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Multi-Item Probabilistic Buffer Stock Inventory Model with Zero Lead-time and Varying Holding Cost under a Restriction

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ABSTRACT: This paper developed a probabilistic periodic review model with a constraint which on the storage space for crisp and fuzzy environment using geometric programming approach with the following characteristics: Demand is instantaneous and its distribution is continuous. The costs are considered item cost, order cost and holding cost. The expected total cost will be based on the cost per cycle rather than cost per period. Then a Numerical example is provided.

Keywords: Probabilistic inventory system; Geometric programming; Triangular fuzzy number; Periodic review model;

1. **Introduction:** In general the classical inventory problems are designed by considering that the cost of an item is constant. But in practical situation, for the owner of the store, the holding cost is varying, where the holding cost of an item inversely relates to the length of the cycle.

Abou - El-Ata and Kotb (1997) introduced multi-item deterministic EOQ inventory model with varying holding cost under two restrictions using geometric programming approach. Kotb and Fergany (2011) introduced multi-item EOQ model with both demand-dependent unit cost and varying leading time via Geometric Programming. A considerable amount of inventory control literature concerns the computation of (Q_m, N) policies under Scarf's assumptions as in Feng and Xiao (2000). Abuo-El-Ata et al. (2003)

illustrated Probabilistic multi-item inventory model with varying order cost under two restrictions using geometric programming approach. Fergany (2005) developed a Periodic review probabilistic multi-item inventory system with zero lead time under constraints and varying ordering cost using Lagrange multiplier technique. Chiang (2008) introduced a Periodic review inventory models with stochastic supplier's visit intervals with constant units costs. In this regard, (Q_m, N) policy in which the replenishment is set every N periods to raise the inventory position to the order-up-to-level Q_m – provides an effective means of dampening the planning instability and coping with demand uncertainty. Furthermore, as pointed out by Silver et al. (1998), (Q_m, N) is particularly appealing when items are ordered from the same supplier or require resource sharing.

In such a case all items in a coordinated group can be given the same replenishment period. Periodic review also allows a reasonable prediction of the level of the workload on the staff involved and is particularly suitable for advanced planning environments. For these reasons, as stated by Silver et al. (1998), (Q_m, N) is a popular inventory policy. An important class of stochastic production/inventory control problems assumes a non-stationary demand process. Instead of employing a penalty cost scheme as in Tarim and Kingsman (2006) for $({Q_m}^n, N^n)$ under a penalty cost scheme. The optimal solution to the problem is the (Q_m, N) policy with different values for each period in the time varying demand situation. Other notable works on non-stationary stochastic demand adopt (Q_m, N) or base-stock policies and are due to Iida (1999), Sobel and Zhang (2001). InIida (1999), the periodic review dynamic inventory problem is considered and it is shown that near myopic policies are sufficiently close to optimal decisions for the infinite horizon inventory problem. In Sobel and Zhang (2001), it is assumed that demand arrives simultaneously from a deterministic source and a random source, and proven that a modified (Q_m, N) policy is optimal under general conditions. Shashank et al. (2012) presented a periodic tabular policy for scheduling of a single stage production-inventory system. Tarim and Smith (2008) explained Constraint programming for computing non-stationary (Q_m, N) inventory policies.

The cost parameters in real inventory systems and other parameters such as price, marketing and service elasticity to demand are imprecise and uncertain in nature. So, the notion of fuzziness can be applied to cope with this uncertainty. Since the proposed model is in a fuzzy environment, a fuzzy decision should be made to meet the decision criteria, and the results should be fuzzy. Fuzzy sets introduced by many researchers as a mathematical way of representing impreciseness or vagueness in everyday life. Samadi et al. (2013) illustrated Fuzzy pricing, marketing and service planning in a fuzzy inventory model: A geometric programming approach. Sadjadi et al. (2010) introduced Fuzzy pricing and marketing planning model: a possibilistic geometric programming approach. Liu (2007) used the geometric programming with fuzzy parameters in engineering optimization. Islam (2008) explained multi-objective marketing planning inventory model: A geometric programming approach. Islam and Roy (2007) developed Fuzzy multi-item economic production quantity model under space constraint: A geometric programming approach. Boydetet al. (2007) presenteda tutorial on geometric programming.

This paper develop multi-item probabilistic buffer stock inventory model with zero lead-time and varying holding cost under a restriction for crisp and fuzzy environment using geometric programming approach. The holding cost is considered as varying of the time of review. Furthermore, Demand is instantaneous in that all demand occurs at the beginning of the cycle, decreasing the stock level by that quantity. Our aim is to find the optimal maximum inventory level, the optimal time of review which minimizes the expected total cost.The model is illustrated witha numerical example for evaluation and validation of the results of model.The system is represented graphically in Figure (1).

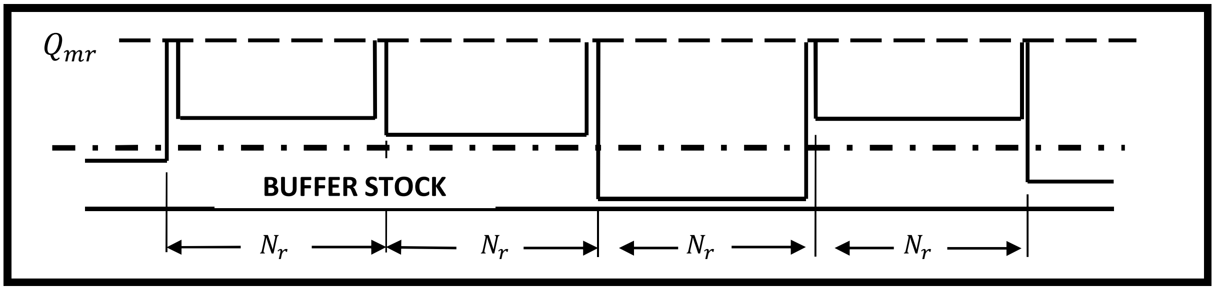

Figure (1): Inventory process with instantaneous demand

2. Notations and assumptions

Parameters for the r^{th} $(r = 1,2,\dots,n)$ item are

C_{pr}	the item cost for r^{th} item.
C_{hr}	the holding cost per unit item per unit time.
N_r	the review time (cycle) (decision variable).
β_r	a constant real number selected toprovide the best fit of estimated expected cost function such that $0 < \beta_r < 1$
$C_{hr}(N_r)$	the varying holding cost for the r^{th}per cycle $= C_{hr}N_r{}^{-\beta_r}$
C_{or}	the order cost per unit item per cycle.
S_r	the required warehouse space per unit of the r^{th} item.
Q_{mr}	maximum inventory level for the r^{th}item, at time$t = 0$(decision variable).
x_r,	The random variable representing the demand of the r^{th} item during the cycle N_r.
x_{ur}	the maximum demand for the r^{th} item during a cycle.
$E(TC)$	expected total average cost function $E\big(TC(Q_{mr},N_r)\big)$ for the r^{th} item
$\tilde{C}_{hr}(N_r)$	the fuzzy varying holding cost for the r^{th}per cycle $= \tilde{C}_{hr}N_r{}^{-\tilde{\beta}_r}$
$\tilde{C}_{or}$	the fuzzy set up cost per unit item per unit time.
k_{wr}	the goal associated to the available warehouse floor space.
τ_r	the fraction of the cycle N_r in which a stockout does not occur $0 < \tau_r < N_r$.

3. The model analysis:

For periodic review inventory model with zero lead time and instantaneous demand

The following assumptions have been considered:

(i) Replenishment is instantaneous.

(ii) The size of the replenishment order depends upon the number of units demanded from the stock at the review time, then$E(x_r) = \overline{D}_r N_r (1)$

(iii)No shortages are to be allowed so the buffer stock must be large enough to provide protection against demand fluctuations for a length of time $N + \tau_r$, then$x_{ur} = \overline{D}_r\,(N_r + \tau_r)(2)$

(iv) Assume that $Q_{mr} = x_{ur}$, thus from Equations (1) and (2):

$$Q_{mr} = \overline{D}_r\,(N_r + \tau_r) = E(x_r) + ss_r (3)$$

where $ss_r = \overline{D}_r\tau_r$ is the buffer stock quantity required to absorb demand fluctuations for the r^{th} item during the cycle N_r.

The expected total cost for the cycle will be the sum of the expected item cost, expected order cost and the expected varying holding cost, i.e.

$$E(TC) = \sum_{r=1}^{n}[E(PC_r) + E(OC_r) + E(HC_r)] \quad (4)$$

- The expected item cost for the cycle is given by

$$E(PC_r) = C_{pr} \int_{x=0}^{\infty} x_r\, f(x_r) dx = C_{pr}\bar{x}_r \quad (5)$$

- The expected order cost for the cycle is given by

$$E(OC_r) = C_{or} \quad (6)$$

- The expected varying holding cost for the cycle is given by

$$E(HC_r) = C_{hr}(N_r)\bar{I} = C_{hr}{N_r}^{-\beta_r}\bar{I}$$

Thus, the expected average amount of inventory is

$$\bar{I} = Q_{mr} - \frac{E(x_r)}{2} = Q_{mr} - \frac{\bar{D}_r N_r}{2}$$

Then, $E(HC_r) = \left[C_{hr}{N_r}^{-\beta_r}\left(Q_{mr} - \frac{\bar{D}_r N_r}{2}\right)\right]$ (7)

Hence the expected total cost for the cycle is given by,

$$E(TC) = \sum_{r=1}^{n}\left[C_{pr}\bar{x}_r + C_{or} + C_{hr}{N_r}^{-\beta_r}\left(Q_{mr} - \frac{\bar{D}_r N_r}{2}\right)\right] (8)$$

The policy variables for this model are Q_{mr} and N_r. Optimization can be performed if we assume that maximum inventory level during the cycle is related to the average demand during the cycle

$$Q_{mr} = \bar{x}_r\, g(N_r) = \bar{D}_r N_r\, g(N_r) (9)$$

where $g(N_r) \geq 1$ is the relational function just mentioned.

Equation (8) can now be written as

$$E(TC(Q_{mr}, N_r)) = \sum_{r=1}^{n}\left[C_{pr}\bar{x}_r + C_{or} + \frac{C_{hr}\bar{D}_r{N_r}^{1-\beta_r}}{2}[2g(N_r) - 1]\right] (10)$$

Consider a limitation on the cost of available warehouse floor space where the items are to be stored, i.e.,

$$S(N_r) = \sum_{r=1}^{n} S_r Q_{mr} = \sum_{r=1}^{n} S_r \bar{D}_r N_r\, g(N_r) \leq k_{wr} (11)$$

The problem is to find the optimal maximum inventory level, the optimal time of review (cycle) so as to minimize the expected total average cost function (10) subject to the storage space cost restriction, hence It may written as

Min $E\left(TC(Q_{mr}, N_r)\right)$ for all $r = 1,2,\ldots,n$

subject to inequality constraints $S(N_r) \leq k_{wr}$.

