كلية التجارة جامعة طنطا

المجلة العلمية

التجارة والتمويل

العدد الأول ١٩٩٩

تحليل نموذج التعادل باستخدام آلية ادنى التكاليف

د. غالب عوض صالح
قسم العلوم المالية والمصرفية
كلية الاقتصاد والعلوم الادارية
جامعة الزيتونة الاردنية
عمان - الاردن

د. عبد الحفيظ بلعربي
قسم العلوم المالية والمصرفية
كلية الاقتصاد والعلوم الادارية
جامعة الزيتونة الاردنية
عمان - الاردن

ملخص:

تعتبر طريقة تحليل التعادل (او طريقة الحجم-التكلفة-الربح) لمنشأة وسيلة لرجال الادارة في صنع القرارات المتعلقة بطرح منتوجات جديدة او تحديد مستوى الانتاج او سياسة التسعير او اختيار بدائل تقنيات الانتاج المختلفة. لذلك يلزم على رجال ادارة المنشأة ان يعرفوا طبيعة علاقة منحنيات التكاليف الكلية والايراد الكلي والربح ؛ لأن طريقة تحليل التعادل تأخذ بعين الاعتبار طبيعة علاقة المنحنيات المذكورة لتوفر لرجال الادارة توجيهات تساعد في صنع قراراتهم على الرغم من وجود جدل حول هذه المسألة بين الاقتصاديين والمحاسبين.

وتهدف هذه الدراسة الى محاولة التوفيق بين وجهتي نظر الاقتصاديين والمحاسبين من خلال تطوير طريقة تحليل التعادل اعتماداً على صيغة "نظرية الازدواجية" للإنتاج والتكاليف .

An Explicit Treatment of C-V-P Models Based on Cost Minimization

Dr.Ghaleb Awad Saleh
Department of Finance and Banking
College of Economics and Business
Al-Zaytoonah University of Jordan
Amman - Jordan

Dr . Abdelhafid Belarbi
Department of Finance and Banking
College of Economics and Business
Al-Zaytoonah University of Jordan
Amman - Jordan

Abstract:

Break-even analysis remains a frequent tool in making operating decisions, involving the introduction of new products, the volume of production, the pricing of production and the selection of alternative production processes. Management must understand the relationship between cost, revenues and profit functions. Break-even analysis takes this interrelationship into account to provide management with useful guidelines for decision making.

However, there remains a divergence between accountancy and economy theorists concerning the behavior of the involved functions.

This paper aims at reconciling such divergence by developing break-even analysis or cost-volume-profit (C-V-P) analysis based on introducing, explicitly, the duality formulation in cost and production theories into the Cost-Volume-Profit (C-V-P) analysis through developing total cost and total revenue functions to fit different models of break-even analysis.

The results of the models lessen the existence of such divergence for the sake of C-V-P appplication by management decision- makers.

An Explicit Treatment of C-V-P Models Based on Cost Minimization

Dr.Ghaleb Awad Saleh
Department of Finance and Banking
College of Economics and Business
Al-Zaytoonah University of Jordan
Amman - Jordan

Dr . Abdelhafid Belarbi
Department of Finance and Banking
College of Economics and Business
Al-Zaytoonah University of Jordan
Amman - Jordan

Introduction:

The analysis of cost-volume-profit (C-V-P) models remains a frequent tool used by management decisions-maker in the context of making choices among alternative profit-planning decisions whether under the conditions of certainty or uncertainty .

There exist, nonetheless , some limitations in the traditional C-V-P model due to some of the model's inherent drawbacks , which critically limit the scope of its usefulness . The deficiency lies in not recognizing the difference and the relationships between sales, demand and production variables. Treatment of the C-V-P model under uncertainty state solved these problems.

The break-even analysis examines the relationship between output , profits and costs. It, also, gives sensitivity analysis with respect to changes in pricing and cost structure that contributes to greater profitability under both operating and financial leverage.

Break-even analysis provides a statistic summary of price, cost (variable and Fixed) and profitability level associated with different levels of activity .

However, there remains a divergence between economy and accountancy theorists concerning the issue of the behavior of the involved functions, mainly, total revenue and total cost functions.

This paper aims at seeking reconciliation between the two views by explicitly introducing the "duality formulation" pertaining to production and cost theory.

The paper consists of an introduction, three sections and a conclusion. Section (1) reviews the literature mainly pertaining to the assumptions of C-V-P analysis. Section (2) presents the duality theory with respect to C-V-P analysis. Section (3)treats the development of different C-V-P models by explicitly introducing the duality theory in the C-V-P analysis. The results of linear and nonlinear models, as well as, Baumol's model are discussed. Finally, the conclusion stipulates the main results of the paper .

