An Analysis of Fizeau Fringes Crossing a Fiber of Nearly Triangular Cross-Section

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Theoretical and experimental treatments of the shape of Fizeau fringes crossing a fiber of nearly triangular shaped cross-section assumed to consist of a single medium are presented. A new mathematical expression is derived and used to estimate the refractive indices and birefringence of nylon 6 (polycaprolactam). The resulting data are used to calculate the volume polarizabilities per unit. Cauchy's dispersion constants, the isotropic refractive indices and the variation of refractive indices due to annealing process. Also other optical parameters obtained are calculated. Illustrations are given using micro-interferograms and curves.

1. Introduction

Nylon-6 (polyamide-6) is one of the most widely used engineering plastics and is also commercially produced in the form of fibers and films. The thermal properties of these fibers or films are affected by the processing conditions, molecular weight and molecular weight distribution of the polymer. The morphology of polyamide has been investigated by various groups and there have been a considerable number of articles published in the recent years on the subject of microstructure characterization of Nylon-6 (polyamide-6) fibers and films by various techniques [1-6].

The refractive index and birefringence values of fibers provide parameters that characterize the structure of polymers on a molecular level. The birefringence depends on the molecular orientation in the polymer fiber, as it contains contributions from polarizabilites of all the molecular units in the sample [7]. Several authors have used interferometric methods to measure the variation of the average refractive indices across fibers of regular and irregular cross sections [8-16].

In this work, using a new derived formula, for the nearly triangular shape of the cross-section of nylon-6 fibers to fit with its real shape which is obtained experimentally by high power optical microscopy. Multiple-beam Fizeau system in transmission have been used to investigate the effect of opto-thermal parameters.

The obtained results are utilized to calculate the mean refractive indices in the parallel $n_a^{\parallel}$ and perpendicular $n_a^{\perp}$ and mean birefringence Δn_a of the thermally treated fibers. Other parameters such as Cauchy's constants, polarizabilites and thermal coefficient are calculated.

2. Theoretical assumption

In case of a single medium fiber the experimental cross-section shape of the fiber, as shown in Fig. (1), can be considered as an area bounded by the lines of intersection of a circle with an equilateral triangle.

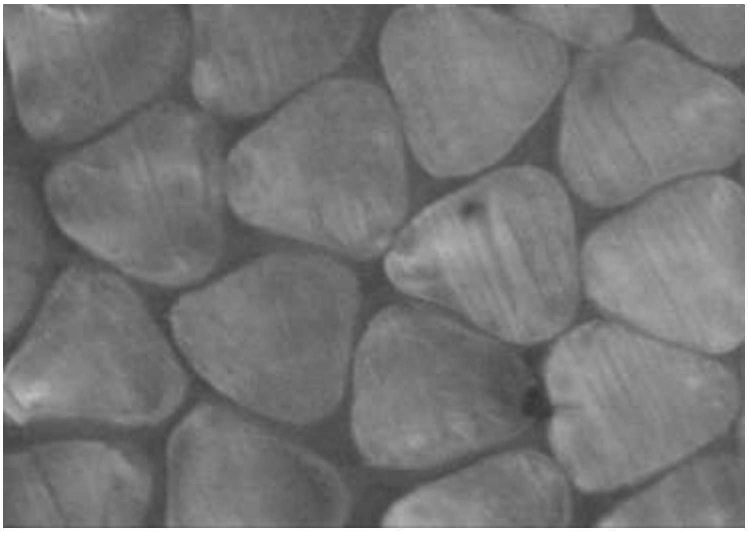


Fig. (1): Optical cross-section of nylon 6.

The center of the circle, which is taken to be the point of intersection of the three symmetrical axes of the triangle, is considered to be the origin of the Cartesian coordinates. The setting of the fiber inside the wedge interferometer along the Z-axis, as shown in Fig. (2), may be such that one of its symmetrical axes makes an angle φ with the Y-axis.

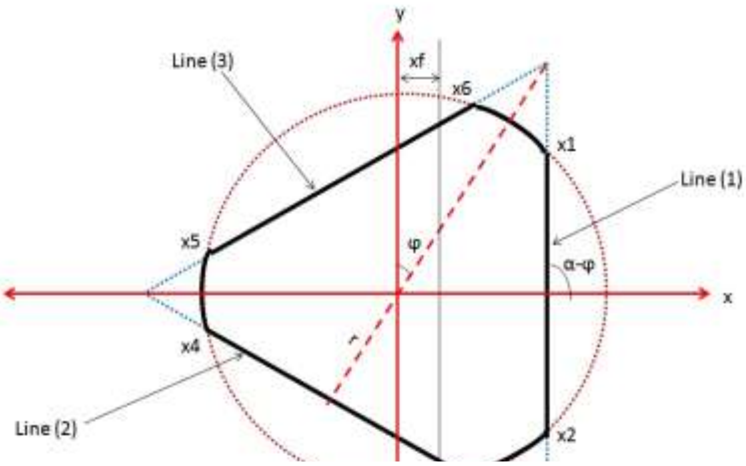


Fig. (2): Circle and triangle building the outer contour of nearly triangular cross-section fiber.

