DJS Vol. 37 (2016) 103-111

Delta Journal of Science
Available online at
https://djs.journals.ekb.eg/

Research Article PHYSICS

# An Optical Model Analysis for p, d and $^4$He Elastically Scattered on $^7$Li Nuclei in a Wide Range of Energies

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**Abstract:** Optical model analysis for p, d and α-particles elastically scattered by $^7$Li nuclei have been performed within the framework of optical model using computer codes (ECIS88 and FRESCO) at different projectile's energies. Good agreement between the theoretical calculations and experimental data was obtained.

**Key words: elastic scattering; Optical Model and volume integral of real and imaginary potential depth.**

## Introduction:

The interaction between two nuclei is a many-body problem which unfortunately has lots of complex mathematical difficulties [1]. Therefore for a many-body system, it is logical to work on simplified models instead of taking into account individual forces between nucleons. Within the framework of Optical Model (OM) [1-3], the many body problem may be replaced by one body problem of mass equals the reduced mass μ. The optical model is one of the most fundamental theoretical models in nuclear reaction theory. The key point of the optical model is how to give the optical model potential. From the phenomenological studies, it is clear that the major part of the nuclear interaction potential can be approximated by a Woods-Saxon form which gives a simple analytic expression, parameterized explicitly by the depth, the radius, and diffuseness of the potential well. In practice it is required to obtain the potential from the analysis of experimental data by varying their parameters to optimize the overall fit to the data, using appropriate (OM) codes. But such analysis cannot give unique values of the all potential parameters; rather it is certain combinations that correspond to a particular set of data. Thus, for example, the fit to that data is insensitive to variations of Vo and ro that keep $v_o r_o^2$ constant, and similarly for $W_D$ $a_D$. Since the potential determination from phenomenological analysis is insufficiently precise to resolve these ambiguities, it is usual to fix the geometrical parameters (radius, diffuseness) to average values and then to adjust the potential depths $V_0$, $W_D$, and $V_{so}$ to fit the data [4-5]

The purpose of the present work is the extraction of reliable information about potential parameters for the interaction of protons, deuterons and α-particles with $^7$Li nuclei at different projectile's energies. Many such analysis of nucleon scattering have now been made and it is found that the potentials are quite similar for all nuclei and vary other slowly with the incident energy. The optical model is thus a successful way for describing of the elastic scattering data in a wide range of conditions, and this provides confirmation of the overall correctness of the derivations of the potential from more fundamental considerations. Elastic scattering of nucleon–nucleus data at intermediate energies are useful tools for testing and analyzing nuclear structure models and intermediate energy reaction theories [6-15]. The elastic scattering of proton-nucleus has been analyzed in order to determine ground state matter densities empirically for comparison with Hartree–Fock predictions [16-18]. The study of spin dependent effect at the intermediate energy proton scattering plays an important role [19].The optical potential has been extensively used in studying the proton-nucleus scattering [20].The nuclear potential may be used as a combination of real part, imaginary part and/or spin orbit potential (if a projectile and/or target nucleus has spin). The imaginary part of potential could be taken in the volume shape if the incident projectile energy is relatively high. While, at low energies, the imaginary potential could be taken as surface which could be expressed in a Gaussian or Wood- Saxon derivative form. At incident particle energy above 20 MeV, a volume term as well as a surface term seems to be necessary. Good agreement with experimental data is achieved with $R_s$=$R_V$ (say $R_I$) and as=av(say aI), fixing four parameters Ws, Wv, $R_I$ and $a_I$ for the imaginary central term.

**Optical Model parameters:**

In a nuclear reaction, the form of a potential, which represents the two-body interaction between the projectile and the target nucleus, must be appropriate to the elastic scattering and the reactions take place between the projectile and the target. Generally, the real part of the interaction potential represents the elastic scattering and the imaginary part corresponds to the absorption (inelastic scattering and the reactions). This complex potential is called optical potential and depends only on the distance between center of mass of colliding nuclei. So, the optical potential can be written down in the form:

$$U_{op}(r) = V_C(r) - V(r) - iW(r) - V_{so}(r) \quad (1)$$

where $V_C$ (r) is the coulomb potential due to a uniform distribution of appropriate size and total charge.

$$V_C(r) = \frac{Z_p Z_t e^2}{2R_C}\left(3 - \frac{r^2}{R_C^2}\right) \quad \text{For } r \le R_C$$

$$V_C(r) = \frac{Z_p Z_t e^2}{r} \quad \text{For } r \geq R_C \quad (2)$$

Where $Z_p$ and $Z_t$ are the charges of projectile and target nucleus, $R_c = r_c A^{1/3}$ is the coulomb radius of the nucleus and $e$ is the charge of electron:

$$R_i = r_i (A_t)^{1/3}, \; i = V, W, C$$

The real volume part has the following form:

$$V(r) = V_o f(r, r_v, a_v) \quad (3)$$

Where $f(r, r_v, a_v) = \left[1 + \exp\left(\frac{r - r_v}{a_v}\right)\right]^{-1}$

Wood-Saxon form factor which involves three parameters $V_O$, $r_V$ and $a_V$.

