Glassy Behaviour of Random Field on Bethe Lattice in an External Magnetic Field

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The thermodynamics of random field Ising model (RFIM) on Bethe lattice is studied by using the replica trick (Bethe lattice is placed in an external magnetic field (B)). A Gaussian distribution for random field with variance $\left\langle h_i^2 \right\rangle = H_{RF}^2$ *is considered. The free-energy (F), magnetization (M), order parameter (q), susceptibility (* χ *) and internal energy (U) are calculated. The phase diagram is investigated at different values of co-ordination number* $z = 3,4,5,6$ *and* $z \to \infty$. *The phase diagram shows several interesting behaviours and present tricritical point at critical temperature* $T_c = J/k$, $H_{RF} = 0$ *for finite co-ordination number (z). The critical temperature (*T_c*) and the corresponding value of* H_{RF} *are found. Finally, we compare our results with the recent results obtained before for spins glasses.*

1. Introduction:

The magnetic random systems with a frustration in the exchange interactions between the spins exhibit a spin-glass phase at low temperature [1, 2]. The Sherrington-Kirkpatrick (SK) model with infinite-ranged interactions was suggested as a mean field model of spin-glasses [3]. Parisi found a stable solution using the replica-symmetry-breaking scheme [4].

The random field Ising model (RFIM) is one of the simplest models of disordered interacting systems. This RFIM describes the behavior of diluted antiferromagnets in strong magnetic fields [5]. In recent years, there have been many theoretical [6] and experimental indications [7]. The essential difficulties in breakdown the behaviour of the RFIM may be related to the existence of many metastable states and associated glassy phase, similar as that found in spin glass materials [8].

Standard approaches, such as the Bragg-Williams mean field theory (MFT), when used to examine the RFIM predicted only the existence of the ferromagnetic (FM) and the paramagnetic (PM) phases, but failed to identify a spin-glass (SG) phase [9]. Very recently, by including fluctuations beyond MFT using a $1/N$ -approach, Mezard and Young [6] were successful in predicting the SG phase, based on an appropriate replica-symmetry-breaking scheme.

The simplest effect of turning on a weak random field is the resulting depression of the critical temperature for uniform ordering, while for sufficiently strong randomness the ordered phase completely disappears. The more sophisticated theoretical schemes have to be used in order to identify the corresponding instability of the high-temperature paramagnetic phase. Such a theory was formulated by Mezard and Young [6], who utilized the self-consistent screening (SCS) approach of Bray [10], and identified the glassy phase by carrying out a replica-symmetry-breaking stability analysis. Similar result were obtained by numerically solving the mean-field equations for a fixed realization of disorder by Lancaster *et* al. [11], confirming the existence of the glassy phase. Mezard and Monasson [12] and de Dominicis *et* al.[13] presented arguments that the glass phase should persist even at weak disorder, everywhere and precede the uniform ordering. Pastor and Dobrosavljevic [14] presented the simplest possible approach that is capable of providing a description of glassy phase. The recent theoretical efforts use the sophisticated numerical approaches to study the behavior of the order parameter under magnetic field and the temperature for the RFIM [15, 16, 17]. The phase diagram has been investigated by Monte-Carlo simulation [18, 19]. The one-dimensional disordered Ising systems are considered to be one of the most important physical problems [20].

The statistical properties of peculiar responses is studied in glassy systems at mesoscopic scales based on a class of mean-field spin-glass models which exhibit one step replica-symmetry-breaking [21]. For the RFI model, Belanger and Nattermann discussed experiments and surveyed the theory only the nearest–neighbor interactions in the absence of an external magnetic field (B) [22, 23]. Belanger pointed that the experiments give a clear evidence for a finite temperature transition in three dimensions and discussed the observed critical behavior noting the difficulties arising from irreversible effects below the critical point. In spin glass models, replica symmetry breaking implies the existence of many pure states of equilibrium. Their hierarchical organization is described by the order parameter (q) [24]. In the SK model, q has a continuous form and it is a marginally stable saddle point of the replica free energy surface in the limit $n \to 0$ [25], where n is the number of replicas.

A Bethe lattice is introduced by Hans Bethe in 1935. It is a connected cycle-free graph, where each node is connected to z neighbors, where z = 2, 3,…, and is called the coordination number. It can be seen as a tree-like structure emanating from a central node. The central node may be called the" root or origin" of the lattice as shown in Fig. (1). The name "Bethe lattice" originates from the fact that Bethes approximation for the Ising model is exact on this lattice [26, 27]. A finite portion of the Bethe lattice is called Cayley tree.

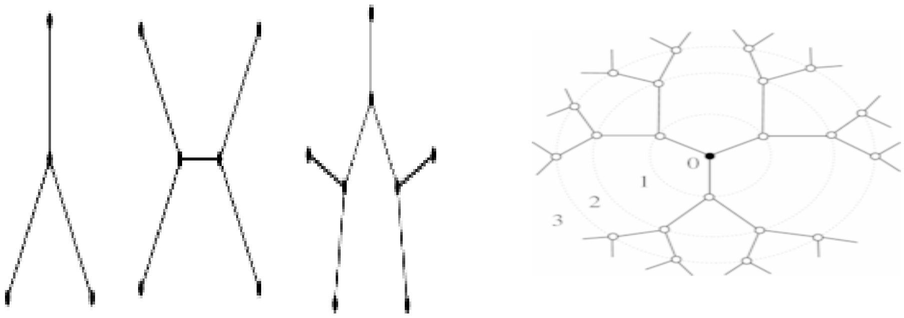

Fig.1: A Bethe lattice with coordination number z = 3.