1-Literature Review

The Break-even analysis is a simplification of the usual short-run analysis in economics with the following restricted assumptions:

1- linearity of the involved curves is maintained through the entire analysis.

2- the implicit assumptions of perfect competition and certainty state are present through the analysis, that is, maitaining the assumption of constant product mix and factor of production mix over time.

Reviewing the literature pertaining to these assumptions leads to framework the inherent drawbacks of the CVP model and how we can extend or remodify the model to overcome these drawbacks.

With respect to the linearity assumption, although it makes the analysis simple and straightforward, it diminishes the usefulness of the model by too much abstracting away from reality. For this reason, Vickers (1960) developed nonlinear model to conform to the behavior of cost and revenue functions . The results of the modeling conveys multiple break-even points for the firm in any one period, in addition to an equilibrium condition pertaining to a certain position of profit level.

Whereas the models of Goggans (1965), Givens (1966), Morrison and Kaczka (1969), by incorporating the nonlinearity assumption into the formal C-V-P models concluded that the linearity assumption in the CVP model is only appropriate for a mid-range of activities for which the break-even parameters are approximately linear.

With respect to assumption (2) the formal C-V-P model implies that the firm is strictly confined to a given configuration of market structure and technology-mainly, perfect competition situation. Thus, market structure should be explicitly incorporated into modeling C-V-P analysis.

From a practical standpoint, the use of cost-volume- profit analysis by business decision-makers ignores the economist generalized theorems of cost and revenue behavior. This refusal has led to the marriage between business decisions-makers (Accountants) and economists, a marriage of convenience. Such a situation is illuminated by the pioneer work of Vickers (1960) :

a) The components of the break-even analysis charts as used in industry are in need of reinterpretation to bring them more closely into line with some significant suggestions of economic theory, and , at the same time, to clarify their true empirical significance.
b) In the new analysis, a key role should be played by the cost accountant, and the advanced techniques of cost accounting should be used as the bridge between theoretical concepts and real world analysis at this point.
c) Economic analysis should, now, take up new lines of development based, principally, on quantitative studies under the auspices of firms and industrial groups.

Joel Dean strongly stated the opinion that : "Break-even analysis... provides an important bridge between business behavior and the theory of the firm".

During the 1930's , while economists were developing the theories of the firm on the assumptions of curvi-linear cost and revenue functions, management decision-makers heavily relied on the linearity assumption of such curves.

By explicitly introducing the duality formulation in production and cost theory into the C-V-P analysis, reconciliation between economy and accountancy theorists is sought, since management decision makers are optimum choice seekers.

2- Duality Theory

One of the conventional approaches to cost and production theory is the use of duality theory. The approach stipulates that the optimum choice of a firm's demand inputs can be analyzed, not only as the problem of choosing the lowest isocost line tangent to the production isoquant, but also, as the problem of choosing the highest production isoquant tangent to a given isocost line. The result of duality theory confirms that the necessary conditions for an output firm's maximization, given a cost constraint, are identical necessary conditions for a firm's cost minimization subject to a given output level constraint. Under the duality approach, the firm chooses the inputs combinations for which the marginal rate of technical substitution between inputs is equal to the ratio of inputs rental rates.

In the short-run analysis, there is only one point which fulfills such conditions. The duality theory guarantees that this point of equilibrium yields the maximum output level of a firm given a cost constraint, and the same point of equlibrium yields the minimum cost of a firm for producing that associated maximum output.

For the sake of break-even analysis, and for the fact that a business firm is an optimum seeker, and taking into consideration such optimum choice by the firm guarranteed by the duality theory, we explicitly derive the minimum total cost function of the firm associated with the highest possible output level, and then derive the repercussions of such formulation on the results of the C-V-P analysis. The paper develops three different models: a linear model, a nonlinear model and the well-known Baumol model.