The imaginary volume part has the following form:

$$W_V(r) = -W_o f(r, r_w, a_w), \quad f(r, r_w, a_w) = \left[1 + \exp\left(\frac{r - r_w}{a_w}\right)\right]^{-1} \quad (4)$$

Real spin-orbit part has the following form:

$$V_{so}(r) = V_{so} \frac{2}{r} \frac{d}{dr}\left[1 + \exp\left(\frac{r - r_{so}A^{1/3}}{a_{so}}\right)\right]^{-1} \quad (5)$$

The spin-orbit term $U_{so}(r) = V_{so}(r) + iW_{so}(r)$ , it is usual to take $W_{so}(r) = 0$, leaving the three parameters $V_{so}$, $r_{so}$, and $a_{so}$. The model thus involves nine parameters although several analysis have been performed using more restricted sets by equating some of the geometrical parameters and/or neglecting one the imaginary terms. The interaction potential can be rewritten as

$$U(r) = V_C(r) - V_o f(r, r_v, a_v) - iW_o f(r, r_w, a_w) + \hbar^2 \frac{2}{r} V_{so} \frac{d}{dr} f(r, r_{so}, a_{so}) \quad (6)$$

**Results and Discussion:**

**Analysis of protons elastically scattering on $^7$Li**

The comparison between the experimental data for protons elastically scattering on $^7$Li at energies (0.45, 0.75, 0.991, 1.0 MeV) [21] (6.15, 10.3, 24.4, 49.65 MeV) [22] is shown in Fig. 1. Analysis for protons elastically scattering on $^7$Li was performed using code ECIS88 where the following parameters were fixed $r_C$=1.3fm, $r_V$=1.17 fm, $r_D$=1.8 fm. At lower energies (0.45, 0.75, 0.991, 1.0 MeV), the spin orbit potential parameters were fixed at $v_{so}$=8.48 MeV, $r_{so}$= 1.10fm and diffuseness parameter $a_{so}$= 0.60fm. While at higher energies (6.15, 10.3, 24.4, 49.65 MeV), $v_{so}$=11.689 MeV, $r_{so}$ was fixed at1.17 fm and $a_{so}$ was fixed at 0.656 fm. The optical potential parameters obtained in calculations are listed in table 1. As shown in Fig. 1, the agreement between theoretical predictions and experimental data is fairly good over the whole angular range which gives clear evidence about the pure potential character of protons elastic scattering on $^7$Li nuclei.

Table 1. Optical potentials parameter for proton elastic Scattering on $^7$Li

| $J_W$ MeV.fm$^3$ | $J_R$ MeV.fm$^3$ | $\chi^2/N$ | $a_D$ fm | $W_D$ MeV | $a_v$ fm | $V_o$ MeV | $E_p$ MeV |
|---|---|---|---|---|---|---|---|
| 279.6 | 597.4 | 4.25 | 0.897 | 3.0 | 0.998 | 59 | 0.45 |
| 101.96 | 665.1 | 7.21 | 0.87 | 1.14 | 1.09 | 57.8 | 0.75 |
| 285.32 | 698.5 | 4.39 | 0.87 | 3.19 | 1.10 | 59.9 | 0.991 |
| 209.58 | 712.4 | 3.00 | 0.74 | 2.89 | 1.147 | 57.3 | 1.0 |
| 159.02 | 386.0 | 0.55 | 1.046 | 1.377 | 0.853 | 46.8 | 6.15 |
| 230.96 | 297.9 | 1.63 | 0.812 | 2.828 | 0.588 | 52.7 | 10.3 |
| 158.6 | 227.1 | 0.99 | 0.19 | 9.706 | 0.576 | 40.9 | 24.4 |
| 362.9 | 146.9 | 3.49 | 0.881 | 3.986 | 0.432 | 32.1 | 49.65 |

