Interferometric Method to Determine the Birefringence for an Anisotropic Material

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A simple interferometric technique was used for measuring the absolute birefringence of an anisotropic material. The sample has constant thickness. It was placed between crossed polarizers and illuminated with a parallel beam of monochromatic light. The resulting interferograms were analyzed to deduce the birefringence and its dispersion across the visible spectrum. A Cauchy formula was applied to fit the experimental data. The same technique was used for a variable thickness with constant rate (a wedge form) of the same sample. Interference fringes was formed in the photographic plate. The value of the birefrengence was also measured.

1. Introduction:

An isotropic optical material has only one refractive index but an anisotropic material may have two or three refractive indices. In case of two refractive indices the material behaves optically as a uniaxial crystal, but for three refractive indices the material behaves optically as a biaxial crystal. The difference between the higher and lower refractive indices is taken as the birefringence "Δn" of the material. Different methods were used for measuring the birefringence. These methods such as interference colors and a Berek compensator [1], Moiré deflection technique [2], double speckle photography [3] and standard compensating method [4] were used for measuring the birefringence. However most of these techniques require expensive and highly sophisticated optical apparatus as well as a large volume sample [5]. Channeled spectra [6] resulting from two-beam interference were used for measuring the anomalous dispersion of various optical materials such as quartz, sapphire and magnesium fluoride [7]. They form the basis of the Roschdestvenski hook method of measuring the oscillator strengths of resonance absorption lines [8]. They were also used to measure the absolute phase shift and dispersion [9]. The birefringence of an anisotropic material such as a transparency sheet film either written type or copier type was deduced. The resulting interferogram was analyzed by a simple approach. The dispersion across the visible spectrum was also discussed [10]. A new kind of interference fringe that equal tangential inclination by curvature-induced birefringence was presented [11]. The change of birefringence induced by applying different redii of curvatures to a Fortepan sheet was measured. The stored birefringence was also deduced. An interferometric method was used to investigate the effect of controlled stress on the optical behavior of a transparent isotropic acrylic and glass samples. The stresses optical coefficient and Young's modulus of elasticity was evaluated. The induced birefringence as well as its dispersion was measured [12]. Measurements of the ordinary and extraordinary refractive indexes of synthetic sapphire were reported [13]. Direct independent measurements of the birefringence were presented. Temperature and piezo-optical coefficients of birefringence were measured in addition to Young's modulus. The ordinary and extraordinary refractive index of two samples of sapphire, that differed in the way each was grown, were measured [14]. The measurements were made over a wavelength range of 477-701 nm and a temperature range of 20-295 K.

In our work, a simple and low cost interferometric technique for measuring the absolute birefringence and its dispersion for constant an anisotropic material is presented. The birefringence of the variable thickness (a wedge form) for the same material was also deduced.

2. Theory:

A plane polarized beam incident in the direction of the normal to the anisotropic material is divided into two beams propagating with different velocities. One with vibration parallel to the surface of the optical axis and the other with vibration perpendicular to it. After emergence from the sample, the two beams are recombined again and obtain by the analyzer. Interference fringes are observed on the photographic plate. Suppose that the refractive indices of the sample for the two beams are ordinary "n_o" and extraordinary "n_e", and its thickness is "t", then the phase difference between the two beams is given by[15]:

$$\varphi = (2\pi/\lambda)\, t\, (n_o - n_e) \qquad (1)$$

and the condition for a dark fringe is [10]:

$$t\, (n_o - n_e) = m\, \lambda \qquad (2)$$

where "*m*" is the order of the interference fringes and "λ" is the wavelength of the light used. It is necessary that the sample has uniform thickness "t" and

$(n_o - n_e)$ over the whole spectrograph slit. After "P" fringes in the direction of shorter wavelengths eq.(2) can be written as:

$$\Delta n\, t\, \kappa = m + P \quad (3)$$

where $\Delta n = |n_o - n_e|$ is the birefringence and $\kappa = 1/\lambda$ is the wave number. The Cauchy dispersion formula for an anisotropic optical material can be written as:

$$no = Ao + Bo\, \kappa 2 \quad (4)$$

and:

$$n_e = A_e + B_e\, \kappa^2 \quad (5)$$

where A and B are constants characterizing the optical material. Then the birefringence is:

$$\Delta n = (A_o - A_e) + (B_o - B_e)\, \kappa^2 \quad (6)$$

Substituting by eq.3 we get:

$$a\, \kappa + b\, \kappa^3 = P + m \quad (7)$$

where:

$$a = t\,(A_o - A_e), \qquad b = t\,(B_o - B_e) \quad (8)$$

By a least-squares fitting of eq.(7), the constants a, b and *m* are determined. By knowing the thickness of the material "t", birefringence "Δn" can be obtained from eq. (6) for different wavelengths.

