

FAULT ANALYSIS AND A PROPOSED ULTRA HIGH SPEED RELAYING SCHEME FOR SIX PHASE SYSTEMS

تحليل أخطاء النظم سداسية الأوجه واقتراح متمم ذو سرعة فائقة

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ملخص البحث

يقدم هذا البحث نموذجاً رياضياً لتمثيل خط نقل سداسي الأوجه باستخدام تقنيات مجال التردد. وقد تم استخدام النموذج المقترح لدراسة وتحليل الأخطاء التي تتعرض لها خطوط النقل سداسية الأوجه سواء كانت أخطاء من داخل النظام أو من خارجه. كما يتضمن البحث اقتراحاً متمم يستخدم الموجات المتحركة للكشف عن الأخطاء التي يتعرض لها النظام بسرعة فائقة وقد تم تطبيق النماذج الرياضية المقترحة للمتمم للكشف عن الأخطاء التي يتعرض لها نظام نقل سداسي الأوجه

ABSTRACT

This paper presents a frequency domain technique for modelling six-phase transmission lines. Fault studies on six phase systems have been conducted using this model. Also, this work proposes an ultra high speed relaying scheme based on the concept of travelling waves. Fault studies on a sample six phase network have also been presented. The paper also discusses a novel and straightforward technique for Laplace transform inversion.

INTRODUCTION

Study of higher phase order systems has gained increased importance in recent years because of ever increasing demands on power. Proper designing of such systems calls for accurate and mathematically convenient modelling of various system elements [1&6]. Mathematical models of generators, transformers etc. have been presented by Tewari and Singh [1&4], and Chaudhary and Singh [2&5]. The present paper presents a frequency domain model for six phase transmission lines. The model can be used for transient analysis of networks containing distributed elements. The advantage of frequency domain analysis is twofold. Firstly, it reduces differential equations to algebraic operations. Secondly, it is more accurate. The major problem in using Laplace transform in frequency domain studies is that the inversion to the time domain is cumbersome. This paper also discusses a new technique for inverting Laplace transforms. Finally, the paper presents an ultra high speed relaying scheme, based on the concept of travelling waves [3 &7], for transmission lines protection.

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FREQUENCY DOMAIN MODELLING OF SIX PHASE TRANSMISSION LINES

Considering a fully transposed six-phase transmission line. Denoting the series self impedance per unit length of phase i ($i=a, b, \dots, f$) by Z_{ii} and the mutual impedance per unit length between phase i and phase j by Z_{ij} , then

$$Z_{aa} = Z_{bb} = Z_{cc} = \dots = Z_{ff} = Z_s$$

$$Z_{ab} = Z_{bc} = Z_{cd} = \dots = Z_{fa} = Z_m$$

where, $Z_{ii} = Z_s$ for all $i = a, b, \dots, f$ and $Z_{ij} = Z_{ji} = Z_m$ for all $i, j = a, b, \dots, f, i \neq j$

Let the phase to ground voltages of the six phases be denoted in the frequency domain by the following column vector:

$$V(s) = \begin{bmatrix} V_a(s) \\ V_b(s) \\ V_c(s) \\ V_d(s) \\ V_e(s) \\ V_f(s) \end{bmatrix}$$

where, s is the laplace transform operator.

Similarly, let $I(s)$ denote the column vector of the six line currents, i.e.,

$$I(s) = \begin{bmatrix} I_a(s) \\ I_b(s) \\ I_c(s) \\ I_d(s) \\ I_e(s) \\ I_f(s) \end{bmatrix}$$

Then using the telegraph equations, for the transform of domain quantities we get,

$$dV/dX = -[Z]I \quad (1)$$

$$dI/dX = -[Y]V \quad (2)$$

Where,

$$[Z] = \begin{bmatrix} Z_{aa} & Z_{ab} & \dots & Z_{af} \\ Z_{ba} & Z_{bb} & \dots & Z_{bf} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{fa} & Z_{fb} & \dots & Z_{ff} \end{bmatrix}$$

is the series impedance matrix for a unit length of the line;

and

$$[Y] = \begin{bmatrix} Y_{aa} & Y_{ab} & \dots & Y_{af} \\ Y_{ba} & Y_{bb} & \dots & Y_{bf} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{fa} & Y_{fb} & \dots & Y_{ff} \end{bmatrix}$$

is the shunt admittance matrix for a unit length of the line. Using the above relations, we get,

$$d^2V/dX^2 = [Z][Y]V \tag{3}$$

Eq.(3) consists of six coupled ordinary differential equations. Since the matrix $[Z][Y]$ is symmetrical, then Eq. (3) can be transformed to six decoupled six-phase symmetrical components by using the following transformation matrix [8].