T

are given

$x^2 + Y_c^2$

Y_1 [cos(

(2)

$Y_2 \cos\varphi = -x \sin\varphi - r$ (3)

$Y_3 \cos(\square)[(\alpha + \varphi) = - x \sin(\square)[(\alpha + \square) - r$

(4)

Where: *R* is the radius of the circle, $\alpha = 120°$ and *r* is equal to one third of the triangle height.

The x-positions of the two points of intersection of the line given by Eq. (2) with the circle are:

$$x_{1,2} = r \sin(\square)[(\alpha - \varphi) \pm [\cos(\square)\alpha - \varphi)\sqrt{R^2 - \square^2}$$

(5)

The x- positions of the two points of intersection of the line given by Eq. (3) with the circle are:

(6)

The x- positions of the two points of intersection of the line given by Eq. (4) with the circle are:

$$x_{5,6} = -r\,\sin(\alpha + \varphi) \pm \cos(\alpha + \varphi)\sqrt{R^2 - r^2}$$ (7)

To get the bounded shape of the fiber cross-section inside the circle, as shown in Fig. (2), the parameter *r* must verify the following inequality:

$$R > r > \frac{R}{2}$$ (8)

A value of *r = 0.7R* was found to be the best value which makes the given theoretical fiber cross-section shape to fit with the experimental cross-section.

Let the fiber be inserted inside a wedge interferometer of angle and a parallel beams of monochromatic light of wavelength λ are incident normal on the wedge, as shown in Fig. (3).

The optical path length O.P.L. of a beam crossing the fiber will be given by:

$$O.P.L. = n_L t_N + (n_F - n_L)|(Y_1 - Y_2)|$$ (9)

where: n_L is the refractive index of the liquid inside the wedge,

n_F is the refractive index of the fiber,

t_N is the thickness of the wedge at which the beam is crossing it and

$|(Y_1 - Y_2)|$ is the fiber thickness at which the beam is crossing it.

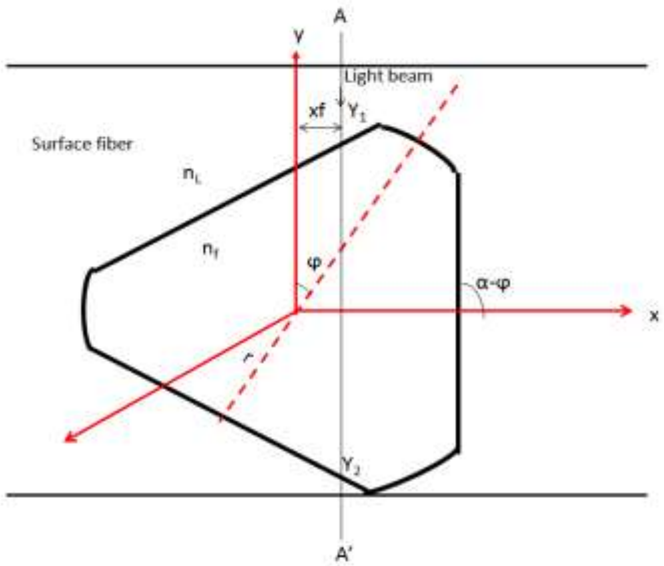


Fig. (3): Cross-section in the nylon-6 fiber of refractive index n_F, immersed in a liqiud of refractive index n_L inside a silvered wedge, where AA' light beam passes through the fiber parallel to the y-axis.

Fizeau straight lines of interference fringes will be obtained, parallel to the edge of the wedge, modulated by a shift in the region where the beams are crossing the fiber. The shape of the shift depends on the cross-sectional shape of the fiber and its position inside the wedge.

In the region where the beams are not crossing the fiber, the condition of interference for the straight line fringes is given by:

$$N\lambda = 2n_L t_N \qquad (10)$$

where: N is the order of the straight line fringes, starting from the edge of the wedge, which is formed at wedge thickness t_N.

In the region where the beams are crossing the fiber, the fringes will be shifted along the Z-axis towards to or apart from the edge of the wedge depending on $n_F > n_L$ or $n_F < n_L$ respectively.

The condition of interference of the shifted fringes is given by:

$$N\lambda = 2n_L(t_N \mp \delta t_N) + 2(n_F - n_L)|(Y_1 - Y_2)| \qquad (11)$$

Relations (10) and (11) give:

$$\pm\mu_L \delta t_N = (n_F - n_L)|(Y_1 - Y_2)| \qquad (12)$$

Since $t_N = Z \tan\varepsilon$ it gives:

$$\delta t_N = \delta z \tan\varepsilon \qquad (13)$$

where δz represents the fringe shift.

The successive fringe spacing ΔZ is given by:

$$\Delta Z = \frac{\lambda}{2n_L \tan\varepsilon} \qquad (14)$$

Thus from Eq. (12), (13) and (14) the following relation will be given:

$$\pm\frac{\delta z}{\Delta Z} = 2\left(\frac{n_F - n_L}{\lambda}\right)|(Y_1 - Y_2)| \qquad (15)$$

Equation (15) shows that the shape of the shift depends on $|(Y_1 - Y_2)|$ which can be determined from the intersection points of the beam represented by $x = x_F$, as shown in Fig. (2), with the outlines of the fiber cross-section which are given by Eq. (1), (2), (3) and (4).