3. Birefringence for an anisotropic plate of changing thickness

Suppose that a birefringent plate of uniform thickness is placed between two crossed polarizers and illuminated by a parallel beam of monochromatic light normal to it. The plate is uniformly illuminated and the luminous intensity distribution for crossed polarizers is given by[16] :

$$I_{\perp\, max} = E^2 \sin^2 \varphi/2 \quad (9)$$

where E is the amplidude of the light incident on the plate.

Now, if the plate has a variable thickness, the plate will no longer show uniform illumination and variations of intensity will follow the variations in thickness according to eq. (2). In a region where one has $(n_o - n_e)\, t = m\lambda$, the intensity after crossed polarizers will be zero, where "*m*" is an integral number.

One will observe a dark fringe. Now move to a neighboring region where one has $(n_o - n_e)\, t^{/} = (m+1)\, \lambda$; then one is in at the next dark fringe. In passing from one dark or bright fringe to the next dark or bright fringe, the thickness varies by an amount equals to
$\lambda/(n_o - n_e)$ [17].

4. Experimental work and discussion

The optical set-up to produce two-beam white light interference fringes in transmission is shown in Fig.1. S_o is a source of white light (Tungsten filament), L_1 is a condensing lens with a short focal lens 3cm to form a minimized image of the source on the pinhole C, the pinhole increases the spatial coherence of the light source. L_2 is an achromatic collimating lens of focal length 5cm to produce a parallel beam of light. N_1 is a linear polarizer to give a linearly polarized beam of light. S is an anisotropic (uniaxial) sample to be investigated. A developed Fortepan photographic plate with thickness 0.18mm as an example of stored birefringence is used in our experiment. N_2 is a linear analyzer which is crossed with N_1. L_3 is an achromatic imaging lens of focal length 15cm, where its focal plane coincides with the slit of a grating spectrograph H, which has nearly a linear dispersion of 1nm / mm. The fringes are seen on the spectral plane of the spectrograph as colored two-beam fringes.

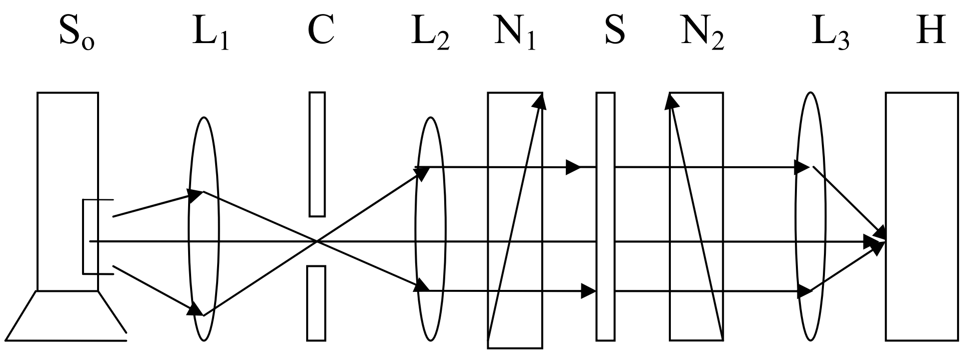

Fig.(1): Optical set-up for measuring the birefringence dispersion of an anisotropic optical material.