Consider the following relationship $g(N_r) = \frac{{N_r}^2 + \tau_r}{{N_r}^2}$ (12)

Substituting from Equation (12) into Equation (10) gives

$$E(TC) = \sum_{r=1}^{n}\left[C_{pr}\bar{x} + C_{or} + \frac{C_{hr}\bar{D}_r{N_r}^{-(\beta_r+1)}}{2}\left({N_r}^2 + 2\tau_r\right)\right]$$

$$E(TC) = \sum_{r=1}^{n}\left[C_{pr}\bar{x}_r + C_{or} + \frac{C_{hr}\bar{D}_r{N_r}^{1-\beta_r}}{2} + C_{hr}\bar{D}_r{N_r}^{-(\beta_r+1)}\tau_r\right] (13)$$

The terms $\sum_{r=1}^{n} C_{pr}\bar{x}_r$, $\sum_{r=1}^{n} C_{or}$ are constants and hence can be postponed without any effect, hence:

$$\min E(TC) = \sum_{r=1}^{n}\left[\frac{C_{hr}\bar{D}_r{N_r}^{1-\beta_r}}{2} + C_{hr}\bar{D}_r{N_r}^{-(\beta_r+1)}\tau_r\right] (14)$$

$$\text{Subject to: } \sum_{r=1}^{n}\frac{S_r\bar{D}_r N_r}{k_{wr}} + \sum_{r=1}^{n}\frac{S_r\bar{D}_r\tau_r}{N_r k_{wr}} \leq 1 \quad (15)$$

Applying the geometric programming techniques to Equations (14) and (15), the enlarged pre-dual function could be written in the following form:

$$G(\underline{W}) = \prod_{r=1}^{n}\left[\begin{array}{c}\left(\frac{C_{hr}\bar{D}_r{N_r}^{1-\beta_r}}{2\,W_{1r}}\right)^{W_{1r}}\left(\frac{C_{hr}\bar{D}_r{N_r}^{-(\beta_r+1)}\tau_r}{W_{2r}}\right)^{W_{2r}} \\ \times\left(\frac{S_r\bar{D}_r N_r}{k_{wr}W_{3r}}\right)^{W_{3r}}\left(\frac{S_r\bar{D}_r\tau_r}{N_r k_{wr}W_{4r}}\right)^{W_{4r}}\end{array}\right]$$

$$G(\underline{W}) = \prod_{r=1}^{n}\left[\begin{array}{c}\left(\frac{C_{hr}\bar{D}_r}{2\,W_{1r}}\right)^{W_{1r}}\left(\frac{C_{hr}\bar{D}_r\tau_r}{W_{2r}}\right)^{W_{2r}} \\ \times\left(\frac{S_r\bar{D}_r}{k_{wr}W_{3r}}\right)^{W_{3r}}\left(\frac{S_r\bar{D}_r\tau_r}{k_{wr}W_{4r}}\right)^{W_{4r}} \\ \times {N_r}^{(1-\beta_r)W_{1r} - (\beta_r+1)W_{2r} + W_{3r} - W_{4r}}\end{array}\right],$$

Also, the dual function is given by:

$$g(\underline{W}) = \prod_{r=1}^{n}\left[\begin{array}{c}\left(\frac{C_{hr}\bar{D}_r}{2\,W_{1r}}\right)^{W_{1r}}\left(\frac{C_{hr}\bar{D}_r\tau_r}{W_{2r}}\right)^{W_{2r}} \\ \times\left(\frac{S_r\bar{D}_r}{k_{wr}W_{3r}}\right)^{W_{3r}}\left(\frac{S_r\bar{D}_r\tau_r}{k_{wr}W_{4r}}\right)^{W_{4r}}\end{array}\right] \quad (16)$$

where $\underline{W} = W_{jr},\ 0 < W_{jr} < 1,\ j = 1,2,3,4$, $r = 1,2,\dots,n$ are the weights and can be chosen to yield the following normal and orthogonal conditions:

$$W_{1r} + W_{2r} = 1$$

$$(1-\beta_r)W_{1r} - (\beta_r + 1)W_{2r} + W_{3r} - W_{4r} = 0\,,$$

$$r = 1,2,\dots,n\,.$$

Solving the above Equations, we get:

$$\begin{cases} W_{1r} = \dfrac{(\beta_r + 1) - W_{3r} + W_{4r}}{2} \\ W_{2r} = \dfrac{(1-\beta_r) + W_{3r} - W_{4r}}{2}, \end{cases} \quad (17)$$

Substituting from Equations (17) into Equation (16), we get:

$$g(W_{3r}, W_{4r}) = \prod_{r=1}^{n}\left[\begin{array}{c}\left(\frac{C_{hr}\bar{D}_r}{(\beta_r+1) - W_{3r} + W_{4r}}\right)^{\frac{(\beta_r+1)-W_{3r}+W_{4r}}{2}} \\ \times\left(\frac{2\,C_{hr}\bar{D}_r\,\tau_r}{(1-\beta_r) + W_{3r} - W_{4r}}\right)^{\frac{(1-\beta_r)+W_{3r}-W_{4r}}{2}} \\ \left(\frac{S_r\bar{D}_r}{k_{wr}W_{3r}}\right)^{W_{3r}}\left(\frac{S_r\bar{D}_r\tau_r}{k_{wr}W_{4r}}\right)^{W_{4r}}\end{array}\right] \quad (18)$$

By taking the logarithm of both sides of (18), we have:

$$\begin{aligned}\ln g(W_{3r}, W_{4r}) = \sum_{r=1}^{n}\Bigg[&\frac{(\beta_r+1) - W_{3r} + W_{4r}}{2}\{\ln[C_{hr}\bar{D}_r] \\ &- \ln[(\beta_r+1) - W_{3r} + W_{4r}]\} \\ &+ \frac{(1-\beta_r) + W_{3r} - W_{4r}}{2}\{\ln[2\,C_{hr}\bar{D}_r\tau_r] \\ &- \ln[(1-\beta_r) + W_{3r} - W_{4r}]\} \\ &+ W_{3r}\left\{\ln\left(\frac{S_r\bar{D}_r}{k_{wr}}\right) - \ln W_{3r}\right\} \\ &+ W_{4r}\left\{\ln\left(\frac{S_r\bar{D}_r\tau_r}{k_{wr}}\right) - \ln W_{4r}\right\}\Bigg]\end{aligned}$$

Equate the first partial derivatives of $\ln g(W_{3r}, W_{4r})$ with respect to W_{3r} *and* W_{4r} respectively to zero we can calculate $W_{3r}{}^*$ and $W_{4r}{}^*$ which maximize $g(W_{3r}, W_{4r})$ as follows:

$$\begin{aligned}\frac{\partial \ln g(W_{3r}, W_{4r})}{\partial W_{3r}} = &-\frac{1}{2}\{\ln[C_{hr}\bar{D}_r] - \ln[(\beta_r+1) - W_{3r} + W_{4r}]\} + \frac{1}{2} \\ &+ \frac{1}{2}\{\ln[2\,C_{hr}\bar{D}_r\tau_r] - \ln[(1-\beta_r) + W_{3r} - W_{4r}]\} \\ &- \frac{1}{2} + \left\{\ln\left(\frac{S_r\bar{D}_r}{k_{wr}}\right) - \ln W_{3r}\right\} - 1 = 0\end{aligned}$$

Then we get,

$$W_{3r}{}^2 = \left(\frac{2\,\tau\,[(\beta_r+1) - W_{3r} + W_{4r}]}{[(1-\beta_r) + W_{3r} - W_{4r}]}\right)\left(\frac{S_r\bar{D}_r}{k_{wr}\,e}\right)^2 \quad (19)$$

$$\begin{aligned}\frac{\partial \ln g(W_{3r}, W_{4r})}{\partial W_{4r}} = &\frac{1}{2}\{\ln[C_{hr}\bar{D}_r] - \ln[(\beta_r+1) - W_{3r} + W_{4r}]\} \\ &-\frac{1}{2} - \frac{1}{2}\{\ln[2\,C_{hr}\bar{D}_r\tau_r] - \ln[(1-\beta_r) + W_{3r} - W_{4r}]\} + \frac{1}{2} \\ &+ \left\{\ln\left(\frac{S_r\bar{D}_r\tau_r}{k_{wr}}\right) - \ln W_{4r}\right\} - 1 = 0\end{aligned}$$

Then we get,

$$W_{4r}{}^2 = \left(\frac{[(1-\beta_r) + W_{3r} - W_{4r}]}{2\,\tau\,[(\beta_r+1) - W_{3r} + W_{4r}]}\right)\left(\frac{S_r\bar{D}_r\tau_r}{k_{wr}\,e}\right)^2 \quad (20)$$

Multiplying Equations (19) and (20) and simplifying them, we get:

$$W_{3r}W_{4r} = \left(\frac{S_r\bar{D}_r}{k_{wr}\,e}\right)\left(\frac{S_r\bar{D}_r\tau_r}{k_{wr}\,e}\right) \quad (21)$$

$$W_{3r}W_{4r} = B_1{}^2\tau_r, \quad \textit{where} \quad B_1 = \left(\frac{S_r\bar{D}_r}{k_{wr}\,e}\right)$$

$$\begin{aligned}f(W_{3r}) = \left[(1-\beta_r) + W_{3r} - B_1{}^2\tau_r W_{3r}{}^{-1}\right]W_{3r}{}^2 \\ - 2\,\tau_r\big[(\beta_r+1) - W_{3r} \\ + B_1{}^2\tau_r W_{3r}{}^{-1}\big]B_1{}^2 = 0 \quad (22)\end{aligned}$$

$$\begin{aligned}f(W_{4r}) = 2\,\tau_r\left[(\beta_r+1) - B_1{}^2\tau_r W_{4r}{}^{-1} + W_{4r}\right]W_{4r}{}^2 \\ - \big[(1-\beta_r) + B_1{}^2\tau_r W_{4r}{}^{-1} \\ - W_{4r}\big]B_1\tau_r{}^2 = 0 \quad (23)\end{aligned}$$

It could be easily proved that $f_j(0) < 1$ and $f_j(1) > 0$, $\forall j = 3,4$, and this means that there exists a root $w_{jr} \in (0,1), j = 3,4$. Numerical methods (such as dichotomous search) could be used to calculate these roots for eachitem.