The position of x_F with respect to the regions between the intersection points, given by Eq. (5), (6) and (7) defines the two outlines equations of the fiber cross-section from which

$|(Y_1 - Y_2)|$ can be calculated.

The shape of the fringe shift, which is proportional to $|(Y_1 - Y_2)|$, was calculated for different values of φ. By comparing the calculated shapes of the fringe shift with that which is obtained experimentally, the orientation angle φ of the fiber inside the wedge could be determined.

Through the calculated shape of the fringe shift and that obtained experimentally, the parameter R could be calculated from the width of the fringe shift measured from the starting point of the shift up to its end of shifting.

Through Eq. (15), by measuring the ratio of $\delta z / \Delta Z$, at any point $x = x_F$ as a fraction of R, n_F could be determined.

3. Experimental Verification

3.1 Nylon-6 chemistry and characteristics

Nylon-6 (supplied by Kafr El-Dawar co., Egypt.) is poly-caprolactom filament and staple, the nylon-6(from the lactom of ε -amino caproic acid gives a linear polyamide) with general formula $-$ [N(CH_2) $_2$CO$-$]$_n$. By trade definition, nylon is any long chain synthetic polyamide which has recurring amide groups as an integral part of the main polymer chain. Nylons are synthesized in many ways, including the condensation of diamines with diacids, the polymerization of amino acids, and the ring-opening polymerization of lactams. The varied synthesis routes lead to many different types of nylons whose structures and properties differ widely. The various aliphatic nylons are named by a numerical system based on the number of carbon atoms in the monomer [5] from which they are synthesized.

For convenience, nylons may be classified into six groups according to

the number of CH_2groups in the monomeric units. Nylon-6 Group F, from –

amino acids with odd numbers of CH_2 groups (even numbers in the conventional expression).

3.2 Interferometric determination of Nylon-6 refractive indices

The optical set-up for producing multiple-beam Fizeau fringes in transmission is described in detail in Refs. [8, 10].Figure (4) shows a microinterferogram of multiple-beam Fizeau fringes in transmission for Nylon-6 fiber using plane-polarized light of wavelength $\lambda = 546.1$ nm vibrating parallel to the fiber axis. Notice that the shift is towards the smaller wedge thickness, i.e. $n_L < n_F^{\parallel}$.

Figure (5) shows multiple-beam Fizeau fringes in transmission for light vibrating perpendicular to the fiber axis, and the shift is towards the greater wedge thickness, i.e. $n_L > n_F^{\perp}$.

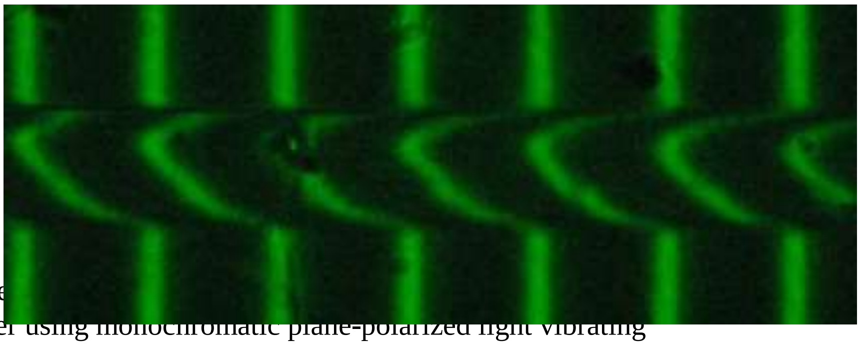


Fig. (4): Interfe
to nylon 6 fiber using monochromatic plane-polarized light vibrating
parallel to the fiber axis.

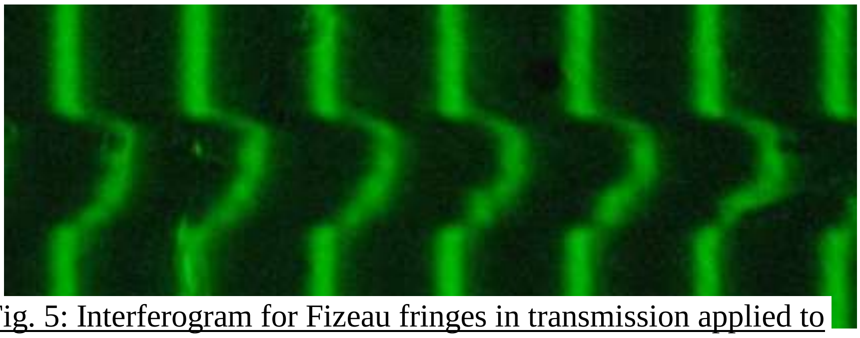


Fig. 5: Interferogram for Fizeau fringes in transmission applied to nylon 6 fiber using monochromatic plane-polarize light viberating perpendicular to the fiber axis.

The principal refractive indices and the birefringence were determined interferometrically, applying the new derived formula Eq. (15).The results are tabulated in tables (1-3).

