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A Method for Improving Rough Set Approximation Accuracy in terms of j-Neighborhood Spaces

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Abstract: This paper aims to introduce an effective method for improving rough set approximation accuracy. Considering the j-neighborhood space, the lower and upper approximation operators are defined and their fundamental properties are obtained. The approximations are constructed in four different approaches. Comparison between the accuracy of these four types of approximations is imposed and the best one is defined

Keywords: *Rough Sets; j-Neighborhood Spaces; The Lower and Upper Rough Set Approximations.*

1. Introduction

Rough set theory is a powerful tool for dealing with uncertainty, granularity, and incompleteness of knowledge in information systems. It is a mathematical approach which deals with vagueness by a pair of exact sets called the lower and upper approximation sets. These approximations correspond to minimal (resp. maximal) exact set contained in (resp. containing) the rough set. It was proved that the pair of lower and upper approximation operators induced by a reflexive and transitive binary relation is exactly a pair of interior and closure operators of a topology [6].

In Pawlak's original rough set theory [11], equivalence relation is a core concept which seems to be a very stringent condition that limits the application domain of the rough set theory. To solve this problem, several authors have generalized the notion of approximation operators by using arbitrary binary relations [10, 13, 14, 15, 18, 19, 20, and 22].

Lin [9] and Yao [21] studied rough sets using neighborhood systems for the interpretation of granules. M.E. Abd El-Monsef et al. [2] introduced mixed neighborhood systems to approximate rough sets. Lashin et al. [8] used a topology generated by right neighborhoods as a subbase and defined the lower and upper approximation operators by the interior and closure operators of this topology.

In 2014, Abd El-Monsef et al. [1] introduced the concept the j-neighborhood space which represents a generalized type of neighborhood spaces. Accordingly, we use this concept to define different types of the lower and upper approximation operators based on general binary relation. The lower and upper approximation operators are defined and their fundamental properties are obtained. The approximations are constructed in four different approaches. Comparison between the accuracy of these four types of approximations is imposed and the best one is defined.

2. j-Neighborhood Spaces

In this section, we give an exposition of the needed definitions. Also, we introduce a definition the lower and upper approximation operators in the j-neighborhood space and a definition of accuracy of the approximations of rough sets.

Definition 1 Let $\mathcal{R}$ be an arbitrary binary relation on a non-empty finite set U.
The j-neighborhood of $x \in U$ $(N_j(x))$,

$$j = r, \ell, i, u, \langle r\rangle, \langle \ell\rangle, \langle i\rangle, \langle u\rangle,$$

can be defined as follows:

(i) r-neighborhood [4]:
$$N_r(x) = \{y \in U \mid xRy\},$$
(ii) ℓ-neighborhood [5]:
$$N_\ell(x) = \{y \in U \mid yRx\},$$
(iii) $\langle r\rangle$-neighborhood [3]:
$$N_{\langle r\rangle}(x) = \cap_{x\in N_r(y)} N_r(y),$$
(iv) $\langle \ell\rangle$-neighborhood [3]:
$$N_{\langle \ell\rangle}(x) = \cap_{x\in N_\ell(y)} N_\ell(y),$$
(v) i-neighborhood [1]:
$$N_i(x) = N_r(x) \cap N_\ell(x),$$
(vi) u-neighborhood [1]:
$$N_u(x) = N_r(x) \cup N_\ell(x),$$
(vii) $\langle i\rangle$-neighborhood [1]:
$$N_{\langle i\rangle}(x) = N_{\langle r\rangle}(x) \cap N_{\langle \ell\rangle}(x),$$
(viii) $\langle u\rangle$-neighborhood [1]:
$$N_{\langle u\rangle}(x) = N_{\langle r\rangle}(x) \cup N_{\langle \ell\rangle}(x).$$

Example 1 Let $U = \{a, b, c, d, e\}$ and $\mathcal{R} = \{(a, b), (a, d), (b, b), (d, c), (d, e), (e, d), (e, e)\}$.

Thus we get

$N_r(a) = \{b, d\}$, $N_\ell(a) = \emptyset$, $N_i(a) = \emptyset, N_u(a) = \{b, d\}$. $N_r(b) = \{b\}$, $N_\ell(b) = \{a, b\}, N_i(b) = \{b\}, N_u(b) = \{a, b\}$.

$N_r(c) = \emptyset$, $N_\ell(c) = \{d\}, N_i(c) = \emptyset, N_u(c) = \{d\}$. $N_r(d) = \{c, e\}, N_\ell(d) = \{a, e\}, N_i(d) = \{e\}, N_u(d) = \{a, c, e\}$. $N_r(e) = \{d, e\}, N_\ell(e) = \{d, e\}, N_i(e) = \{d, e\}, N_u(e) = \{d, e\}$.

$N_{\langle r\rangle}(a) = \emptyset, N_{\langle \ell\rangle}(a) = \{a\}, N_{\langle i\rangle}(a) = \emptyset, N_{\langle u\rangle}(a) = \{a\}$.

$N_{\langle r\rangle}(b) = \{b\}, N_{\langle \ell\rangle}(b) = \{a, b\}, N_{\langle i\rangle}(b) = \{b\}, N_{\langle u\rangle}(b) = \{a, b\}$.

$N_{\langle r\rangle}(c) = \{c, e\}, N_{\langle \ell\rangle}(c) = \emptyset, N_{\langle i\rangle}(c) = \emptyset, N_{\langle u\rangle}(c) = \{c, e\}$.