3- The Models :

Case 1 :The Linear Model

Consider the following production function : $Q = L\,\overline{K}$ .... (1)
where : Q = output, L = Labor factor, $\overline{K}$ = Fixed capital factor

Equation (1) can be rewritten as :

$$L = \frac{Q}{\overline{K}} \qquad \text{...(2)}$$

If (w) is the wage rate of labor and (r) is the cost of capital, then the total cost function (TC) can be written as :

$$TC = wL + r\overline{K} \qquad \text{..................................(3)}$$

substituting equation (2) into equation (3)

$$TC = w\frac{Q}{K} + r\,\overline{K} \qquad \text{...........................(4)}$$

since total revenue (TR) is defined as :
TR = PQ
where P: output price per unit
Q: output level

Cost-volume-profit analysis stipulates that at break-even point total revenue (TR) equals total cost (TC) then:

TR = TC

$$PQ = w\frac{Q}{\overline{K}} + r\overline{K} \qquad \text{............................ (5)}$$

Rearranging terms in equation (5) gives :

$$\left(p - \frac{w}{\bar{\bar{K}}}\right) Q = r\,\bar{K}$$

Hence, the break-even point of output level, denoted by ($Q_{e^{\bullet}}$), is given as:

$$Q_{e^{\bullet}} = \frac{r\,\bar{K}}{p - \frac{w}{\bar{\bar{K}}}} \quad \text{.........................} \quad (6)$$

Equation (6) coincides with the well-known, conventional result of the break-even analysis, where, the underlying assumption is that total variable cost must be a constant percentage of total revenue.

This condition is not different from assuming a constant price and constant average variable cost, as it is stipulated by the linearity assumption of CVP model . That is to say, if the products' average variable costs represent different percentage of each product's price (ie , the case of multi-product plant), where the product price and the unit variable costs may not be available, hence, a difficultly arises to establish the break-even point of the plant; then the total variable cost has to bear a constant relationship to total revenue changes.

Yet, the results stipulate that given revenue alternative possibilities, the level of production and sales, at which an enterprise hits its break-even point , will depend as much on the variable cost ratio, as on the level of fixed cost.

Case 2 : Case of Curvilinearity of Total Cost and Total Revenue Functions

Consider the general Cobb-Douglass production function ($Q=AL^{\propto}K^{\beta}$), which is widly used by economists to estimate long-run cost functions, where engineering data are not available or insufficient. The cost function for the general Cobb-Douglass production function is useful in a situation in which the exact relationship between capital and labor is not known, but might be empirically estimated from the data. Moreover, the estimates of ($\propto$) and (β) provide a direct estimate of the returns to scale of the underlying production process.

For the sake of break-even analysis, we proceed to find the conditional inputs demands and the long-run cost function for the general Cobb-Douglass function by opting a cost-minimization setting, and then, applying the results to break-even analysis with a minimum total cost. Then the problem is set as follows:

$$\underset{(L,K)}{\text{Minimize}} : \; wL + rK$$

$$\text{subject to: } AL^{\propto} K^{\beta} = Q^{o}$$

setting the Lagrangian function :

$$\underset{(L,K,\lambda)}{\text{Maximize}} : f = wL + rK + \lambda (Q^{o} - AL^{\propto}K^{\beta})$$

Deriving the First-Order conditions with respect to L, K , λ, where, λ is the Lagrangian multiplier, which simply corresponds to the marginal cost.

$$\frac{\partial f}{\partial L} = f_L = w - \lambda A \propto L^{\propto-1} K^{\beta} = O \quad \text{...............(1)}$$

$$\frac{\partial f}{\partial K} = f_K = r - \lambda A\beta L^{\propto} K^{\beta-1} = O \quad \text{...............(2)}$$

$$\frac{\partial f}{\partial \lambda} = f_{\lambda} = Q^{o} - AL^{\propto}K^{\beta} = O \quad \text{...............(3)}$$

Solving, simultaneously, the set of equations 1 , 2, 3 yields the optimal labor and capital quantities (L*, K*) respectively, which are associated with the minimum total cost :

$$L^* = A^{-1/\propto+\beta} \left[\frac{\propto r}{\beta w}\right]^{\beta/\propto+\beta} Q^{o\ (1/\propto+\beta)} \quad \text{........... (4)}$$

$$K^* = A^{-1/\propto+\beta} \left[\frac{\propto r}{\beta w}\right]^{-\propto/\propto+\beta} Q^{o\ (1/\propto+\beta)} \quad \text{.......... (5)}$$

Substituting equations (4) and (5) into :

TC = wL* + rK* , yields

$$TC = A^{-1/\propto+\beta} \left[\frac{\propto}{\beta}^{\beta/\propto+\beta} + \frac{\propto}{\beta}^{-\propto/\propto+\beta} \right] (w^{\propto} r^{\beta} Q^{o})^{1/\propto+\beta}$$

For a Cobb-Douglass technology production function, we set A = 1, and by using the constant-returns to scale assumption that ($\propto + \beta = 1$) then the total cost function is reduced to :

$$TC = c\, w^{\propto}\ r^{\beta}\ Q^{o}$$

where: $c = \propto^{-\propto} (\beta)^{\propto-1}$, is a constant term .