$$[T] = 1/\sqrt{6} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a & a^2 & -1 & -a & -a^2 \\ 1 & a^2 & -a & 1 & a^2 & -a \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & a & a^2 & -1 & -a & -a^2 \\ 1 & a^2 & -a & 1 & a^2 & -a \end{bmatrix} \tag{4}$$

Where $a = \exp. (j 60^\circ)$

The decoupled equations have the form:

$$d^2 V^{(m)} / dX^2 = \sigma_m V^{(m)} \quad m=0,1,\dots,5 \tag{5}$$

where $V^{(m)}$ is the m^{th} mode voltage and σ_m is the m^{th} eigen value of the $[Z][Y]$ matrix. σ_m is usually called the propagation vector of the m^{th} mode.

The modal voltages and currents are related to the corresponding phase values by the relations :

$$\begin{bmatrix} V^{a,b,c,d,e,f} \\ I^{a,b,c,d,e,f} \end{bmatrix} = [T] \begin{bmatrix} V^{0,1,2,3,4,5} \\ I^{0,1,2,3,4,5} \end{bmatrix} \tag{6}$$

From Eq.(5)

$$\begin{aligned} V_s^{(m)} &= \cosh \sigma_m^1 Z_0^m \sinh \sigma_m^1 / \sigma_m^m & V_r^{(m)} \\ I_s^{(m)} &= \sigma_m^m \sinh \sigma_m^1 / Z_0^m & \cosh \sigma_m^1 & I_r^{(m)} \end{aligned} \tag{7}$$

$m = 0,1, \dots, 5.$

$V_s^{(m)}$ & $I_s^{(m)}$ and $V_r^{(m)}$ & $I_r^{(m)}$ are the sending and receiving end voltages and currents.

Z_0^m is the line surge impedance for the mode m.

From Eqns. (6) and (7),

$$\begin{bmatrix} V_s^{a,b,c,d,e,f} \\ I_s^{a,b,c,d,e,f} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_r^{a,b,c,d,e,f} \\ I_r^{a,b,c,d,e,f} \end{bmatrix} \quad (8)$$

Where,

$$A = [T^{*T}] [\cosh \sigma l] [T]; \quad B = [T^{*T}] [Z \sinh \sigma l / \sigma] [T];$$

$$C = [T^T] [\sigma \sinh \sigma l / Z] [T]; \quad D = [T^T] [\cosh \sigma l] [T];$$

are 6x6 sub-matrices.

FAULT SIMULATION

Various types of faults are simulated in references [7&8] by connecting appropriate voltage source $V_{FF}^{a,b,c,d,e,f}$ at fault location. Considering the general six phase system shown in Fig.2. Let the prefault voltage at fault location be denoted by a column vector V_{PF} . Then the fault on any phase is simulated according to [7&8] by connecting a fault voltage source V_{FF} in opposition to V_{PF} as shown in fig.(2). Actual values of V_{FF} depend upon the types of faults being considered. To calculate the voltage and current components caused by the fault, we ground all voltage sources except V_{FF} .