There are two special properties that make the Bethe lattice particularly suited for theoretical investigations. One is its self-similar structure which may lead to recursive solutions and it plays an important role in statistical and condensed-matter physics. Because of some problems involving disorder and/or interactions can be solved exactly when defined on a Bethe lattice, e.g., Ising models [26-28]. In a recent publication, Bruinsma [29] has obtained a recursion relation for the random-field distribution functions in RFIM on a Bethe lattice. He has concentrated on the $T=0$ properties that follow from this recursion relation. Hartztein and Entin-Wohlman [30] explored the finite-temperature phase diagram by analyzing the limit of the distribution averages. They obtained first-and second-order transition lines and a tricritical point for RFIM on a Bethe lattice. Castelliani and *et* al. [31] derived the zero-temperature phase diagram of spin glass models with a generic fraction of ferromagnetic interactions on the Bethe lattice. They used the cavity method at the level of one-step replica-symmetry-breaking (1RSB) and found three phases namely, a replica-symmetric (RS) ferromagnetic one, a magnetized spin glass one (mixed phase) and a unmagnetized spin glass one[31]. Liers *et* al. [32] studied the Ising spin glass on random graphs with fixed connectivity z with mean μ and unit variance. They exactly computed ground states by using a sophisticated branch-and-cut method for z=4, 6. It is located the spin-glass/ferromagnet phase transition at $\mu = 0.77 \pm 0.02$ (z = 4) and $\mu = 0.56 \pm 0.02$ (z =6). Finally, Broges and Silva [33] studied the RFIM within differential operator method and obtained the phase diagrams for coordination numbers z = 4, 6 and showed several interesting behaviors, presenting tricritical points (only for z = 6).

In this work, we present the thermodynamic properties for the RFIM on Bethe lattice by using the replica trick for different coordination numbers (z). Gaussian distribution of the local random field is considered. The phase diagram, the critical temperature and the thermodynamic magnetic parameters (magnetization, order parameter and susceptibility) are investigated. The mean field theory (MFT) enables us to identify the ferromagnetic (FM), paramagnetic (PM) and spin-glass (SG) phases and allow studying its properties for high disorder. We also study the effect of the static external magnetic field (B) on the same thermodynamic parameters for the RFIM. The paper is organized as follows. In Sec. 2, we describe the details of the model and apply the replica trick to obtain the free-energy (F). We calculate in section 3 the magnetization (M) and order parameter (q) by self-consistently. We deduced the equation of the critical temperature at which the phase transitions occur. In section 4, we analyze our results for RFIM on Bethe lattice in the presence of an external magnetic field. Finally, we discuss and compare our results with other numerical simulations obtained before.

2. The model:

We Consider a Bethe lattice of N Ising spins $S_i = \pm 1$ which are coupled by finite constant interactions. The Hamiltonian of the random field Ising model (RFIM) is given by

$$\mathcal{H} = -\sum_{ij} J_{ij} S_i S_j - \sum_{i=1}^{N} H_i S - B\sum_{i=1}^{N} S_i \qquad (1)$$

where $J_{ij} = J/z$ are uniform ferromagnetic interactions between nearest-neighbor sites, rescaled with the coordination number (z)and B is the external magnetic field applied on our system. The random field variables $\{H_i\}$ are Gaussian distributed, with zero mean and variance $\left[H_i^2\right]_{av} = H_{RF}^2$ and we take H_{RF} as initial constant values

Thermodynamic properties of the model can be calculated from the ensemble averaged free energy per spin

$$F(\beta) = -\frac{1}{\beta N}\lim_{N\to\infty} \ln Z(\beta) \qquad (2)$$

With the partition function

$$Z = \sum_{\{S_i\}} \exp\left(-\beta \mathcal{H}(S_i)\right), \qquad (3)$$

where $\sum_{\{S_i\}}$ means the sum over all states of configurations, $\beta = 1/kT$, where k is the Boltzmann constant and T is the absolute temperature. We calculate the average quenched free energy. It is very difficult to compute $[\ln Z]_{av}$ directly. In order to overcomes this difficulty, we use the replica trick [1] since the summation over the disorder systems

$$\ln Z = \lim_{n\to 0}\left(\frac{Z^n - 1}{n}\right) \qquad (4)$$

where Z^n is the partition function of the α^{th} replica, $\alpha = 1,....,n$, where n is the number of replicas. We calculate Z^n by replicating the system n times. Thus we introduce n identical replicas of the original system. The resulting partition function takes the form

$$Z^n = Tr_{\{S_i^\alpha\}} \exp\left(\frac{\beta J}{z}\sum_{ij}\sum_{\alpha} S_i^\alpha S_j^\alpha + \frac{1}{2}(\beta H_{RF})^2 \sum_{ij}\sum_{\alpha\beta} S_i^\alpha S_i^\beta S_j^\alpha S_j^\beta + B\sum_{i,\alpha} S_i^\alpha\right) \qquad (5)$$

where α , β =1,…, n, where the trace extends over all states of a single replicated spin $\{S_i^\alpha\}$. By expanding first term of equation (5) in powers of the interaction J/z, hence, one can obtain the partition function as follows:

$$Z^n = Tr_{\{S_i^\alpha\}} \exp\left[\frac{\beta J}{z}\sum_{ij}\sum_{\alpha} S_i^\alpha S_j^\alpha + \left(\frac{\beta^2 J^2}{z^2} + \frac{1}{2}(\beta H_{RF})^2\right) \sum_{ij}\sum_{\alpha\beta} S_i^\alpha S_i^\beta S_j^\alpha S_j^\beta + B\sum_{i,\alpha} S_i^\alpha\right] \qquad (6)$$