At the beginning of the experiment, the grating spectrograph is calibrated using four wavelengths, cadmium lines (643.847, 508.582, 479.992, 467.816 nm). The position of each spectral line on the photographic film is measured with an image processing system. Then the relation between the wavelengths and their positions on the spectral plane is found by means of a least squares fitting by the following equation:

$$\lambda = 6438.47 - 367.572\, X + 34.52\, X^2 - 1.772\, X^3 \quad (10)$$

where X is the distance between the orders of fringes m, m+1, m+2...etc and the calibration red line (λ = 643.847 nm) of cadmium lamp. Fig. (2) is a reproductive example of the resulting white light interference fringes formed with a Fortepan photographic plate. The standard lines of a cadmium spectral lamp are superimposed as a wavelength marker. Since the fringes are due to two-beam interference, the widths of the bright and dark fringes are equal. The resulting two-beam interference fringes are serially numbered from zero to m and their positions on the photographic film are measured. Then the corresponding wavelength for each fringe is deduced from the calibration curve of the grating spectrograph. Thus we have "*m*" and "κ" for each interference fringe. A least squares fitting of these data for eq.(7), gives the values of "*a*", "*b*" and "*m*". By knowing the thickness "t" of the sample sheet under test hence, "Δn" is obtained. The order of interference of this red line is deduced from knowing the order "*m*". The sample thickness can be directly measured by an interferometric method. In our experiment, we measured the thickness "t" by means of a micrometer screw. Then by substituting the value of thickness t in eq.(8) to deduce ($A_o - A_e$) and ($B_o - B_e$). Finally, substituting these values in eq.(6), "Δn" is found for different wavelengths across the visible spectrum. Table.1 shows the fitting parameters of a Cauchy dispersion function for the investigated sample. Fig.(3) represent the variation of "Δn" for the transparent Fortepan photographic film with the wavelength in the range from 500 to 600 nm across the visible spectrum applying Cauchy function. This figure shows that the birefringence "Δn" is inversely proportional to the wavelength "λ", the rate of increase of it is greater at shorter wavelengths.

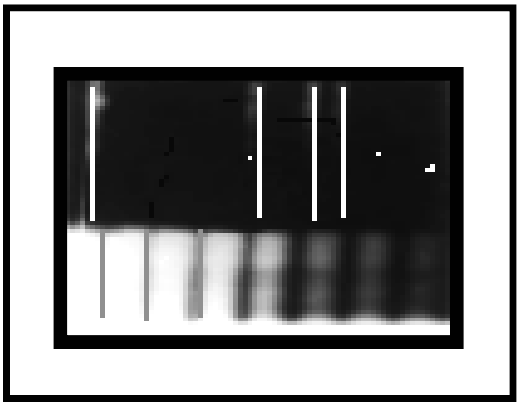

Fig. (2): Reproduction of the interferogram of white light interference fringes formed by a Fortepan photographic plate. The standard lines of a cadmium spectral lamp are superimposed as wavelength marker.

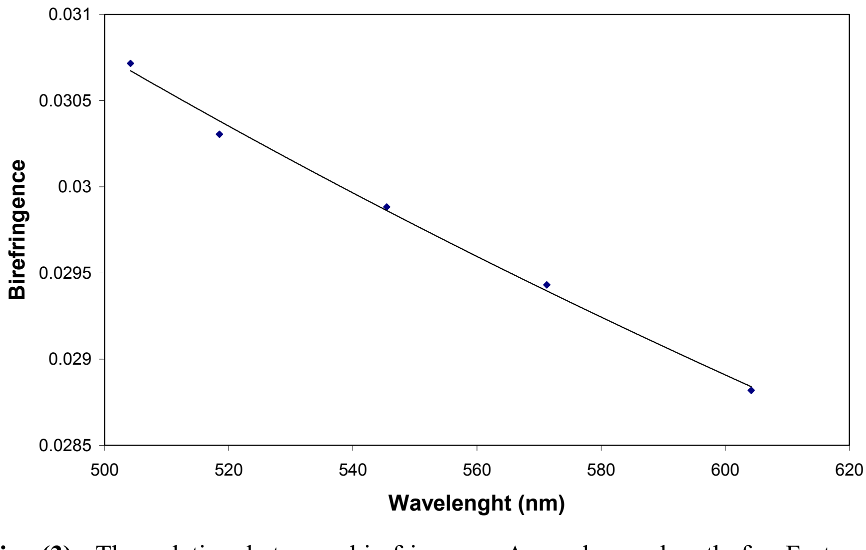

Fig. (3): The relation between birefringence Δn and wavelength for Fortepan transparent photographic film of thickness 0.22 mm. The continuous line is the Cauchy function where the dots are the experimental values.

Table (1): Fitting parameters of a Cauchy dispersion function.