INVERSION TO TIME DOMAIN

We know that $U(s) = \int_0^{\infty} e^{-st} u(t) dt$ is the Laplace transform of $u(t)$. A closed form integration can be approximately replaced by a summation, that is:

$$\int_{-1}^1 f(r) dr \approx \sum_{i=1}^N w_i \cdot f(r_i)$$

It can be shown that any function $f(r)$ in the range $(-1, 1)$ can be expressed in terms of a set of Legendre polynomials in this region, i.e.,

$$f(r) \approx \sum_{k=0}^N b_k P_k(r)$$

Furthermore, a fairly accurate determination of the integral is done if the points r_i and the coefficients w_i are chosen according to Gauss criterion which suggests that r_i for $i=1, \dots, N$ should be chosen as the zeros of Legendre polynomial $P_N(r)$ and

$$w_i = \int_{-1}^1 P_N(r) dr / (r - r_i) P_N'(r_i), \quad i=1, 2, \dots, N$$

The infinite time interval $0 \leq t \leq \infty$ of the Laplace domain can easily be mapped in a one to one fashion into $0 \leq x \leq 1$ by a transformation such as $x = e^{-t}$. Similarly, a simple change of variable $x = (1+r)/2$ shifts the region $-1 \leq r \leq 1$ to $0 \leq x \leq 1$, therefore, the last equation can be written as:

$$\int_0^1 2f(2x-1) dx \approx \sum_{i=1}^N w_i f(2x_i-1)$$

Denoting $f(2x-1)$ as $g(x)$, then

$$\int_0^1 g(x) dx \approx \sum_{i=1}^N w_i g(x_i)$$

where w_i is a new set of weights and x_i 's are the zeros of the shifted Legendre polynomials.

Use will be made of some of the above mentioned properties while applying the inverse Laplace transform to get the function $u(t)$. We introduce a new variable $x = e^{-t}$ and express the above integral as

$$U(s) = \int_0^1 x^{s-1} u(-\ln x) dx = \int_0^1 x^{s-1} g(x) dx$$

To use numerical integration, the above equation is expressed as follows:

$$\int_0^1 x^{s-1} g(x) dx \approx \sum_{i=1}^N w_i x_i^k g(x_i) = F(k+1); \quad k=0,1,2,\dots,(N-1)$$

w_i are known for $i=1,2,\dots,N$; $x_i = \exp(-t_i)$; $F(k+1)$ can be evaluated for $k=0,1,2,\dots,(N-1)$.

Hence, the elements of matrix $M = \{w_i x_i^k\}$ are known. Thus the unknown function $g(x_i) = u(-\log x_i)$ can be found by inverting the matrix M , i.e.,

$$\{g(x_i)\} = M^{-1} \{F(k+1)\}$$

Hence, the values of $u(-\log x_i)$ are known, values of $u(x)$ for x other than the zeros of shifted Legendre polynomials can be found by using the property that

$$L(u(at)) = F(s/a)/a$$

Where L is the Laplace operator.

AMPLITUDE COMPARISON RELAYING SCHEME

a) Principle of Operation:

Consider a transmission line connecting two large power systems as shown in Fig.3. Once a fault occurs on one of the phase of the line at point F to both line ends. Subsequent reflections of these waves take place at both line ends. These reflections are shown by the Bewley diagram of Fig. 4. We are interested only in fault generated components.

$$\begin{aligned} \text{Define} \quad S_1 &= V_{fs} - R_{fs} I_{fs} \\ S_2 &= V_{fs} + R_{fs} I_{fs} \end{aligned}$$

where V_{fs} & i_{fs} = fault generated voltage and current at relay location;

R = line surge impedance.

Then from table 1, for internal faults $|S_1| < |S_2|$, from $t = \tau^-$ till next wave appears at relay location. But for external fault, $V_{fs} = \mu_t V_{ff}$ and $i_{fs} = \mu_t V_{ff} / Z'_0$ where μ_t = refraction coefficient. Then

$$S_1 = (1 - Z_0 / Z'_0) \mu_t V_{ff}$$

$$S_2 = (1 + Z_0 / Z'_0) \mu_t V_{ff}$$

Hence, for external faults, $|S_1| < |S_2|$ till the next reflected wave appears at the relay location.

