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Application of Multilevel Models in Analysis of Factors Affecting Academic Performance of Graduated Students in Higher Education in Saudi Arabia: A Case Study

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Abstract: Multilevel modeling provides a powerful framework for exploring how average relationships vary across hierarchical structures. The major objective of this paper is to use the multilevel models to illustrate how differences among universities in their organizational characteristics and types might influence the distribution of academic achievements of graduated students within and among universities .In order to explore these differences a two level intercepts and slopes as outcomes model will be developed and applied using student level and university level data .The proposed model will be used to analyze graduated students performance at departments of business administration in eight universities in Saudi Arabia.

Key Wards: Multilevel Modeling; Empirical Bayes Estimates; Random Coefficient model; Intercepts and Slopes as Outcomes Model.

1-Introduction

Over the past 20 years ,fitting multilevel models to data with a hierarchical or nested structure has become increasingly common for statisticians in many applications(e.g. Bryk and Raudenbush,1992 ;Goldstien,1999; Dunson ,2003; Dominici et al ,2004;Bailey & Hewson ,2004; Steele, F. ,2008 and Browne et al.,2009).The main purpose of fitting such models is to partition the variation in a response variable as a function of levels in the hierarchy and relate this variability to the descriptions of the data structure . In education , for example , multilevel or hierarchical modeling can be used to calculate the proportion of variation in an observation that is explained by the variability between students , classes and schools in a 3 level nested structure (Bock ,2003). Other examples of hierarchically structured data, where data in one cluster are more similar than data across clusters, are: repeated observations on a subject over time; samples of subjects in different geographical, political, cultural or administrative units; and test results of students in different schools.

In educational research , single level models are the traditional linear methods which measured relationships among student level variables ,but ignored the actual ways in which student were allocated to schools (or universities) and the influence of the school factors upon the students . These types of analyses result in two problems . The first is that the resulting statistical conclusions are often biased and overly optimistic. Traditional linear models offer a simple view of a complex situation that is statistically weak in its interpretation. This leads to the second concern that these models generally assume the same effects across groups , which fails to explicitly incorporate schools in the statistical model, so that very little can be said about the influence of schools on student level variables (Goldstein ,2001). However , multilevel analysis accounts for these clusters .The multilevel analysis not only estimates the model coefficients at each level , but also predict the random effects associated with each sampling unit at each level . This can be empirically verified if the variance is partitioned so that the researcher can determine what proportion of variance is attributed to the individual and which proportions is attributed to the group (Heck &Thomas ,2000) . It is hypothesized that if effects do differ across groups , differences can be explained with multilevel modeling. Individual student grouped together in a school or classroom share common experiences , which make their results more homogeneous than those of a random sample of students drawn from the population of all schools . This greater homogeneity is naturally modeled by positive within school correlation among student results in same school (McCaffrey et al ,2004).

Analysis that explicitly models the manner in which students are grouped within school has several advantages(Osborne,2000) .Firstly, it enables the researcher to obtain improved estimation of individual effects at each level of analysis .Secondly ,by modeling cross level effects (how variables measured at one level affect relations occurring at another) using the clustering of information it provides correct standard errors , confidence intervals and significance tests , which will generally be more conservative than the traditional regression analysis . Thirdly , multilevel analysis allows for an investigation of how student performance is influenced at the individual level as well as the school level. Goldstein & Thomas (1996)briefly discussed the differences of using individual subject areas as the criterion compared with using total examination scores. They argued that by using total examination scores ,fine distinctions and detailed rank orderings were statistically invalid and the underlying relationships could be masked .

Raudenbush &Bryk(2002)pointed out that , there are differentiating effects of the distribution of academic achievement in schools from different sectors and additional research implying that the regional location of schools affects academic achievement (Thrupp,2001). Browne et al .(2005) considered multilevel models where the level 1 variance depends on predictor variables .They examined two cases using a data set from educational research ;in the first case the variance at level 1 of a test score depends on continuous intake score predictor , and in the second case the variance is assumed to differ according to gender . Also, they used two simulation experiments to compare two maximum likelihood methods based on iterative generalized least squares with two MCMC methods.