Now it is to be verified that any roots W_{3r}^*, W_{3r}^*calculated from Equation (22) and (23) maximizeg (W_{3r}^*, W_{3r}^*) respectively. Apply the following conditions:

$$\frac{\partial^2 \ln g(W_{3r}, W_{4r})}{\partial {W_{3r}}^2} = -\frac{1}{4}\left[\frac{1}{W_{1r}} + \frac{1}{W_{2r}}\right] - \frac{1}{W_{3r}} < 0$$

$$\frac{\partial^2 \ln g(W_{3r}, W_{4r})}{\partial {W_{4r}}^2} = -\frac{1}{4}\left[\frac{1}{W_{1r}} + \frac{1}{W_{2r}}\right] - \frac{1}{W_{4r}} < 0$$

$$\frac{\partial^2 \ln g(W_{3r}, W_{4r})}{\partial W_{3r}\,\partial W_{4r}} = \frac{\partial^2 \ln g(W_{3r}, W_{4r})}{\partial W_{4r}\,\partial W_{3r}} = \frac{1}{4}\left[\frac{1}{W_{1r}} + \frac{1}{W_{2r}}\right] > 0$$

It is clear that: $\left[\frac{\partial^2 \ln g(W_{3r}, W_{4r})}{\partial {W_{jr}}^2}\right] < 0$

Therefore from the Hesian matrix we have:

$$\Delta = \left(\frac{\partial^2 \ln g(W_{3r}, W_{4r})}{\partial W_{3r}\,\partial W_{4r}}\right)^2 - \left(\frac{\partial^2 \ln g(W_{3r}, W_{4r})}{\partial {W_{3r}}^2}\right)\left(\frac{\partial^2 \ln g(W_{3r}, W_{4r})}{\partial {W_{4r}}^2}\right)$$
$$= -\frac{1}{4}\left[\frac{1}{W_{1r}} + \frac{1}{W_{2r}}\right]\left[\frac{1}{W_{3r}} + \frac{1}{W_{4r}}\right] - \frac{1}{W_{3r}W_{4r}} < 0$$

Therefore the roots W_{3r}^*, W_{4r}^* calculated from Equations (22), (23) maximize the dual function $g(W_{3r}, W_{4r})$. Hence the optimal solution is $W_{jr}^*, j = 1,2,3,4$ where W_{1r}^*, W_{2r}^* are calculated by substituting the values of W_{3r}^* and W_{4r}^* in Equation (17).

To find the optimal review time(cycle)$N_r{}^*$, the optimal maximum inventory level Q_{mr}^* and the minimum expected total cost $\min E(TC)$ use the following relations due to Duffin and Peterson's theorem (1966) of geometric programming as follows:

$$\frac{C_{hr}\bar{D}_r {N_r}^{1-\beta_r}}{2} = W_{1r}^*\ \ g(W_{3r}^*, W_{4r}^*)$$

and

$$C_{hr}\bar{D}_r {N_r}^{-(\beta_r+1)}\tau_r = W_{2r}^*\ \ g(W_{3r}^*, W_{4r}^*)$$

hence,

$$N_r^* = \left(\frac{2\ \tau_r W_{1r}^*}{W_{2r}^*}\right)^{\frac{1}{2}} \qquad (24)$$

$$Q_{mr}^* = \bar{D}_r {N^*}_r\ g(N_r) = \bar{D}_r {N^*}_r \left(\frac{{N^*}_r{}^2 + \tau_r}{{N^*}_r{}^2}\right)$$

$$Q_{mr}^* = \bar{D}_r {N^*}_r \left[1 + \frac{\tau_r}{{N^*}_r{}^2}\right] \qquad (25)$$

Substituting the value of N_r^* in Equation (13), after restore the constant terms, we get:

$$min\ E(TC) = \sum_{r=1}^{n}\left[\begin{matrix} C_{pr}\bar{x}_r + C_{or} + \dfrac{C_{hr}\bar{D}_r {N^*}_r{}^{1-\beta_r}}{2} \\ + C_{hr}\bar{D}_r {N^*}_r{}^{-(\beta_r+1)}\tau_r \end{matrix}\right]$$

$$min\ E(TC) = \sum_{r=1}^{n}\left[\begin{matrix} C_{pr}\bar{x}_r + C_{or} + \dfrac{C_{hr}\bar{D}_r {N^*}_r{}^{-(\beta_r+1)}}{2} \\ \left({N^*}_r{}^2 + 2\tau_r\right) \end{matrix}\right] \qquad (26)$$

which is the optimal minimum expected total cost.

4. The model when All parameters are Fuzzy Numbers

The inventory cost coefficients, elasticity parameters and other coefficients in the model are fuzzy in nature. Therefore, the decision variables and the objective function should be fuzzy as well, and we are interested in deriving the membership functions of $E(TC)$ by solving the model via geometric programming According to the assumptions, we have the following formulation for the fuzzy inventory model:

$$E(\widetilde{TC}) = \sum_{r=1}^{n}\left[\begin{matrix} \tilde{C}_{pr}\bar{x}_r + \tilde{C}_{or} + \dfrac{\tilde{C}_{hr}\bar{D}_r {N_r}^{1-\tilde{\beta}_r}}{2} \\ + \tilde{C}_{hr}\bar{D}_r {N_r}^{-(\tilde{\beta}_r+1)}\tilde{\tau}_r \end{matrix}\right] \qquad (27)$$

The terms $\sum_{r=1}^{n} \tilde{C}_{pr}\bar{x}_r$, $\sum_{r=1}^{n} \tilde{C}_{or}$are constants and hence can be postponed Restores without any effect, hence Equation (27) tends to

$$E(\widetilde{TC}) = \sum_{r=1}^{n}\left[\frac{\tilde{C}_{hr}\bar{D}_r N_r^{\,1-\tilde{\beta}_r}}{2} + \tilde{C}_{hr}\bar{D}_r N_r^{\,-(\tilde{\beta}_r+1)}\tilde{\tau}_r\right] \quad (28)$$

Subject to: $$\sum_{r=1}^{n}\frac{\tilde{S}_r\bar{D}_r N_r}{\tilde{k}_{wr}} + \sum_{r=1}^{n}\frac{\tilde{S}_r\bar{D}_r\tilde{\tau}_r}{N_r\tilde{k}_{wr}} \le 1 \quad (29)$$

To solve this inventory model using geometric programming technique, we should find the right and the left shape functions of the objective function and decision variables, by find the upper bound and the lower bound of the objective function, i.e. $\widetilde{TC}^L(\propto)$ and $\widetilde{TC}^R(\propto)$. Recall that $\widetilde{TC}^L(\propto)$ and $\widetilde{TC}^R(\propto)$ represent the largest and the smallest values (The left and right $\propto$ cuts)of the optimal objective function $\widetilde{TC}(\propto)$.

Consider the model when all parameters are triangular fuzzy numbers (TFN) as given below

$C_{pr} = (C_{pr} - a_{1r},\ C_{pr}\ , C_{pr} + a_{2r}),$

$C_{or} = (C_{or} - a_{3r},\ C_{or},\ C_{or} + a_{4r}),$

$C_{hr} = (C_{hr} - a_{5r}\ , C_{hr}, C_{hr} + a_{6r}),$

$S_r = (S_r - a_{7r},\ S_r,\ S_r + a_{8r})$,

$\beta_r = (\beta_r - a_{9r},\ \beta_r,\ \beta_r + a_{10r}),$

$K_{wr} = (K_{wr} - a_{11r},\ K_{wr},\ K_{wr} + a_{12r})$

and $\tau_r = (\tau_r - a_{13r},\ \tau_r\ ,\ \tau_r + a_{14r}).$

where a_{ir} , $i = 1,2,\ldots\ldots,14$ are arbitrary positive numbers under the following restrictions:

$0 \le a_{1r} \le C_{pr}, a_{2r} \ge 0,\ 0 \le a_{3r} \le C_{or}, a_{4r} \ge 0$,
$0 \le a_{5r} \le C_{hr}, a_{6r} \ge 0, 0 \le a_{7r} \le S_r, a_{8r} \ge 0,$
$0 \le a_{9r} \le \beta_r, a_{10r} \ge 0\ ,\ 0 \le a_{11r} \le K_{wr}, a_{12r} \ge 0$

and $0 \le a_{13r} \le \tau_r, a_{14r} \ge 0$

The left and right limits of $\propto cuts$ of $C_{pr}, C_{or}, C_{hr}, S_r, \beta_r, K_{wr}$ and τ_r are given by

$\tilde{C}_{pr_L}(\propto) = C_{pr} - (1-\propto)a_{1r}\tilde{C}_{pr_R}(\propto) = C_{pr} + (1-\propto)a_{2r},$

$\tilde{C}_{or_L}(\propto) = C_{or} - (1-\propto)a_{3r}, \tilde{C}_{or_R}(\propto) = C_{or} + (1-\propto)a_{4r},$

$\tilde{C}_{hr_L}(\propto) = C_{hr} - (1-\propto)a_{5r}, \tilde{C}_{hr_R}(\propto) = C_{hr} + (1-\propto)a_{6r},$

$\tilde{S}_{r_L}(\propto) = S_r - (1-\propto)a_{7r}, \tilde{S}_{r_R}(\propto) = S_r + (1-\propto)a_{8r}$

$\tilde{\beta}_{r_L}(\propto) = \beta_r - (1-\propto)a_{9r}, \tilde{\beta}_{r_R}(\propto) = \beta_r + (1-\propto)a_{10r}$,

$\tilde{K}_{wr_L}(\propto) = K_{wr} - (1-\propto)a_{11r}, \tilde{K}_{wr_R}(\propto) = K_{wr} + (1-\propto)a_{12r}$,

and $\tilde{\tau}_{r_L}(\propto) = \tau_r - (1-\propto)a_{13r},\ \tilde{\tau}_{r_R}(\propto) = \tau_r(1-\propto)a_{14r}.$

where

$\tilde{C}_{pr} = C_{pr} + \frac{1}{4}(a_{2r} - a_{1r}), \tilde{C}_{or} = C_{or} + \frac{1}{4}(a_{4r} - a_{3r}),$

$\tilde{C}_{hr} = C_{hr} + \frac{1}{4}(a_{6r} - a_{5r}), \tilde{S}_r = S_r + \frac{1}{4}(a_{8r} - a_{7r})$

$\tilde{\beta}_r = \beta_r + \frac{1}{4}(a_{10r} - a_{9r}), \tilde{K}_{wr} = K_{wr} + \frac{1}{4}(a_{12r} - a_{11r})$

and $\tilde{\tau}_r = \tau_r + \frac{1}{4}(a_{14} - a_{13})$.