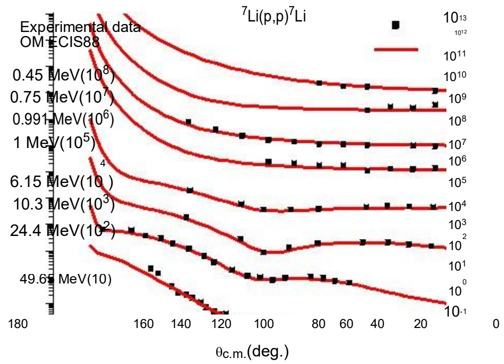



Fig. 1 comparison between calculated and experimental angular distributions for protons elastically scattered from $^7$Li at different energies.

We have calculated the real volume integral using the following equation:

$$J_R(E) = -\left(1/A_p A_t\right)\int V(r)4\pi \quad (7)$$

Where AP and At mass values of the incident particle and the target nucleus. The value of the real volume integral should be close to the corresponding value of the nucleon-nucleon potential of the interaction. The volumetric integral of the imaginary part of the optical potential determined as:

$$J_W(E) = -\left(1/A_p A_t\right)\int [W_v(E.r) + W_s]dr \quad (8)$$

$J_R$ and $J_W$ are supposed to be independent of the projectiles and target and are useful to compare different sets of optical potential parameters for different nuclei [23]. As expected the relation between volume integral of both real potential Vo and imaginary potential WD depths with proton energy Ep is linear as shown in figures 2 and 3 respectively.

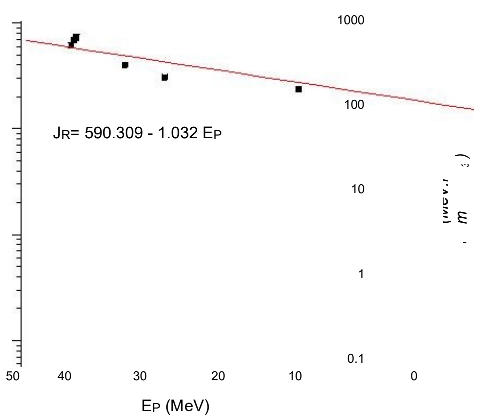



Fig. 2 relation between volume integral of real potential depth and proton energy.

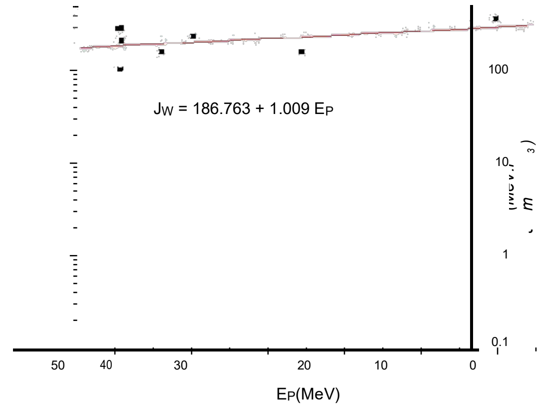



Fig. 3 relation between volume integral of imaginary potential depth and proton energy.

The strength of parameters listed in table 1 can be represented by the following equations:

$$V_o = 57.23 - 0.55E_p$$

$$J_R(E) = 590.309 - 1.032\, E_p$$

$$J_W(E) = 186.763 + 1.009\, E_p$$

**Analysis of deuterons elastically scattered by $^7$Li**

Deuterons have several special features that complicate the way they are scattered by nuclei. They are very loosely bound, and are therefore easily broken up when they encounter the nuclear field. The scattering is thus rather sensitive to the nuclear structure and it is correspondingly more difficult to define overall optical potentials. The charge and mass centers of the deuteron are significantly separated, and this gives rise to forces tending to twist and break the deuteron even before it encounters the nuclear field. The nucleon potentials are to be taken at an energy one half of deuteron energy. Since the well depth for the nucleon scattering is roughly 50 MeV, this leads to a central potential of deuterons of about 100 MeV. While experimental cross-sections can be described with several discrete values (e.g., 50 MeV, 100 MeV and 150 MeV), the above argument leads one to prefer 100 MeV deep potential **[6]**.

