Dependence of Displacement Measurements on Surface Roughness

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The effect of surface roughness on the accuracy of displacement determination of an object in digital double exposure technique was studied. From the Young's fringes, it was observed, that the higher the rms of the roughness and the smaller the displacement of the surface the greater is the accuracy of the displacement determination.

1. Introduction:

Digital double exposure speckle photography [9] is a simple powerful method for measuring in-plane displacement and surface rotation [2]. The basic technique is to take a double exposure photograph of an object displaced between the exposures with a divergent laser beam. In this paper the doubled exposured in –plane displaced rough surface was recorded on the computer memory rather than on a photographic film. The far-field diffraction pattern, which is the Fourier transform of the doubly exposured image, consists of a speckled diffraction halo, modulated by cosine-squared fringes (Young's fringes) [10]. The fringes spacing is inversely proportional to the displacement of the speckle pattern, and lie perpendicular to the displacement vector. Since the determination of the displacement is a function of the fringe spacing [7], therefore the visibility of the fringes which is a function of the surface roughness, is an important factor in the accuracy of its determination [3].

This work deals with finding a relation between the accuracy of the displacement measurements and the surface roughness.

2. Theoretical Principle of Double Exposure Technique

The measurement of rms surface roughness [1] can be done in many different ways. One approach is to observe the fringe visibility of the test surface using the double exposure technique. Let the test surface in plane (x,y) be illuminated with a laser beam and the speckle pattern is recorded on a photographic plate H [8]. The speckle on H is described by a function D(x,y),which represents the intensity of the light on H. The transmitted amplitude t(x,y) of the negative H is given by

$$t(x,y) = a - bD(x,y) \tag{1}$$

where a and b are constants.

When two equal exposures are made, for the object, which is shifted by a distance x_0 between the exposures, the final recorded $D_t(x,y)$ is the sum of recorded D-values in each exposure and is equal to

$$D_t(x,y) = D(x,y) + D(x - x_0, y) \tag{2}$$

transmission of the negative is given by:

$$t(x,y) = a - bD(x,y) \otimes [\delta(x,y) + \delta(x - x_0, y)] \tag{3}$$

When the negative H, which is placed in the front of the focal plane of a lens, and is illuminated with a collimated beam of wavelength λ, the Fourier transform of the amplitude transmission t(x,y) of H is obtained in the back focal plane of the lens (Fourier transform plane) and is given by

$$\tilde{t}(u,v) = a\delta(u,v) - b\tilde{D}(u,v)[1 + \exp(i\pi u x_0 / \lambda)] \tag{4}$$

where the tilded variables represent the Fourier transform and (u,v) are the angular coordinates of a point in the focal plane. The first term $a\delta(u,v)$ is the direct part of the spectrum; it represents homogenous illumination which can be attributed to point source located at infinity when diffraction effects are neglected. $\tilde{D}(u,v)$ in the second term is the Fourier transform of $D(x,y)$. It is modulated by $[1 + \exp(i\pi u x_0 / \lambda)]$. Since $D(x,y)$ contains fine structures, its transform $\tilde{D}(u,v)$ spreads out considerably in the focal plane. Like $D(x,y)$, the transform $\tilde{D}(u,v)$ has speckle patterns. If the DC part of the spectrum is neglected, then the intensity in the Fourier transform plane is

$$I = \left|\tilde{D}(u,v)\right|^2 [1 + \exp(i\pi u x_0 / \lambda)]^2 = \left|\tilde{D}(u,v)\right|^2 \cos^2(\frac{\pi u x_0}{\lambda}) \tag{5}$$

The diffused background $\left|\tilde{D}(u,v)\right|^2$ is modulated by $\cos^2(\pi u x_0 / \lambda)$, which represents Young's fringes. The angular distance between two consecutive bright (or dark) fringes is equal to λ/x_0 .

The experimental work is devoted to study the effect of the surface roughness on the visibility V of I given by eq. (5).