By using the following inequalities (the replicas mean field theory [1])

$$\sum_{ij} s_i^\alpha s_j^\alpha \cong z\sum_i \left(s_i M - \frac{M^2}{2}\right) \quad ; \quad M_i^\alpha = \langle s_i^\alpha \rangle = M \,\forall\, i$$

$$\sum_{(ij)} s_i^\alpha s_j^\alpha s_i^\beta s_j^\beta \cong z\sum_{ij} \left(q s_i^\alpha s_i^\beta - \frac{q^2}{2}\right) \quad ; \quad q_i^{\alpha\beta} = \langle s_i^\alpha s_i^\beta \rangle = q \,\forall\, i$$

We can formally average over disorder and the free-energy, in thermodynamic limit may be expressed as:

$$\beta F = -\lim_{N\to\infty}\lim_{n\to 0}(Nn)^{-1}\left\{ Tr\exp\left[\begin{array}{l} \frac{\beta J}{2z}\sum_{\alpha}\left(\sum_i S_i^{\alpha}\right)^2 + \\ \left(\frac{\beta^2 J^2}{2z^2}+\frac{1}{4}(\beta H_{RF})^2\right)\sum_{(\alpha\beta)}\left(\sum_i S_i^{\alpha}S_i^{\beta}\right)^2 + \beta B\sum_{\alpha}\sum_i S_i^{\alpha} \end{array} \right] - 1 \right\} \quad (7)$$

where $(\alpha\beta)$ refers to combinations of α and β with $\alpha\neq\beta$. Note that the exchange terms in the exponent are in the form $\lambda\left(\sum_i O_i\right)^2$, where O_i is local intensive operator, which leads to physical thermodynamic consequence only if $\lambda \propto N^{-1}$. By using the Hubbard Stratononvitch identity on the squared terms which is given by:

$$\exp(\lambda a^2) = \left(\frac{N}{2\pi}\right)^{\frac{1}{2}} \int dx \quad \exp\left(-\frac{Nx^2}{2} + (2\lambda N)^{\frac{1}{2}} ax\right) \quad (8)$$

and using the single-spin property,

$$Tr_n \exp(g\sum_i S_i^{\alpha}) = \exp(N \ln Tr_n \exp g(S^{\alpha})) \quad (9)$$

It is assumed that the limit $n \to 0$ and the thermodynamic limit $N \to \infty$ can be interchanged. We can drop terms which vanish in the thermodynamic limit. For $N \to \infty$, the integral in Eq. (8) can be done by the Steepest descents method (The integral being dominated by the region of maximum integrand). Since the replicas are indistinguishable.

Then we can obtain the final form of free–energy as follows:

$$\beta F = -\left[\begin{array}{l} \frac{1}{2}\beta J M^2 + \frac{1}{2z}(\beta J)^2\left(q - M^2 - \frac{q^2}{2}\right) \\ -\frac{1}{\sqrt{2\pi}}\int \exp\left(-\frac{x^2}{2}\right)\ln\left(2\cosh\left(\beta B + \beta J M + \left((\beta H_{RF})^2 + \frac{\beta^2 J^2}{z}(q - M^2)\right)^{\frac{1}{2}} x\right)\right) dx \end{array} \right] \quad (10)$$

3. The thermodynamic parameters

The magnetization (*M*), order parameter (*q*), susceptibility (χ) and internal energy (*U*) are calculated as functions of temperature (*T*) and random field (H_{RF}).

By differentiating equation (10) with respect to *B* we find that the magnetization:

$$M(T)=\frac{1}{\sqrt{2\pi}}\int\exp\left(-\frac{x^2}{2}\right)\tanh\left(\beta B+\beta JM+\left((\beta H_{RF})^2+\frac{\beta^2J^2}{z}\left(q-M^2\right)\right)^{\frac{1}{2}}x\right)dx \qquad (11)$$

It is a self-consistent equation which can be solved with the saddle-point conditions. The mean-square of frozen moment (*q*) per site is given by

$$q(T)=\frac{1}{\sqrt{2\pi}}\int\exp\left(-\frac{x^2}{2}\right)\tanh^2\left(\beta B+\beta JM+\left((\beta H_{RF})^2+\frac{\beta^2J^2}{z}\left(q-M^2\right)\right)^{\frac{1}{2}}x\right)dx \qquad (12)$$

In the $z\to\infty$ limit and for *B*=0, one can obtain the straightforward generalization equation of the well–known Bragg–Williams's condition to include the effect of random fields. This is sufficient to determine the ferromagnetic (FM) phase boundary which is determined by setting *m*=0.
We can find zero-temperature magnetization (the ground-state magnetization $M(T=0)$):

$$M(0)=erf\left[\frac{1}{\sqrt{2}}\left(\frac{J}{H_{RF}}M(0)+\frac{B}{H_{RF}}\right)\right] \qquad (13)$$