Watkins A.(2008) discussed whether social conditions in a school's attendance area affect the likelihood of students bringing weapons to school. He pointed out that ,the level of economic disadvantage, residential mobility, and violent crime in a school's attendance area are unrelated to student-level weapon carrying. The school effectiveness research paradigm describes educational research concerned with exploring differences within and between schools. Researches in this area focus on obtaining knowledge about relationships between explanatory and outcome variables using a variety of statistical models(Goldstein,2003). Often ,the outcome variables are the examination results, attendance, etc., the explanatory variables are related to the characteristic of the student intake, like demographic and socio-economic background, sex, ability, etc, or to the school and teacher process, like class size, student/staff ratio, resources, etc. In order to reach a conclusion about the importance of explanatory variables and the estimated effectiveness of individual schools(or universities), the general framework for analysis are multilevel models with outcome regressed on student intake and school (or university) variables. The use of such models is particularly appropriate because of the hierarchical structure of schooling systems(Tate ,2004). We refer to a hierarchy as consisting of units grouped at different levels. Thus, students may be the level 1 units grouped within schools (or universities) that are the level 2 units.

The objective of this paper is to introduce a multi-level model designed to be used in understanding the variation among graduated students performance(students graduated in academic year, 2008/2009) at business administration departments in eight universities in Saudi Arabia (namely: King Faisal , Prince Mohamed Bin Fahd,. King Khalid ,Taif , Taibah, Prince Sultan Bin Abdel Aziz universities, Imam University at Alhasa and King Fahd University of Petroleum & Minerals) based on several student(level -1) and university(level-2) variables .

This paper is organized as follows: the proposed multi-level model is introduced in Section 2. In section 3, the estimation of the model parameters using iterated generalized least squares(IGLS) and empirical Bayes methods will presented . This is followed by a case study in section 4. Finally , the conclusion of the paper will be presented in section 5.

2-Model Design

The first level of the proposed model (student level) examines the relationship between overall academic achievement and four parameters: an intercept and three regression coefficients. Three predictor variables were included in level one of the model:(1) gender status(X_1) , (2)Average years of parents education(X_2) and (3) student mark in secondary school(X_3). The choice on where to locate these predictor variables is very important .In the simple model , the intercept β_{0j} is defined as the expected outcome for a student attending university j who has a value of zero on X_{ij} If this is not meaningful , then the researcher can transform X_{ij} to make the intercept β_{0j} more meaningful by group – mean centering $(X_{ij} - \bar{X}_{1.j})$, grand – mean centering $(X_{ij} - \bar{X}_{1..})$ or locating it on another metric that makes sense to the researcher. In education studies, Raudenbush & Bryk (2002) and others recommend group – mean centered for all level -1 variables and grand – mean centered for all level -2 variables. When variables are group – mean centered , the interpretation of the within group slopes is the expected outcome for a student whose value is equal to the university average on all predictors . Alternately , when variables are grand - mean centered , the intercept is the expected outcome for a student whose value on the predictor is equal to the grand mean of the total sample .

Level one model :

$$Y_{ij}= \beta_{0j} + \beta_{1j}(X_{1ij} - \bar{X}_{1.j}) + \beta_{2j}(X_{2ij} - \bar{X}_{2.j}) + \beta_{3j}(X_{3ij} - \bar{X}_{3.j}) + e_{ij} \qquad (1)$$

More succinctly,

$$Y_{ij}= \beta_{0j} + \textstyle\sum_{j=1}^{3} \beta_{qj}(X_{qij} - \bar{X}_{q.j}) + e_{ij} \quad ,$$

$$= \beta_{0j} + \textstyle\sum_{j=1}^{3} \beta_{qj} X^{*}_{qij} + e_{ij} \quad ,i=1,2,\ldots, n_j ,\textstyle\sum_j n_j = n \text{ , j=1,2,...,J, q=1,2,3} \qquad (2)$$

where Y_{ij} is the graduation mark of student i in university j which is asşumed to be normally distributed y ~ (Xβ , V); and the errors e_{ij} are assumed to be normally distributed with mean 0 and variance σ^2 .