$N_{\langle r\rangle}(d) = \{d\}, N_{\langle \ell\rangle}(d) = \{d\}, N_{\langle i\rangle}(d) = \{d\}, N_{\langle u\rangle}(d) = \{d\}$. $N_{\langle r\rangle}(e) = \{e\}, N_{\langle \ell\rangle}(e) = \{e\}, N_{\langle i\rangle}(e) = \{e\}, N_{\langle u\rangle}(e) = \{e\}$.

Definition 2 [1] Let $\mathcal{R}$ be an arbitrary binary relation on a non-empty finite set U and the map $\xi_j: U \longrightarrow P(U)$ be a mapping which assigns for each x in U its j-neighborhood in $P(U)$, where $P(U)$ is the power set of U. The triple $(U, \mathcal{R}, \xi_j)$ is called a j-neighborhood space.

Definition 3 Let (U, R, ξ_j) be a j-neighborhood space and $A \subseteq U$. The j-lower and j-upper approximations of A are defined respectively by

$$\underline{R}_j(A) = \{p \in A:\ N_j(p) \neq \emptyset,\ N_j(p) \subseteq A\},$$

$$\overline{R}_j(A) = A \cup \{p \in A^c:\ N_j(p) \cap A \neq \phi\}$$

where $j = r, \ell, i, u, \langle r\rangle, \langle \ell\rangle, \langle i\rangle, \langle u\rangle$.

Definition 4 Let (U, R, ξ_j) be a j-neighborhood space and $A \subseteq U$. The j-boundary, j-positive and j-negative regions of A are defined respectively by

$$B_j(A) = \overline{R}_j(A) - \underline{R}_j(A),$$

$$POS_j(A) = \underline{R}_j(A),$$

$$NEG_j(A) = U - \overline{R}_j(A),$$

where $j = r, \ell, i, u, \langle r\rangle, \langle \ell\rangle, \langle i\rangle, \langle u\rangle$.

Definition 5 Let (U, R, ξ_j) be a j-neighborhood space. The j-accuracy of the approximations of $A \subseteq U$ is defined by

$$\alpha_j(A) = \frac{|\underline{R}_j(A)|}{|\overline{R}_j(A)|}, where\ |\overline{R}_j(A)| \neq 0,$$

where $j = r, \ell, i, u, \langle r\rangle, \langle \ell\rangle, \langle i\rangle, \langle u\rangle$.

It is clear that, $0 \le \alpha_j(A) \le 1$ and if $\alpha_j(A) = 1$ then A is called j-definable (exact) set. Otherwise, it is called j-rough.

Example 2 Let $U = \{a, b, c, d\}$ and $\mathcal{R} = \{(a, c), (b, b), (c, a), (d, a)\}$. Thus we get

$N_r(a) = \{c\}, N_\ell(a) = \{c, d\}, N_i(a) = \{c\}, N_u(a) = \{c, d\}$.
$N_r(b) = \{b\}, N_\ell(b) = \{b\}, N_i(b) = \{b\}, N_u(b) = \{b\}$.

$N_r(c) = \{a\}, N_\ell(c) = \{a\}, N_i(c) = \{a\}, N_u(c) = \{a\}$.

$N_r(d) = \{a\}, N_\ell(d) = \phi, N_i(d) = \phi, N_u(d) = \{a\}$.

$N_{\langle r\rangle}(a) = \{a\}, N_{\langle \ell\rangle}(a) = \{a\}, N_{\langle i\rangle}(a) = \{a\}, N_{\langle u\rangle}(a) = \{a\}$.

$N_{\langle r\rangle}(b) = \{b\}, N_{\langle \ell\rangle}(b) = \{b\}, N_{\langle i\rangle}(b) = \{b\}, N_{\langle u\rangle}(b) = \{b\}$.

$N_{\langle r\rangle}(c) = \{c\}, N_{\langle \ell\rangle}(c) = \{c, d\}, N_{\langle i\rangle}(c) = \{c\}, N_{\langle u\rangle}(c) = \{c, d\}.$

$N_{\langle r\rangle}(d) = \phi, N_{\langle \ell\rangle}(d) = \{c, d\}, N_{\langle i\rangle}(d) = \phi, N_{\langle u\rangle}(d) = \{c, d\}.$

Applying Definition 3, we have the following tables

Table 1 : $\underline{R}_r(A), \overline{R}_r(A), \underline{R}_\ell(A), \overline{R}_\ell(A), \underline{R}_u(A), \overline{R}_u(A), \underline{R}_i(A)$ and $\overline{R}_i(A)$ for all $A \subseteq U$.