The critical temperature where the magnetization vanishes is easily computed by differentiating the right hand side of equation (11) with respect to *M* by putting both *M* and *B* is zeros. The FM phase boundary is obtained by locating at a given value of H_{RF} where the magnetization *m* vanishes. The temperature phase diagram is given by

$$\frac{kT}{J}=\frac{1}{\sqrt{2\pi}}\int\exp\left(\frac{-x^2}{2}\right)\sec h^2\left(\left((\beta H_{RF})^2+\frac{\beta^2J^2}{z}q\right)^{\frac{1}{2}}x\right)dx \qquad (14)$$

From this equation, we can plot the phase diagram. By differentiating the right hand side of Eq. (14) with respect to temperature at $T=T_C=0$. The zero solution occurs when $\frac{H_{RF}}{J} \geq \sqrt{\frac{2}{\pi}}$ which means that the system becomes SG for all *T*. The identification of the SG phase is more difficult. Similarly as for spin glass models in a uniform external field, in our case the Edward-Anderson order parameter (q) in non zero for any temperature and thus cannot be used to determine the phase boundary. In our case, the random magnetic field plays a role of a source conjugate to the order parameter, locally breaking the up-down symmetry. The situation is similar as in spin-glass models in a uniform external field [24], where the replica symmetric order parameter q remains nonzero for any temperature and thus cannot be used to identify glassy freezing. We follow Mezard and Young [6] and look for instability to replica symmetry breaking (RSB) within the paramagnetic phase. To this, we can set m=0, and note that remaining equation for q is in fact identical to that describing the Sherrington-Kirkpatrick model in presence of magnetic fields. This model is also described with the first two terms of the Hamiltonian of Eq. (1), but this time with J. J_{ij}'s are being Gaussian random variables with zero mean and variance

$$\langle J_{ij}^2 \rangle = J^2 / zN$$

By differentiating equation (11) with respect to the external magnetic field (B) and taking the limit $B \to \infty$, the susceptibility (χ **)** is given by

$$\chi(T) = \frac{1-q}{kT - J(1-q)} \tag{15}$$

We apply Adaptive Simpson quadrate method to compute the integral in Eq.(14) within MATLAB package. We are able to obtain numerical results for the internal energy (U) by differentiating Eq.(14) with respect to β with applying central difference method.

4. Results and discussion

The thermodynamic parameters obtained above are calculated where the Gaussian distribution of the local random field is considered. In the absence of external magnetic field (B), we found that the free-energy (*F)* is constant at low temperature and then linearly increases with increasing temperature, whatever the random field and the coordination number (z) are, as shown in Fig.2. This result agrees with that obtained by [34]

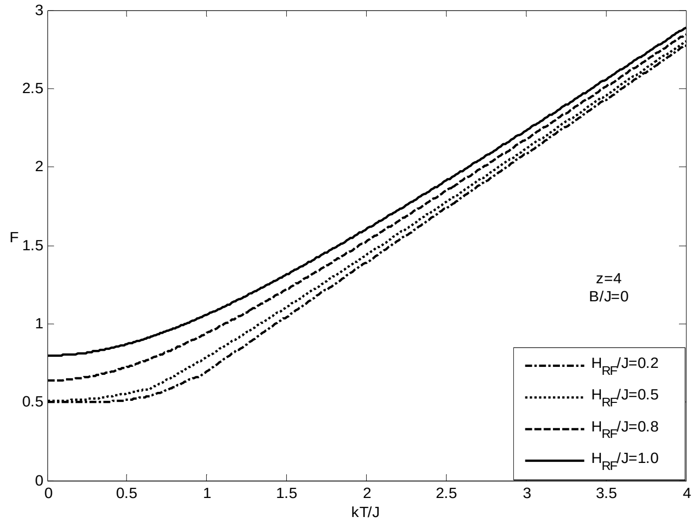

Fig.2: The free energy (*F*) versus *kT/J* at different strengths of random field (H_{RF}/J) and at z = 4.

It is clear that Eqs.(11) and (12) are self-consistently. For this reason, we solved it numerically by using Newton Raphson method (NRM). The value of *M* and *q* are obtained as a functions of temperature (*T*) and random field (H_{RF}) at different values of *z* and *B*.

When *M* and *q* are a functions of random field (H_{RF} / J). We noted that *M* and $q^{1/2}$ curves are almost coinciding for weak random field but on increasing H_{RF}. The difference between them increases until the critical temperature (T_C) at which the system become SG (i.e. $M = 0, q \neq 0$) whatever coordination number (*z*) as in Fig.(3). We also found that the curve of both *M* and $q^{1/2}$ separated. We noticed that $q^{1/2}$ curve is constant at $T = 0$. On increasing the temperature the curve of $q^{1/2}$ decreases until the critical temperature and then increases on increasing H_{RF} to converges to one. But at $kT / J = 1$, the system become SG for all values of H_{RF}. In the presence of *B*, the effect of the weak external magnetic field is observed as shown in Fig.3 (b). The system has a slow phase transition from FM to SG. But at $B / J = 1$, we note that M and $q^{1/2}$ curves are constant at low H_{RF} and decreasing on increasing H_{RF}. The difference between the M and $q^{1/2}$ curves are decreasing until the system becomes pure ferromagnet at $B / J = 1$ as shown in Fig.3(d). The behavior of *M* and $q^{1/2}$ curves is the same wherever value of *z* is.