The comparison between the experimental data for deuterons elastic scattering by $^7$Li at energies (4, 7, 8, 9, 10)[24], (12)[25], (14.7) [26]) MeV is shown in Fig. (4). Analysis for deuterons elastic scattering $^7$Li was performed in the forward angular range ($\theta \leq 90$) using code ECIS88, while the following parameters were fixed $r_C$=1.3fm, $r_V$=1.25fm, $r_D$=1.325 fm, $v_{so}$= 6.76 MeV, $r_{so}$=1.07 fm and $a_{so}$=0.66 fm. We noted that at energies 10MeV and 12MeV there is $^5$He transfer reaction according to this configurations $^7$Li(d,$^7$Li)d. There is anomalous in the backward angle scattering which will be calculated .The optical potential parameters used in calculations are listed in table 2.

Table 2: Optical Potentials Parameters for deuteron elastic scattering on $^7$Li at different energies

| $J_W$ V.fm$^3$ | $J_R$ MeV.fm$^3$ | $\chi2$ /N | $a_D$ fm | $W_D$ MeV | $a_v$ fm | $V_o$ MeV | $E_d$ MeV |
|---|---|---|---|---|---|---|---|
| .73 | 524.65 | 10 | 0.99 | 0.466 | 0.83 | 116.43 | 4 |
| 9.7 | 373.92 | 4.42 | 0.92 | 1.986 | 0.756 | 91.658 | 7 |
| 7.7 | 375.12 | 1.74 | 0.995 | 2.282 | 0.781 | 88.919 | 8 |
| .68 | 400.17 | 0.95 | 0.806 | 3.765 | 0.836 | 88.093 | 9 |
| 08.9 | 395.7 | 1.26 | 0.416 | 10.43 | 0.792 | 92.41 | 10 |
| 75.5 | 397.4 | 2.92 | 0.957 | 5.485 | 0.727 | 101.269 | 12 |
| 181.1 | 285.2 | 3.7 | 1.18 | 4.00 | 0.755 | 70.0 | 14.7 |

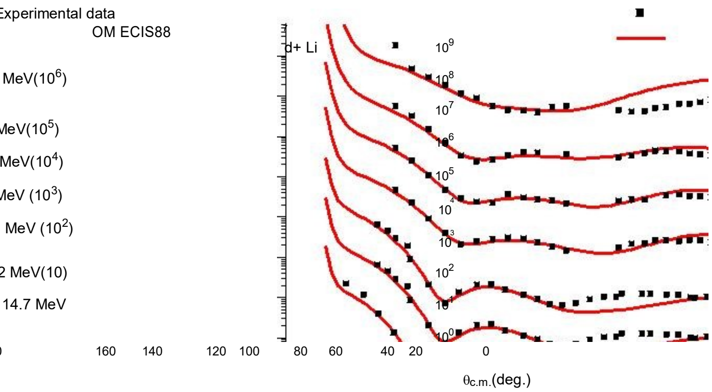



Fig. 4 comparison between calculated and experimental angular distributions for deuterons elastically scattered from $^7$Li at different energies.

As expected the relation between volume integral of both real potential $V_o$ and imaginary potential $W_D$ depth with deuteron energy $E_d$ is linear as shown in figures 5 and 6 respectively. The strength of parameters listed in table 2 can be represented by the following equations:

$$V_o = 125.21 - 3.87\, E_d\,,$$

$$W_D = -0.65 + 0.40\, E_d\,,$$

$$J_R(E) = 547.183 - 16.66\, E_d\,,$$

$$J_W(E) = -54.726 + 16.915\, E_d\,.$$

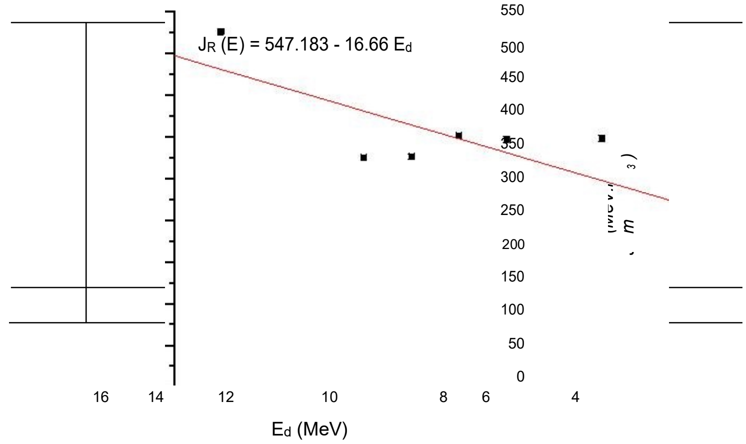



Fig. 5 relation between volume integral of real potential depth and deuterons energy.