3. Experimental Method and Results:

Figure (1) shows the experimental arrangement [11]. Spatially coherent light from an unpolarized He-Ne laser source is employed after being expanded and collimated for illuminating a transparent test surface. To study the effect of surface roughness on the accuracy of the determination of the displacement, different test surfaces with different rms surface roughness were used. The test surfaces were prepared by polishing the test glasses with emery powders. The roughness of the test surface was precisely mechanically measured beforehand by using a stylus instrument. The sensor of the webcam camera has received directly the two speckle pattern before and after displacing the object by means of a precision micrometer.

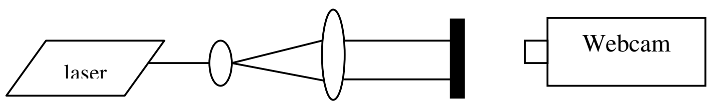

Fig. (1): The Experimental Arrangement

To study the effect of surface roughness on the accuracy of the displacement [11], the obtained signals were digitally stored on the H.D of a pc-computer. The Fourier transform of the added doubly exposed speckle pattern was calculated by means of software program (Image J program).

Fig.(2): The Fourier transform of the added doubly exposed speckle pattern obtained from a rough surface having rms of 9.6 µm and displaced 150 µm.

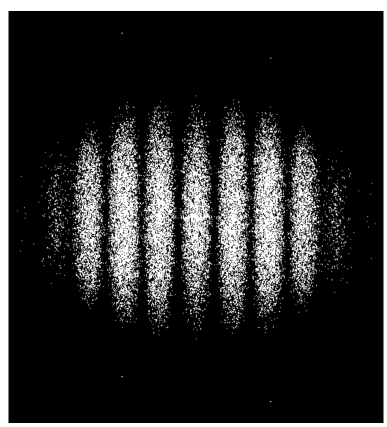

Figure (2) shows a sample of the obtained Fourier transform of a 150 µm displaced rough surface having a rms roughness of 9.6 µm. The figure shows, as expected, the fringes of Young double slit. By varying the displacement of the same sample the visibility for each displacement was calculated according to the relation,

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \qquad (6)$$

where I_{max} and I_{min} were obtained from the plotted profile of the Young's fringes given in (Fig.3).

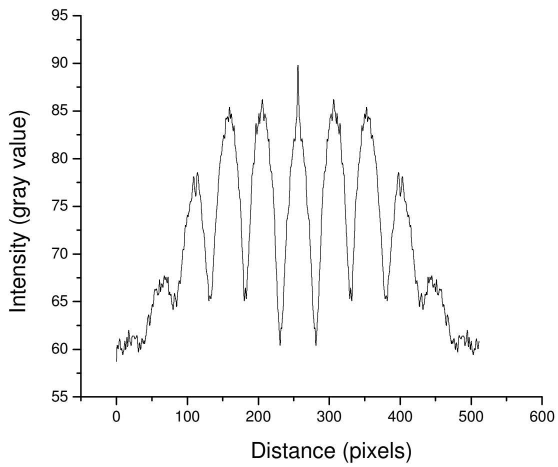

Fig. (3): the plotted profile of fig.2.

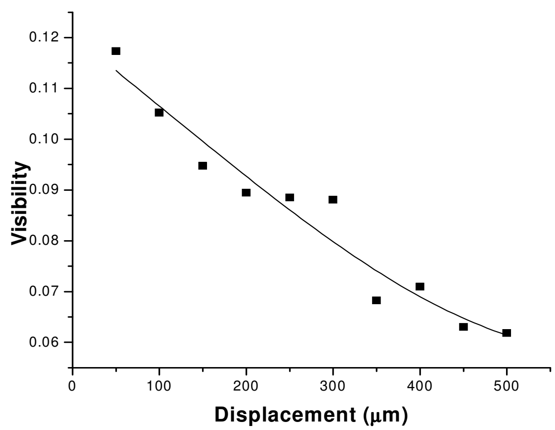

Fig. (4): The relationship between the visibility and the displacement for rough surface having roughness of rms 2.48 µm.

Figure (4) shows the visibility as a function of the displacement of a surface having a roughness of rms 2.48 µm. The figure shows that the visibility decreases as the displacement increases. This behavior is due to the decrease of the fringe spacing by increasing the displacement of the object. According to [13] when the fringe spacing becomes comparable to the speckle size, the fringe visibility decreases and drops to zero when they become equal.