Furthermore, the magnetization is increasing on increasing the coordination number (z).

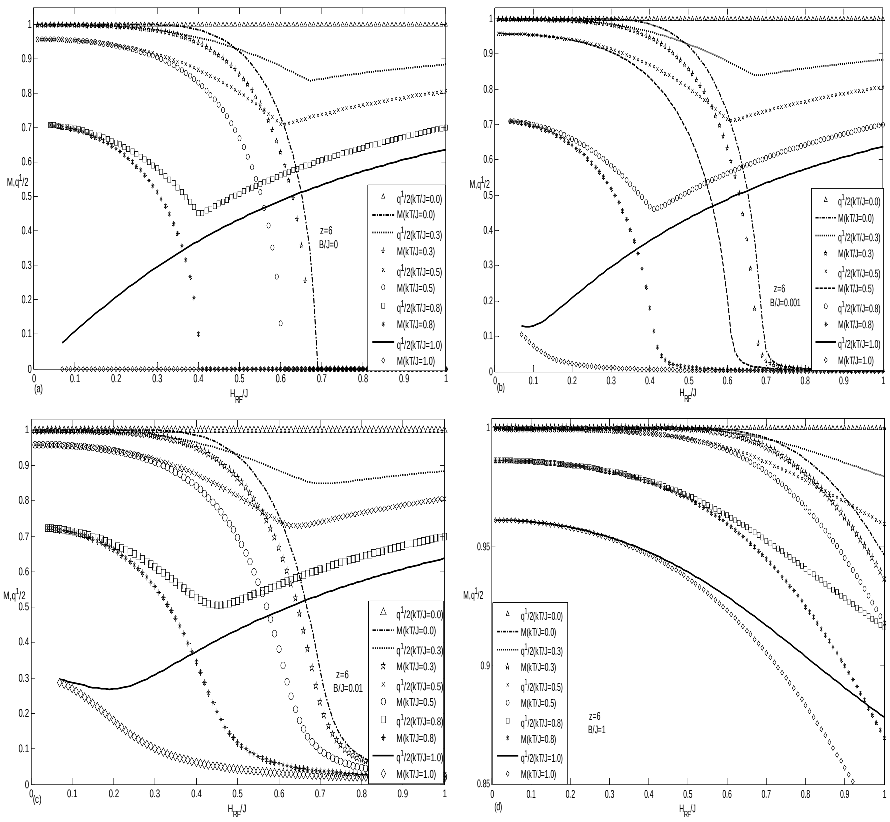

Fig. (3): The magnetization (M) and the order parameter ($q^{1/2}$) versus $\dfrac{H_{RF}}{J}$ at different values of $\dfrac{kT}{J}$ in the presence of magnetic field **(** $\dfrac{B}{J}$) at z = 6: (a) B/J = 0.0, (b) B/J = 0.001, (c) B/J = 0.01 and (d) B/J = 0.0.

But when M and q are as a function of $kT\ /\ J$. We noted that M and $q^{1/2}$ curves are almost constant at low temperature. In the absence of B, M and $q^{1/2}$ decreases on increasing temperature until the critical temperature (T_C) at which the system become SG (i.e. $M = 0$, $q \neq 0$) whatever coordination number (z) as in Fig.4(a). In presence of B, we find that M and $q^{1/2}$ linearly increase with increasing the external magnetic field (B). Also, we find that M = $q^{1/2}$ as $B\ /\ J = 1$. This result agrees with those obtained by [35, 36]. Because they found that a finite external field removes sharp phase transitions by allowing M and $q^{1/2}$ to be nonzero at all temperatures. On increasing H_{RF}, the curve of M and $q^{1/2}$ are separated. This separation decreases with increasing B. The M and $q^{1/2}$ curves are almost coinciding until the system becomes pure ferromagnet at B/J =1 as shown in Fig. (4b). The separation between M- and $q^{1/2}$ - curves is increasing at $H_{RF} \geq \sqrt{2/\pi}J$ with as shown in Fig. (4c,d). The behavior of M and $q^{1/2}$ curves is the same wherever value of z is. Furthermore, the magnetization is increasing on increasing the coordination number (z).

The ground-state of magnetization on a Bethe lattice decreases and tends to zero as increasing the random field (H_{RF}) as shown in Fig, (5). The maximum value of M (T = 0) is equal to one at H_{RF} = 0.11. We found that there are groups of lines ($n(l)$). Each group contains a number of lines these lines increase on increasing the coordination number (z) as shown in Fig. (5). These groups are growing at 0.1 of the random field (H_{RF}). The number of lines and the interval of the coordination number (z) are given in Table (1).