β_{0j} is the mean graduation marks of all students in university j,

β_{1j} is the average effect of gender status on graduation marks in university j,

β_{2j} is average effect of years of parent education factor on graduation marks in university j,

β_{3j} is average effect of student mark in secondary school on graduation marks in university j. Within level one model, each university can have a different average achievement (i.e ,intercept) and a different impact of three variables on average graduation marks (i.e., slope).The second level of the model(university level) examines the effects of three university level variables on level – one relationships.

Level two model,

$$\beta_{qj}=\lambda_{q0} + \textstyle\sum_{j=1}^{3} \lambda_{qk} \; (W_{kj} - \bar{W}_{k.}) + u_{qj}$$

$$=\lambda_{q0} + \textstyle\sum_{j=1}^{3} \lambda_{qk} \; W^{*}_{kj} + u_{qj} \qquad (3)$$

where q=0,1,2,3 , n_j ,k = 1,2,3

W_{1j} is student staff ratio per university j,

W_{2j} is the graduation rate of department of business administration in university j,

W_{3j} is the type of university j (0 =public and 1 = private),

λ_{0q} are level -2 intercept/slopes to model β_{0j} ,

λ_{1q} are level -2 intercept/slopes to model β_{1j} ,

λ_{2q} are level -2 intercept/slopes to model β_{2j},

λ_{3q} are level -2 intercept/slopes to model β_{3j},

u_{qj} are level -2 random effects. The variances and covariance matrix of the level - 2 random effects is given by:

$$\Omega = \begin{bmatrix} \theta_{00} & \\ \theta_{10} & \theta_{11} \end{bmatrix} \quad (4)$$

At level -2(university) , the (θ) represents the variances of the intercepts and slopes and covariance between them. Also , it assumed that the university level residuals follow multivariate normal distribution with variances (θ_{00},θ_{11}) and covariance (θ_{10}).This dependency violates the assumption in ordinary regression of independent errors across observations, but can be handled using a multilevel modeling (Heck & Thomas ,2000). Another consideration of multilevel modeling is that, missing data at level one can be handled but there cannot be missing data at level – two (university).

The mixed model is ,

$$Y_{ij} = \lambda_{00} + \sum_{j=1}^{3} \lambda_{q0} X_{qij}^{*} + \sum_{j=1}^{3} \lambda_{0q} W_{kj}^{*}$$
$$+ \sum_{j=1}^{9} \lambda_{qk} X_{qij}^{*} W_{kj}^{*} + \sum_{j=1}^{3} u_{1j} X_{qij}^{*} + u_{0j} + e_{ij} \quad (5)$$

The part of the equation ,$\sum_{j=1}^{9} \lambda_{qk} X_{qij}^{*} W_{kj}^{*}$, represents the cross – level interaction between level one X_{qij} and level two W_{kj} variables .The error term $u_{1j} X_{qij}^{*} + u_{0j} + e_{ij}$,accommodates the relationship between u_{1j} and u_{0j} which are common to every level one observation within each level two unit. We must notice that , there are some theoretical assumptions of such model require consideration. The first assumption is that at level one , errors are normally distributed and are homogeneous , that is , $\text{Var}(e_{ij}) = \sigma^2$

Raudenbush and Bryk (2002) suggested that statistical evidence recommends that the estimation of the fixed effects , and their standard errors will be robust to violations of this assumption . In education research , it is commonly to called model (5) as intercepts and slopes as outcomes model.