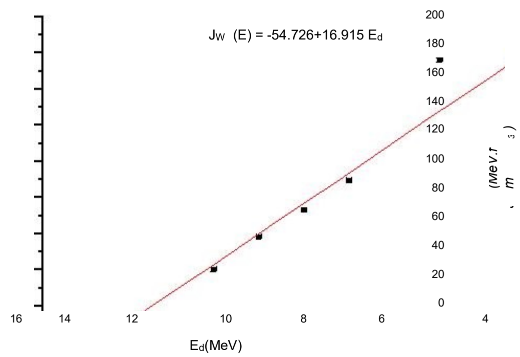



Fig. 6: The relation between volume integral of imaginary potential depth and deuterons energy

**Analysis of $^4$He+$^7$Li elastic scattering**

An alpha particle is a tightly bound spinless structure, and this simplifies the description of how it is scattered by nuclei. Its charge and mass ensure that a substantial number of partial waves contribute when the energy is above the Coulomb barrier, so that the diffraction and parameterized scattering amplitude models are very successful in many respects. The elastic scattering of α-particles is particularly sensitive to the potential in the region of the nuclear surface, and may thus be used to study the radii of nuclei [6].

The comparison between the experimental data for $^4$He elastic scattering by $^7$Li at energies taken from MeV [27], (18 MeV, 26 MeV) [29], 3.69 MeV, 3.69 and 29.4 MeV [26] is shown in Fig. (5). Analysis for $^4$He elastic scattering by $^7$Li was performed in the whole angular range using code FRESCO [30] , while the following parameters were fixed $r_c$=1.28 fm ,$r_V$=1.245 fm, $r_D$=1.7fm.Optical potential parameters used in calculations are listed in table 3.

Table 3: Optical Potentials Parameters for α-particles elastic scattering on $^{7}Li$ at different energies

| $J_W$ MeV.fm$^3$ | $J_R$ MeV.fm3 | $a_D$ fm | $W_D$ MeV | $a_V$ fm | $V_o$ MeV | $E_\alpha$ MeV |
|---|---|---|---|---|---|---|
| 54.224 | 523.878 | 0.545 | 4.798 | 0.879 | 109.643 | 3.69 |
| 94.055 | 561.404 | 0.50 | 9.194 | 0.836 | 124.518 | 5 |
| 65.55 | 357.962 | 0.789 | 3.672 | 0.876 | 75.222 | 18 |
| 83.175 | 333.044 | 0.903 | 3.883 | 0.673 | 91.964 | 26 |
| 65.689 | 374.875 | 0.750 | 3.931 | 0.756 | 92.627 | 29.4 |

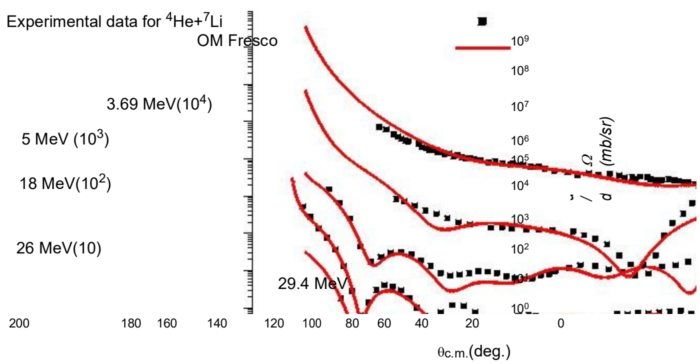



Fig. 7 comparison between calculated and experimental angular distributions for α-particles elastically scattered from $^{7}Li$ at different energies.

As expected the relation between volume integral of both real potential depth $V_o$ and imaginary potential depth $W_D$ with α-particles energy $E_\alpha$ is linear as shown in figures 8 and 9 respectively. The strength of parameters listed in table 3 can be represented by the following equations:

$$V_o = 115.994 - 1.06\, E_\alpha ,$$

$$J_R(E) = 555.419 - 7.752\, E_\alpha$$

$$W_D = 7.052 - 0.1212\, E_\alpha ,$$

$$J_W(E) = 53.509 + 0.721\, E_\alpha$$

The comparison of calculated angular distributions with experimental ones for some cases is shown in Fig. 7. As it is seen there is a good agreement between theory and experiment in the *whole angular range* at all energies.