Table (1)**:**

Number of lines $n(l)$	Interval of coordination number (z**)**
1	$2 \leq z \leq 9$
2	$10 \leq z \leq 11$
2	$12 \leq z \leq 13$
3	$14 \leq z \leq 16$
6	$17 \leq z \leq 22$
12	$23 \leq z \leq 35$
64	$36 \leq z \leq 99$
∞	$z \geq 100$

The critical temperature at which occurs phase transition from FM to SG is decreasing as random field increases. By differentiating the r. h. s of Eq. (18) with respect to T at $T = T_C = 0$, we find that the zero solution of equation occurs as a unique solution when $H_{RF} / J \geq \sqrt{2/\pi}$. The identification of the SG phase

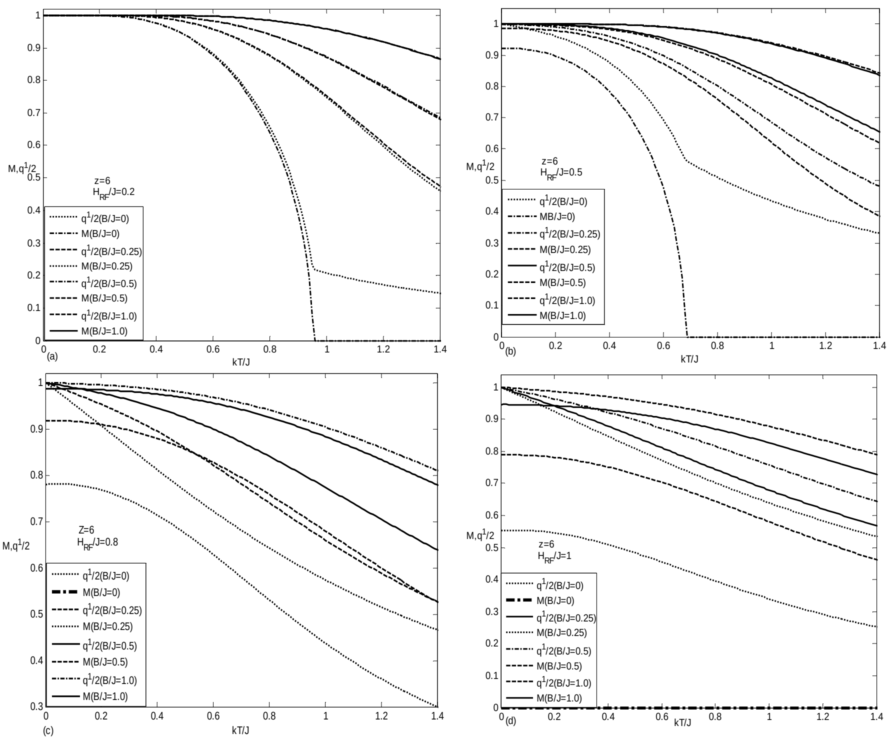

Fig. (4): The magnetization (M) and the order parameter ($q^{1/2}$) versus kT/J at different values of B/J in the presence of H_{RF}/J = 0.2, 0.5, 0.8 and 1.

is more difficult. Similarly as for spin glass models in a uniform external field, in our case the Edward-Anderson order parameter (q) in non zero for any temperature and thus cannot be used to determine the phase boundary in our case, the random magnetic field plays a role of a sourse conjugate to the order parameter, locally breaking the up-down symmetry the situation is similar as in spin-glass models in a uniform external field [24], where the replica symmetric order parameter q remains non zero for any temperature and thus cannot be used to identify glassy freezing. We follow Mezard and Young [6] and look for instability to replica symmetry breaking (RSB) with in the paramagnetic phase. To this, we can set M = 0, and note that remaining equation for q is in fact identical to that describing the Sherrington-Kirkpatrick model in presence of magnetic fields. This model is also described with the first two terms of the Hamiltonian of Eq. (1), but this time with J_{ij}'s being Gaussian random variables with zero mean and variance $\langle J_{ij}^2 \rangle = J^2 / zN$.

From the phase diagram in Fig. (7), we obtained that: (i) the glass phase transition do not exist for $z = \infty$. This result is the same that obtained by [28], (ii) the T_C decreases as H_{RF} increases in the boundary of phase transition from FM to PM, (iii) our RF system behaves as PM as $H_{RF} / J \geq \sqrt{2/\pi}$, whatever the T is, (iv) we note that there is a phase transition from FM to SG when $B = 0$ as shown in Fig. (6) and (v) The critical temperature (T_C) increases on increasing the coordination number (z)

But when the coordination number is finite, we note that ferromagnetic (FM), paramagnetic (PM) and spin-glass (SG) phase are found as shown in Fig.7. at z=3, 4, 5, 6. The critical temperature decreases as H_{RF} increases in the boundary of phase transition from FM to SG. The Phase diagram shows several interesting behaviours and present tricritical point at critical temperature $T_C = J/k$ and $H_{RF} = 0$.

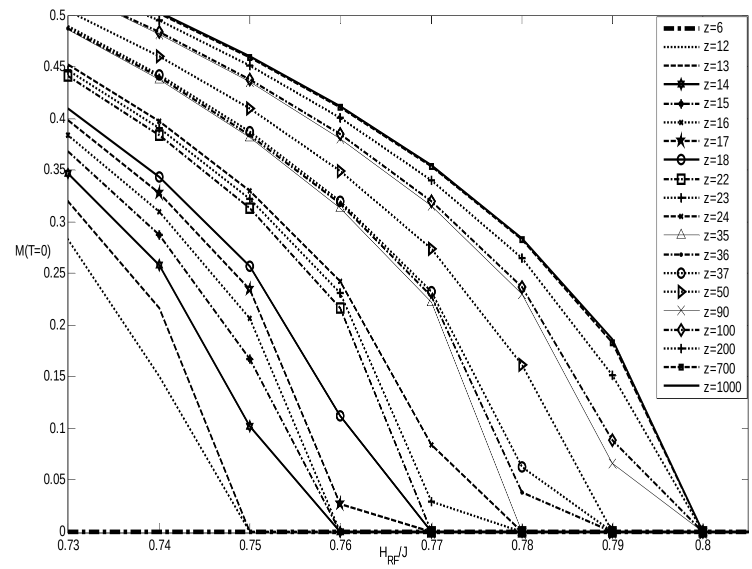

Fig.(5): Ground-state magnetization *(M(T =0))* versus H_{RF}/J with GD for the RFIM at different coordination number *z*.