A	$\underline{R}_r(A)$	$\overline{R}_r(A)$	$\underline{R}_\ell(A)$	$\overline{R}_\ell(A)$	$\underline{R}_u(A)$	$\overline{R}_u(A)$	$\underline{R}_i(A)$	$\overline{R}_i(A)$
$\{a\}$	$\emptyset$	$\{a,c,d\}$	$\emptyset$	$\{a,c\}$	$\emptyset$	$\{a,c,d\}$	$\emptyset$	$\{a,c\}$
$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$
$\{c\}$	$\emptyset$	$\{a,c\}$	$\emptyset$	$\{a,c\}$	$\emptyset$	$\{a,c\}$	$\emptyset$	$\{a,c\}$
$\{d\}$	$\emptyset$	$\{d\}$	$\emptyset$	$\{a,c\}$	$\emptyset$	$\{a,d\}$	$\emptyset$	$\{d\}$
$\{a,b\}$	$\{b\}$	U	$\{b\}$	$\{a,b,c\}$	$\{b\}$	U	$\{b\}$	$\{a,b,c\}$
$\{a,c\}$	$\{a,c\}$	$\{a,c,d\}$	$\{c\}$	$\{a,c\}$	$\{c\}$	$\{a,c,d\}$	$\{a,c\}$	$\{a,c\}$
$\{a,d\}$	$\{d\}$	$\{a,c,d\}$	$\emptyset$	$\{a,c,d\}$	$\{d\}$	$\{a,c,d\}$	$\emptyset$	$\{a,c,d\}$
$\{b,c\}$	$\{b\}$	$\{a,b,c\}$	$\{b\}$	$\{a,b,c\}$	$\{b\}$	$\{a,b,c\}$	$\{b\}$	$\{a,b,c\}$
$\{b,d\}$	$\{b\}$	$\{b,d\}$	$\{b\}$	$\{a,b,d\}$	$\{b\}$	$\{a,b,d\}$	$\{b\}$	$\{b,d\}$
$\{c,d\}$	$\emptyset$	$\{a,c,d\}$	$\emptyset$	$\{a,c,d\}$	$\emptyset$	$\{a,c,d\}$	$\emptyset$	$\{a,c,d\}$
$\{a,b,c\}$	$\{a,b,c\}$	U	$\{b,c\}$	$\{a,b,c\}$	$\{b,c\}$	U	$\{a,b,c\}$	$\{a,b,c\}$
$\{a,b,d\}$	$\{b,d\}$	U	$\{b\}$	U	$\{b,d\}$	U	$\{b\}$	U
$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c\}$	$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c\}$	$\{a,c,d\}$
$\{b,c,d\}$	$\{b\}$	U	$\{b\}$	U	$\{b\}$	U	$\{b\}$	U
U	U	U	$\{a,b,c\}$	U	U	U	$\{a,b,c\}$	U
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Table 2: $\underline{R}_{\langle r\rangle}(A), \overline{R}_{\langle r\rangle}(A), \underline{R}_{\langle \ell\rangle}(A), \overline{R}_{\langle \ell\rangle}(A), \underline{R}_{\langle u\rangle}(A), \overline{R}_{\langle u\rangle}(A), \underline{R}_{\langle i\rangle}(A)$ and $\overline{R}_{\langle i\rangle}(A)$ for all $A \subseteq U$.

A	$\underline{R}_{\langle r\rangle}(A)$	$\overline{R}_{\langle r\rangle}(A)$	$\underline{R}_{\langle \ell\rangle}(A)$	$\overline{R}_{\langle \ell\rangle}(A)$	$\underline{R}_{\langle u\rangle}(A)$	$\overline{R}_{\langle u\rangle}(A)$	$\underline{R}_{\langle i\rangle}(A)$	$\overline{R}_{\langle i\rangle}(A)$
$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$
$\{c\}$	$\{c\}$	$\{c\}$	$\emptyset$	$\{c,d\}$	$\emptyset$	$\{c,d\}$	$\{c\}$	$\{c\}$
$\{d\}$	$\emptyset$	$\{d\}$	$\emptyset$	$\{c,d\}$	$\emptyset$	$\{c,d\}$	$\emptyset$	$\{d\}$
$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$
$\{a,c\}$	$\{a,c\}$	$\{a,c\}$	$\{a\}$	$\{a,c,d\}$	$\{a\}$	$\{a,c,d\}$	$\{a,c\}$	$\{a,c\}$
$\{a,d\}$	$\{a\}$	$\{a,d\}$	$\{a\}$	$\{a,c,d\}$	$\{a\}$	$\{a,c,d\}$	$\{a\}$	$\{a,d\}$
$\{b,c\}$	$\{b,c\}$	$\{b,c\}$	$\{b\}$	$\{b,c,d\}$	$\{b\}$	$\{b,c,d\}$	$\{b,c\}$	$\{b,c\}$
$\{b,d\}$	$\{b\}$	$\{b,d\}$	$\{b\}$	$\{b,c,d\}$	$\{b\}$	$\{b,c,d\}$	$\{b\}$	$\{b,d\}$
$\{c,d\}$	$\{c\}$	$\{c,d\}$	$\{c,d\}$	$\{c,d\}$	$\{c,d\}$	$\{c,d\}$	$\{c\}$	$\{c,d\}$
$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b\}$	U	$\{a,b\}$	U	$\{a,b,c\}$	$\{a,b,c\}$
$\{a,b,d\}$	$\{a,b\}$	$\{a,b,d\}$	$\{a,b\}$	U	$\{a,b\}$	U	$\{a,b\}$	$\{a,b,d\}$
$\{a,c,d\}$	$\{a,c\}$	$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c\}$	$\{a,c,d\}$
$\{b,c,d\}$	$\{b,c\}$	$\{b,c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$	$\{b,c\}$	$\{b,c,d\}$
U	$\{a,b,c\}$	U	U	U	U	U	$\{a,b,c\}$	U
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Table 3: $\alpha_r(A), \alpha_\ell(A), \alpha_u(A), \alpha_i(A), \alpha_{\langle r\rangle}(A), \alpha_{\langle \ell\rangle}(A), \alpha_{\langle u\rangle}(A)$ and $\alpha_{\langle i\rangle}(A)$ for all $A \subseteq U$.