Using approximated value of TFN which observe in Figure 2.

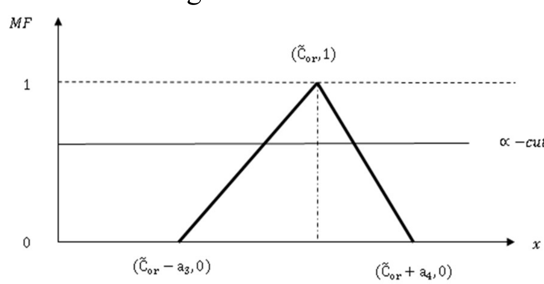

Figure 2: Order cost as triangular fuzzy number

By using the Geometric Programming approach, the above fuzzy system of Equations (28), (29) reduces to

$$\tilde{G}(\underline{W}) = \prod_{r=1}^{n}\left[\begin{array}{c}\left(\frac{\tilde{C}_{hr}\bar{D}_r}{2\,W_{1r}}\right)^{W_{1r}}\left(\frac{\tilde{C}_{hr}\bar{D}_r\tilde{\tau}_r}{W_{2r}}\right)^{W_{2r}} \\ \times\left(\frac{\tilde{S}_r\bar{D}_r}{\tilde{K}_{wr}W_{3r}}\right)^{W_{3r}}\left(\frac{\tilde{S}_r\bar{D}_r\tilde{\tau}_r}{\tilde{K}_{wr}W_{4r}}\right)^{W_{4r}} \\ \times N_r^{\,(1-\tilde{\beta}_r)W_{1r}-(\tilde{\beta}_r+1)W_{2r}+W_{3r}-W_{4r}}\end{array}\right] \quad (30)$$

hence,

$$\tilde{g}(W_{3r}, W_{4r}) = \prod_{r=1}^{n}\left[\begin{array}{c}\left(\frac{\tilde{C}_{hr}\bar{D}_r}{(\tilde{\beta}_r+1) - W_{3r} + W_{4r}}\right)^{\frac{(\tilde{\beta}_r+1)-W_{3r}+W_{4r}}{2}} \\ \times\left(\frac{2\,\tilde{C}_{hr}\bar{D}_r\tilde{\tau}_r}{(1-\tilde{\beta}_r) + W_{3r} - W_{4r}}\right)^{\frac{(1-\tilde{\beta}_r)+W_{3r}-W_{4r}}{2}} \\ \times\left(\frac{\tilde{S}_r\bar{D}_r}{\tilde{K}_{wr}W_{3r}}\right)^{W_{3r}}\left(\frac{\tilde{S}_r\bar{D}_r\tilde{\tau}_r}{\tilde{K}_{wr}W_{4r}}\right)^{W_{4r}}\end{array}\right] \quad (31)$$

Due to Duffin and Peterson's theorem (1966), the optimal values are given by:

$$N_r^* = \left(\frac{2\ \tilde{\tau}_r W_{1r}^*}{W_{2r}^*}\right)^{\frac{1}{2}} \quad (32)$$

$$Q^*_{mr} = \overline{D}_r N^*_r \left[1 + \frac{\tilde{\tau}_r}{{N^*_r}^2}\right] \quad (33)$$

Then we can determine the optimal minimum expected total cost as follows:

$$min\ E(\widetilde{TC}) = \sum_{r=1}^{n} \left[\begin{array}{c} \widetilde{C}_{pr}\overline{x}_r + \widetilde{C}_{or} + \dfrac{\widetilde{C}_{hr}\overline{D}_r {N^*_r}^{1-\widetilde{\beta}_r}}{2} \\ +\widetilde{C}_{hr}\overline{D}_r {N^*_r}^{-(\widetilde{\beta}_r+1)}\tilde{\tau}_r \end{array}\right] \quad (34)$$

5. The Model with Continuous Distributions

Suppose that the demand for a particular item follows some Continuous distribution such as:

The demand follows uniform distribution over range from zero to b then,

$$min\ E(TC) = \sum_{r=1}^{n}\left[C_{pr}\frac{{Q_m}^2}{2b} + C_{or} + \frac{C_{hr}\overline{D}_r {N^*_r}^{-(\beta_r+1)}}{2}\left({N^*_r}^2 + 2\tau_r\right)\right]$$

The demand follows negative exponential distribution when $0 < x < Q_{mr},\ \theta > 0$ then,

$$min\ E(TC) = \sum_{r=1}^{n}\left[\begin{array}{c}\dfrac{C_{pr}}{\theta}\left(1 - e^{-\theta Q_{mr}}\left(1 + \theta Q_{mr}\right)\right) + C_{or} \\ + \dfrac{C_{hr}\overline{D}_r {N^*_r}^{-(\beta_r+1)}}{2}\left({N^*_r}^2 + 2\tau_r\right)\end{array}\right]$$

The demand follows exponential distribution when $x \geq 0$ then,

$$min\ E(TC) = \sum_{r=1}^{n}\left[\begin{array}{c}\dfrac{C_{pr}}{\theta}\left(\theta^2 - \theta e^{-\frac{Q_{mr}}{\theta}}\left(\theta + Q_{mr}\right)\right) + C_{or} \\ + \dfrac{C_{hr}\overline{D}_r {N^*_r}^{-(\beta_r+1)}}{2}\left({N^*_r}^2 + 2\tau_r\right)\end{array}\right]$$

The demand follows Cauchy distribution when $-\infty < x < \infty$ then,

$$min\ E(TC) = \sum_{r=1}^{n}\left[\begin{array}{c}\dfrac{C_{pr}}{2\pi}\ln\left(1 + {Q_m}^2\right) + C_{or} \\ + \dfrac{C_{hr}\overline{D}_r {N^*_r}^{-(\beta_r+1)}}{2}\left({N^*_r}^2 + 2\tau_r\right)\end{array}\right]$$

The demand follows Weibull distribution when the variable x and the parameters η and σall are positive real numbers then,

$$min\ E(TC) = \sum_{r=1}^{n}\left[\begin{array}{c}C_{pr}\left(1 - e^{-\left(\frac{x}{\sigma}\right)^{\eta}}\right) + C_{or} \\ + \dfrac{C_{hr}\overline{D}_r {N^*_r}^{-(\beta_r+1)}}{2}\left({N^*_r}^2 + 2\tau_r\right)\end{array}\right]$$

The demand follows Rayleigh distribution when the variable$x$is real positive valuesand α a real positive parameter then,

$$min\ E(TC) = \sum_{r=1}^{n}\left[\begin{array}{c}C_{pr}\left(Q_m e^{-\frac{x^2}{2\alpha^2}} - \alpha\sqrt{\frac{\pi}{2}}e^{-\frac{x}{\sqrt{2}\alpha}}\right) + C_{or} \\ + \dfrac{C_{hr}\overline{D}_r {N^*_r}^{-(\beta_r+1)}}{2}\left({N^*_r}^2 + 2\tau_r\right)\end{array}\right]$$

The demand follows triangular distribution when the variable x is bounded to the interval $x - \Gamma \leq x \leq x + \Gamma$ and the location and scale parameters $\gamma$and $\gamma = \Gamma$ $(\Gamma > 0)$ all are real numbers.

$$min\ E(TC) = \sum_{r=1}^{n}\left[\begin{array}{c}\dfrac{C_{pr}}{2\,\Gamma^2}\left(\Gamma {Q_m}^2 - \dfrac{1}{3}\left(\gamma^2|\gamma|\dfrac{(Q_m-\gamma)^2(2Q_m+\gamma)}{sign(Q_m-\gamma)}\right)\right) \\ + C_{or} + \dfrac{C_{hr}\overline{D}_r {N^*_r}^{-(\beta_r+1)}}{2}\left({N^*_r}^2 + 2\tau_r\right)\end{array}\right]$$

6. Numerical example

A businessman decides to open a Boutique, say XYZ, in Egypt, say, where he wishes to exclusively showcase indigenously made fabrics of three items (e.g. ties, shirts and trousers) of India. As the businessman has to travel to the places where these bolts of fabrics are made to collect his orders, he wishes to follow a periodic review "order up to Q_m". Sufficient information is not available for this kind of a business. But from what is available, he estimates that the parameters are given in Table 1, Table2.The demand is occurs at the begging of the cycle. Also the optimal solutions of the crisp environment and triangular fuzzy number TFN are given in Tables 3, 4, 5, 6, 7, 8 and 9.

It is desired to determine to optimal values ${Q_m}^*, N^*$ and the minimum total cost in the following cases:

When demand follows

1- Uniform distribution for$0 \leq x \leq b$,

2- negative exponential distribution for $x \geq 0$.

3- Exponential distribution for $x \geq 0$.

4- Cauchy Distribution for $-\infty < x < \infty$

5- Weibull Distribution for $x \geq 0$.

6- Rayleigh Distribution for $x \geq 0$.

7- Triangular Distribution for$\mathrm{x} - \Gamma \leq \mathrm{x} \leq \mathrm{x} + \Gamma$.