Table 1: Parallel refractive index of the nylon-6 fibers.

T(°C)	$\frac{dz^{\parallel}}{\Delta z}$	n_L $\pm 5 \times 10^{-4}$	$n^{\parallel}$
30	0.927	1.5325	1.5373
90	0.859	1.5343	1.5393
130	0.283	1.5386	1.5411
160	0.456	1.5386	1.5428
190	2.053	1.5452	1.5563

Table 2: Perpendicular refractive index of the nylon-6 fibers.

T(°C)	$\frac{dz^{\perp}}{\Delta z}$	n_L $\pm 5 \times 10^{-4}$	$n^{\perp}$
30	0.299	1.5325	1.5299
90	0.435	1.5343	1.5316
130	0.89	1.5386	1.5336
160	0.667	1.5386	1.5324
190	0.540	1.5452	1.5420

3.3 Determination of the polarizability P and Cauchy´s constants A and B

The resultant data of the refractive indices for light vibrating parallel and perpendicular to the fiber axis for different wavelengths were utilized in calculating the above optical parameters according to the following equations.

$$\frac{n_a^2 - 1}{n_a^2 + 2} = \frac{4}{3}\pi \text{ P} \qquad (16)$$

$$n_a = A + \frac{B}{\lambda^2} \qquad (17)$$

$$\frac{dn_a}{d\lambda} = -2\frac{B}{\lambda^3} \qquad (18)$$

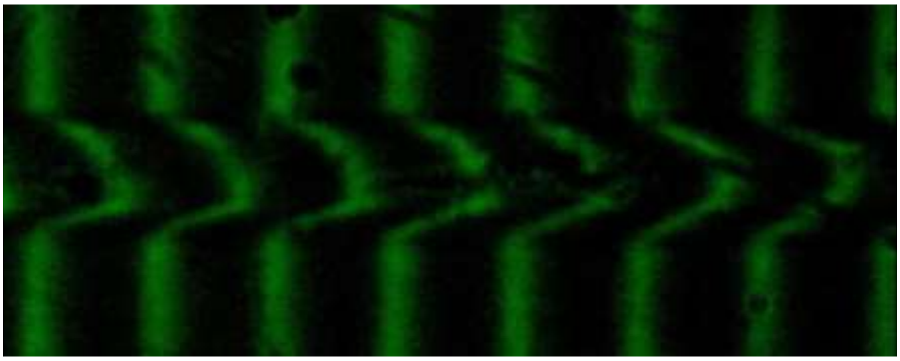
The isotropic refractive index for Nylon-6 (n_{iso}) was calculated from the experimental values, using the formula.

$$n_{iso} = \frac{1}{3}\left(n_f^{\parallel} + 2n_f^{\perp}\right) \tag{19}$$

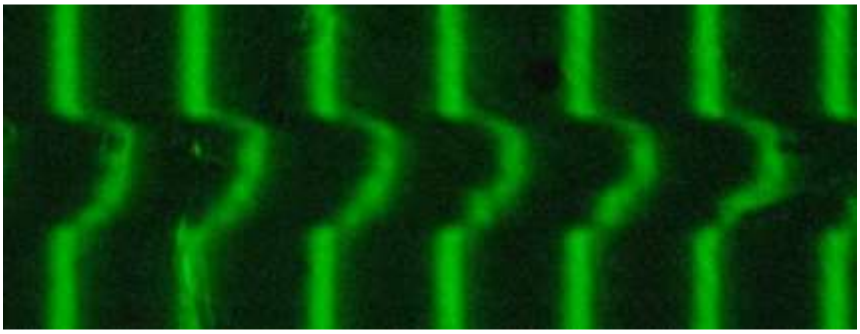
Table 3: Birefringence of the nylon 6 fibers.

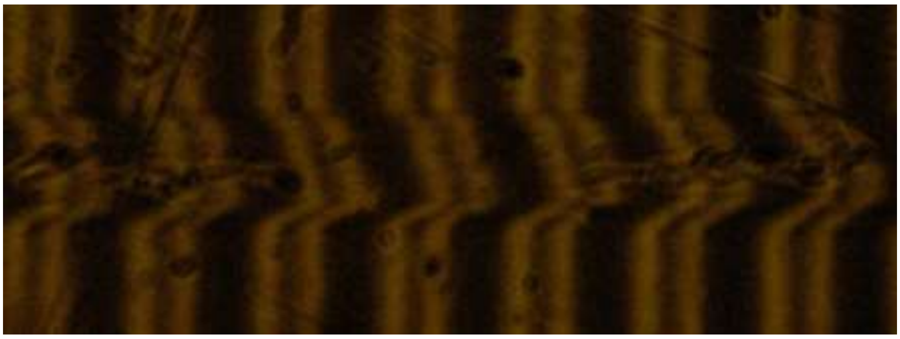
T(°C)	$n_f^{\parallel}$	$n_f^{\perp}$	Δn
30	1.5373	1.5299	0.0074
90	1.5393	1.5316	0.0077
130	1.5411	1.5336	0.0075
160	1.5428	1.5324	0.0104
190	1.5563	1.5420	0.0143

(a)



(b)





(c)

Fig. (6): Multiple-beam Fizeau fringes in transmitted light of wavelength (a) 540 nm, (b) 546.1nm and (c)577,579nm vibrating perpendicular to the fiber axis.