At the break-even point :

$$TR = PQ = TC = wL^{*} + rK^{*}$$

then the corresponding output level (Q_e) which gives the break-even point:

$$Q_e = \frac{c\, w^{\propto}\ r^{\beta} Q^{o}}{P}$$

which yields multiple break-even points with curvi-linear total cost and total revenue functions.

Case 3 : Baumol's Model :

The standard literature of the firm's theory stipulates that the objective of the firm is to maximize its profit, being jusfied as its "raison d'etre" under any form. However, with the evolutionary forms of management in today's corporation, in which ownership and management are separate entities, therefore, they seek different objectives. Baumol proposed a solution to the objectives of the two different agents by a rational behavior, where the corporation management is pursuing the alternative goal of maximizing the sales revenue.

The total revenue is often taken as a vital indicator of the competitive position of the firm whithin the industry. Such index reveals the actual performance of the firm and is a sign of the managerial success which entails a conceivable remuneration. To avoid possible ownership discontent, management seeks to maximize sales revenue, with the condition that profit does not fall below a predetermined level .

Then the treatment of cost-volume-profit analysis under baumol's model is as follows:

Baumol's model of sales maximization assumes that managers may pursue sales maximization if they think that their own compensation and / or their professional remuneration depend more on sales volume than on profits. However, they constrain their behavior to realize certain prescribed level of profit which satisfies owners' objectives.

The mathematical presentation of Baumol's model is proceeded as :

Maximize :

$$R = R(Q)$$

Subject to :

$$R(Q) - C(Q) \geq \Pi^{o}$$

$Q \geq o$

where : $R(Q)$ = Sales Revenue

Q : Produced and sold units

$C(Q)$: Total Cost

then $R(Q) - C(Q)$ is the profit function, which is greater than a predetermined level of profit (Π^{o}).

Then we can rewrite the model as :

Maximize: $R = R(Q)$

Subject to: $C(Q) - R(Q) \leq -\Pi^{o}$

and $Q \geq o$

By assuming concavity of $R(Q)$ function and convexity of $C(Q)$ function, then, we can solve the model as :

Forming the Lagrangjan function :

$$L = R(Q) + \lambda[-\Pi^{o} - C(Q) + R(Q)]$$

Setting the Kuhn - Tucker conditions :

$$\frac{\partial L}{\partial Q} = R'(Q) - \lambda\, C'(Q) + \lambda\, R'(Q) \leq o \text{ .... (i)}$$

$$\frac{\partial L}{\partial \lambda} = - \prod^{o} - C(Q) + R(Q) \geq o \text{ ....(ii)}$$

adding the non-negativity and the complementary slackness conditions. Since R(O) = O , and C (O) > O , that is, a zero output would give $\frac{\partial L}{\partial \lambda} = - \prod^{o} - C(O) < o$ which violates the second condition (ii) .

Thus, (Q) must be positive , ie , (Q>O) .

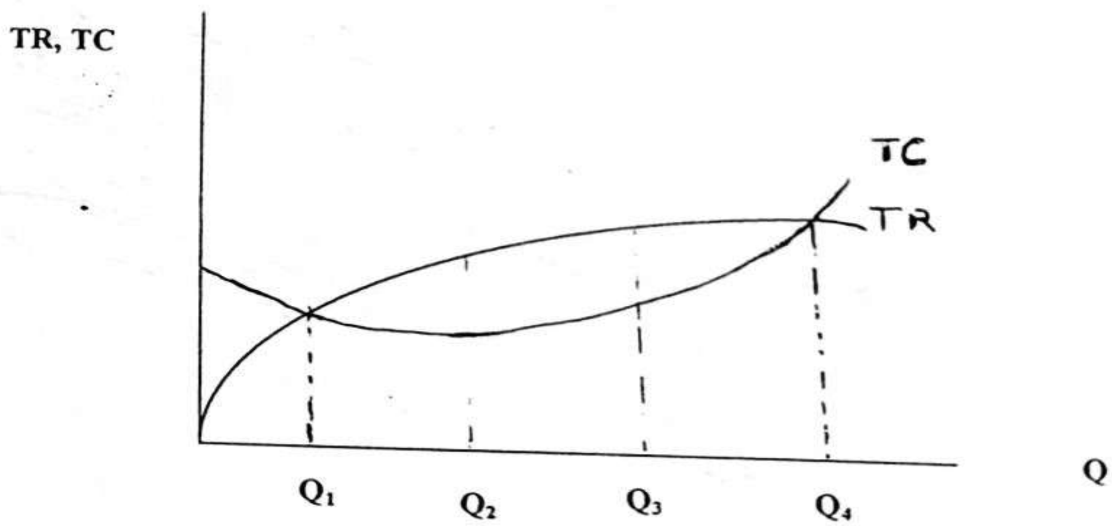
The positivity of (Q) yields by the complementary- slackness assumption that :

$\frac{\partial L}{\partial Q} = \dot{O}$, which, satsfies equation (i) as an equality .

