The Effect of Anharmonic Potential on the Planar Channeling Radiation of Positrons in the Disordered fcc Single Crystals

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Planar channeling radiation characteristics like, transition energies and transition probabilities were investigated against the planar channeling potential with higher anharmonic terms. The calculations have been performed, by using first-order perturbation theory, to fcc single crystals the lattices and disordered lattices by dumb-bell configuration (DBC) or by body-centered interstitial (BCI) , and has been executed for relativistic positrons at energies (10 - 500) MeV channeled in copper single crystal in the planar direction (100). It has been found that, by using higher anharmonic terms in the channeling potential, a new transition line at $\Delta n = \pm 5$. has been detected. In addition, the calculations showed that, the effect of higher anharmonic terms on channeling radiation characteristics is very significant for transitions at higher bound states, and the significance decreases for transitions at lower bound states.

1. Introduction:

In the case of planar channeling of relativistic positively charged particles through crystals, the charged particles are influenced by a transverse planar potential field of the crystal. Due to this field, the charged particles execute a periodic motion in a definite eigenstates, and radiation results from spontaneous transitions between these states. This radiation was predicted by Kumakhov [1,2] and is called now channeling radiation. He also developed the theory of this phenomena [1-5]. The maximum number of bound states of the transverse motion and the eigenfunctions of the channeling particle is determined from the planar channeling potential field in the crystal. A suitable expression , based on Lindhard's model, for the net planar channeling potential $V(x)$ has been used [6], and to be more controllable, this expression has been expanded in a series form around $x = 0$, where x is measured from the midpoint between the two planes, and the series expansion includes x^2, x^4, x^6, x^8,…… terms.

In previous calculations, the eigenvalues of the bound states and the eigenfunctions of the channeled particles have been obtained in harmonic approximation of the planar potential (up to x^2 term in the series expansion) and in anharmonic approximation (up to x^4 term) by using the first-order perturbation theory [7,8]. It has been found that, in the harmonic approximation, the allowed transitions occur at $\Delta n = \pm 1$, while in the case of anharmonic up to x^4 term, the allowed transitions occur at $\Delta n = \pm 1, \pm 3$. As an extension to that previous work, this work was devoted to calculations extended to include higher anharmonics up to x^6 term. Now, in the present calculations for anharmonics up to x^6 term we obtained , in addition to the transitions $\Delta n = \pm 1, \pm 3$, a new transition lines at $\Delta n = \pm 5$.

As one of the most important applications of channeling phenomena is the crystal defect studies, so this work was concerned to investigate the radiation characteristics in disordered lattices. The calculations have been executed by using a planar potential function including harmonic and higher anharmonic terms, as a more accurate expression of channeling potential, for relativistic channeled positrons (10-500) MeV in copper single crystal in the planar direction (100) in normal lattice and disordered lattice by dumb-bell configuration (DBC) or by body centered interstitial (BCI). The channeling potential in fcc single crystals disordered by DBC or by BCI has been calculated, in previous work [9].

2. Radiation Characteristics from channeled positrons

The channeled particle is assumed to travel in the x-z plane with only the z component of velocity relativistic where we choose a coordinate frame in which the x axis is normal to the crystalline planes between which the particle is channeling. The Shrödinger wave equation governing the transverse motion of a particle with rest mass m_0 is

$$-\frac{\hbar^2}{2m_0\gamma}\frac{d^2\psi(x)}{dx^2}+V(x)\psi(x)=E\psi(x) \qquad (1)$$

where E is the energy of the transverse motion, $\gamma = (1-\beta^2)^{-1/2}$ where $\beta = v/c$ and v is the particle velocity and c is the velocity of light.
We consider the net planar channeling potential [6]

$$V(x) = 2\pi z_1 z_2 e^2 Ca^2 Nd_p \left(\frac{1}{l+a-x} + \frac{1}{l+a+x} \right) \quad (2)$$

This can be expanded around $x = 0$, where x is measured from the midpoint between the two planes and $l = d_p/2$. We get

$$V(x) = V_0 + \frac{1}{2}k_1 x^2 + \frac{1}{4}k_2 x^4 + \frac{1}{6}k_3 x^6 + \frac{1}{8}k_4 x^8 + \cdots\cdots \quad (3)$$

where

$$V_0 = \frac{4\pi z_1 z_2 e^2 Ca^2 Nd_p}{(l+a)}$$

$$k_1 = \frac{2V_0}{(l+a)^2}\ ,\ k_2 = \frac{4V_0}{(l+a)^4}\ ,\ k_3 = \frac{6V_0}{(l+a)^6}\ ,\ldots\ldots\ldots,\ k_n = \frac{2nV_0}{(l+a)^{2n}},\ldots..$$

In the above equations d_p is the interplanar distance, a is the Thomas-Fermi screening radius, C is the Lindhard constant given by $\sqrt{3}$, z_1 and z_2 are the charge numbers of the channeled particle and the crystal atoms respectively, and N is the atomic bulk density, $N = n_c/(\text{lattice constant})^3$, where n_c the number of atoms per unit cell.