Parameters	Item 1	Item 2	Item 3
$\boldsymbol{\beta_r}$	$(0,1)$	$(0,1)$	$(0,1)$
$\boldsymbol{C_{pr}}$	11	9	14
$\boldsymbol{C_{or}}$	15	13	17
$\boldsymbol{\bar{D}}$	7	5	9
$\boldsymbol{S_r}$	5	4	6
$\boldsymbol{C_{hr}}$	4	3	5
$\boldsymbol{K_{wr}}$	102	51.9	36.2
$\boldsymbol{\tau_r}$	1.9	1.5	0.9
$\boldsymbol{b_r}$	45	50	55
θ_r	6	5	7
η_r	7	4.5	8.5
σ_r	6	5	7
α_r	4	3	6.5
$\gamma_r = \Gamma_r$	13	11	14

Table 1: Crisp values of the parameters

Parameters	Item 1	Item 2	Item 3
$\tilde{C}_{pr}$	(8,11,12)	(7,9,10)	(12,14,15)
$\tilde{C}_{or}$	(12,15,16)	(10,13,15)	(14,17,19)
$\tilde{S}_r$	(3,5,5.5)	(3,4,4.5)	(4,6,7)
$\tilde{C}_{hr}$	(2,4,4.5)	(2,3,3.3)	(3,5,6)
$\tilde{K}_{wr}$	(94,102,104)	(42.8,51.9,53)	(32,36.2,40)
$\tilde{\tau}_r$	(1.7, 1.9, 1.95)	(1.2, 1.5, 1.7)	(0.8, 0.9, 0.95)

Table 2:Input imprecise data for shape parameters

$\boldsymbol{\beta_r}$	Item 1				Item 2				Item 3				$Min\ E(TC)$
	N_1	Q_{m1}	$E(TC)_1$	$E(TC_1) = E(TC)/Q_{m1}$	N_2	Q_{m2}	$E(TC)_2$	$E(TC_2) = E(TC)/Q_{m2}$	N_3	Q_{m3}	$E(TC)_3$	$E(TC_3) = E(TC)/Q_{m3}$	
0.1	2.20956	21.4862	145.177	6.75673	1.94171	13.5711	69.517	5.12242	S1.25852	17.7628	143.952	8.10412	19.98327
0.2	2.35372	22.1267	143.435	6.48246	2.0699	13.9729	68.7665	4.92143	1.31598	17.9989	143.417	7.96809	19.37198
0.3	2.50934	22.8656	141.914	6.20646	2.20829	14.4377	67.9958	4.70959	1.37575	18.2694	142.491	7.7994	18.71545
0.4	2.67792	23.712	140.808	5.93828	2.35823	14.9715	67.2824	4.49403	1.43795	18.5746	141.225	7.60313	18.03544
0.5	2.86112	24.6763	140.336	5.68706	2.52129	15.5811	66.7151	4.28179	1.50271	18.9147	139.683	7.38489	17.35374
0.6	3.06076	25.7707	140.739	5.46121	2.69915	16.2744	66.3941	4.07967	1.57017	19.2902	137.936	7.15058	16.69146
0.7	3.27882	27.0081	142.287	5.26832	2.89366	17.0602	66.4322	3.89399	1.64046	19.7018	136.065	6.90622	16.06853
0.8	3.51739	28.4029	145.278	5.1149	3.10684	17.9482	66.9553	3.73047	1.71372	20.15	134.154	6.65773	15.5031
0.9	3.77876	29.971	150.045	5.00635	3.34082	18.949	68.104	3.59406	1.79009	20.6357	132.291	6.4108	15.01121
$\tilde{\beta}_r$	$\tilde{N}_1$	$\tilde{Q}_{m1}$	$E(\widetilde{TC})_1$	$E(\widetilde{TC}_1) = E(\widetilde{TC})_1/Q_{m1}$	$\tilde{N}_2$	$\tilde{Q}_{m2}$	$E(\widetilde{TC})_2$	$E(\widetilde{TC}_2) = E(\widetilde{TC})_2/Q_{m2}$	$\tilde{N}_3$	$\tilde{Q}_{m3}$	$E(\widetilde{TC})_3$	$E(\widetilde{TC}_3) = E(\widetilde{TC})_3/Q_{m3}$	$Min\ E(\widetilde{TC})$
0.1	2.1807	21.2434	133.814	6.29908	1.9058	13.3988	58.8633	4.39317	1.2585	17.6732	134.88	7.63188	18.32413
0.2	2.3262	21.8879	132.531	6.055	2.0316	13.7881	58.0367	4.2092	1.3171	17.9183	134.489	7.50568	17.76988
0.3	2.4836	22.6347	131.462	5.80801	2.16725	14.2392	57.146	4.01329	1.3782	18.1992	133.73	7.34813	17.16943
0.4	2.65465	23.4938	130.794	5.56719	2.31419	14.7578	56.2563	3.81197	1.44179	18.5161	132.652	7.16416	16.54332
0.5	2.84117	24.477	130.739	5.34133	2.4739	15.3506	55.4424	3.61174	1.50813	18.8694	131.317	6.95924	15.91231
0.6	3.04522	25.5979	131.539	5.13867	2.64805	16.0253	54.7884	3.41886	1.57733	19.2599	129.795	6.73913	15.29666
0.7	3.2691	26.8716	133.464	4.96674	2.8385	16.7906	54.3883	3.23921	1.6495	19.6881	128.164	6.5097	14.71565
0.8	3.5151	28.3145	136.821	4.83218	3.047	17.656	54.3466	3.07809	1.7249	20.155	126.508	6.27675	14.18702
0.9	3.7859	29.9447	141.957	4.74063	3.2762	18.6319	54.7792	2.94008	1.8036	20.6613	124.915	6.04584	13.72655

Table 3: The results of crisp and fuzzy values for uniform distribution

β_r	Item 1				Item 2				Item 3				$Min\ E(TC)$
	N_1	Q_{m1}	$E(TC)_1$	$E(TC_1) = E(TC)/Q_{m1}$	N_2	Q_{m2}	$E(TC)_2$	$E(TC_2) = E(TC)/Q_{m2}$	N_3	Q_{m3}	$E(TC)_3$	$E(TC_3) = E(TC)/Q_{m3}$	
0.1	2.20956	21.4862	96.2276	4.47858	1.94171	13.5711	52.8995	3.89794	1.25852	17.7628	105.795	5.95599	14.33251
0.2	2.35372	22.1267	91.4138	4.13139	2.0699	13.9729	51.0424	3.65297	1.31598	17.9989	104.186	5.78844	13.5728
0.3	2.50934	22.8656	86.2357	3.77142	2.20829	14.4377	48.951	3.39049	1.37575	18.2694	102.011	5.58367	12.74558
0.4	2.67792	23.712	80.7934	3.40728	2.35823	14.9715	46.6678	3.11711	1.43795	18.5746	99.314	5.34677	11.87116
0.5	2.86112	24.6763	75.1877	3.04695	2.52129	15.5811	44.238	2.83921	1.50271	18.9147	96.1491	5.08331	10.96947
0.6	3.06076	25.7707	69.5184	2.69758	2.69915	16.2744	41.7085	2.56283	1.57017	19.2902	92.5765	4.79914	10.05955
0.7	3.27882	27.0081	63.8824	2.36531	2.89366	17.0602	39.1272	2.29348	1.64046	19.7018	88.6626	4.50023	9.15902
0.8	3.51739	28.4029	58.3715	2.05512	3.10684	17.9482	36.5414	2.03594	1.71372	20.15	84.4778	4.19244	8.06171
0.9	3.77876	29.971	53.0699	1.77071	3.34082	18.949	33.9973	1.79415	1.79009	20.6357	80.0945	3.88135	7.44621
$\widetilde{\beta}_r$	$\widetilde{N}_1$	$\widetilde{Q}_{m1}$	$E(\widetilde{TC})_1$	$E(\widetilde{TC}_1) = E(\widetilde{TC})_1/Q_{m1}$	$\widetilde{N}_2$	$\widetilde{Q}_{m2}$	$E(\widetilde{TC})_2$	$E(\widetilde{TC}_2) = E(\widetilde{TC})_2/Q_{m2}$	$\widetilde{N}_3$	$\widetilde{Q}_{m3}$	$E(\widetilde{TC})_3$	$E(\widetilde{TC}_3) = E(\widetilde{TC})_3/Q_{m3}$	$Min\ E(\widetilde{TC})$
0.1	2.1807	21.2434	88.5292	4.16737	1.9058	13.3988	46.3326	3.45796	1.2585	17.6732	97.8013	5.53387	13.1592
0.2	2.3262	21.8879	84.3278	3.85271	2.0316	13.7881	44.6643	3.23935	1.3171	17.9183	96.3202	5.37551	12.46757
0.3	2.4836	22.6347	79.7678	3.52414	2.16725	14.2392	42.7678	3.00353	1.3782	18.1992	94.2929	5.18116	11.70883
0.4	2.65465	23.4938	74.9388	3.18973	2.31419	14.7578	40.6819	2.75664	1.44179	18.5161	91.7608	4.95574	10.90211
0.5	2.84117	24.477	69.9317	2.85704	2.4739	15.3506	38.4482	2.50467	1.50813	18.8694	88.7743	4.70466	10.06637
0.6	3.04522	25.5979	64.8376	2.53293	2.64805	16.0253	36.1102	2.25332	1.57733	19.2599	85.3912	4.43363	9.21988
0.7	3.2691	26.8716	59.7455	2.22337	2.8385	16.7906	33.7125	2.00782	1.6495	19.6881	81.6753	4.14845	8.37964
0.8	3.5151	28.3145	54.7412	1.93333	3.047	17.656	31.2996	1.77275	1.7249	20.155	77.6942	3.85483	7.56091
0.9	3.7859	29.9447	49.9049	1.66657	3.2762	18.6319	28.9153	1.55193	1.8036	20.6613	73.5181	3.55825	6.77675

Table 4: The results of crisp and fuzzy values for negative exponential distribution

β_r	Item 1				Item 2				Item 3				$Min\ E(TC)$
	N_1	Q_{m1}	$E(TC)_1$	$E(TC_1) = E(TC)/Q_{m1}$	N_2	Q_{m2}	$E(TC)_2$	$E(TC_2) = E(TC)/Q_{m2}$	N_3	Q_{m3}	$E(TC)_3$	$E(TC_3) = E(TC)/Q_{m3}$	
0.1	2.20956	21.4862	151.975	7.07313	1.94171	13.5711	85.0254	6.26517	1.25852	17.7628	174.387	9.81751	24.01431
0.2	2.35372	22.1267	147.837	6.6814	2.0699	13.9729	83.8022	5.9975	1.31598	17.9989	173.434	9.63577	23.17625
0.3	2.50934	22.8656	143.376	6.2704	2.20829	14.4377	82.4046	5.70759	1.37575	18.2694	171.994	9.4143	22.25665
0.4	2.67792	23.712	138.68	5.8485	2.35823	14.9715	80.8678	5.40144	1.43795	18.5746	170.106	9.15803	21.27455
0.5	2.86112	24.6763	133.833	5.42352	2.52129	15.5811	79.2278	5.08486	1.50271	18.9147	167.818	8.87235	20.24862
0.6	3.06076	25.7707	128.92	5.00259	2.69915	16.2744	77.5206	4.76335	1.57017	19.2902	165.182	8.56298	18.32892
0.7	3.27882	27.0081	124.021	4.592	2.89366	17.0602	75.7805	4.44195	1.64046	19.7018	162.258	8.23572	17.26967
0.8	3.51739	28.4029	119.211	4.19712	3.10684	17.9482	74.0394	4.12517	1.71372	20.15	159.111	7.89629	16.21858
0.9	3.77876	29.971	114.558	3.82228	3.34082	18.949	72.3261	3.81687	1.79009	20.6357	155.803	7.55015	15.1893
$\widetilde{\beta}_r$	$\widetilde{N}_1$	$\widetilde{Q}_{m1}$	$E(\widetilde{TC})_1$	$E(\widetilde{TC}_1) = E(\widetilde{TC})_1/Q_{m1}$	$\widetilde{N}_2$	$\widetilde{Q}_{m2}$	$E(\widetilde{TC})_2$	$E(\widetilde{TC}_2) = E(\widetilde{TC})_2/Q_{m2}$	$\widetilde{N}_3$	$\widetilde{Q}_{m3}$	$E(\widetilde{TC})_3$	$E(\widetilde{TC}_3) = E(\widetilde{TC})_3/Q_{m3}$	$Min\ E(\widetilde{TC})$
0.1	2.1807	21.2434	141.1345	6.6437	1.9058	13.3988	77.2921	5.7686	1.2585	17.6732	164.92	9.3316	21.7439
0.2	2.3262	21.8879	137.6016	6.2867	2.0316	13.7881	76.2345	5.5290	1.3171	17.9183	164.1129	9.1589	20.9746
0.3	2.4836	22.6347	133.7538	5.9092	2.16725	14.2392	75.01	5.2678	1.3782	18.1992	162.8407	8.9477	20.1247
0.4	2.65465	23.4938	129.667	5.51923	2.31419	14.7578	73.6475	4.99041	1.44179	18.5161	161.138	8.70262	19.21226
0.5	2.84117	24.477	125.418	5.12394	2.4739	15.3506	72.1831	4.70229	1.50813	18.8694	159.05	8.42898	18.25521
0.6	3.04522	25.5979	121.082	4.73014	2.64805	16.0253	70.649	4.40859	1.57733	19.2599	156.627	8.13228	17.27101
0.7	3.2691	26.8716	116.7282	4.3439	2.8385	16.7906	69.077	4.114	1.6495	19.6881	153.9252	7.8182	16.2761
0.8	3.5151	28.3145	112.4261	3.9706	3.047	17.656	67.4972	3.8229	1.7249	20.155	151.0053	7.4922	15.2857
0.9	3.7859	29.9447	108.2383	3.615	3.2762	18.6319	65.9360	3.5389	1.8036	20.6613	147.929	7.1597	14.3136