Table-1. Relay Input Signals for an Internal Fault

Time	V_{fs}	i_{fs}	$ S_1 $	$ S_2 $
$t < \tau^-$	0	0	0	0
$t = \tau^-$	V_{ff}	$-V_{ff} / Z_0$	$2 V_{ff} $	0
$t \geq \tau^+$ till another wave pair arrives	$(1 + \mu_s) V_{ff}$	$-(1 - \mu_s) V_{ff} / Z_0$	$2 V_{ff} $	$2 \mu_s V_{ff} $

b) Extension to Six Phase Systems:

Since the modal components, $V^{(m)}$ and $I^{(m)}$ are decoupled, then each pair of these, for $m=0,1,\dots,5$, satisfies the above equations.

$$S_1^{(m)} = V_{fs}^{(m)} - Z_0^{(m)} i_{fs}^{(m)}$$

$$S_2^{(m)} = V_{fs}^{(m)} + Z_0^{(m)} i_{fs}^{(m)}$$

For internal faults, $|S_1^{(m)}| > |S_2^{(m)}|$

For external faults, $|S_2^{(m)}| > |S_1^{(m)}|$

TESTING ON A SAMPLE POWER SYSTEM

A 3-bus sample power system is shown in the Fig.5. The transmission lines are 200 and 100 km long with a surge impedance of 50 Ω each. The ends of the line are connected to large power systems which have source impedance equal to one-fourth of the line surge impedance. For simplicity a dead δ L-G fault is considered. The plots of fault generated voltages and relay signals are shown in figures (6) & (7). R_f is the fault impedance and ϕ_0 is the fault initiation angle.

From Fig. 6, it can be seen that for an internal fault with $R_f = 100 \Omega$ and $\phi_0 = 0$ the operating quantity increases positively steady with time up to 5 msec. and oscillate with damping around the steady state value for time more than 5 msec.

From Fig.7, for external fault with $R_f = 100 \Omega$ and $\phi_0 = 0$, the operating quantity has all the time negative value, for any time of fault. The value of the relay signal difference has a fast trajectory toward the negative axis up to 1.5 msec. time. For the time more than 1.5 msec. the value of relay signal oscillate around the steady state value which have a negative value.

CONCLUSIONS

The frequency domain modelling of a multiphase transmission line has been studied in this paper. The model is especially suited for transient analysis of the transmission line. A simple inversion algorithm from the s-domain to the time domain has also been discussed. Finally, this paper concludes with the presentation of an ultra high speed relaying scheme for transmission line protection based on travelling wave phenomenon.

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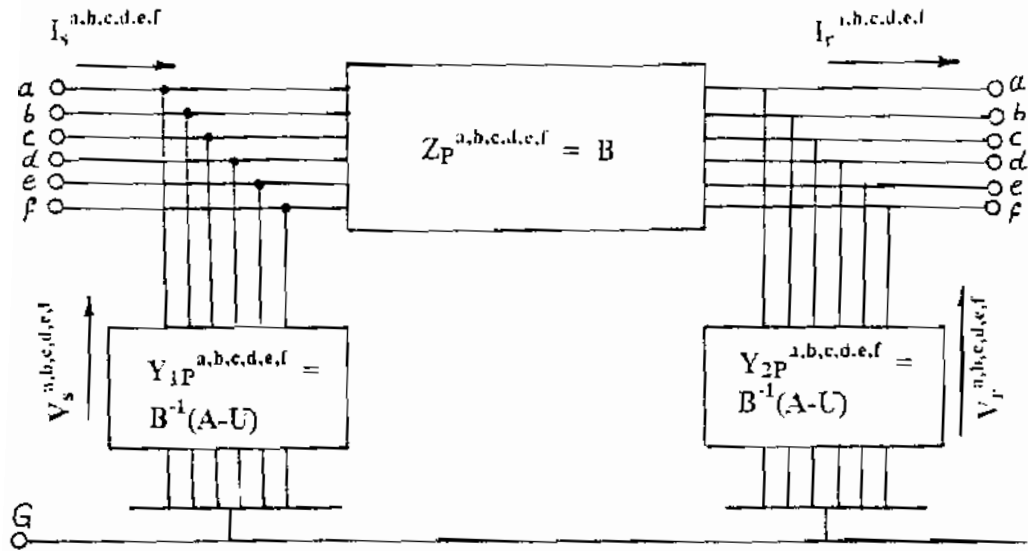


