
$c = -2$ and $d = 2$,
and Y be a continuous random variable has a triangular distribution with a probability density function.

$$f_2(y) = \frac{1}{4} - \frac{1}{16}|y|, -4 < -4 + 2\sqrt{3} \le y \le 4 - 2\sqrt{3} < 4$$

,where y representing the position of the located target on the second line L_2, $g = 4 - 2\sqrt{3}$, and $e = -4 + 2\sqrt{3}$, and the two lines L_1 and L_2 are interested in the origin. We have:

$$F_1(x) = \begin{cases} \frac{X^2 + 8X + 12}{32} & ,x < 0 \\ \frac{-X^2 + 8X + 12}{32} & ,x \ge 0 \end{cases}$$

And

$$F_2(y) = \begin{cases} \frac{Y^2 + 8Y + 4}{32} & ,y < 0 \\ \frac{-Y^2 + 8Y + 4}{32} & ,y \ge 0 \end{cases}$$

From the properties of the symmetric distribution, we can say that its cumulative distribution function satisfies.

$v_1(c,d) - F_1(x) = F_1(-x)$ and $v_2(e,g) - F_2(y) = F_2(-y)$ (2.15)

from (2.9) and using (2.15) we can get:

$$E[D(\Phi)] - 4\sum_{i=1}^{\infty} a_i\,[v_1(c,d) - F_1(a_{i-1})] + 4\sum_{j=1}^{\infty} k_j\,[v_2(e,g) - F_2(k_{j-1})]$$

Also (2.13) and (2.15) become

$$a_{i+1} = \frac{[v_1(c,d) - F_1(a_{i-1})]}{f_1(a_i)}, i \ge 1 \qquad (2.16)$$

And

$$k_{j+1} - \frac{[v_2(e,g) - F_2(k_{j-1})]}{f_2(k_j)}, j \ge 1 \qquad (2.17)$$

By substituting in (2.16) and (2.17) we can get:

$$a_{i+1} = \frac{2[v_1(c,d) - \left[\frac{-a_{i-1}^2 + 8a_{i-1} + 12}{32}\right]]}{\frac{1}{4} - \frac{1}{16}a_{i-1}}$$

$a_{i-1} < a_i$ for $i \ge 1$ (2.18)

And

$$k_{j+1} = \frac{2[v_2(e,g) - \left[\frac{-k_{j-1}^2 + 8k_{j-1} + 4}{32}\right]]}{\frac{1}{4} - \frac{1}{16}k_{j-1}},$$

$k_{j-1} < k_j$ for $j \ge 1$ (2.19)

Then the optimal search plan satisfies (2.18) and (2.19).

In table 1 we generate the steps a_i, $i \ge 1$ and k_j , $j \ge 1$of the searchers which give the optimal search plan by using (2.18) and (2.19) and then ewe can find the optimal expected time. We can see from table 1 that $E[D(\Phi^*)] = 3.00809002$.

Table 1

a_1	a_2	a_3	a_4	k_1	k_2	k_3	k_4	$E[D(\Phi)]$
2				$4-2\sqrt{3}$				3.26794919
1.5	2			0.4	$4-2\sqrt{3}$			3.07680780
1	2			0.2	$4-2\sqrt{3}$			3.00809002
0.5	1.7142857	1.804687488	2	0.15	0.240625	0.37539484	$4-2\sqrt{3}$	3.297022378

References:

1) A.A. Mohamed,H.A Fergany, M.A. El-Hadidy, On the Coordinated Search Problem on the Plane, Istanbul University Journal of the School of Business Administration, Cilt/ Vol:xx, Sayı/No:x, 2011, xx-xx Business Administration, Cilt/Vol :xx,Sayı/No:x, 2012, xx-xx.

2) A. B. El-Rayes, Abd El-moneim A. Mohamed, H. Fergani, On the generalized linear search problem,Delta Journal, No. 6. Second Part (1993), 1-10.

3) Abd El-moneim A. Mohamed, Hamdy M. Abou Gabal and Mohamed. A. El-Hadidy ,Coordinated search for a randomly located target on the plan,European Journal of pure and applied mathmatics,Vol. 2, No. 1, 2009, (97-111).

4) Abd El-moneim A. Mohamed, Hamdy M. Abou Gabal, Generalized optimal 22 search paths for a randomly located target, In: Annual Conference, Cairo ISSR, 35, Math. Statistics (2000), 17-19.

5) Abd El-moneim A. Mohamed, Hamdy M. Abou Gabal, Linear search with multiple searchers for a randomly moving target, In: International Conference for Statistices, Computer Science and Its Application, Egypt (2003), 115-124.

6) Abd El-moneim A. Mohamed, Hamdy M. Abou Gabal, Multiplicative linear search problem, Egyptian Statistical Journal, 48, No. 1 (2004), 34-45.

7) Abd El-moneim A. M. Teamah, Hamdy M. Abou Gabal and W.A. Afifi, On the Coordinated search problem, International Journal of Applied Mathematics, 5 (2007), 627-636.

8) Abd El-moneim A. M. Teamah, Hamdy M. Abou Gabal and W.A. Afifi, Double coordinate search problem .International journal of contemporary Mathematical Science 2013.

9) Diana J. Reyniers, Coordinated two searchers for an object hidden on an interval, Journal of the Operational Research Society, 46 (1995), 1386-1392.

10) Diana J. Reyniers, Coordinated search for an object on the line, European Journal of the Operational Research, 95 (1996), 663-670.

11) Hamdy M. Abou Gabal (2004) : Search Plans With Multiple-Searchers for Lost Targets, Ph. D. Thesis, Tanta University.

12) L.D Stone. Theory of Optimal Search, volume 118 of Mathematics in science and Engineering. Academic Press, New York, 1975.

13) F. Bourgault, T. Furukawa, and H.F. Durrant-Whyte, Optimal search for a lost target in a Bayesian world. S. Yaut et al. (EDS.):Field and Service Robotics (2006), 209-222.

14) F. Bourgault, et al., Coordinated decentralized search for a lost target in a Bayesian world, Proc. IEEE/RSJ Int. Conf. Intel. Robot. Sys., (2003).

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ولاء عبدالله ابراهيم عفيفى

قسم الرياضيات- كليه العلوم – جامعه طنطا

نهتم فى هذا البحث بدراسه مشكله البحث التنسيقى فى حاله اذا كان الهدف المفقود على احد خطين متقاطعين فيكون الهدف يتبع توزيع متماثل على احد الخطين او توزيع متماثل اخر على الخط الثانى , و حينئذ يبدا الاربع باحثين معاً من نقطة ما (نقطه التقاطع), وقد تم فرض جميع الحالات التى يمكن ان يتقاطع فيها الخطين , و قد تم إيجاد الاستراتيجية المثلى التي تجعل القيمة المتوقعة للجهد المبذول اقل ما يمكن و ذلك لاى توزيع متماثل, و جدير بالذكر ان الدراسات السابقه اقتصرت على دراسة مشكله البحث التنسيقى في حاله إذا كان موضع الهدف يتبع توزيع متماثل او غير متماثل لكن على الخطوط الغير متقاطعه.