Parameter	Fortepan Photographic Plate
$(A_o - A_e)$	0.024853
$(B_o - B_e)$ nm^2	50.88716

To estimate the discrepancy between the experimental and expected fitting dispersion relations a chi-square test is used. We adopted the formula:

$$\chi^2 = \sum_i \frac{(\Delta n_{ex\,i} - \Delta n_{th\,i})^2}{\Delta n_{th\,i}} \qquad (11)$$

where Δn_{ex} and Δn_{th}. are the experimental birefringence and expected fitting one, respectively. The test for a transparent photographic plate is ($\chi^2 = 1.2 \times 10^{-8}$). This means that there is a good agreement between the experimental and expected fitting birefringence. The relative error in measuring the birefringence is found by differentiating eq.(2) such that:

$$\delta(\Delta n) / (\Delta n) = \delta P / P + \delta \lambda / \lambda + \delta t / t \qquad (12)$$

The error in measuring the wavelength $\delta\lambda$ depends on the errors in locating the interference fringe. This error depends on the resolution limit of the reading instrument and the accuracy of locating the peak of the fringe. The resolution of the reading instrument used was (1μm). The accuracy of locating the peak depends on its sharpness. The errors in measuring the wavelength is ($\delta\lambda$ = 0.01nm) and that in the order is (δp= 0). Hence, with (δt=10 nm), (t = 2mm) and for (λ =546.1 nm), the relative error in finding the birefringence is $\delta(\Delta n) / (\Delta n) = 3x10^{-5}$.

Another method to determine the natural birefringence or stored Δn for the same an anisotropic material can be used. It depends on the change of the material thickness gradually with constant rate. The same optical set-up as in Fig.(1) is used with replacing the sample S by a sample of variable thickness t as in Fig.(4), the source S_o by He-Ne laser source with wavelength (632.8nm) and the grating spectrograph H by a photographic film. The fringes are seen on the photographic film as shown in Fig.(5). In the field of view, the resulting two-beam interference fringes are serially numbered from zero to "*m*" corresponding to the variations of the increased thickness. The relation between the thickness t of the sample and the fringe order "*m*" is illustrated in Fig.(6). By knowing the wavelength of the light used, birefringence "Δn" is obtained from eq.(2) as follows:

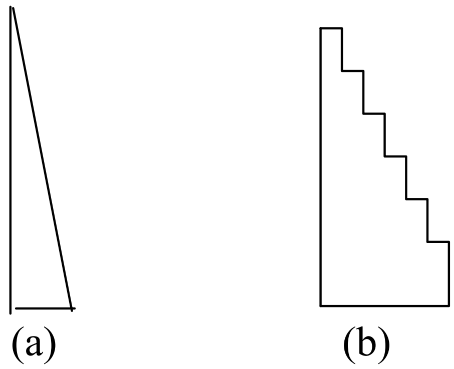

Fig. (4): A sample of an anisotropic material of changing thickness. (a) Aimed and (b) Actual.

$$\Delta n = (\Delta m / \Delta t) \lambda = (\text{slope}) \lambda \tag{13}$$

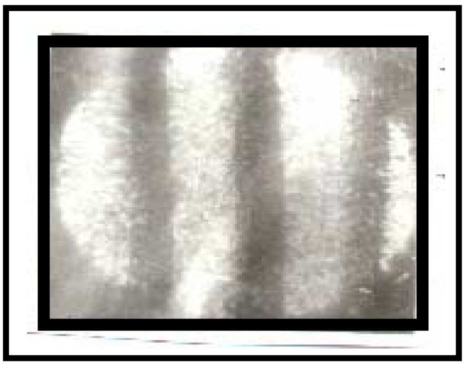

Fig. (5): The fringes seen on the photographic film due to a sample of different thickness