3-Parameters Estimation of the Model

Model (5) requires the estimation of fixed coefficients (λ_{qk}) and variances and covariance which referred as random parameters. There are several statistical methods to carry out estimation in multilevel models.. For example Markov Chain Monte Carlo(MCMC) methods (Browne & Goldstein ,2010)and two standard methods are called IGLS (iterated generalized least squares) and RIGLS (residual, or restricted, IGLS) in MLwiN package(This package can be download from http://www.cmm.bristol.ac.uk/MLwiN). The IGLS method yields maximum likelihood estimates. IGLS is an iterative procedure based on estimating the random and fixed parts of the multilevel model alternately assuming the estimates for the other part are correct. This involves iterating between two GLS model fitting steps until the estimates converge to ML point estimates(Goldstein ,2003)). Our goal is to find the beast estimator of $\hat{\beta}_{qj}$ in model (3) . In order to increase the accuracy of estimating β_{qj} in an intercepts and slopes as outcomes model , empirical Bayes estimators can be computed that shrink the estimates toward predicted values of β_{qj} .Empirical Bayes estimates are more beneficial than OLS regression or ANCOVA , because unlike OLS it take into account group membership even when the number of groups(i.e. universities) are large , and produces relatively stable estimates even when sample sizes per university are small or moderate(Bauer, 2003) .ANCOVA does take group membership into consideration, but this tends to be impractical when the number of universities in the sample is large .After computing the estimates $\hat{\beta}_{qj}$ using IGLS method in MLwiN program, the empirical Bayes estimates can be calculated as follows (Raudenbush & Bryk ,2002):

$$\beta^{*}_{qj} = \Psi_j \hat{\beta}_{qj} + (I - \Psi_j) W_{kj} \hat{\lambda}_{qk} \qquad (6)$$

Where $\Psi_j = \Omega/(\Omega + V_j)$ is the ratio of the parameter dispersion matrix for β_{qj} (i.e Ω) relative to the total dispersion matrix for $\hat{\beta}_{qj}$,which contains error and parameter distribution (i.e. $\Omega + V_j$) . Raudenbush & Bryk (2002) suggested that, Ψ_j could be considered a multivariate reliability matrix.

4-A Case Study

The data for the current study , including some student and university data for the 2008/2009 academic year. The university level data used in this study are eight universities (namely: King Faisal , Prince Mohamed Bin Fahd,. King Khalid ,Taif , Taibah, Prince Sultan Bin Abdel Aziz universities ,Imam University at Alhasa and King Fahd University of Petroleum & Minerals). Two of them (Prince Mohamed Bin Fahd and Prince Sultan Bin Abdel Aziz universities) are private and the others are public universities. In addition to university level data, the study also incorporates student level information .So, the study sample includes (492) students for 8 departments of business administration in eight universities. At student level (level -1) three variables were included : (1) gender status is a binary variable coded , 1= male and 0 = female , (2) average years of parents education ,and (3) student mark in secondary school. At university level (level -2) three variables were included:(1) student/ staff ratio per university , (2) graduation rate of the university and (3) type of the university(coded ,1=private and 0=public). Three aspects of multilevel models will be examined: one way ANOVA with random effects model , a random coefficient model and intercepts & slopes as outcomes model.

(4-1)One way ANOVA with Random Effects Model

The first step in the present analysis was to use one way ANOVA with random effects model to fit our dataset .This model provides useful information about how much variation in the outcome variable lies within and between universities. In one way ANOVA model(with no predictors), the equations are:

$$Y_{ij}= \beta_{0j} + e_{ij} \quad , \quad e_{ij} \sim N(0, \sigma^2) \tag{7}$$

Where , $\beta_{0j}= \lambda_{00} + u_{0j}$, $u_{0j} \sim N(0, \theta_{00})$, $i=1,....492$, $j=1,...,8$

The average university mean (intercept) was estimated as $\lambda_{00,} = 1127.35$. The level - 1 variance was estimated at $\hat{\sigma}^2 = 476.19$ and the variance among the universities means was estimated at $\theta_{00} = 73.25$ .In multilevel modeling it is commonly to use the intraclass correlation coefficient(ICC) . The ICC tells us how much variance in(Y_{ij}) is accounted for by variations among level -2 units. This statistic is calculated as(Goldstein , 2003):

$$ICC = \widehat{\theta_{00}}/(\widehat{\theta_{00}} + \hat{\sigma}^2) \tag{8}$$

From equation (8) the ICC is 13.33%,which means that about 13.33% of variance in the outcome variable can be attributed to differences between universities(level -2) and the remaining to differences between students(level -1).