Table 5: The results of crisp and fuzzy values for exponential distribution

β_r	Item 1				Item 2				Item 3				$Min\ E(TC)$
	N_1	Q_{m1}	$E(TC)_1$	$E(TC_1) = E(TC)/Q_{m1}$	N_2	Q_{m2}	$E(TC)_2$	$E(TC_2) = E(TC)/Q_{m2}$	N_3	Q_{m3}	$E(TC)_3$	$E(TC_3) = E(TC)/Q_{m3}$	
0.1	2.20956	21.4862	105.394	4.90521	1.94171	13.5711	60.0995	4.42848	1.25852	17.7628	117.795	6.63156	15.96525
0.2	2.35372	22.1267	100.58	4.54567	2.0699	13.9729	58.2424	4.16825	1.31598	17.9989	116.186	6.45515	15.16907
0.3	2.50934	22.8656	95.4024	4.17231	2.20829	14.4377	56.151	3.88919	1.37575	18.2694	114.011	6.24051	14.30201
0.4	2.67792	23.712	89.9601	3.79386	2.35823	14.9715	53.8678	3.59802	1.43795	18.5746	111.314	5.99282	13.3847
0.5	2.86112	24.6763	84.3543	3.41843	2.52129	15.5811	51.438	3.30131	1.50271	18.9147	108.149	5.71774	12.43748
0.6	3.06076	25.7707	78.6851	3.05328	2.69915	16.2744	48.9085	3.00525	1.57017	19.2902	104.576	5.42122	11.47975
0.7	3.27882	27.0081	73.0491	2.70471	2.89366	17.0602	46.3272	2.71551	1.64046	19.7018	100.663	5.10931	10.52953
0.8	3.51739	28.4029	67.5381	2.37786	3.10684	17.9482	43.7414	2.43709	1.71372	20.15	96.4778	4.78797	9.60292
0.9	3.77876	29.971	62.2365	2.07656	3.34082	18.949	41.1973	2.17411	1.79009	20.6357	92.0945	4.46287	8.71354
$\widetilde{\beta}_r$	$\widetilde{N}_1$	$\widetilde{Q}_{m1}$	$E(\widetilde{TC})_1$	$E(\widetilde{TC}_1) = E(\widetilde{TC})_1/Q_{m1}$	$\widetilde{N}_2$	$\widetilde{Q}_{m2}$	$E(\widetilde{TC})_2$	$E(\widetilde{TC}_2) = E(\widetilde{TC})_2/Q_{m2}$	$\widetilde{N}_3$	$\widetilde{Q}_{m3}$	$E(\widetilde{TC})_3$	$E(\widetilde{TC}_3) = E(\widetilde{TC})_3/Q_{m3}$	$Min\ E(\widetilde{TC})$
0.1	2.1807	21.2434	96.9292	4.56279	1.9058	13.3988	53.3326	3.98039	1.2585	17.6732	109.587	6.20074	14.74392
0.2	2.3262	21.8879	92.7278	4.23649	2.0316	13.7881	51.6643	3.74703	1.3171	17.9183	108.106	6.03326	14.01678
0.3	2.4836	22.6347	88.1678	3.89525	2.16725	14.2392	49.7678	3.49513	1.3782	18.1992	106.079	5.82876	13.21914
0.4	2.65465	23.4938	83.3388	3.54727	2.31419	14.7578	47.6819	3.23096	1.44179	18.5161	103.547	5.59225`	12.37048
0.5	2.84117	24.477	78.3317	3.20022	2.4739	15.3506	45.4482	2.96068	1.50813	18.8694	100.56	5.32926	11.49016
0.6	3.04522	25.5979	73.2376	2.86108	2.64805	16.0253	43.1102	2.69013	1.57733	19.2599	97.177	5.04556	10.59677
0.7	3.2691	26.8716	68.1455	2.53597	2.8385	16.7906	40.7125	2.42472	1.6495	19.6881	93.461	4.74707	9.70776
0.8	3.5151	28.3145	63.1412	2.23	3.047	17.656	38.2996	2.16922	1.7249	20.155	89.4799	4.43959	8.83881
0.9	3.7859	29.9447	58.3049	1.94709	3.2762	18.6319	35.9153	1.92763	1.8036	20.6613	85.3038	4.12867	8.00339

Table 6: The results of crisp and fuzzy values for Weibull distribution

β_r	Item 1				Item 2				Item 3				$Min\ E(TC)$
	N_1	Q_{m1}	$E(TC)_1$	$E(TC_1)=E(TC)/Q_{m1}$	N_2	Q_{m2}	$E(TC)_2$	$E(TC_2)=E(TC)/Q_{m2}$	N_3	Q_{m3}	$E(TC)_3$	$E(TC_3)=E(TC)/Q_{m3}$	
0.1	2.20956	21.4862	105.138	4.89329	1.94171	13.5711	58.5784	4.3164	1.25852	17.7628	116.624	6.5656	15.77529
0.2	2.35372	22.1267	100.427	4.53874	2.0699	13.9729	56.8045	4.06534	1.31598	17.9989	115.073	6.39331	14.99739
0.3	2.50934	22.8656	95.3639	4.17063	2.20829	14.4377	54.8064	3.79605	1.37575	18.2694	112.964	6.18322	14.1499
0.4	2.67792	23.712	90.0486	3.7976	2.35823	14.9715	52.6267	3.51512	1.43795	18.5746	110.341	5.94043	13.25315
0.5	2.86112	24.6763	84.5822	3.42766	2.52129	15.5811	50.3108	3.22896	1.50271	18.9147	107.257	5.67056	12.32718
0.6	3.06076	25.7707	79.0646	3.06801	2.69915	16.2744	47.9055	2.94362	1.57017	19.2902	103.771	5.37948	11.39111
0.7	3.27882	27.0081	73.5926	2.72484	2.89366	17.0602	45.4588	2.66461	1.64046	19.7018	99.9513	5.07321	10.46266
0.8	3.51739	28.4029	68.2577	2.40319	3.10684	17.9482	43.0179	2.39678	1.71372	20.15	95.8666	4.75764	9.55761
0.9	3.77876	29.971	63.1441	2.10684	3.34082	18.949	40.6288	2.14411	1.79009	20.6357	91.5892	4.43838	8.68933
$\tilde{\beta}_r$	$\tilde{N}_1$	$\tilde{Q}_{m1}$	$E(\widetilde{TC})_1$	$E(\widetilde{TC}_1)=E(\widetilde{TC})_1/Q_{m1}$	$\tilde{N}_2$	$\tilde{Q}_{m2}$	$E(\widetilde{TC})_2$	$E(\widetilde{TC}_2)=E(\widetilde{TC})_2/Q_{m2}$	$\tilde{N}_3$	$\tilde{Q}_{m3}$	$E(\widetilde{TC})_3$	$E(\widetilde{TC}_3)=E(\widetilde{TC})_3/Q_{m3}$	$Min\ E(\widetilde{TC})$
0.1	2.1807	21.2434	96.647	4.54951	1.9058	13.3988	51.8185	3.86738	1.2585	17.6732	108.414	6.13438	14.55127
0.2	2.3262	21.8879	92.5452	4.22815	2.0316	13.7881	50.2294	3.64297	1.3171	17.9183	106.993	5.97116	13.84228
0.3	2.4836	22.6347	88.0972	3.89213	2.16725	14.2392	48.4222	3.40063	1.3782	18.1992	105.034	5.77135	13.06411
0.4	2.65465	23.4938	83.3924	3.54956	2.31419	14.7578	46.4354	3.1465	1.44179	18.5161	102.577	5.5399	12.23596
0.5	2.84117	24.477	78.5222	3.208	2.4739	15.3506	44.311	2.88659	1.50813	18.8694	99.6731	5.28225	11.37684
0.6	3.04522	25.5979	73.5774	2.87436	2.64805	16.0253	42.0923	2.62661	1.57733	19.2599	96.3794	5.00415	10.50512
0.7	3.2691	26.8716	68.6474	2.55465	2.8385	16.7906	39.824	2.3718	1.6495	19.6881	92.7594	4.71144	9.63789
0.8	3.5151	28.3145	63.8177	2.25389	3.047	17.656	37.5506	2.12679	1.7249	20.155	88.8807	4.40986	8.79054
0.9	3.7859	29.9447	59.1682	1.97592	3.2762	18.6319	35.3157	1.89544	1.8036	20.6613	84.8129	4.10491	7.97627