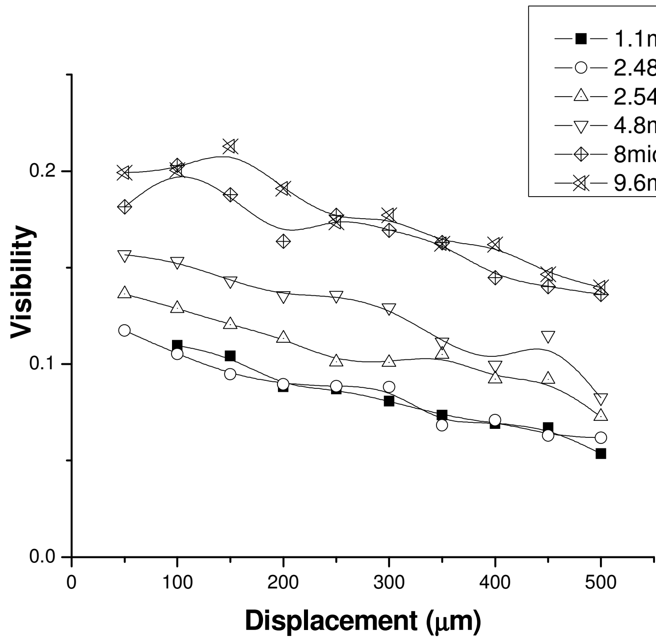

Fig. (5): The relation between the visibility and displacement with different surface roughness as a parameter.

Figure (5) shows the same relation as that demonstrated in Fig. (4) but with the roughness as a parameter. From relation (The angular distance = λ/x_0) it evident that the displacement of the object can be obtained from the fringe spacing and since its determination dependence on the location of the max of the fringe and this inturn dependence on the sharpness of contrast, therefore it is evident, as seen from the figure that the accuracy of the determination of the displacement increases by decreasing the displacement and increasing the surface roughness.

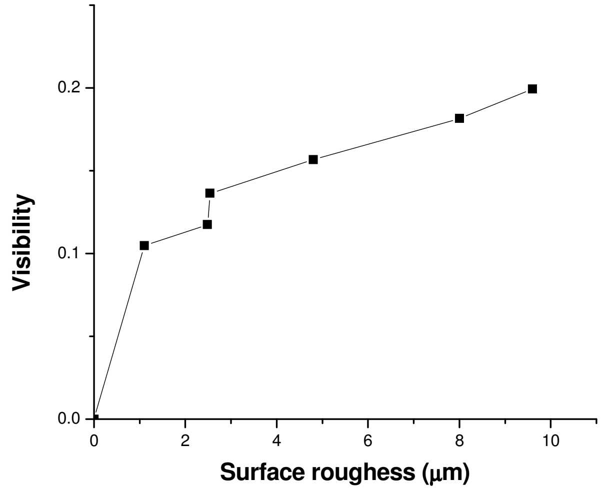

Fig. (6): The visibility versus surface roughness for a constant displacement equal to 50 μm

Figure (6) shows the relation between the visibility and the surface roughness at 50 μm displacement. This behavior can be explained as follows: in case of a surface without roughness with infinite dimension, the transmitted wave is a plane one as the illuminating wave was plane. This will lead to a humongous illumination and the visibility of the fringes in this case will be equal to zero [1]. As the dimension of the surface become finite, one gets the diffraction patterns of a single obstacle, which shows a bad visibility as the dimensions are great compared to the wave length of the illuminating radiation. By introducing roughness on the surface more and more, spherical waves will be transmitted as the rms of the roughness increases. These waves will superimpose to give great difference between maximum and minimum leading to increase of the visibility.

4. Conclusion:

The presented work shows that the visibility of the Young's fringes in the used technique is increased by increasing the roughness and decreasing the displacement. Since the accuracy of the determination of the displacement increases by increasing the visibility [3, 6], therefore the accuracy increases by increasing the roughness.

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