Solving equation (I) as an equality yields :

$$R'(Q) = \frac{\lambda}{1 + \lambda}\, C'(Q) \text{ .... (iii)}$$

We can give an interpretation to the result of equation (iii) by referring to the following diagram :



The diagram represents a convex C(Q) function and a concave R(Q) function, as it has been assumed.

According to result (iii) , the value of (λ) can take either zero or positive value.

If $\lambda = O$, then equation (iii) is reduced to $R'(Q) =$; the firm will push its output level to the point where the marginal revenue vanishes, that is, the case of pure sales maximization, which means that the firm will proceed to an output level that maximizes total-revenue curve .

According to the diagram, the firm will seek an output level in the closed interval [Q2 , Q3] . Such behavior is not feasible under our assumptions . Accordingly , λ must take a positive value ($\lambda > O$). By using the assumption of complementary-slackness condition yields :

$$\frac{\partial L}{\partial \lambda} = o,$$

that is, the profit constraint is to be sastisfied as an equality equation resulting in the firm behavior attempting to earn just the predetermined level of profit ($\prod^{o}$) . The positive value of (λ) renders result (iii) :

$R'(Q) < C'(Q)$, since $\frac{\lambda}{1+\lambda} < 1$, such result will ensure that sales-maximization model would generally give a higher output level than the profit maximizing rule $R'(Q) = C'(Q)$.

With respect to break-even anlysis, and as the diagram shows, Baumol's model entails multiple break-even points, such as, points (A) and (B), with the cited assumptions .

Conclusion :

Since a firm, seeks an optimum choice, and by deriving the minimum total cost function associated with the maximum output level as revealed by the duality theory of production and cost theories and appling that to the C-V-P analysis, the results of the suggested models yield that : the result of the linear model coincides with the conventional result of the C-V-P analysis ignoring the issue of divergence between economy and accountancy theorists.

However the cases of nonlinear and Baumol models explicitly reveal the existence of multiple break-even points as stipulated by economic theory, and the solution remains at the hand of the management decision maker who seeks an optimum choice.

References :

1-Alpha, C. Chiang." Fundamental Methods of Mathematical Economics, McGraw-Hill Company, 1984.

2-Douglas, Vickers. "On the Economics of Break-even", The Accounting Review, 1960, p.p. 405-412.

3-Hands , Wade . Introductory Mathematical Economics , Heath . USA . 1991.

4-Henderson,J., and Quandt,R.,. Microeconomics Theory , Mc Graw Hill , USA. 1980 .

5-Hilton,Roland . Managerial Accounting , Mc Graw Hill, USA . 1994 .

6-Hirshey, Mark and Pappas , James. Fundamentals of Managerial Economics, Dryden, 1992 .

7-Horace, R. Givens. " An Application of Curvilinear Break-even Analysis", The Accounting Review, Jan. 1966, p.p. 141-143 .

8-Horngren, C., Foster, G., and Datar, S., Cost Accounting , Prentice Hall. U.S.A. 1994 .

9-Joel, Dean. "Managerial Economics", New York, Prentice Hall, 1951.

10-McGuigan,J., and Moyer,C., " Managerial Economics", West Publishing Company, 1993.

11-Polimeni,F., Fabozzi, F.,and Adlelberg, A., Cost Accounting , Mc-Graw Hill, 1991.

12-Silberberg, Eugene . The Structure of Economics, Mc-Graw Hill, 1990.

13-Townsend, Harry . Foundation of Business Economics, Routledge , UK. 1995.

14-Travis, P. Goggans. "Break-even Analysis with Curvilinear Function, The Accounting Review, Oct.1965, p.p. 867-871.

15-Varian, Hall . Microeconomic Analysis , Norton, USA . 1990 .

16-William, J. Baumol. " On the Theory of Oligopoly, " Econometrica, August 1958, p.p. 187-198.