A	$\alpha_r(A)$	$\alpha_\ell(A)$	$\alpha_u(A)$	$\alpha_i(A)$	$\alpha_{\langle r\rangle}(A)$	$\alpha_{\langle \ell\rangle}(A)$	$\alpha_{\langle u\rangle}(A)$	$\alpha_{\langle i\rangle}(A)$
$\{a\}$	0	0	0	0	1	1	1	1
$\{b\}$	1	1	1	1	1	1	1	1
$\{c\}$	0	0	0	0	1	0	0	1
$\{d\}$	0	0	0	0	0	0	0	0
$\{a,b\}$	1/4	1/3	1/4	1/3	1	1	1	1
$\{a,c\}$	2/3	1/2	1/3	1	1	1/3	1/3	1
$\{a,d\}$	1/3	0	1/3	0	1/2	1/3	1/3	1/2
$\{b,c\}$	1/3	1/3	1/3	1/3	1	1/3	1/3	1
$\{b,d\}$	1/2	1/3	1/3	1/2	1/2	1/3	1/3	1/2
$\{c,d\}$	0	0	0	0	1/2	1	1	1/2
$\{a,b,c\}$	3/4	2/3	1/2	1	1	1/2	1/2	1
$\{a,b,d\}$	1/4	1/4	1/2	1/4	2/3	1/2	1/2	2/3
$\{a,c,d\}$	1	2/3	1	2/3	2/3	1	1	2/3
$\{b,c,d\}$	1/4	1/4	1/4	1/4	2/3	1	1	2/3
U	1	3/4	1	3/4	3/4	1	1	3/4

3. The rough set approximations using three types of neighborhood systems

In this section, we introduce three types of neighborhood systems of any element p in a j-neighborhood space $(\mathrm{U}, \mathcal{R}, \xi_j)$, namely, the k-neighborhood system of p , s- neighborhood system of p and t- neighborhood system of p. Making use of these three neighborhood systems, we define the lower and upper rough set approximations. Comparisons between the accuracy of these three types of approximations are obtained.

We proved that the best approximations are those based on the t- neighborhood systems. Moreover, the approximations based on the t- neighborhood systems are more accurate than the approximations which introduced in Definition 3.

Definition 6 Let $(\mathrm{U}, \mathcal{R}, \xi_{\mathrm{j}})$ be a j-neighborhood space, and $p \in U$. Then

(i) the k- neighborhood system of p, s- neighborhood system of p and t- neighborhood system of p are defined respectively by

$$NS_k(p) = \{N_r(p), N_\ell(p), N_u(p), N_i(p)\},$$

$$NS_s(p) = \{N_{\langle r\rangle}(p), N_{\langle \ell\rangle}(p), N_{\langle u\rangle}(p), N_{\langle i\rangle}(p)\},$$

$$NS_t(p) = \begin{Bmatrix} N_r(p), N_\ell(p), N_u(p), N_i(p) \\ , N_{\langle r\rangle}(p), \\ N_{\langle \ell\rangle}(p), N_{\langle u\rangle}(p), N_{\langle i\rangle}(p) \end{Bmatrix},$$

(ii) every element of $NS_k(p)$, $NS_s(p)$ and $NS_t(p)$ is called $N_k(p)$, $N_s(p)$ and $N_t(p)$ respectively.

Definition 7 Let $(\mathrm{U}, \mathcal{R}, \xi_{\mathrm{j}})$ be a j-neighborhood space and $\mathrm{A} \subseteq \mathrm{U}$. Then

(i) The k-lower and k-upper approximations of A are defined respectively by

$$\underline{\mathcal{R}}_k(A) = \{p \in A\colon \exists\, N_k(p) \neq \phi, N_k(p) \subseteq A\},$$

$$\overline{\mathcal{R}}_k(A) = A \cup \{p \in A^c\colon \forall\, N_k(p), N_k(p) \cap A \neq \phi\},$$

(ii) The s-lower and s-upper approximations of A are defined respectively by

$$\underline{\mathcal{R}}_s(\mathrm{A}) = \{\mathrm{p} \in \mathrm{A}\colon \exists \mathrm{N_s(p)} \neq \phi, \mathrm{N_s(p)} \subseteq \mathrm{A}\},$$

$$\overline{\mathcal{R}}_s(A) = A \cup \{p \in A^c\colon \forall\, N_s(p), N_s(p) \cap A \neq \phi\},$$

(iii) The t-lower and t-upper approximations of A are defined respectively by

$$\underline{\mathcal{R}}_t(A) = \{p \in A\colon \exists\, N_t(p) \neq \phi, N_t(p) \subseteq A\}$$

$$\overline{\mathcal{R}}_t(A) = A \cup \{p \in A^c\colon \forall N_t(p), N_t(p) \cap A \neq \phi\}.$$

Definition 8 Let $(\mathrm{U}, \mathcal{R}, \xi_{\mathrm{j}})$ be a j-neighborhood space, and $\mathrm{A} \subseteq \mathrm{U}$. Then:

(i) The boundary, positive and negative regions of A using k-neighborhood system are defined respectively by

$$B_k(A) = \overline{\mathcal{R}}_{\mathrm{k}}(\mathrm{A}) - \underline{\mathcal{R}}_{\mathrm{k}}(\mathrm{A}),$$

$$POS_k(A) = \underline{\mathcal{R}}_{\mathrm{k}}(\mathrm{A}),$$

$$NEG_k(A) = \mathrm{U} - \overline{\mathcal{R}}_{\mathrm{k}}(\mathrm{A}),$$

(ii) The boundary, positive and negative regions of A using s-neighborhood system are defined respectively by

$$B_s(A) = \overline{\mathcal{R}}_{\mathrm{s}}(\mathrm{A}) - \underline{\mathcal{R}}_{\mathrm{s}}(\mathrm{A}),$$

$$POS_s(A) = \underline{\mathcal{R}}_{\mathrm{s}}(\mathrm{A}),$$

$$NEG_s(A) = \mathrm{U} - \overline{\mathcal{R}}_{\mathrm{s}}(\mathrm{A}),$$

(iii) The boundary, positive and negative regions of A using t-neighborhood system are defined respectively by

$$B_{\mathrm{t}}(\mathrm{A}) = \overline{\mathcal{R}}_{\mathrm{t}}(\mathrm{A}) - \underline{\mathcal{R}}_{\mathrm{t}}(\mathrm{A}),$$

$$POS_t(A) = \underline{\mathcal{R}}_{\mathrm{t}}(\mathrm{A}),$$

$$NEG_t(A) = \mathrm{U} - \overline{\mathcal{R}}_{\mathrm{t}}(\mathrm{A}).$$

Definition 9 Let (U, R, ξ_j) be a j-neighborhood space and $\mathrm{A} \subseteq \mathrm{U}$. Then the accuracy of the approximations of a subset A using k-neighborhood system, s-neighborhood system and t-neighborhood system are defined respectively by

$$\alpha_k(A) = \frac{|\underline{\mathcal{R}}_k(A)|}{|\overline{\mathcal{R}}_k(A)|},$$

$$\alpha_s(A) = \frac{|\underline{\mathcal{R}}_s(A)|}{|\overline{\mathcal{R}}_s(A)|},$$

$$\alpha_t(A) = \frac{|\underline{\mathcal{R}}_t(A)|}{|\overline{\mathcal{R}}_t(A)|}.$$

Where $|\overline{\mathcal{R}}_{\mathrm{k}}(\mathrm{A})|, |\overline{\mathcal{R}}_{\mathrm{s}}(\mathrm{A})|$ and $|\overline{\mathcal{R}}_{\mathrm{t}}(\mathrm{A})| \neq 0$.

It is Obvious that,$0 \leq \alpha_{\mathrm{k}}(\mathrm{A}) \leq 1$,$0 \leq \alpha_{\mathrm{s}}(\mathrm{A}) \leq 1$ and$0 \leq \alpha_{\mathrm{t}}(\mathrm{A}) \leq 1$.

Table 4: $\underline{R}_k(A), \overline{R}_k(A), \underline{R}_s(A), \overline{R}_s(A), \underline{R}_t(A), \overline{R}_t(A)$ $for\ all\ A \subseteq U$.

A	$\underline{R}_k(A)$	$\overline{R}_k(A)$	$\underline{R}_s(A)$	$\overline{R}_s(A)$	$\underline{R}_t(A)$	$\overline{R}_t(A)$
$\{a\}$	$\emptyset$	$\{a,c\}$	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$	$\{b\}$
$\{c\}$	$\emptyset$	$\{a,c\}$	$\{c\}$	$\{c\}$	$\{c\}$	$\{c\}$
$\{d\}$	$\emptyset$	$\{d\}$	$\emptyset$	$\{d\}$	$\emptyset$	$\{d\}$
$\{a,b\}$	$\{b\}$	$\{a,b,c\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$
$\{a,c\}$	$\{a,c\}$	$\{a,c\}$	$\{a,c\}$	$\{a,c\}$	$\{a,c\}$	$\{a,c\}$
$\{a,d\}$	$\{d\}$	$\{a,c,d\}$	$\{a\}$	$\{a,d\}$	$\{a,d\}$	$\{a,d\}$
$\{b,c\}$	$\{b\}$	$\{a,b,c\}$	$\{b,c\}$	$\{b,c\}$	$\{b,c\}$	$\{b,c\}$
$\{b,d\}$	$\{b\}$	$\{b,d\}$	$\{b\}$	$\{b,d\}$	$\{b\}$	$\{b,d\}$
$\{c,d\}$	$\emptyset$	$\{a,c,d\}$	$\{c,d\}$	$\{c,d\}$	$\{c,d\}$	$\{c,d\}$
$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$	$\{a,b,c\}$
$\{a,b,d\}$	$\{b,d\}$	U	$\{a,b\}$	$\{a,b,d\}$	$\{a,b,d\}$	$\{a,b,d\}$
$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c,d\}$	$\{a,c,d\}$
$\{b,c,d\}$	$\{b\}$	U	$\{b,c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$	$\{b,c,d\}$
U	U	U	U	U	U	U
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Moreover, if $\alpha_k(A) = 1$ (respectively $\alpha_s(A) = 1$ or $\alpha_t(A) = 1$), then A is called k-definable(respectively s-definable or t-definable) set. Otherwise, it is called k-rough (respectively s-rough or t- rough) set.

Example 3 According to Example 2, we have the neighborhood systems

$NS_k(a) = \{\{c\},\{c,d\}\}, NS_k(b) = \{\{b\}\}, NS_k(c) = \{\{a\}\}, NS_k(d) = \{\emptyset,\{a\}\}$

$NS_s(a) = \{\{a\}\}, NS_s(b) = \{\{b\}\}, NS_s(c) = \{\{c\},\{c,d\}\}, NS_s\{d\} = \{\emptyset,\{c,d\}\}$

$NS_t(a) = \{\{a\},\{c\},\{c,d\}\}, NS_t(b) = \{\{b\}\}, NS_t(c) = \{\{a\},\{c\},\{c,d\}\}$,

$NS_t(d) = \{\emptyset,\{a\},\{c,d\}\}$.