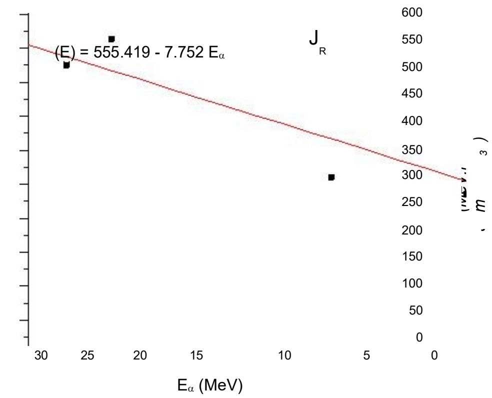
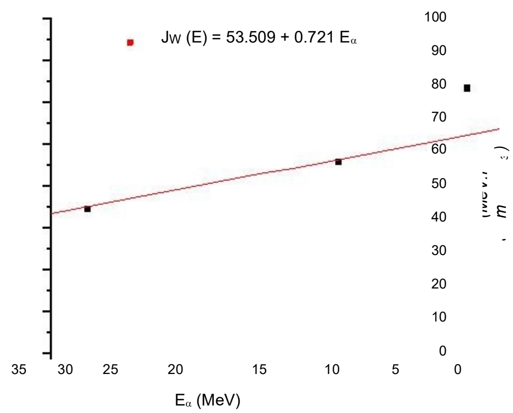



Fig. 8 relation between volume integral of real potential depth and α-particles energy




. Fig. 9 relation between volume integral of imaginary potential depth and α-particles energy.

Search of optimal optical parameters (OP) were carried out with the use of ECIS88 Code by means of the minimization of the **χ2/N** value:

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{\sigma_{iT}(\Theta_i) - \sigma_{iE}(\Theta_i)}{(\Delta\sigma_i)_E}\right]^2 \quad (9)$$

Where $\sigma_{iT}(\Theta_i)$ and $\sigma_{iE}(\Theta_i)$ — calculated and experimental values of differential cross sections for the given angle $(\Theta_i)$, respectively; $(\Delta\sigma_i)_E$ - the experimental error; N= (P-F), P- the number of measured points and F-number of varying parameters which equal 4 in this paper.

**Conclusion:**

An analysis of protons, deuterons and α-particles elastically scattered by $^{7}Li$ in a wide energy range has been performed within the framework of the standard optical model. Optical potential parameters were achieved on the base of best agreement between theoretical and experimental angular distribution with physical meaning. Linear relationship between volume integral of both real potential $J_R$ and imaginary potential $J_W$ with incident particle energies have been obtained. Good agreement between theory and experiment in the whole angular range for protons and α-particles at all energies has been obtained while for deuterons in forward angular range.

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**الولخص العشبً**

**ححلٍل الٌْوصج البصشي للخشخج الوشى للبشّحًى ّالزًّخشّى ّخسٍواث ألفا علًَّْاة الٍِِلٍْم-7 فًَ هذي ّاسع هي الطاقاث**

**أحوذ حواد عاهش – أحوذ عواس – ابشاٍُن ابشاٍُن بٌُذق – فخحً الحسٌٌٍّ**

**قسن الفٍِضٌّاء – كلٍت العلْم – خاهعت طٌَطا – هصش**

**فًَُزا البحث قوٌّا باسخخذام الٌْوٌرج الضْئً باعخباسٍ هي أشٍِش الٌْوارج الٌٌٌّْت الخًَّدحج فًَّصف الخفاعالث الٌٌٌّّْتالوباششة لخحلٍل السخطاسة الوشًت للدسٍواث الوشحٌْت هثل البشحًٌّّ الزٌّخشًّ ّكزلك خسٍواث ألفا علًَّْاة الٍِِلٍْم-7**

**ٌّرلك باسخخذام أكٌْاد حاسٌْبٍت للٌْوٌرج البصشيٌّ رلك لحل هعادلت شٌّشّدٌدش ٌّاسخٌْباط خِذ الخفاعل بٍي االٌَّْت الوخفاعلت ٌّالزي هي خاللَ أهكي اسخٌْباط الخٌْصٌّع الضأّي للوقطع الوسخعشض الخفاضلً لالسخطاسة الوشًت لِز الخفاعالث، ٌّحوج هقاسًت الٌْخائح الٌْظشٌّت بوا ٌّقابلِا هي القٍِاساث العولٍِت الوٌْشٌْسة فًَ الزٌّسّاث العولٍِت. ٌّلقذ حبٍِ هيٌُ ِز الوقاسًت ٌّبشكل عام أى الٌْوٌرج الضْئً قادس على حقذٌٌْن حٌْصٍِف ٌّاحخ لٌٌِْاحٌُِز الخفاعالث.**