Figure (6) Shows multiple-beam Fizeau fringes in transmission for light vibrating perpendicular to the fiber axis using monochromatic light of wavelengths 540, 546.1,577 and 579 nm, respectively.

Table (4) gives the results of the experimental determination of the refractive indices and birefringence for different wavelengths of light. Figures (7-9) show the relationships between $n_{\parallel}$, $n_{\perp}$, Δn and $1/\lambda^2$; they were constructed to evaluate the constants A and B of Cauchy´s formula, and the results are given in table (5).

According to Eq. (16), (18) and (19), the polarizability and isotropic refractive indices at different wavelengths can be obtained.

Table 4:Values of the mean refractive indices and birefringence at different wavelengths for the nylon-6 fibers at 30°C.

λ (nm)	N		Δn
	$n_f^{\parallel}$	$n_f^{\perp}$	
540	1.5452	1.5365	0.0087
546.1	1.5446	1.5357	0.0089
577	1.5422	1.5332	0.009
579	1.5420	1.5329	0.0091
632.8	1.5391	1.5279	0.0112

Table 5: Values of Cauchy's constants for the nylon-6 fibers.

Constant A		Constant B	
$A^{\parallel}$	$A^{\perp}$	$B^{\parallel}$ (nm^2)	$B^{\perp}$ (nm^2)
1.5226	1.5056	6.54×10^3	9.05×10^3

The average dispersive coefficients are given by:

$$\frac{dn^{\parallel}}{d\lambda} = -8.03 \times 10^{-5} nm^{-1}$$

And

$$\frac{dn^{\perp}}{d\lambda} = -1.11 \times 10^{-4} nm^{-1}$$

The results are tabulated in Tables (6) and (7).

Table 6: Values of the polarizability of the nylon-6 fibers at 30°C applying the Lorentz-Lorenz formula.

λ (nm)	*P*		Δ*p*
	$p^{\parallel}$	$p^{\perp}$	
540	0.0755	0.0745	0.001
546.1	0.0754	0.0744	0.00103
577	0.0751	0.0741	0.00104
579	0.0751	0.074	0.00105
632.8	0.0748	0.0745	0.0013

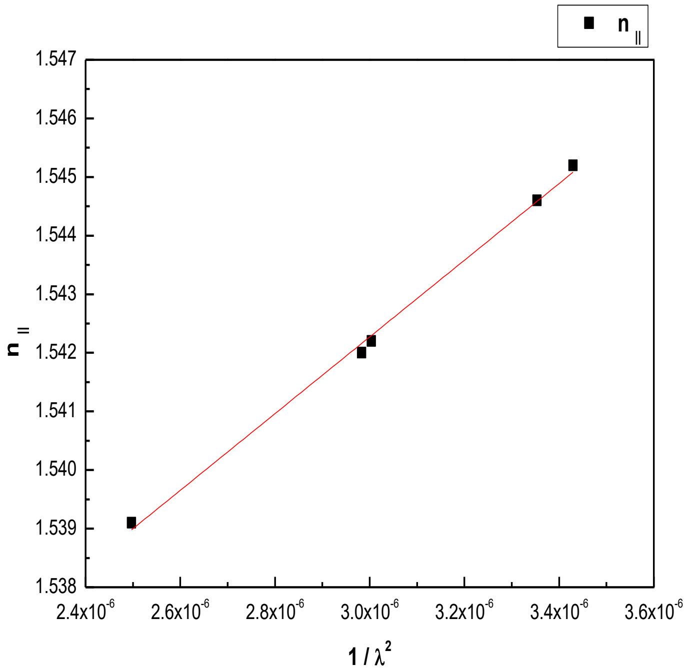


Fig. (7): Variation of the mean refractive index, $n_f^{\parallel}$, with $1/\lambda^2$.

Table 7: Values of the isotropic refractive indices at different wavelengths for the nylon-6 fiber.

λ (nm)	n_{iso}
540	1.5394
546.1	1.5387
577	1.5362
579	1.5359
632.8	1.5316

3.4 Determination of the thermal coefficient of the refractive indices for nylon-6 fibers.

Figure (10(a)) shows an interferogram for light perpendicular to the fiber axis using monochromatic light of wavelength λ=546.1 nm. The temperature of the experiment was 90°C. The experiment was then repeated at 130°C and the fringes were recorded photographically Fig. (10(b)).

Figures (11) and (12) show the effect of the annealing temperature on $n^{\parallel}$ and $n^{\perp}$ for times of annealing 2 hours respectively. The figures show that both of $n^{\parallel}$ and $n^{\perp}$ are increasing with the annealing temperature and show also that the values of $n^{\parallel}$ are larger than that of $n^{\perp}$. It means that the density of the molecular chains which are parallel to the fiber axis exceeds that which is perpendicular to the fiber axis.