Proposition 1. Let $(U,\mathcal{R},\xi_j)$ be a j-neighborhood space and $A \subseteq U$. Then

(i) $\underline{\mathcal{R}}_j(A) \subseteq \underline{\mathcal{R}}_t(A)$,

(ii) $\overline{\mathcal{R}}_t(A) \subseteq \overline{\mathcal{R}}_j(A)$,

(iii) $\mathcal{B}_t(A) \subseteq \mathcal{B}_j(A)$,

(iv) $\alpha_j(A) \leq \alpha_t(A)$.

Proof. (i) Let $p \in \underline{\mathcal{R}}_j(A)$, then $p \in A$ such that $N_j(p) \neq \emptyset$, $N_j(p) \subseteq A$. Thus $p \in A$ such that $\exists\, N_t(p) \neq \emptyset$, $N_t(p) \subseteq A$. Hence $p \in \underline{\mathcal{R}}_t(A)$ and so $\underline{\mathcal{R}}_j(A) \subseteq \underline{\mathcal{R}}_t(A)$.

(ii) Let $p \notin \overline{\mathcal{R}}_j(A)$, then $p \in A^C$ and $N_j(p) \cap A = \emptyset$. Thus $p \in A^C$ and $\exists\, N_t(p)$, $N_t(p) \cap A = \emptyset$. So, $p \notin \overline{\mathcal{R}}_t(A)$. Therefore, $\overline{\mathcal{R}}_t(A) \subseteq \overline{\mathcal{R}}_j(A)$.

(iii) Using (i) and (ii) we have $\mathcal{B}_t(A) \subseteq \mathcal{B}_j(A)$.

(iv) $\underline{\mathcal{R}}_j(A) \subseteq \underline{\mathcal{R}}_t(A) \Rightarrow |\underline{\mathcal{R}}_j(A)| \leq |\underline{\mathcal{R}}_t(A)|$ and $\overline{\mathcal{R}}_t(A) \subseteq \overline{\mathcal{R}}_j(A) \Rightarrow |\overline{\mathcal{R}}_t(A)| \leq |\overline{\mathcal{R}}_j(A)|$, then we have $\left|\frac{\underline{\mathcal{R}}_j(A)}{\overline{\mathcal{R}}_j(A)}\right| \leq \left|\frac{\underline{\mathcal{R}}_t(A)}{\overline{\mathcal{R}}_t(A)}\right| \Rightarrow \alpha_j(A) \leq \alpha_t(A)$. ■

Proposition 2 . Let (U,R,ξ_j) be a j-neighborhood space and $A \subseteq U$. Then

(i) $\underline{\mathcal{R}}_k(A) \subseteq \underline{\mathcal{R}}_t(A)$.

(ii) $\overline{\mathcal{R}}_t(A) \subseteq \overline{\mathcal{R}}_k(A)$.

(iii) $\mathcal{B}_t(A) \subseteq \mathcal{B}_k(A)$.

(iv) $\alpha_k(A) \leq \alpha_t(A)$.

Proof. The proof is similar to the proof of Proposition 1 .

Proposition 3. Let $(U,\mathcal{R},\xi_j)$ be a j-neighborhood space and $A \subseteq U$. Then

(i) $\underline{\mathcal{R}}_s(A) \subseteq \underline{\mathcal{R}}_t(A)$.

(ii) $\overline{\mathcal{R}}_t(A) \subseteq \overline{\mathcal{R}}_s(A)$.

(iii) $\mathcal{B}_t(A) \subseteq \mathcal{B}_s(A)$.

(iv) $\alpha_s(A) \leq \alpha_t(A)$.

Proof. The proof is similar to the proof of Proposition 1

Remark 1 Let (U,R,ξ_j) be a j-neighborhood space and $A,B \subseteq U$, then the following are not necessarily true.

(1) $\underline{\mathcal{R}}_j(A) = \underline{\mathcal{R}}_t(A)$,

(2) $\overline{\mathcal{R}}_t(A) = \overline{\mathcal{R}}_j(A)$,

(3) $\mathcal{B}_t(A) = \mathcal{B}_j(A)$,

(4) $\alpha_j(A) = \alpha_t(A)$,

(5) $\underline{\mathcal{R}}_k(A) = \underline{\mathcal{R}}_t(A)$,

(6) $\overline{\mathcal{R}}_t(A) = \overline{\mathcal{R}}_k(A)$,

(7) $\mathcal{B}_t(A) = \mathcal{B}_k(A)$,

(8) $\alpha_k(A) = \alpha_t(A)$,

(9) $\underline{\mathcal{R}}_s(A) = \underline{\mathcal{R}}_t(A)$,

(10) $\overline{\mathcal{R}}_t(A) = \overline{\mathcal{R}}_s(A)$,

(11) $\mathcal{B}_t(A) = \mathcal{B}_s(A)$,

(12) $\alpha_s(A) = \alpha_t(A)$.

The following example is employed as a counter example to show this remark.

Example 4 According to Examples 2 and 3, If $A = \{a,b,d\}$, the twelve equalities in the above remark are not satisfied.