By using the series expansion of the potential function $V(x)$ as given by Eq.(3), we can solve Eq.(1) to get the eigenvalues and eigenstates of the channeled particle. Therefrom, the frequency of the emitted radiation can be determined and also the probability of transition from state i to state j is determined from the matrix element of the x- position operator x_{ij} (the dipole matrix element). Accordingly, the energy of the emitted radiation due to transitions between different states may be calculated.

Solution of Eq.(1) in harmonic oscillator approximation gives the eigenstates and the eigenvalues respectively as [8]:

$$\psi_n(x) = \left(\frac{\alpha}{\sqrt{\pi}\, 2^n n!} \right)^{1/2} H_n(\alpha x) \exp(-\alpha^2 x^2/2) \quad (4)$$

$$E_n = V_0 + (n + \frac{1}{2})\hbar\omega_0 \quad (5)$$

where

$\alpha = \left(\frac{\gamma m_0 k_1}{\hbar^2}\right)^{1/4}$ and $\omega_0 = \left(\frac{k_1}{\gamma m_0}\right)^{1/2}$, where ω_0 is the angular frequency of the particle motion and $H_n(\alpha x)$ is the Hermite polynomials of the n$^{\underline{th}}$ degree.

The frequency of the emitted radiation is significantly increases due to Doppler effect [4,7] and is given (in the laboratory frame) as a function of θ, the angle of emission with respect to the incident beam direction, by :

$$\omega(\theta) = \frac{\omega_0}{1 - \beta\cos\theta} \tag{6}$$

where $\beta^2 = v^2/c^2 = (v_x^2 + v_z^2)/c^2$, v_x and v_z are the *x*- and *z*-components of the particle velocity. In the dipole approximation [10] where $\frac{v_x^2}{c^2} \langle\langle 1 - \frac{v_z^2}{c^2} \langle\langle 1$ we have $\beta = \frac{v}{c} \approx \frac{v_z}{c} \approx 1$ and $\gamma = (1 - \frac{v_z^2}{c^2})^{-1/2} \rangle\rangle 1$.

The non-zero matrix elements in the harmonic oscillator approximation are $x_{n,n\pm 1}$ where

$$x_{n,n-1} = (n\hbar\omega_0 / 2k_1)^{1/2} \tag{7}$$

where $n = 0,1,2,\ldots,n_{max}$. Here n_{max} is the largest value of n for which the energy eigenvalue (in harmonic oscillator approximation) is less than the interplanar barrier potential. Then n_{max} can be calculated using the equation

$$(n_{max} + \frac{1}{2})\hbar\omega_0 = \frac{1}{2} k_1 x_{max}^2 \tag{8}$$

where $x_{max} = d_p/2 - a$

From Eqs.(5) and (6), the emitted photon energy in the laboratory frame is given by:

$$\hbar\omega_{n,n-1} = \frac{\hbar\omega_0}{1 - \beta\cos\theta} \tag{9}$$

From the above formal treatment we can calculate the transition probability from state i to state j per unit time for spontaneous emission from the equation [8]

$$p_{ij} = \frac{4e^2\omega_{ij}^3}{3\hbar c^3}\left|\langle i|x|j\rangle\right|^2 \qquad (10)$$

where $\omega_{ij} = (E_i - E_j)/\hbar$, is the emitted photon frequency in the laboratory frame in the forward direction ($\theta = 0$) and $\langle i|x|j\rangle$ is the dipole matrix element.

3. The effect of anharmonic potential

From the fitted planar potential given by Eq. (3), we can investigate the effect of anharmonic terms on the planar potential. It can be seen that, the effect of x^4 term exceeds the harmonic approximation by ≈ 18 % for Cu (100) at $x = x_{max}$ ($x_{max} = d_p/2 - a$) , and if, in addition , we include also x^6 term , this exceeds the harmonic potential by ≈ 27.4 %. While at $x = x_{max}/2$, the total effect of x^4 and x^6 terms exceeds the harmonic potential by ≈ 1.7 % . Thus we expect the higher-lying quantum states to show significant effects due to the potential anharmonicity while the effect on the lower quantum states can be neglected. Now we will investigate the anharmonic effects by using first–order perturbation theory with an applied perturbation $\frac{1}{4}k_2x^4$ and $(\frac{1}{4}k_2x^4 + \frac{1}{6}k_3x^6)$ respectively. In the following discussion, the fitted channeling potential as given by Eq. (3), up to x^4 term will be denoted by anharmonic(a) potential, and up to x^6 term by anharmonic(b) potential.