Table 7: The results of crisp and fuzzy values for Cauchy distribution

β_r	Item 1				Item 2				Item 3				$Min\ E(TC)$
	N_1	Q_{m1}	$E(TC)_1$	$E(TC_1)=E(TC)/Q_{m1}$	N_2	Q_{m2}	$E(TC)_2$	$E(TC_2)=E(TC)/Q_{m2}$	N_3	Q_{m3}	$E(TC)_3$	$E(TC_3)=E(TC)/Q_{m3}$	
0.1	2.20956	21.4862	222.15	10.3392	1.94171	13.5711	106.818	7.87099	1.25852	17.7628	220.54	12.4158	30.62599
0.2	2.35372	22.1267	221.147	9.99458	2.0699	13.9729	108.347	7.75408	1.31598	17.9989	221.981	12.333	30.08166
0.3	2.50934	22.8656	219.757	9.61084	2.20829	14.4377	110.083	7.6247	1.37575	18.2694	223.263	12.2206	29.45614
0.4	2.67792	23.712	217.79	9.18482	2.35823	14.9715	112.058	7.48477	1.43795	18.5746	224.412	12.0817	28.75129
0.5	2.86112	24.6763	214.922	8.70964	2.52129	15.5811	114.284	7.3348	1.50271	18.9147	225.461	11.9199	27.96434
0.6	3.06076	25.7707	210.641	8.17367	2.69915	16.2744	116.74	7.17324	1.57017	19.2902	226.448	11.739	27.08591
0.7	3.27882	27.0081	204.167	7.55948	2.89366	17.0602	119.351	6.99586	1.64046	19.7018	227.407	11.5425	26.09784
0.8	3.51739	28.4029	194.351	6.84264	3.10684	17.9482	121.959	6.79502	1.71372	20.15	228.373	11.3336	24.97126
0.9	3.77876	29.971	179.535	5.99029	3.34082	18.949	124.286	6.55893	1.79009	20.6357	229.371	11.1152	23.66442
$\tilde{\beta}_r$	$\tilde{N}_1$	$\tilde{Q}_{m1}$	$E(\widetilde{TC})_1$	$E(\widetilde{TC}_1)=E(\widetilde{TC})_1/Q_{m1}$	$\tilde{N}_2$	$\tilde{Q}_{m2}$	$E(\widetilde{TC})_2$	$E(\widetilde{TC}_2)=E(\widetilde{TC})_2/Q_{m2}$	$\tilde{N}_3$	$\tilde{Q}_{m3}$	$E(\widetilde{TC})_3$	$E(\widetilde{TC}_3)=E(\widetilde{TC})_3/Q_{m3}$	$Min\ E(\widetilde{TC})$
0.1	2.1807	21.2434	206.884	9.73873	1.9058	13.3988	97.3231	7.26354	1.2585	17.6732	209.351	11.8457	28.84797
0.2	2.3262	21.8879	206.51	9.4349	2.0316	13.7881	98.8707	7.17075	1.3171	17.9183	210.993	11.7753	28.38095
0.3	2.4836	22.6347	205.81	9.09267	2.16725	14.2392	100.625	7.06675	1.3782	18.1992	212.503	11.6765	27.83592
0.4	2.65465	23.4938	204.592	8.70833	2.31419	14.7578	102.617	6.9534	1.44179	18.5161	213.905	11.5524	27.21413
0.5	2.84117	24.477	202.531	8.27437	2.4739	15.3506	104.864	6.83129	1.50813	18.8694	215.232	11.4064	26.51206
0.6	3.04522	25.5979	199.108	7.77832	2.64805	16.0253	107.356	6.69915	1.57733	19.2599	216.514	11.2417	25.71917
0.7	3.2691	26.8716	193.518	7.2016	2.8385	16.7906	110.033	6.55327	1.6495	19.6881	217.786	11.0618	24.81667
0.8	3.5151	28.3145	184.558	6.51815	3.047	17.656	112.765	6.38678	1.7249	20.155	219.075	10.8695	23.77443
0.9	3.7859	29.9447	170.466	5.69269	3.2762	18.6319	115.312	6.18899	1.8036	20.6613	220.402	10.6674	22.54908

Table 8: The results of crisp and fuzzy values for triangular distribution

β_r	Item 1				Item 2				Item 3				$Min\ E(TC)$
	N_1	Q_{m1}	$E(TC)_1$	$E(TC_1)=E(TC)/Q_{m1}$	N_2	Q_{m2}	$E(TC)_2$	$E(TC_2)=E(TC)/Q_{m2}$	N_3	Q_{m3}	$E(TC)_3$	$E(TC_3)=E(TC)/Q_{m3}$	
0.1	2.20956	21.4862	149.54	6.95981	1.94171	13.5711	84.9343	6.25846	1.25852	17.7628	211.187	11.8893	25.10757
0.2	2.35372	22.1267	144.726	6.54081	2.0699	13.9729	83.0793	5.94576	1.31598	17.9989	210.147	11.6755	24.16207
0.3	2.50934	22.8656	139.548	6.10298	2.20829	14.4377	80.9892	5.60955	1.37575	18.2694	208.573	11.4165	23.12903
0.4	2.67792	23.712	134.106	5.65562	2.35823	14.9715	78.7067	5.2571	1.43795	18.5746	206.495	11.1171	22.02982
0.5	2.86112	24.6763	128.5	5.20742	2.52129	15.5811	76.2773	4.8955	1.50271	18.9147	203.95	10.7826	20.88552
0.6	3.06076	25.7707	122.831	4.76631	2.69915	16.2744	73.748	4.53154	1.57017	19.2902	200.982	10.4189	19.71675
0.7	3.27882	27.0081	117.195	4.33925	2.89366	17.0602	71.1666	4.1715	1.64046	19.7018	197.646	10.0319	18.54265
0.8	3.51739	28.4029	111.684	3.93213	3.10684	17.9482	68.5809	3.82104	1.71372	20.15	193.999	9.62771	17.38088
0.9	3.77876	29.971	106.382	3.54951	3.34082	18.949	66.0368	3.48497	1.79009	20.6357	190.104	9.21236	16.24684
$\tilde{\beta}_r$	$\tilde{N}_1$	$\tilde{Q}_{m1}$	$E(\widetilde{TC})_1$	$E(\widetilde{TC}_1)=E(\widetilde{TC})_1/Q_{m1}$	$\tilde{N}_2$	$\tilde{Q}_{m2}$	$E(\widetilde{TC})_2$	$E(\widetilde{TC}_2)=E(\widetilde{TC})_2/Q_{m2}$	$\tilde{N}_3$	$\tilde{Q}_{m3}$	$E(\widetilde{TC})_3$	$E(\widetilde{TC}_3)=E(\widetilde{TC})_3/Q_{m3}$	$Min\ E(\widetilde{TC})$
0.1	2.1807	21.2434	125.909	5.92695	1.9058	13.3988	77.4764	5.78232	1.2585	17.6732	201.089	11.3782	23.08747
0.2	2.3262	21.8879	121.707	5.56048	2.0316	13.7881	75.8105	5.49828	1.3171	17.9183	200.203	11.1731	22.23186
0.3	2.4836	22.6347	117.147	5.17556	2.16725	14.2392	73.9156	5.19101	1.3782	18.1992	198.804	10.9238	21.29037
0.4	2.65465	23.4938	112.318	4.78077	2.31419	14.7578	71.8307	4.8673	1.44179	18.5161	196.916	10.6349	20.28297
0.5	2.84117	24.477	107.311	4.38417	2.4739	15.3506	69.5974	4.53385	1.50813	18.8694	194.573	10.3115	19.22952
0.6	3.04522	25.5979	102.217	3.99318	2.64805	16.0253	67.2596	4.19708	1.57733	19.2599	191.816	9.95936	18.14962
0.7	3.2691	26.8716	97.1249	3.61441	2.8385	16.7906	64.8619	3.86299	1.6495	19.6881	188.695	9.5842	17.0616
0.8	3.5151	28.3145	92.1206	3.25348	3.047	17.656	62.4491	3.53699	1.7249	20.155	185.265	9.19199	15.98246
0.9	3.7859	29.9447	87.2843	2.91485	3.2762	18.6319	60.0648	3.22376	1.8036	20.6613	181.586	8.78868	14.92729