Considering a j-neighborhood space (U,R,ξ_j), Propositions 1, 2, and 3 prove that the approximations of sets using the operators $\underline{\mathcal{R}}_t$ and $\overline{\mathcal{R}}_t$ are more accurate than the approximations obtained by using the operators $\underline{\mathcal{R}}_j$, $\overline{\mathcal{R}}_j$ or $\underline{\mathcal{R}}_k$, $\overline{\mathcal{R}}_k$ or $\underline{\mathcal{R}}_s$, $\overline{\mathcal{R}}_s$. For this reason, we study the properties of $\underline{\mathcal{R}}_t$ and $\overline{\mathcal{R}}_t$ in the next proposition.

Proposition 4 Let (U,R,ξ_j) be a j-neighborhood space and $A,B \subseteq U$. Then

$(\mathcal{L}_1)$ $\underline{\mathcal{R}}_t(A) \subseteq A$.

$(\mathcal{L}_2)$ $\underline{\mathcal{R}}_t(U) \subseteq U$.

$(\mathcal{L}_3)$ $\underline{\mathcal{R}}_t(\emptyset) = \emptyset$.

$(\mathcal{L}_4)$ $A \subseteq B \Rightarrow \underline{\mathcal{R}}_t(A) \subseteq \underline{\mathcal{R}}_t(B)$.

$(\mathcal{L}_5)$ $\underline{\mathcal{R}}_t(A \cap B) \subseteq \underline{\mathcal{R}}_t(A) \cap \underline{\mathcal{R}}_t(B)$.

$(\mathcal{L}_6)$ $\underline{\mathcal{R}}_t(A \cup B) \supseteq \underline{\mathcal{R}}_t(A) \cup \underline{\mathcal{R}}_t(B)$.

$(\mathcal{L}_7)$ $\underline{\mathcal{R}}_t(A) = \left(\overline{\mathcal{R}}_t(A^C)\right)^C$.

$(\mathcal{U}_1)$ $A \subseteq \overline{\mathcal{R}}_t(A)$.

$(\mathcal{U}_2)$ $\overline{\mathcal{R}}_t(U) = U$.

$(\mathcal{U}_3)$ $\overline{\mathcal{R}}_t(\emptyset) = \emptyset$.

$(\mathcal{U}_4)$ $A \subseteq B \Rightarrow \overline{\mathcal{R}}_t(A) \subseteq \overline{\mathcal{R}}_t(B)$.

$(\mathcal{U}_5)$ $\overline{\mathcal{R}}_t(A \cap B) \subseteq \overline{\mathcal{R}}_t(A) \cap \overline{\mathcal{R}}_t(B)$.

$(\mathcal{U}_6)$ $\overline{\mathcal{R}}_t(A \cup B) \supseteq \overline{\mathcal{R}}_t(A) \cup \overline{\mathcal{R}}_t(B)$.

$(\mathcal{U}_7)$ $\overline{\mathcal{R}}_t(A) = \left(\underline{\mathcal{R}}_t(A^C)\right)^C$.

$(\mathcal{LU})$ $\underline{\mathcal{R}}_t(A) \subseteq \overline{\mathcal{R}}_t(A)$.

Proof. The proof of $(\mathcal{L}_1), (\mathcal{L}_2), (\mathcal{L}_3), (\mathcal{U}_1), (\mathcal{U}_2)$ and $(\mathcal{U}_3)$ follows directly from Definition 7.

$(\mathcal{L}_4)$ Let $A \subseteq B$ and $p \in \underline{\mathcal{R}}_t(A)$, then $p \in A$ *such that* $\exists\ N_t(p) \neq \emptyset, N_t(p) \subseteq A$. Thus $p \in A \subseteq B$ *such that* $\exists\ N_t(p) \neq \emptyset, N_t(p) \subseteq A \subseteq B$. Hence $p \in \underline{\mathcal{R}}_t(B)$ and so $\underline{\mathcal{R}}_t(A) \subseteq \underline{\mathcal{R}}_t(B)$. Therefore, $A \subseteq B \Longrightarrow \underline{\mathcal{R}}_t(A) \subseteq \underline{\mathcal{R}}_t(B)$.

$(\mathcal{U}_4)$ *Let* $A \subseteq B$ *and* $p \in \overline{\mathcal{R}}_t(A)$, *then we have:*

(1) $p \in A \Longrightarrow p \in A \subseteq B \Longrightarrow p \in B \subseteq \overline{\mathcal{R}}_t(B) \Longrightarrow p \in \overline{\mathcal{R}}_t(B)$

(2) $p \in A^C$. *Then* $p \in \overline{\mathcal{R}}_t(A) \Longrightarrow \forall\ N_t(p), N_t(p) \cap A \neq \emptyset$. *Since* $A \subseteq B$, *we have* $\forall\ N_t(p), N_t(p) \cap B \neq \emptyset$ *and hence we have two cases:*

(i) $p \in B - A \Longrightarrow p \in B \Longrightarrow p \in \overline{\mathcal{R}}_t(B)$.

(ii) $p \in B^C$. *So* $\forall\ N_t(p), N_t(p) \cap B \neq \emptyset \Longrightarrow p \in \overline{\mathcal{R}}_t(B)$.

Hence, by (1) *and* (2), *we have* $A \subseteq B \Longrightarrow \overline{\mathcal{R}}_t(A) \subseteq \overline{\mathcal{R}}_t(B)$.