By applying the perturbation $\frac{1}{4}k_2x^4$, the first-order wave functions and energy eigenvalues (laboratory frame) are

$$\begin{aligned}|\psi_n\rangle = |n\rangle + \frac{\varepsilon}{48}\{&[n(n-1)(n-2)(n-3)]^{1/2}|n-4\rangle \\ &+4(2n-1)[n(n-1)]^{1/2}|n-2\rangle \\ &-4(2n+3)[(n+1)(n+2)]^{1/2}|n+2\rangle \\ &-[(n+1)(n+2)(n+3)(n+4)]^{1/2}|n+4\rangle\ \}\end{aligned} \qquad (11)$$

and

$$E_n = V_0 + \hbar\omega_0\left[(n+\frac{1}{2}) + \frac{\varepsilon}{4}(2n^2+2n+1)\right] \qquad (12)$$

where $|j\rangle$ labels the eigenfunctions of the harmonic oscillator (given by Eq. (4)) and $\varepsilon = 3\hbar k_2 / 4\gamma m_0 \omega_0 k_1$.

To the first order in ε, it can be shown from Eq. (11) that, the non-zero dipole matrix elements are $x_{n,n\pm 1}$, $x_{n,n\pm 3}$. Where

$$x_{n,n-1} = \left(\frac{\hbar^2 n^2}{4\gamma m_0 k_1}\right)^{1/4}\left(1-\frac{\varepsilon}{2}n\right) \qquad (13)$$

and

$$x_{n.n-3} = \frac{\varepsilon}{12}\left(\frac{\hbar^2}{4\gamma m_0 k_1}\right)^{1/4}\left[n(n-1)(n-2)\right]^{1/2} \qquad (14)$$

From Eq. (12), the emitted photon energies, in the laboratory frame, are

$$\hbar\omega_{n,n-1} = \frac{\hbar\omega_0}{1-\beta\cos\theta}(1+n\varepsilon) \qquad (15)$$

and

$$\hbar\omega_{n,n-3} = \frac{3\hbar\omega_0}{1-\beta\cos\theta}[1+(n-1)\varepsilon] \qquad (16)$$

The maximum number of bound states can be calculated from the equation

$$\hbar\omega_0\left[(n_{\max}+\frac{1}{2}) + \frac{\varepsilon}{4}(2n_{\max}^2+2n_{\max}+1)\right] = \frac{1}{2}k_1 x_{\max}^2 + \frac{1}{4}k_2 x_{\max}^4 \qquad (17)$$

Now by applying the perturbation $(\frac{1}{4}k_2 x^4 + \frac{1}{6}k_3 x^6)$, the first-order wave functions and energy eigenvalues (laboratory frame) are given by

$$\begin{aligned}|\psi_n\rangle = |n\rangle &+ \frac{\varepsilon_1}{288}\left[n(n-1)(n-2)(n-3)(n-4)(n-5)\right]^{1/2}|n-6\rangle \\ &+ \frac{1}{192}\left[n(n-1)(n-2)(n-3)\right]^{1/2}\left[4\varepsilon + 3\varepsilon_1(2n-3)\right]|n-4\rangle \\ &+ \frac{1}{96}\left[n(n-1)\right]^{1/2}\left[8\varepsilon(2n-1) + 15\varepsilon_1(n^2-n+3)\right]|n-2\rangle \\ &- \frac{1}{96}\left[(n+1)(n+2)\right]^{1/2}\left[8\varepsilon(2n+3) + 15\varepsilon_1(n^2+3n+3)\right]|n+2\rangle \\ &- \frac{1}{192}\left[(n+1)(n+2)(n+3)(n+4)\right]^{1/2}\left[4\varepsilon + \varepsilon_1(2n+5)\right]|n+4\rangle \\ &- \frac{\varepsilon_1}{288}\left[(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)\right]^{1/2}|n+6\rangle\end{aligned} \tag{18}$$

and

$$E_n = V_0 + \hbar\omega_0\left[(n+\frac{1}{2}) + \frac{\varepsilon}{4}(2n^2+2n+1) + \frac{5\varepsilon_1}{48}(4n^3+6n^2+8n+3)\right] \tag{19}$$

where $\varepsilon_1 = k_3\hbar^2/k_1^2\gamma\, m_0$.