Figures 13 and 14 show that $\frac{dn_{\parallel}}{dT}$ and $\frac{dn_{\perp}}{dT}$ are nearly constant in the range T=30°C up to 160°C. but in the range of 160°C up to 190°C increases with increasing the temperature.

4. Discussion

Interferometric studies have made it possible to determine the principal refractive indices of nylon-6 fiber and its birefringence, which are valuable parameters in correlating the structural properties of the fiber with its mechanical properties and, for man-made fibers, with their method of production. The results obtained throw light on the variation of some optical parameters with wavelength, i.e. optical parameters are not a material constant but depend on the wave interactions, orientation and the cross-sectional shape.

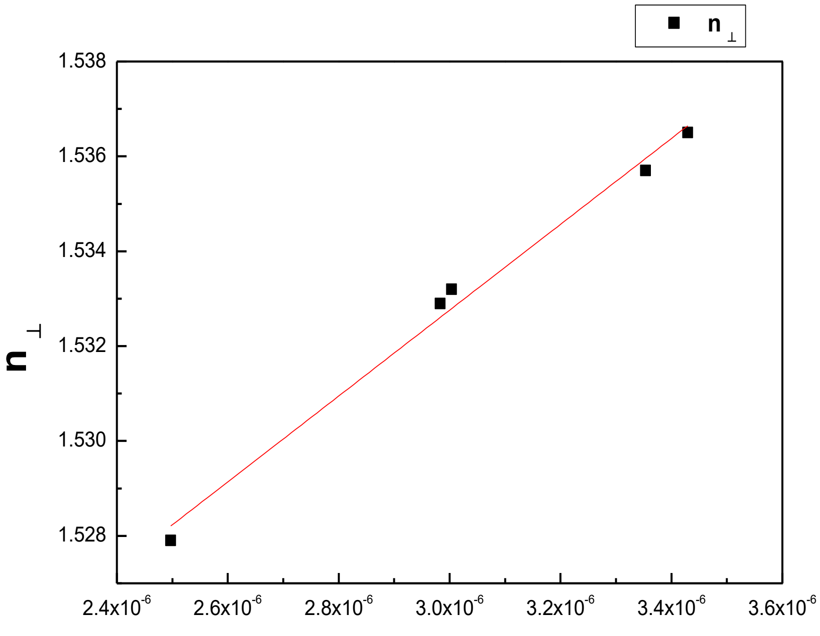


Fig. (8): Variation of the mean refractive index, $n_f^{\perp}$, with $^1/_{\lambda^2}$.

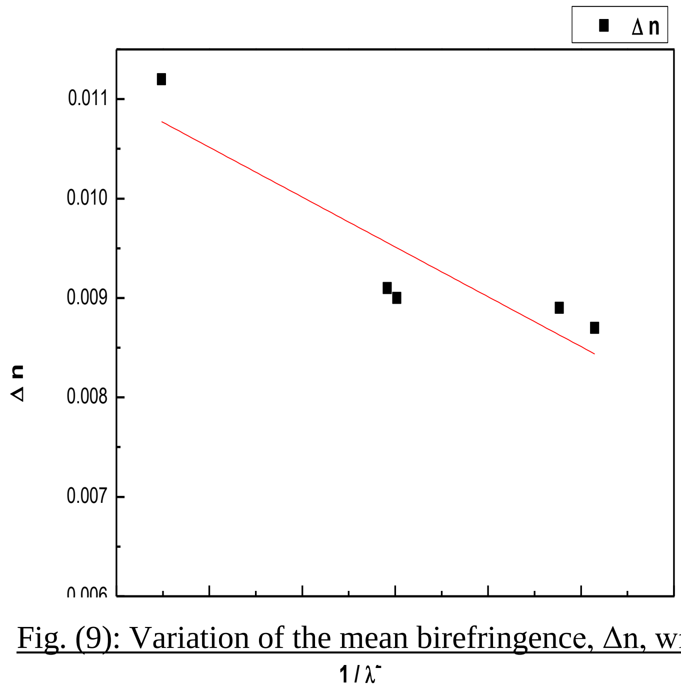
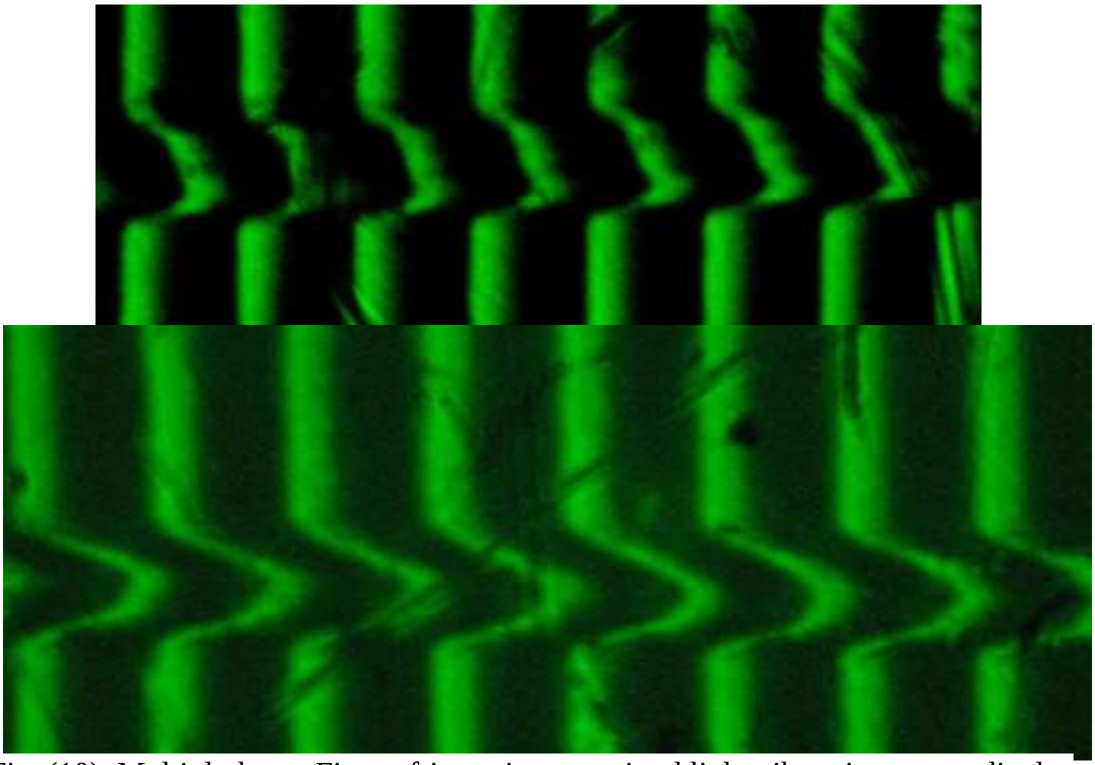