$(\mathcal{L}_5)$ *Let* $p \in \underline{\mathcal{R}}_t(A \cap B) \Longrightarrow p \in (A \cap B), \exists\ N_t(p) \neq \emptyset, N_t(p) \subseteq (A \cap B)$

$\Longrightarrow p \in A, \exists\ N_t(p) \neq \emptyset, N_t(p) \subseteq A \ \wedge\ p \in B, \exists\ N_t(p) \neq \emptyset, N_t(p) \subseteq B$

$\Longrightarrow p \in \underline{\mathcal{R}}_t(A) \ \wedge\ p \in \underline{\mathcal{R}}_t(B) \Longrightarrow p \in \underline{\mathcal{R}}_t(A) \cap \underline{\mathcal{R}}_t(B)$.

$(\mathcal{U}_5)$ $(A \cap B) \subseteq A \Longrightarrow \overline{\mathcal{R}}_t(A \cap B) \subseteq \overline{\mathcal{R}}_t(A)$ *and* $(A \cap B) \subseteq B$

$\Longrightarrow \overline{\mathcal{R}}_t(A \cap B) \subseteq \overline{\mathcal{R}}_t(B)$. *So* $\overline{\mathcal{R}}_t(A \cap B) \subseteq \overline{\mathcal{R}}_t(A) \cap \overline{\mathcal{R}}_t(B)$.

$(\mathcal{L}_6)$ $A \subseteq (A \cup B) \Longrightarrow \underline{\mathcal{R}}_t(A) \subseteq \underline{\mathcal{R}}_t(A \cup B)$ *and* $B \subseteq (A \cup B)$

$\Longrightarrow \underline{\mathcal{R}}_t(B) \subseteq \underline{\mathcal{R}}_t(A \cup B)$. *Hence* $\underline{\mathcal{R}}_t(A \cup B) \supseteq \underline{\mathcal{R}}_t(A) \cup \underline{\mathcal{R}}_t(B)$.

$(\mathcal{U}_6)$ *Let* $p \notin \overline{\mathcal{R}}_t(A \cup B)$, *then* $p \notin (A \cup B)$ *and* $p \in (A \cup B)^C, \exists\ N_t(p), N_t(p) \cap (A \cup B) = \emptyset$. *So* $p \in (A^C \cap B^C), \exists\ N_t(p), (N_t(p) \cap A) \cup (N_t(p) \cap A) = \emptyset$. *Thus*

$p \in A^C, \exists\ N_t(p), N_t(p) \cap A = \emptyset \ \wedge\ p \in B^C, \exists\ N_t(p), N_t(p) \cap B = \emptyset$

$\Longrightarrow p \notin \overline{\mathcal{R}}_t(A) \ \wedge\ p \notin \overline{\mathcal{R}}_t(B) \Longrightarrow p \notin \left(\overline{\mathcal{R}}_t(A) \cup \overline{\mathcal{R}}_t(B)\right)$.

$(\mathcal{L}_7)$ *Let* $p \in \underline{\mathcal{R}}_t(A) \Leftrightarrow p \in A, \exists\ N_t(p) \neq \emptyset, N_t(p) \subseteq A \Leftrightarrow p \in (A^C)^C, \exists\ N_t(p) \neq \emptyset, N_t(p) \cap A^C = \emptyset$

$\Leftrightarrow P \notin \overline{\mathcal{R}}_t(A^C) \Leftrightarrow P \in \left(\overline{\mathcal{R}}_t(A^C)\right)^C$.

Hence $\underline{\mathcal{R}}_t(A) = \left(\overline{\mathcal{R}}_t(A^C)\right)^C$.

$(\mathcal{U}_7)$ *By substituting* A^C *for* A *in* $(\mathcal{L}_7)$ *we have* $\overline{\mathcal{R}}_t(A) = \left(\underline{\mathcal{R}}_t(A^C)\right)^C$.

$(\mathcal{LU})$ *Obviously, by* $(\mathcal{L}_1)$ *and* $(\mathcal{U}_1)$ *we get* $\underline{\mathcal{R}}_t(A) \subseteq \overline{\mathcal{R}}_t(A)$. ■

4. Conclusions

In this paper, we introduce three types of neighborhood systems in a j-neighborhood space, namely, the k- neighborhood system, s- neighborhood system and t- neighborhood system. Using these three neighborhood systems, we define the lower and upper rough set approximations. Comparisons between the accuracy of these three types of approximations are superimposed.

Propositions 1, 2, and 3 prove that the approximations of sets using the operators $\underline{\mathcal{R}}_t$ and $\overline{\mathcal{R}}_t$ are more accurate than the approximations obtained by using the operators $\underline{\mathcal{R}}_j$, $\overline{\mathcal{R}}_j$ or $\underline{\mathcal{R}}_k$, $\overline{\mathcal{R}}_k$ or $\underline{\mathcal{R}}_s$, $\overline{\mathcal{R}}_s$. That is, the best approximations are those based on the t- neighborhood systems.

Considering the j-neighborhood space, this study provides a method to improve the accuracy of rough set approximations by using the t- neighborhood system.

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"طريقه لتحسين دقة المجموعات التقريبية بدلالة فراغات الجوار من النوع j "

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ملخص البحث

يختص هذا البحث بتقديم طريقة لتحسين دقة المجموعات التقريبية باستخدام فراغات الجوار من النوع j ولقد تم تعريف مؤثرات التقريب السفلية والعلوية وتم دراسة خواصها الاساسية. ولقد تم تكوين التقريبيات باستخدام اربعة اساليب مختلفة ولقد تم عمل المقارنات بين هذه الاساليب وتحديد افضلها من حيث الدقه