To the first order in ε and ε_1, it can be shown from Eq. (18) that, the non-zero dipole matrix elements are $x_{n,n\pm1}$ *and* $x_{n,n\pm3}$, $x_{n,n\pm5}$ and Where

$$x_{n,n-1} = \left(\frac{\hbar^2 n^2}{4\gamma\, m_0 k_1}\right)^{1/4}\left\{1 - \frac{1}{16}\left[8\varepsilon n + 5\varepsilon_1(2n^2-1)\right]\right\} \tag{20}$$

$$x_{n,n-3} = \frac{1}{48}\left(\frac{\hbar^2}{4\gamma\, m_0 k_1}\right)^{1/4}\left[n(n-1)(n-2)\right]^{1/2}\left[4\varepsilon + \varepsilon_1(n^2+13n-8)\right] \tag{21}$$

$$x_{n,n-5} = \frac{\varepsilon_1}{48}\left(\frac{\hbar^2}{4\gamma m_0 k_1}\right)^{1/4}\left[n(n-1)(n-2)(n-3)(n-4)\right]^{1/2}\left[7n + (n-2)^{1/2} - 11\right] \tag{22}$$

From Eq. (19), the emitted photon energies, in the laboratory frame, are

$$\hbar\omega_{n,n-1} = \frac{\hbar\omega_0}{1-\beta\cos\theta}\left[1 + n\varepsilon + \frac{5}{8}\varepsilon_1(2n^2+1)\right] \tag{23}$$

$$\hbar\omega_{n,n-3} = \frac{3\hbar\omega_0}{1-\beta\cos\theta}\left[1 + \varepsilon(n-1) + \frac{5}{24}\varepsilon_1(6n^2-12n+13)\right] \tag{24}$$

$$\hbar\omega_{n,n-5} = \frac{5\hbar\omega_0}{1-\beta\cos\theta}\left[1 + \varepsilon(n-2) + \frac{5}{8}\varepsilon_1(2n^2-8n+13)\right] \tag{25}$$

The maximum number of bound states can be calculated from the equation

$$\begin{aligned}\hbar\omega_0\left[(n_{\max}+\frac{1}{2})+\frac{\varepsilon}{4}(2n_{\max}^2+2n_{\max}+1)+\frac{5\varepsilon_1}{48}(4n_{\max}^3+6n_{\max}^2+8n_{\max}+3)\right]&\\ =\frac{1}{2}k_1x_{\max}^2+\frac{1}{4}k_2x_{\max}^4+\frac{1}{6}k_3x_{\max}^6&\end{aligned} \tag{26}$$

The maximum number of bound states calculated by using harmonic, anharmonic(a) and anharmonic(b) potentials for positron channeled in the (100) planar direction in copper was given in Table (1) at different values of positron energy. And to illustrate the effect of lattice disorder, the maximum number of bound states was given in Table (2) at different values for positron energy by using anharmonic(b) potential for normal lattice, disordered lattice by DBC or BCI .

Table (1): Maximum number of bound states, n_{max}, for channeled positrons in Cu single crystal in the planar direction (100) at different values for positron energy for normal lattice by using harmonic, anharmonic(a) and anharmonic(b) potential.

Positron energy (MeV)	10	15	20	25	50	100	200	500
n_{max} for harmonic	3	4	5	5	7	10	15	23
n_{max} for anharmonic(a)	4	5	6	6	9	13	18	29
n_{max} for anharmonic(b)	4	5	6	7	10	13	19	30

Table (2): Maximum number of bound states, n_{max}, for channeled positrons in Cu single crystal in the planar direction (100) at different values for positron energy by using anharmonic(b) potential for normal lattice, disordered lattice by DBC or BCI .

Positron energy (MeV)	10	15	20	25	50	100	200	500
n_{max} for normal lattice	4	5	6	7	10	13	19	30
n_{max} for BCI lattice	5	6	7	8	11	15	21	34
n_{max} for DBC lattice	6	7	9	10	13	19	27	42

To illustrate the anharmonic effects on the channeling radiation characteristics, the emitted photon energies due to different allowed transitions were plotted for harmonic, anharmonic(a) and anharmonic(b) potentials respectively. For positrons with $E = 50$ MeV channeled in copper in the planar direction (100) are presented the emitted photon energy in the forward direction due to the allowed transitions *n,n-1* in Fig.(1) and for transions *n,n-3* in Fig.(2) respectively. Its evident from Fig.(1) and Fig.(2) that the higher quantum states (i.e., states at $n = n_{max}$) is highly affected, while the effect at the lower states due to the potential anharmonicity is very small.