Fig. (9): Variation of the mean birefringence, Δn, with $^1/_{\lambda^2}$.



(a)

(b)

Fig. (10): Multiple-beam Fizeau fringes in transmitted light viberating perpendicular to the fiber axis at temperature of (a) 90 and (b)130°C.

5.

Anew mathematical formula describing nearly triangular cross-sectional shape was derived. This formula can be used for the determination of the mean principal refractive indices and birefringence of Nylon-6 fibers with fair accuracy, compared with previously published data(20-24).

Variations in the shape of the observed fringes are due to the shape of the cross-section and to the mode of the fiber inside the silvered liquid wedge. It appears that analysis of Fizeau fringes of nearly triangular cross-section requires further studies going beyond the assumption that the fiber is a single medium.

Also as there are variations in density due to the change in isotropic refractive index n_{iso} also the crystallinity and crystalline parameters of the fiber material varied systematically with different annealing conditions

We conclude from the results and considerations that there are practical importance for these measurements since they provide acceptable results for opto-thermal parameters. Also reorientation of Nylon-6 fiber may occur not only during manufacturing but also due to the annealing process.

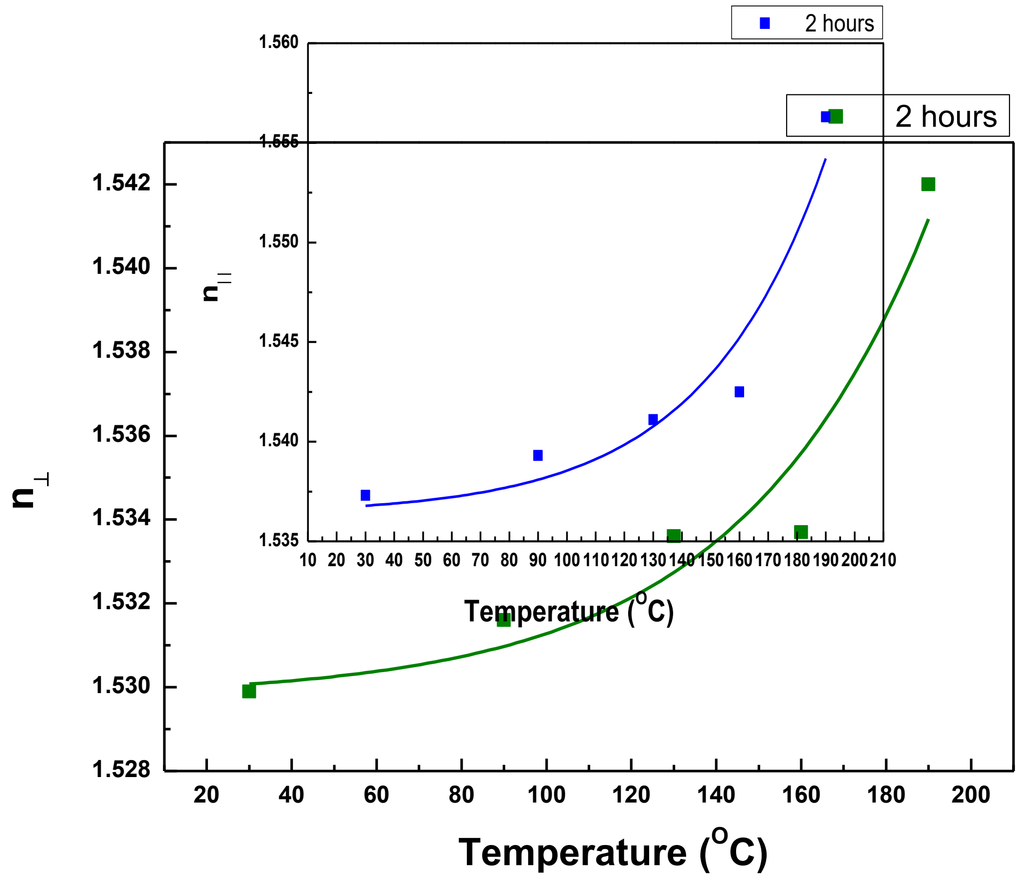
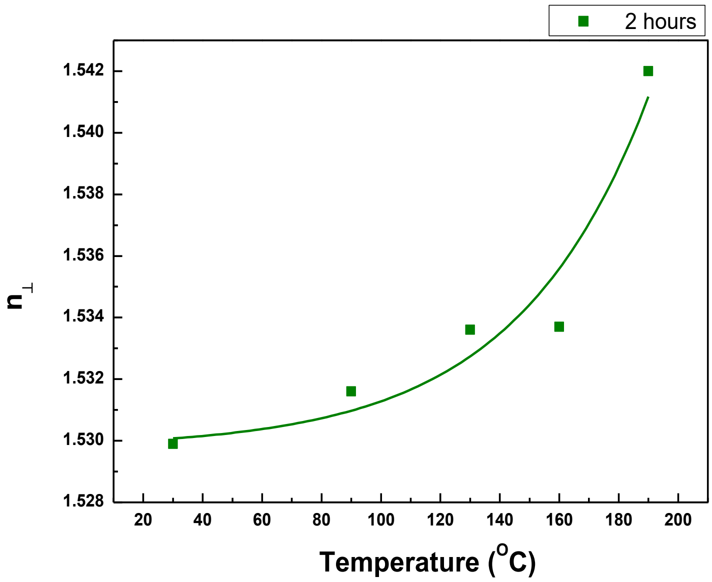