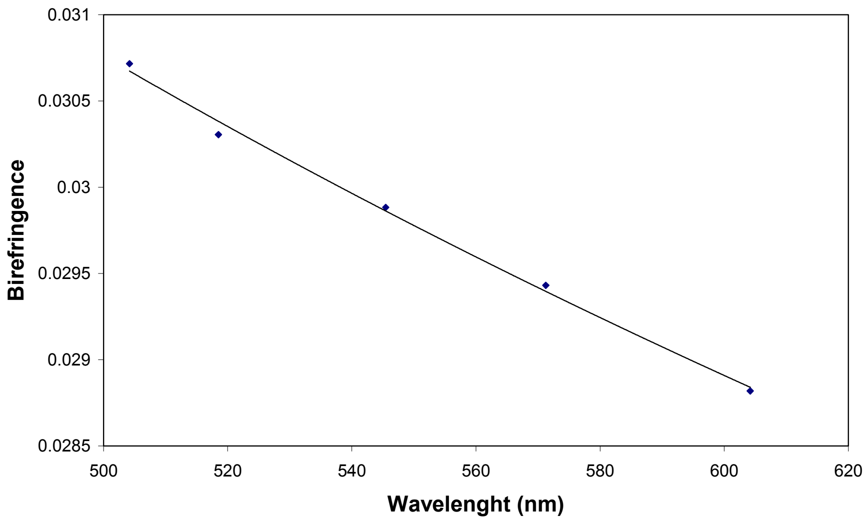

Fig. (6): The relation between the order of fringe m and the variable thickness t of the sample used.

It should be noted that since "Δt" varies in discrete manner, rather than continuous, then "Δm" is also a discrete quantity which may or may not be unity. In either case "Δm" changes gradually and uniformly in fixed steps as long as "Δt" changes in the same manner. This adds a constant step "p" of orders regularly to "m" such that the actual step is:

$$(\Delta m\, p / \Delta t) = p\,(\Delta m / \Delta t) \tag{14}$$

In other words, the real slope is multiplied by a fixed quantity "p", which is to be found. Fortunately "p" is an integral number of the order of 1, 2, 3…etc. The calculated value of "Δn" is reduced to the real value by dividing the value obtained from eq.(14) by any of the given integers, trying to get the near most value of the published data. Here, we have found that "p =1". The resulted value is "Δn = 0.035" for the given wavelength of used source.

5. Conclusion:

In this work, we describe a simple and accurate interferometric method for measuring the natural birefringence and its dispersion across the visible region of spectrum. Although the theoretical and experimental background of the method is nearly known, the fields of application and data processing approach are firstly presented. The method is applicable for liquid and solid samples of fixed thickness all of over the slit of the spectrograph. A two– term of Cauchy dispersion function is most suitable to give accurate values of the birefringence dispersion because it is simple and time saving. The natural birefringence Δn for an anisotropic material also evaluated for the sample used by changing thickness gradually by the same method.

References:

1. F.B.Bloss, "*An Introduction to the Methods of Optical Crystallography*", Holt Rinehart and Winston, New York, p.142, (1961).

2. D. F. Heller, O. Kafri, J. Krasinski, *Appl. Optics*, **24**, 3037, (1985).

3. L. M. Bernardo, O. D. D. Soares, *Appl. Optics*, **26**, 769, (1987).

4. P. L.Y. Chuanzeng, L. Guohua, *Appl. Optics*, **29**, 4546,(1990).

5. L.Y. Zheng, S. Xiyu, S. Lianke, *Appl. Optics*, **31**, 2968, (1992).

6. M. Born, E. W. Wolf, Principles of Optics, Pergamon, Oxford, p.265, (1983).

7. V. Chandrsekharam, H. Damage, *Appl. Optics*, **8**, 671, (1969).

8. R,W.D.ditchburn, "Light", Acadamic Press, London, p. 553, (1976).

9. R.J. Sandeman, *Appl. Optics*, **10**, 1087, (1971).

10. M. Medhat, S. Y. El-Zaiat, *Optics Communications*, **141**, 145 (1997).

11. M. Medhat, N.I. Hendawy and A.A. Zaki, *Opt. and Laser Tech*., **35**, 31 (2003).

12. M. Medhat, N.I. Hendawy and A.A. Zaki, accepted for publication in *Egypt J. Phys*. (2003).

13. A. Myron Jeppesen, *Journal of the Optical Society of America*, **48**, (9), 629, (1958).

14. A. C. DeFranzo and B. G. Pazol, *Applied Optics*, **32**, (13), 1 (1993).

15. A. Francis Jenkins and E. Harvey White, "*Fundamentals of Optics*", McGraw-Hill, Inc. 474 (1976).

16. Max Born and Emil Wolf "*Principles of Optics*" Pergamon Press, p. 694 (1980).

17. M. Francon. "*Optical Image Formation and Processing*". Academic Press, Inc. p. 10 (1979)