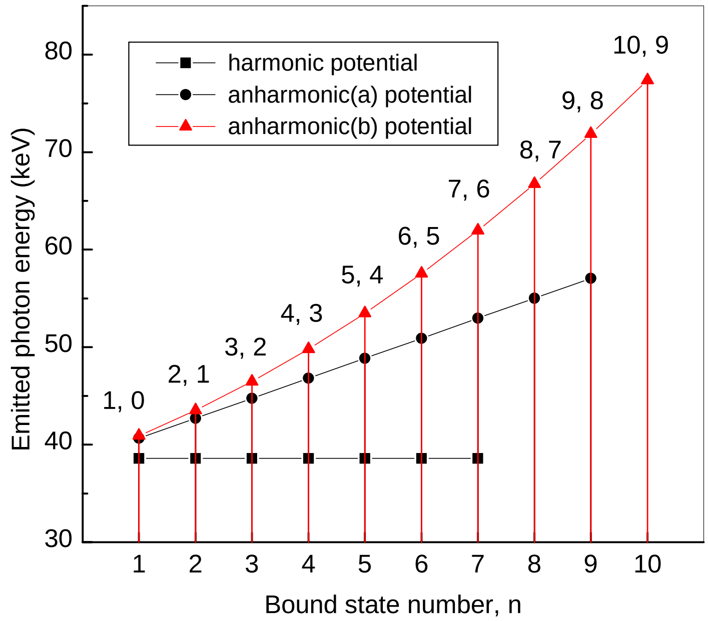

Fig. (1): The emitted photon energy due to the allowed transitions *n,n-1* for positrons with $E = 50$ MeV channeled in copper in the planar direction (100) by using planar potential in harmonic approximation, anharmonic(a) and anharmonic(b) respectively.

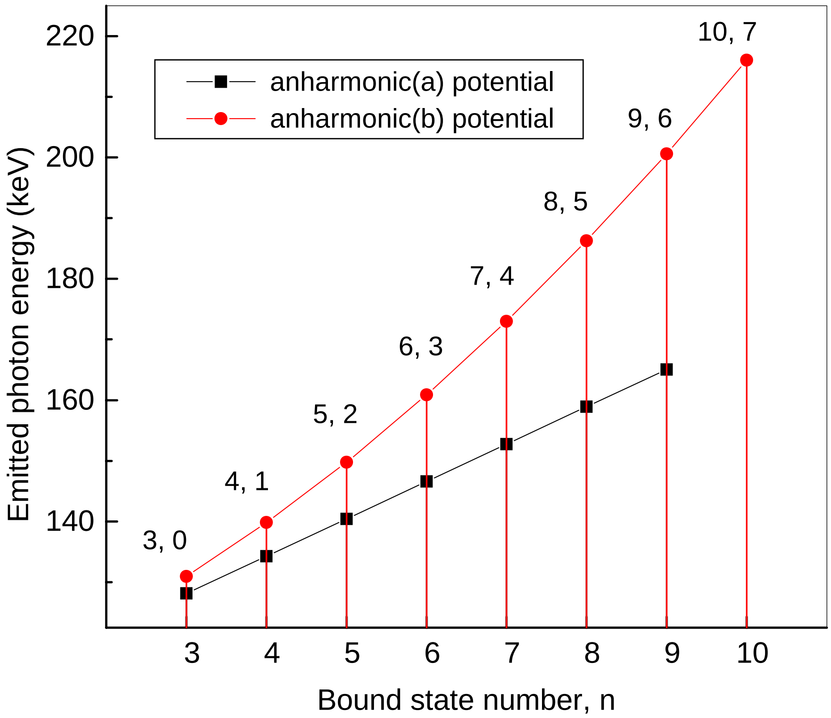

Fig. (2): The emitted photon energy due to the allowed transitions *n,n-3* for positrons with E = 50 MeV channeled in copper in the planar direction (100) by using planar potential in anharmonic(a) and anharmonic(b) approximations respectively.

Now we find that by using the anharmonic(b) potential we obtained a new allowed transitions *n,n-5* given from Eq. (22) with emitted radiation frequencies given from Eq.(25). So, in the following, we will use the anharmonic(b) potential in the emitted radiation calculations in order to cover all possible allowed transitions.

The total emitted photon energy due to the new transitions n,n-5 between higher quantum states are presented in Fig.(3) as a function of positron energy up to 50 MeV. The transition probability per unit time for spontaneous emission as a function of the emitted photon energy (in the forward direction) due to transitions n,n-5 are presented in Fig.(4) for positrons with E = 50 MeV channeled in copper in the planar direction (100). It is found that, the highest transition probability occurs at higher bound states i.e., 10→5 transition line.