(4-2)Random Coefficient Model

The next model designed was the random coefficient model to represent the distribution of overall achievement in each of the eight universities. In this model the overall graduation mark for student i in university $j(Y_{ij})$ was regressed on gender status , average years of parents education ,and student mark in secondary school . Each university distribution of achievement was explained in terms of four parameters : an intercept and three regression coefficient as follows.

$$Y_{ij}= \beta_{0j} + \sum_{j=1}^{3} \beta_{qj} X^{*}_{qij} + e_{ij} \tag{9}$$

Where, $\beta_{0j}= \lambda_{00} + u_{0j}$,

$\beta_{1j}= \lambda_{01} + u_{1j}$,

$\beta_{2j}= \lambda_{02} + u_{2j}$,

$\beta_{3j}= \lambda_{03} + u_{3j}$,

In random coefficient model , each university have a different average achievementβ_{0j} (i.e. intercept) and a different impact of three variables on average academic achievement β_{qj} (i.e. slope).The results from the random coefficient regression model are reported in table (1).

Table(1): Results from the Random Coefficient Model

Fixed effects	**Coefficient**	**S.E.**	**T . ratio**	**P - value**
Intercept , λ_{00}	1123.29	0.31	3618.51	0.001
Gender , λ_{01}	-2.09	0.11	-18.65	0.002
Average years of parents education, λ_{02}	0.69	0.09	6.55	0.001
Secondary school mark , λ_{03}	6.51	0.23	27.48	0.000
Random effects	**Variance Component**	**df**	χ^2	**P- value**
Intercept , u_{0j}	147.24	7	781.07	0.000
Gender , u_{1j}	0.69	7	25.12	0.002

Average years of parents education, u_{2j}	0.88	7	48.17	0.000
Secondary school mark, u_{3j}	0.73	7	68.36	0.000
Level -1 effects, e_{ij}	223.05			

From table (1) , we can see that the average university mean achievement was estimated as (λ_{00} =1123.29) and the average overall secondary school mark differentiation ($\lambda_{03,}$=6.51) and average years of parents education (λ_{02} =0.69) were positively related to the average overall graduation mark . In contrast ,gender status (λ_{01} =-2.09) was negatively related to the average overall graduation mark. This implies that in the average, university male students with similar secondary school mark and average years of parents education scored 2.09 points ,lower on the graduation mark compared with female students. The level -1 variance was reduced from 476.19 in one way ANOVA model to 223.58 in random coefficient model (after taking into account these three level -1 variables). The proportion of variance in level-1 explained by random coefficient model can be calculated as follows(Bock ,2003):

$$\text{Proportion of variance explained} = \frac{\hat{\sigma}^2(ANOVA) - \hat{\sigma}^2(\text{random coefficient mode})}{\hat{\sigma}^2(ANOVA)} \qquad (10)$$

Appling equation (10),The proportion of variance explained for level -1(students) was 53.05%. Also from the reported T.ratios and χ^2 values in table (1) we can see that ,each of the level -1 variables and level -2 variances were statistically significant. The estimated level -2 variances for the random coefficient model provide empirical evidence about variability in the relationship between outcome variable and level-1 variables across universities.

(4-3)Intercepts and Slopes as Outcomes Model

The results from the random coefficient model indicated that, each of the level -1 predictors had statistically significant relationship with the outcome variable(Y_{ij}).Further ,there was statistical evidence provided by the χ^2 test to indicate that there was sufficient variability among universities .So , it is important to build an explanatory model to account for this variability .This model is called the intercepts and slopes as outcomes model, which includes predictors at both levels 1 and 2 in the model . All three level -2 variables as defined in section 2 were fitted to this model as indicated in equation(5). We must notice that, some of the estimated effects were trivially small ,so the final model was estimated excluding (λ_{12}, λ_{13}, λ_{23}). The results for the reduced model are reported in table (2).