Fig.1 Frequency Domain PI Model of Six Phase Transmission Line

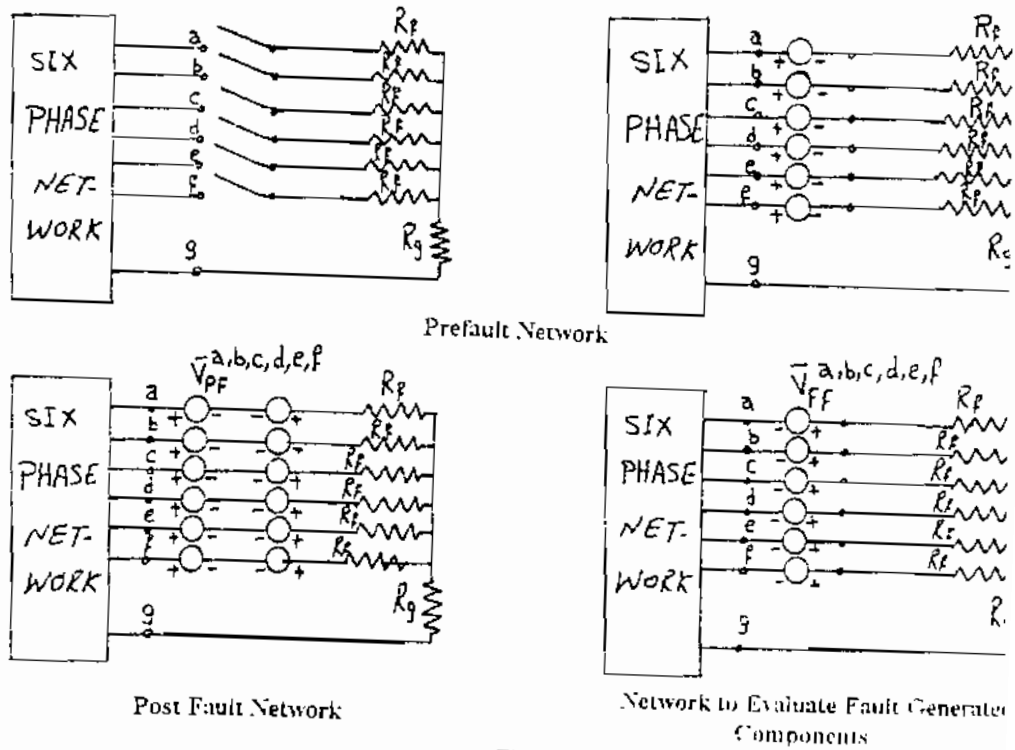


Fig.2

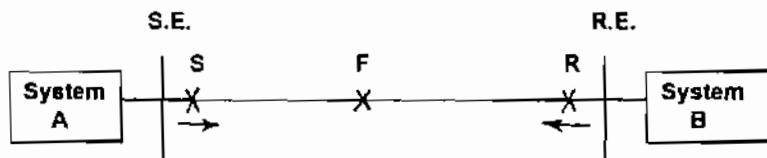


Fig.3 A Transmission Line Interconnecting Two Power Systems

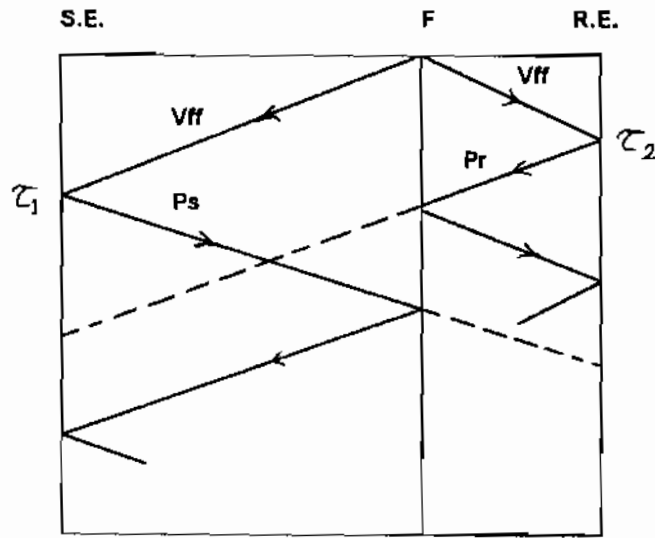


Fig.4 Lattice Diagram for an Internal Fault