Fig. (11): Variation of the mean refractive index, $n_f^{\prime}$, with Temperature.



(b)

Fig. (12): Variation of the mean refractive index, $n_f^{\perp}$, with Temperature.

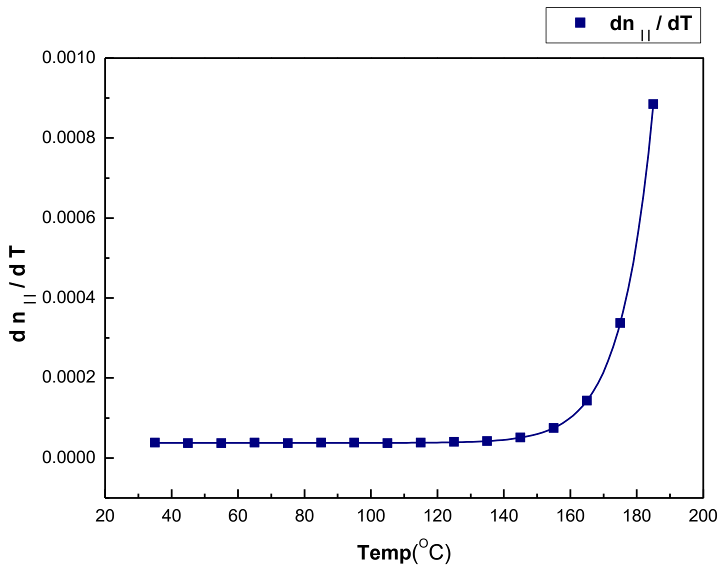


Fig. (13): Variation of $\frac{dn_{\parallel}}{dT}$ with Temperature.

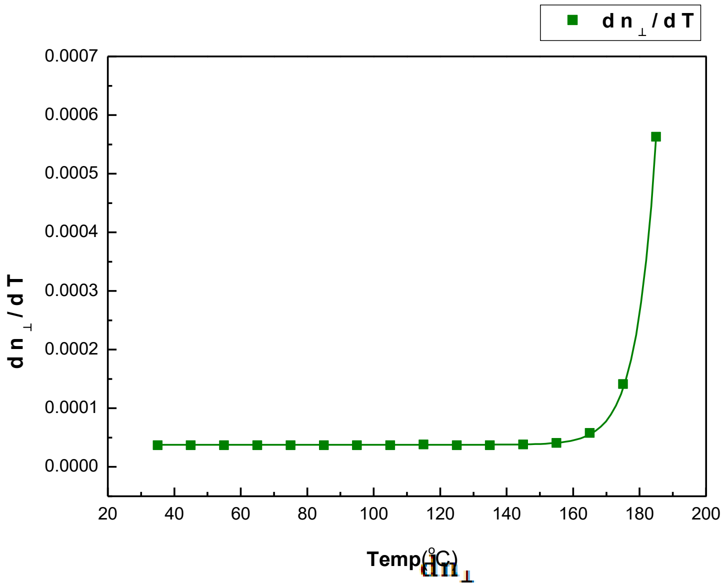


Fig. (14): Variation of $\frac{dn_{\perp}}{dT}$ with Temperature.

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