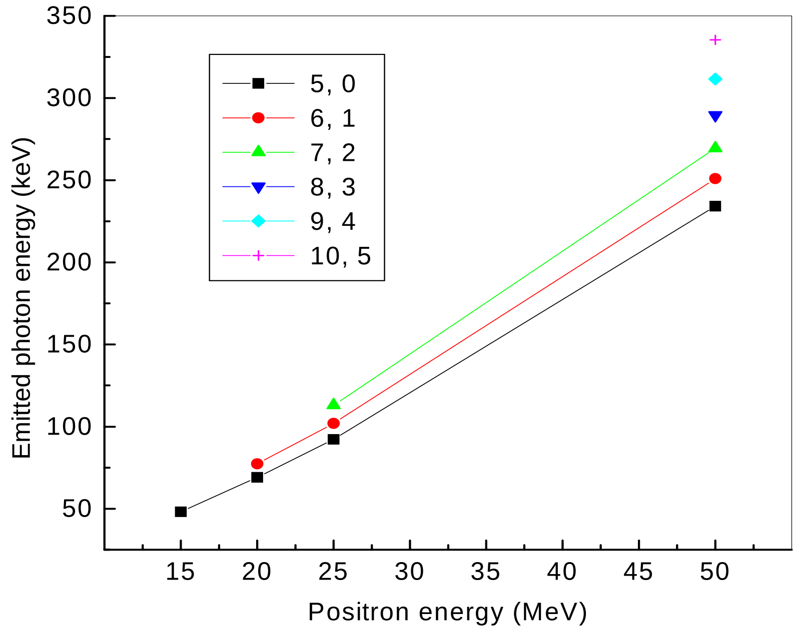

Fig. (3): The emitted photon energy in the forward direction due to *n,n-5* transitions between higher quantum states of transverse motion in the planar direction (100) in copper as a function of positron energy up to 50 MeV.

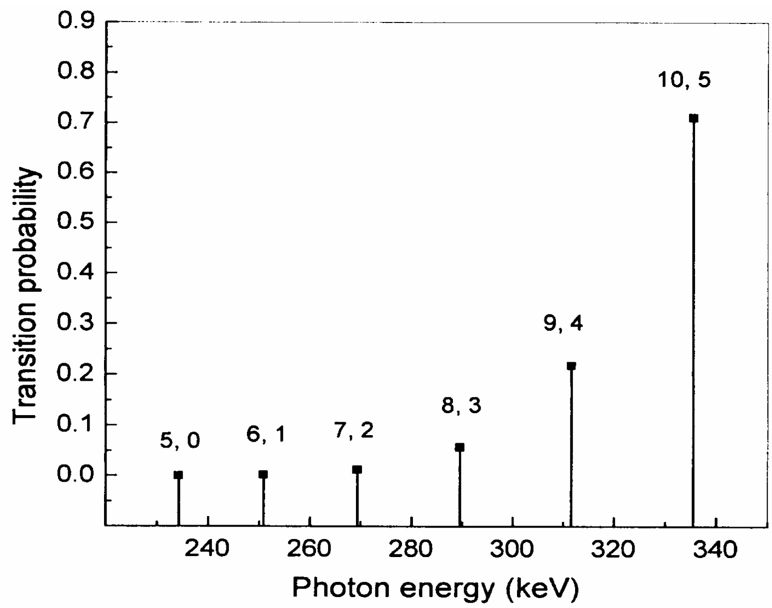

Fig. (4): The emitted photon energy in the forward direction due to *n,n-7* transitions between higher quantum states of transverse motion in the planar direction (100) in copper as a function of positron energy up to 50 MeV.

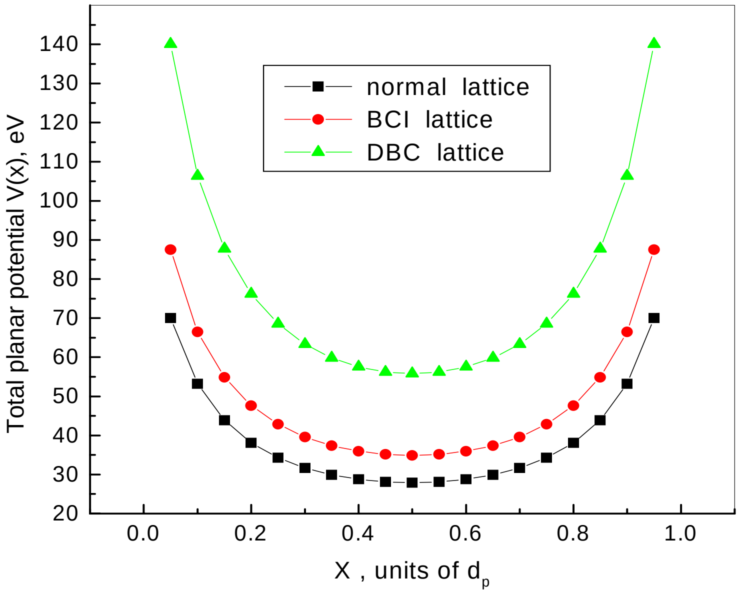

Fig. (5): Transition probability per unit time for spontaneous emission as a function of the emitted photon energy due to transitions *n,n-5* and *n, n-7* (normalized to the total transition probabilities for transitions Δn =5 and Δn = 7 respectively) for positrons with E = 50 MeV channeled in copper in the planar direction (100).