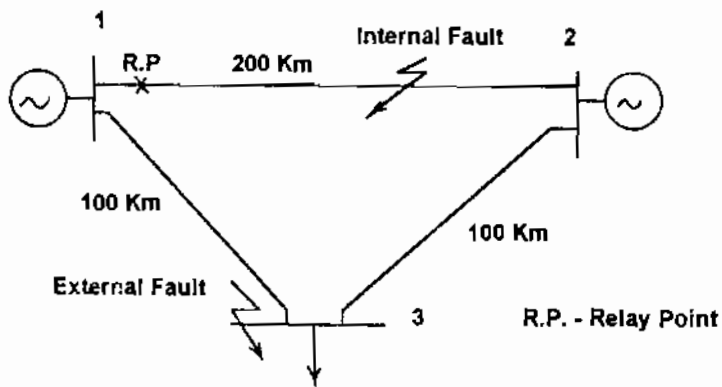


Fig.5 A sample 3- Bus Power System

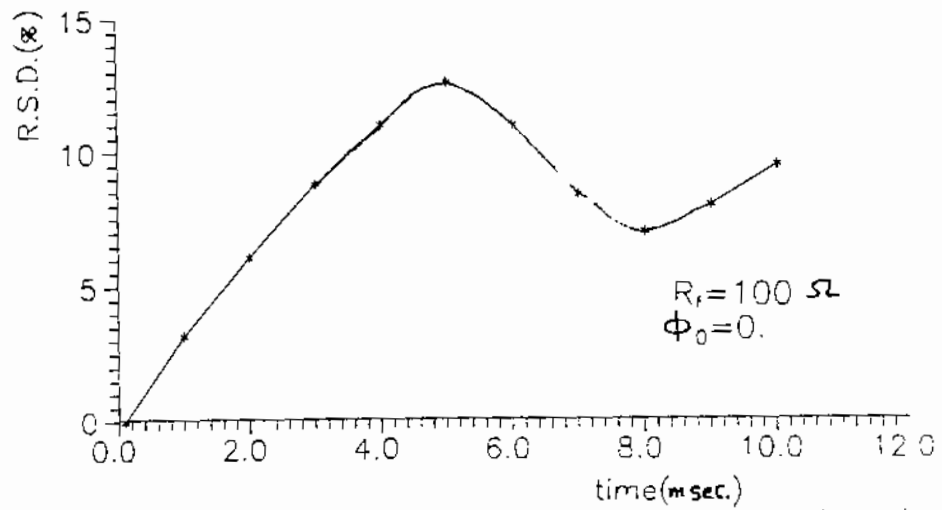


Fig6 Relay Signal Difference (R.S.D.) $|S_1| - |S_2|$
for Internal Fault on Sample Network.

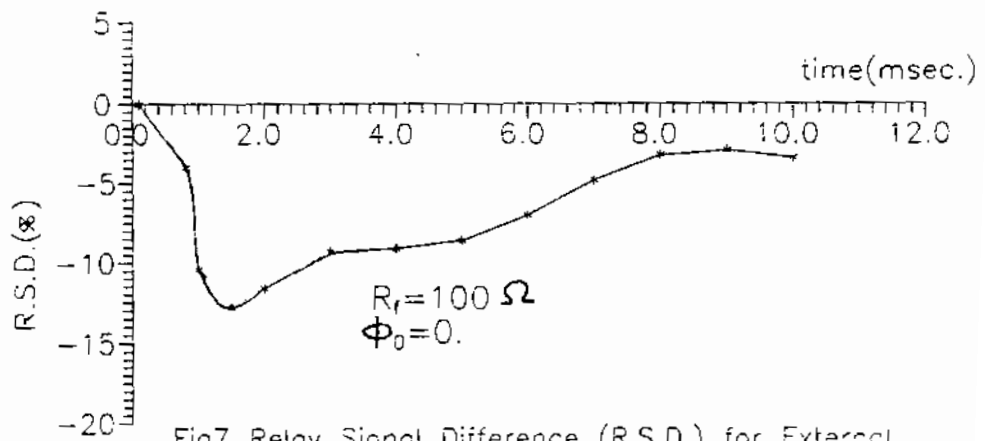


Fig7 Relay Signal Difference (R.S.D.) for External
Fault on Sample Power System Network