In the absence of external magnetic field, we find that the susceptibility (χ) has a sharp cusp at z = 6 as shown in Fig.8 (a). This cusp decreases with increasing temperatures and vanishes at the critical temperature $T_C = J/k$. The susceptibility tends to zero at high temperature and random field. In the presence of external magnetic field, χ shows a rounded peak. This peak decreases and moves at a high temperature as shown in Fig.8 (b,c,d). The rounded peak moves to left on increasing *B*. The maximum of the peak lowers with increasing *B*. This result agrees with that obtained by Bannora *et al*. [36] as well as agrees with that obtained by Ismail [37]. He found that real and imaginary parts of the dynamic susceptibility display maxima at $T = T_C$. These maxima can be described by an Arrhenius law. The susceptibility shows the similar behaviour the coordination number (*z*) is.

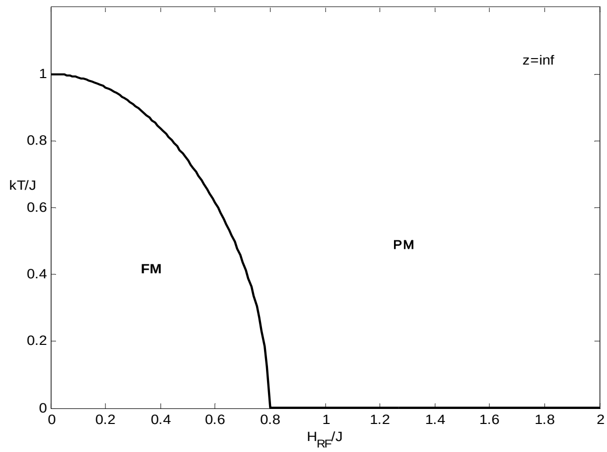

Fig.(6): Magnetic phase diagram for RFIM at $z = \infty$ and *B*=0.

We note that the internal energy (*U*) is almost constant at low temperature. With the further increase of temperature, *U* increases as *T* increases as shown in Fig.9. In the absence of *B*, *U* decreases on increasing H_{RF} and has a singular point at T_c as shown in Fig.(9). This singularity decreases on increasing the coordination number (*z*). Then it linearly decreases as *T* increases at $H_{RF} / J \geq \sqrt{2/\pi}$ whatever *z* is. This result agrees with that obtained by [36].

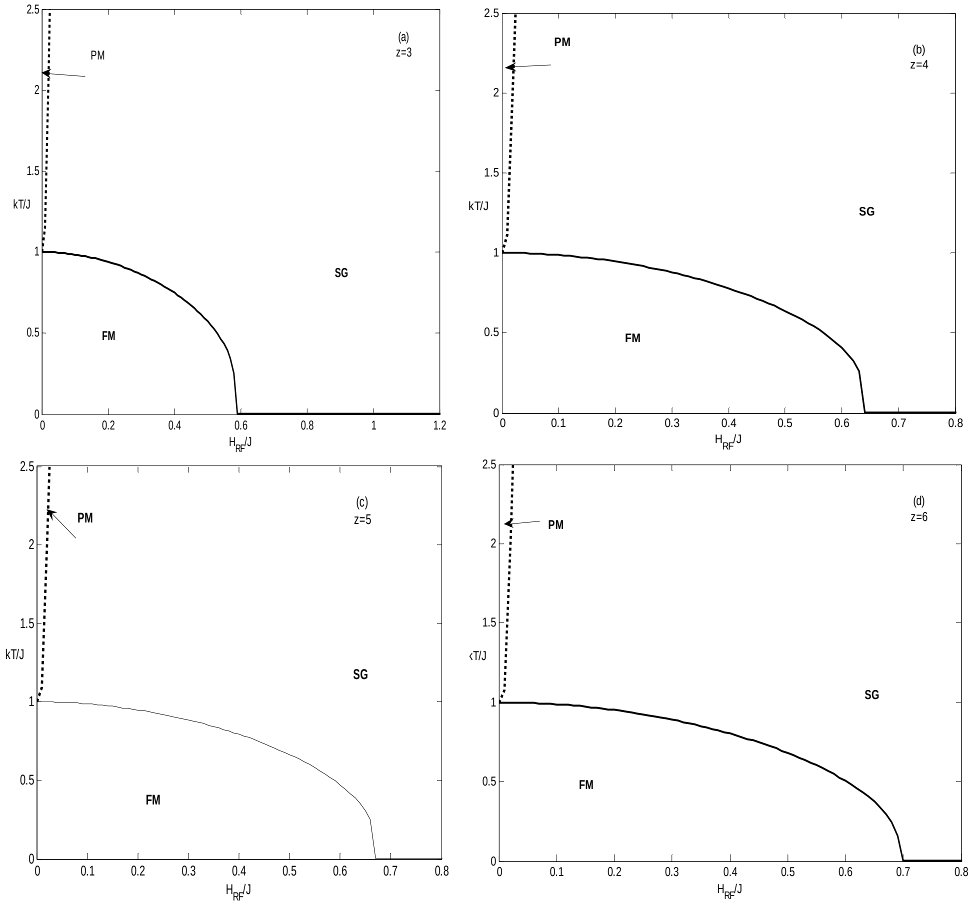

Fig.(7): Magnetic phase diagram for RFIM with different coordination number (z) at B = 0: (a) z = 3, (b) z = 4, (b) z = 5 and (d) z = 6.