4. Radiation characteristics in the disordered lattices:

It may occur to some atoms of the material to be forced to move from its original lattice sites into one of the natural interstices of the lattice and leave behind a vacancy. This results in a reduction in the local atomic separation in a zone around the interstitial atom and the lattice become disordered. The three most probable types according to the order of their configuration stability are [11]:

(1) Dumb-bell configuration (DBC). If the extra atom is accommodated by sharing a site with another atom, the axis of the pair lying along a ⟨100⟩ direction.
(2) Body-centered interstitial (BCI). Here, the extra atom occupies the largest space in the fcc unit cell.
(3) Crowdion. In this case, a long-range relaxation occurs along a ⟨100⟩ close backed row. Neighboring rows are little affected.

Channeling potential (axial and planar) for the first two kinds , which are more stable configurations than the last one, has been calculated in previous work for α-particles channeled through Cu single crystal [9]. In this work, we will consider planar channeling radiation from Cu single crystal disordered by

DBC or BCI. The normal lattice of copper single crystal is fcc structure. In the planar direction (100), the interplanar distance, $d_p = ½ \times$ (lattice constant), and the number of atoms per unit cell, $n_c = 4$. For the disordered lattice by DBC we find that n_c is twice that of the normal lattice. Therefore, in the planar direction (100), the planar atomic density ($N.d_p$) is twice that of the normal lattice. For the disordered lattice by BCI we find that $n_c = 5$. Therefore, in the planar direction (100), the planar atomic density is (5/4) that of the normal lattice. The total planar channeling potential in normal lattice and disordered lattices by DBC or BCI respectively are shown in Fig.(5) for positrons channeled in copper single crystal in the planar direction (100).

Calculated results of radiation characteristics in disordered lattices by DBC or by BCI were listed in Tables compared with calculated results of normal lattice. For example, Tables (3, 4 and 5) lists the energy of the emitted channeling radiation (in the forward direction) of planar channeled positrons at 50 MeV in copper in the planar direction (100) in normal lattice and disordered lattice by DBC or by BCI for the allowed transitions n,n-1, n,n-3 and n,n-5 by using anharmonic(b) potential respectively. The calculated results of transition probabilities were given in Fig. (6) for transition probability per unit time, normalized to the total probability for transitions n,n-5, for spontaneous emission in normal lattice and disordered lattice by DBC or by BCI.

Table (3). Energy of the emitted channeling radiation (in the forward direction) of planar channeled positrons at 50 MeV in copper in planar direction (100) in normal lattice and disordered lattice by BCI or by DBC for the transitions *n, n-1*.

Allowed transitions *n*, *n*-1	Energy of the emitted radiation (keV)		
	Normal lattice n_{max}=10	BCI lattice n_{max}=11	DBC lattice n_{max}=13
1→ 0	40.9319	45.4602	56.8441
2→ 1	43.5282	47.9990	59.2808
3→ 2	46.4878	50.8627	61.9744
4→ 3	49.8108	54.0514	64.9249
5→ 4	53.4970	57.5650	68.1322
6→ 5	57.5465	61.4036	71.5965
7→ 6	61.9594	65.5672	75.3177
8→ 7	66.7356	70.0557	79.2958
9→ 8	71.8750	74.8691	83.5308
10→ 9	77.3778	80.0075	88.0227
11→ 10	No transition	85.4709	92.7715
12→ 11	No transition	No transition	97.7771
13→ 12	No transition	No transition	103.0397

Table (4): Energy of the emitted channeling radiation (in the forward direction) of planar channeled positrons at 50 MeV in copper in planar direction (100) in normal lattice and disordered lattice by DBC or by BCI for the transitions *n, n-7*.