Table 9: The results of crisp and fuzzy values for Rayleigh distribution

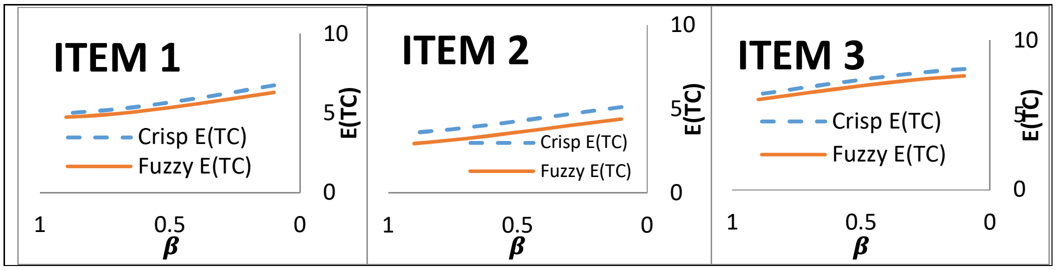

Figure 3: Crisp and fuzzy value for uniform distribution

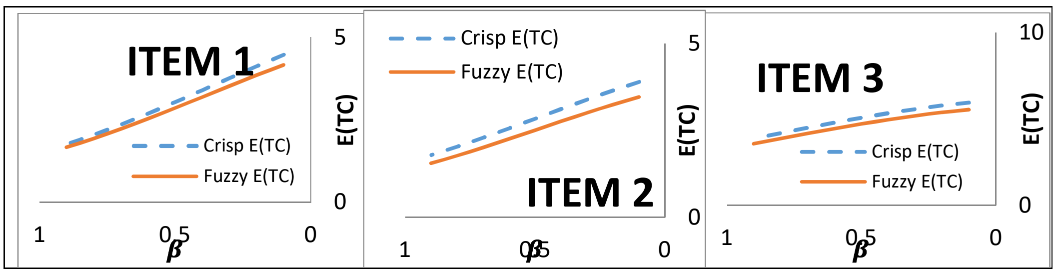

Figure 4: Crisp and fuzzy value for negative exponential

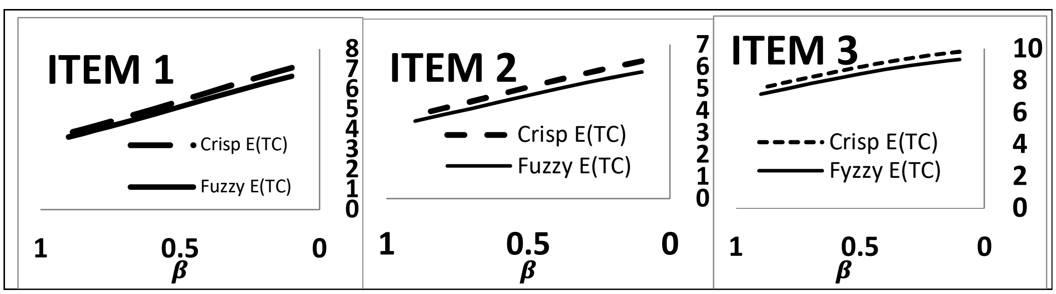

Figure 5: Crisp and fuzzy value for exponential distribution

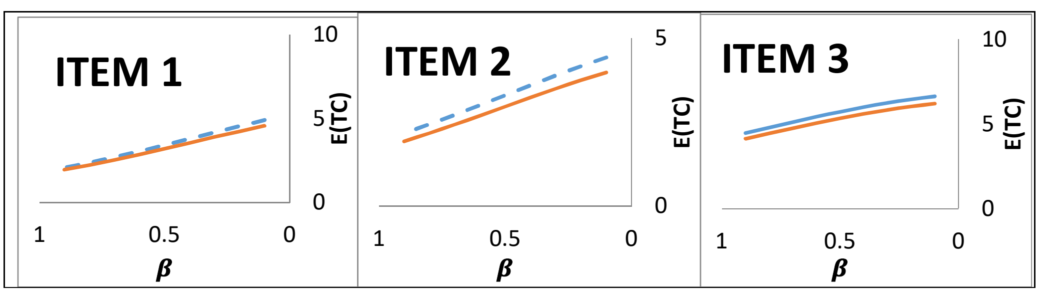

Figure 6: Crisp and fuzzy value for Weibull distribution

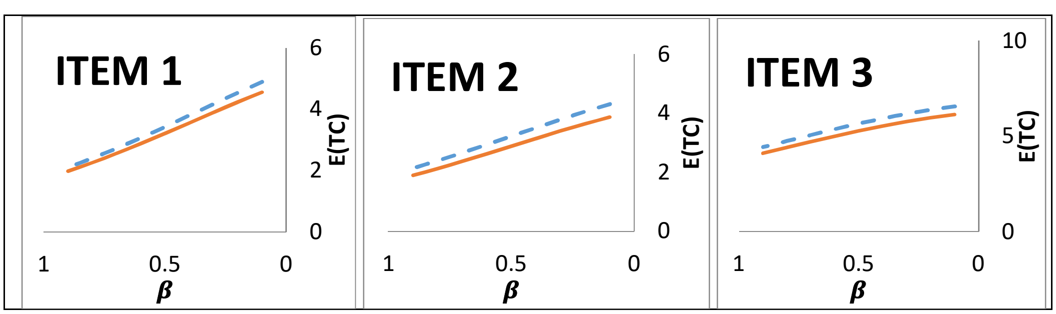

Figure 7: Crisp and fuzzy value for Cauchy distribution

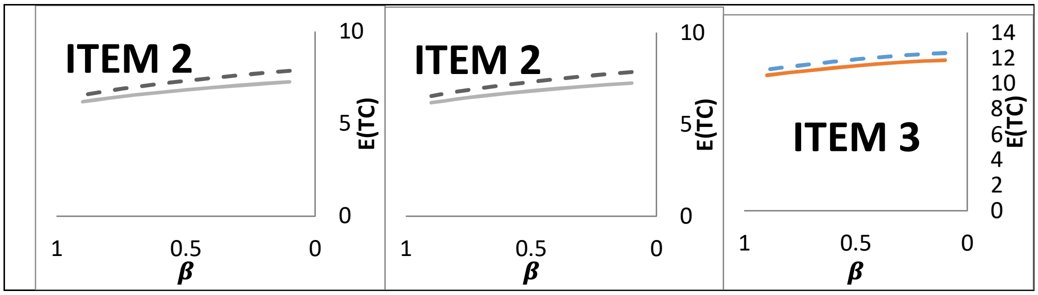

Figure 8: Crisp and fuzzy value for triangular distribution

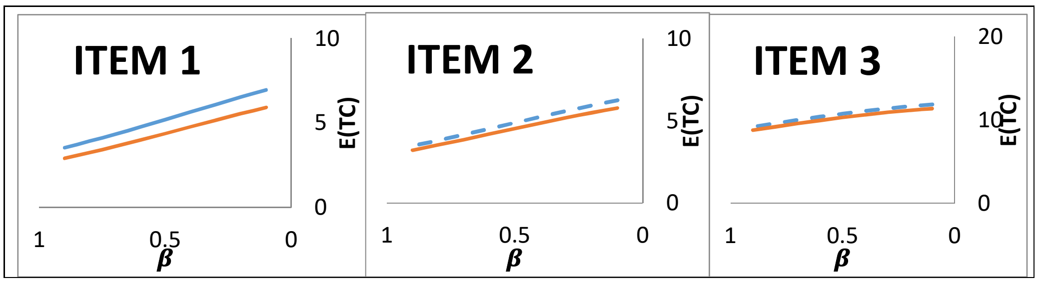

Figure 9: Crisp and fuzzy value for Rayleigh distribution

6. Conclusion

In this paper, we developed a probabilistic Multi-item single-source (MISS) inventory model with varying holding cost under a restriction, which is on storage space. We determine the optimal review period and optimal maximum inventory level that minimized the expected annual total cost under constraint using Geometric programming technique for crisp and TFN environment. When β convergence to 1, the solution approaching to the optimal solution.

References

C. Chiang, Periodic Review Inventory Models with Stochastic Supplier's Visit Intervals, International Journal of Production Economics, 115 (2008) 433-438.

E. A. Silver, D. F. Pyke, R. Peterson, Inventory Management and Production Planning and Scheduling, John Wiley and Sons, New York, 1998.

Farshid Samadi, Abolfazl Mirzazadeh, Mir Mohsen Pedram, Fuzzy Pricing, Marketing and Service Planning In a Fuzzy Inventory Model: A Geometric Programming Approach, Applied

S. Boyd, S .J. Kim, L Vandenberghe, A. Hassibi, a Tutorial on Geometric Programming, Optim. Eng., 8 (2007) 67-127.

Mathematical Modelling Journal, 137 (2013) 6683-6694.

Fergany. H. A., Periodic Review Probabilistic Multi-Item Inventory System with Zero Lead Time under Constraints and Varying Ordering Cost. American Journal of Applied Sciences, 2(8) (2005), 1213-1217.

K. A. M. Kotb, Fergany H. A., Multi-Item EOQ Model with Varying Holding Cost: A Geometric Programming Approach, International Mathematical Forum, Vol. 6, (2011), no. 23, 1135 - 1144

M. J. Soble, R. Q. Zhang, Inventory Polices For Systems with Stochastic and Deterministic Demand, Operations Research, 49 (2001) 157-162.

M.O. Abou-El-Ata, K.A.M. Kotb, Multi-Item EOQ Inventory Model with Varying Holding Cost under Two Restrictions: A Geometric Programming Approach, Production Planning & Control 8 (5) (1997) 608–611.

M.O. Abuo-El-Ata, Hala A. Fergany, Mona F. El-Wakeel, Probabilistic Multi-Item Inventory Model With Varying Order Cost under Two Restrictions: A Geometric Programming Approach, Int. J. Production Economics, 83 (2003) 223–231.

R. J. Duffin, E.L. Peterson, C. Zener, Geometric Programming Theory and Applications, Wiley, New York, 1966.

S. Islam, Multi-Objective Marketing Planning Inventory Model: A Geometric

Programming Approach, Appl. Math, Computer, 205 (2008) 238-246.

S. Islam, T.K. Roy, Fuzzy Multi-Item Economic Production Quantity Model under Space Constraint: A Geometric Programming Approach, Appl. Math. Computer, 184 (2007) 326-335.

S.A. Tarim, B.G. Kingsman, Modeling And Computing (R^n,S^n) Policies For Inventory Systems with Non-Stationary Stochastic Demand, European Journal of Operational Research , 174 (2006) 581–599

S.A. Tarim, B.M. Smith, Constraint Programming For Computing Non-Stationary (R,S) Inventory Policies, European Journal of Operational Research, 189 (2008) 1004–1021

S.J. Sadjadi, M. Ghazanfari, A. Yousefli, Fuzzy Pricing and Marketing Planning Model: A Possibilistic Geometric Programming Approach, Expert Syst. Appl. 37 (2010) 3392–3397.

S.T. Liu, Geometric Programming with Fuzzy Parameters In Engineering Optimization, Int. J. Approx. Reason., 46 (2007) 484-498.

Shashank Grag, D. Krishna Sundar, K. Ravikumar, A Periodic Tabular Policy For Scheduling of A Single Stage Production-Inventory System, Computers & Industrial Engineering, 62 (2012) 21-28.

T. Iida, The Infinite Horizon Non-Stationary Stochastic Inventory Problem: Near Myopic Polices And Weak Ergodicity, European Journal of Operational Research. 116 (1999) 405-422.

Y. Feng, B. Xiao, A New Algorithm For Computing Optimal (s,S) Policies In A Stochastic Single Item/Location Inventory System, IIE Transactions, 32 (2000) 1081–1090.

يعرف المخزون السلعي علي أنه تخزين الخامات وتجهيزها لمواجهة الطلب. ولقد حدثت مشكلة التخزين عندما كان من الضروري الاحتفاظ بالمخزون المادي للبضائع أو للسلع بغية تلبية الطلب علي مدي فترة زمنية محددة. ويعتبر موضوع التحكم في المخزون من أهم الاعتبارات الأساسية في مجالات عدة، السبب في ذلك أهمية المخزون السلعي في مجالات مختلفة في المواقف العملية والاقتصادية مثل علوم الإدارة ونظم الإنتاج وعلوم أخري كثيرة. هذا البحث طور نموذج المخزون السلعي الاحتمالي ذات المراقبة الدورية بزمن تسليم صفري وتكلفة تخزين متغيرة لمصدر واحد والعديد من السلع مع وجود حد أمان تحت قيد علي المساحة التخزينية في حالتي التكلفة الثابتة والمتغيرة باستخدام طريقة البرمجة الهندسية. كما تم اعتبار أن الطلب فوري ويحدث في بداية الدورة. و الهدف هو إيجاد القيمة المثلي لأقصي مستوي للمخزون وزمن المراقبة الأمثل الذي يعطي أقل تكلفة إجمالية سنوية. ثم اتبع بمثال عددي لتوضيح النتائج.