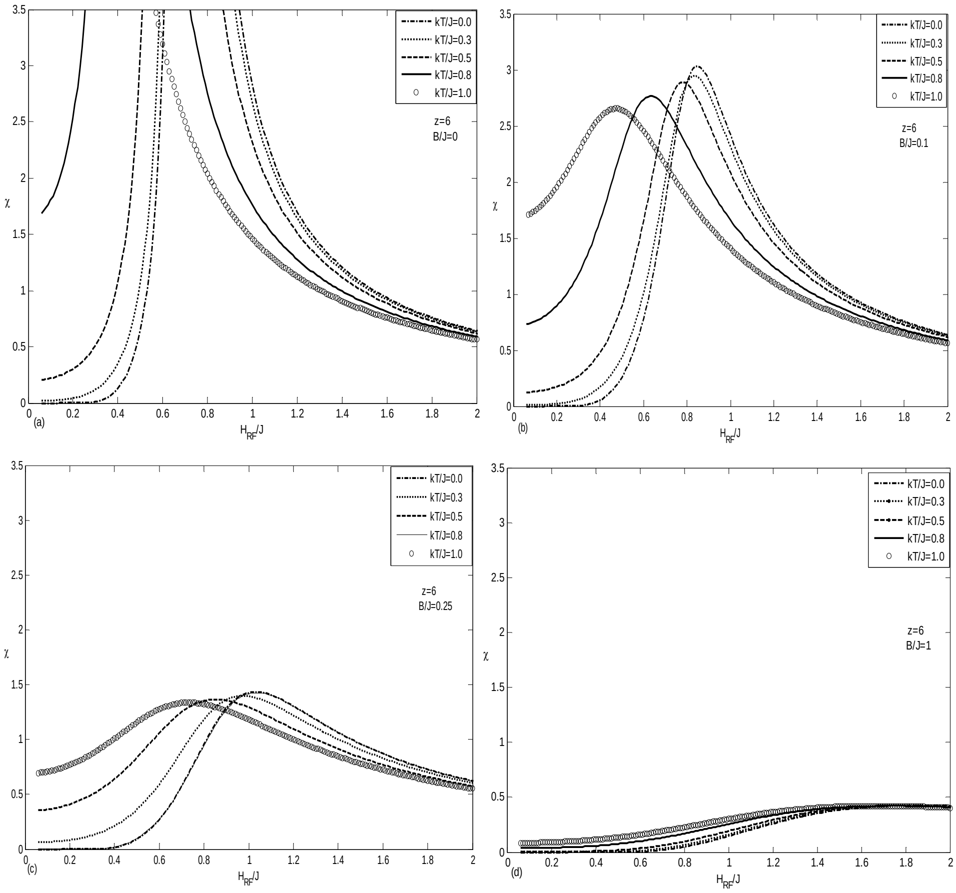

Fig.(8): The susceptibility (χ) versus H_{RF}/J at different values of kT/J in the presence of *B/J* = 0, 0.1, 0.25, 1 and z = 6.

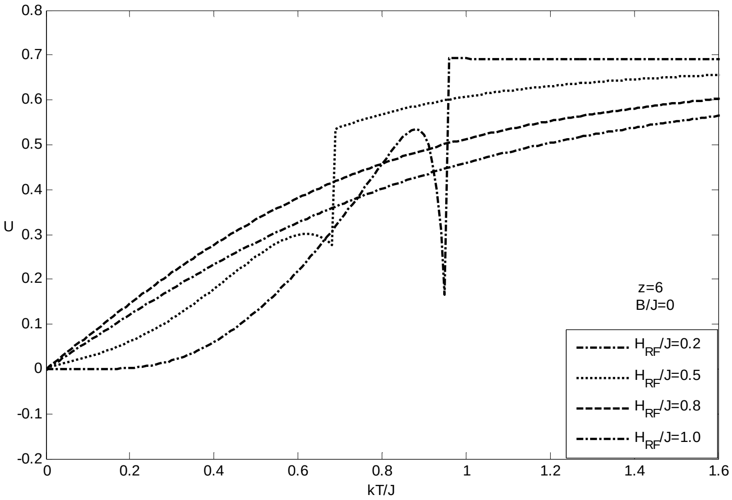

Fig.(9): The internal energy (*U (T)*) versus *kT/J* at different strengths of H_{RF}/J in the absence of *B/J* with GD for the RFIM at z = 6.

5. Conclusions

In summary, we can conclude that in the present Bethe-lattice Ising model with random field, the critical temperature is $T_C = J/k$ at a critical strength $H_{RF}^c / J = \sqrt{2/\pi}$ of the random field. The critical temperature is decreasing on increasing H_{RF}. The Phase diagram shows several interesting behaviours and present tricritical point at critical temperature $T_c = J/k$, $H_{RF} = 0$ and finite number of *z*. When the coordination number is finite, we note that ferromagnetic (FM), paramagnetic (PM) and spin-glass (SG) phase are found and the glass phase transition does not exist for $z = \infty$.

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