Table(2):Results from the Intercepts and Slopes as Outcomes Model

Fixed effects	Coefficient	S.E.	T . ratio	P - value
Intercept model , β_{oj}				
Intercept, λ_{00}	1107.52	0.28	381.28	0.000
Student/Staff ratio, λ_{01}	0.74	0.15	8.33	0.001
Graduation rate, λ_{02}	0.13	0.06	2.06	0.000
University type, λ_{03}	-0.04	0. 07	-0.49	0.000
Secondary school mark model, β_{1j}				
intercept, λ_{10}	6.08	0.25	23.69	0.002
Student/Staff ratio, λ_{11}	0.53	0.11	4.33	0.000
Gender status model, β_{2j}				
Intercept , λ_{20}	-1.74	0.15	-11.07	0.001
Graduation rate, λ_{21}	0.32	0.11	2.39	0.000
Average years of parents education model, β_{3j}				
Intercept λ_{30}	0.78	0.13	5.88	0.023
Student/Staff ratio, λ_{31}	0.12	0.02	5.37	0.001
University type, λ_{32}	0.05	0.01	4.65	0.000
Graduation rate, λ_{33}	0.03	0.01	2.91	0.000
Random effects	**Variance component**	**df**		**P-value**
Intercept , u_{0j}	83.16	4	235.47	0.000
Secondary school mark, u_{1j}	0.49	4	53.08	.001

Gender, u_{2j}	0.37	4	21.69	0.000
Average years of parents education, u_{3j}	0.52	4	33.57	0.000
Level -1 residuals, e_{ij}	223.02			

From table (2) we can see that, Student/Staff is positively related to university mean achievement (λ_{01} =0.74 , t =8.33) and so was the graduation rate (λ_{02} =0.13 , t =2.06).Mean achievement was slightly lower private universities (λ_{03} =0.04 , t =0.49) than public universities . Secondary school mark : when the graduation examination mark was adjusted for the average secondary school mark , there was a moderate positive relationship between secondary school mark and graduation examination mark for different students within a university differs significantly depending on Student/Staff ratios (λ_{11} =0.53 , t =4.33).Gender status :there was a negative relationship between gender and the intercept (λ_{20} = -1.74 , t = -11.07).This implies that , in average , the male students typically score quite a bit lower ; about 1.74 points lower the female students . Also , there is a positive relationship between gender status and graduation score .Average years of parents education : The impact of the average years of parents education on the average graduation examination score was positive . This implies that ,as the number of parents education increases , the graduation examination mark will be increased .Also ,there is a positive relationship between the average years of parents education and the Student/Staff ratio (λ_{31} =0.12 , t =5.37), university type (λ_{32} =0.05 , t =4.65) and the graduation rate (λ_{33} =0.03 , t =2.91). It was positively to these variables with a stronger relationship for Student/Staff ratio and slightly weaker for university type and graduation rate. From table (2)we can see that , the level -1 variance estimate $\hat{\sigma}^2$ was the same as the random coefficient model. This was expected because the level -1 variables did not change in the two models. The variance of intercepts was(θ_{00} = 83.16) in the intercepts and slopes as outcomes model while it was(θ_{00} = 147.24) in random coefficient model .So a large reduction occurred in estimate of(θ_{00}) due to the inclusion of university variables in the intercepts and slopes as outcomes model .Also the significant of(θ_{00}) implies that, there is still some variance among universities in the outcome variable that has not accounted for by level -1 and level -2 predictors.

5- Conclusion

A reasonable amount of the variance between universities and students on the outcomes(overall graduation marks) was explained for our case study by building three aspects of multilevel models : one way ANOVA with random effects, random coefficient regression and intercepts & slopes as outcomes. However , even with the inclusion of student and university variables , significant differences between outcomes remain unexplained. The results of these models demonstrated that , the student and university variables included in the models have statistically significant relationships with the outcome variable.

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