Allowed transitions n, n-3	Energy of the emitted radiation (keV)		
	Normal lattice n_{max}=10	BCI lattice n_{max}=11	DBC lattice n_{max}=13
3→ 0	130.9479	144.3219	178.0992
4→ 1	139.8268	152.9130	186.1800
5→ 2	149.7956	162.4791	195.0314
6→ 3	160.8543	173.0200	204.6536
7→ 4	173.0029	184.5358	215.0465
8→ 5	186.2415	197.0264	226.2100
9→ 6	200.5700	210.4919	238.1443
10→ 7	215.9884	224.9323	250.8492
11→ 8	No transition	240.3475	264.3249
12→ 9	No transition	No transition	278.5712
13→ 10	No transition	No transition	293.5883

Table (5): Energy of the emitted channeling radiation (in the forward direction) of planar channeled positrons at 50 MeV in copper in planar direction (100) in normal lattice and disordered lattice by DBC or by BCI for the transitions *n, n-5*.

Allowed transitions n, n-5	Energy of the emitted radiation (keV)		
	Normal lattice n_{max}=10	BCI lattice n_{max}=11	DBC lattice n_{max}=13
5→ 0	234.2557	255.9383	311.1563
6→ 1	250.8703	271.8817	325.9087
7→ 2	269.3015	289.4499	341.9457
8→ 3	289.5492	308.6428	359.2671
9→ 4	311.6135	329.4606	377.8730
10→ 5	335.4943	351.9031	397.7635
11→ 6	No transition	375.9703	418.9384
12→ 7	No transition	No transition	441.3978
13→ 8	No transition	No transition	465.1417

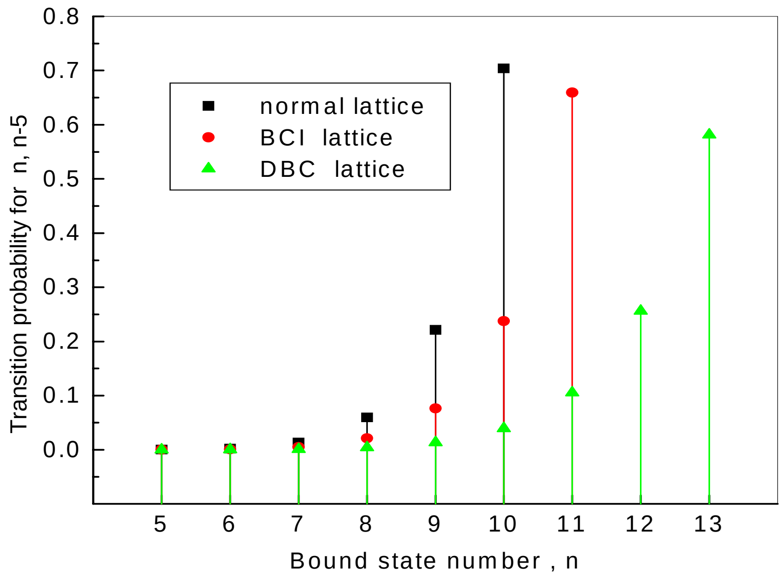

Fig. (6): Transition probability per unit time, normalized to the total probability for transitions $\Delta n = 5$, for spontaneous emission of planar channeled positrons at 50 MeV in copper in planar direction (100) in normal lattice and disordered lattice by DBC or by BCI for the transitions *n, n-5*.

5. Conclusion

In planar channeling of positively charged particles through crystals, the channeled particles execute a periodic motion, due to the transverse planar potential of the crystal, in a definite eigenstates and radiation results from spontaneous transitions between these states. The eigenstates and the eigenfunctions of the channeled particles have been found previously by solving Shrödinger equation governing the transverse motion by using planar potential function in a series expansion form to fit the real channeling potential function. In the present work, we used a more accurate planar potential by including higher anharmonic terms, i.e., x^6 term in the series expansion. Then by using the perturbation theory we obtained, the first-order eigenvalues and eigenfunctions, therefrom, the frequency of the emitted radiation and the dipole matrix elements could be calculated. So, by using the more accurate planar potential we could get in addition to the known transitions $\Delta n = \pm 1, \pm 3$, a new transition line at $\Delta n = \pm 5$.

As a very useful application of channeling radiation in crystal defect studies, calculations of channeling radiation characteristics have been executed to relativistic positrons at energies 10-500 MeV channeled in Cu single crystal, for normal lattice and for lattice disordered by dumb-bell configuration (DBC) or body-centered interstitial (BCI).

The calculations showed that, the maximum number of bound states increases by using higher anharmonic terms in the planar channeling potential, also it increases in the disordered lattices by DBC or BCI, i.e., we realize that the maximum number of bound states increases in the direction of increasing the channeling potential. In addition, the calculations showed that, the effect of higher anharmonic terms in the channeling potential on channeling radiation characteristics is very significant for transitions at higher bound states, and the significance decreases for transitions at lower bound states.

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