ANALYTICAL AND EXPERIMENTAL ANALYSIS OF DOUBLE-EXPOSURE SPECKLE HOLOGRAPHY

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Abstract:

A new analytical formulae representing the intensity distribution of the interference pattern for object in two orthogonal positions, with small displacements, is derived and experimentally confirmed.

Also, holographic interferometry of double-exposure technique with small displacement and interference pattern of two separate identical diffusers displaced with respect to each other, are correlated in a simple formulae and experimentally investigated.

Introduction

Interference effects observed from holographic reconstructions depend on path variation as a function of four variables, but cannot be completely represented by a single two-dimensional pattern [1].

Certainly, it was important to study effects of laser speckle in holography (speckle holography) that eventually brought it to the stage of practical utilization [2, 3]. Single and double-exposure speckle holograms were used to produce circular interference fringes from the reconstructed images as two separate identical diffusers [4, 5]. Comparative studies between speckle and holographic interferometry for two identical diffusers were given by El-Dessouki et al. [6].

This technique is an extrmely powerful one, since any changes in position of elements in the surface of the object observed in the reconstruction give rise to path differences which yield interference effects. Accordingly, the change of position of the surface can be calculated.

In the present work, speckle holography using double-axial and double-lateral exposure technique are studied theoretically. Moreover experiments have been made in measuring axial and lateral displacements individually and simultaneously. The theoretical analysis is given in a simple analytical form as a function of the position of the surface in two orthogonal directions.

Experimental:

1. Axially double-exposure speckle hologram as two identical diffusers.

Single-exposure speckle hologram is used to produce circular interference fringes from the two reconstructed images as two separate identical diffusers. The resultant circular interference fringes can be localized by using converging lens. The condition of interference of these fringes is given in ref. [5] as:

$$R_K^2 = \frac{K\lambda f^2}{L} \qquad (1)$$

where R_K is the radius of the resultant fringe of order K; λ the wavelength of the source; f the focal length of the converging lens; and L the axial separation between the diffused object and the holographic plate.

For an axially double-exposure hologram, the diffused object is axially shifted say in Z-direction by a small displacement $\xi_0(\Delta L)$ as shown in Fig. 1. Hence the radii of the resultant interference pattern will change by ΔR according to the direction of displacement. This can be represented mathematically by differentiating eq. (1),

$$\frac{dR_K}{dL} = -\frac{K\lambda f^2}{2R_K L^2} \qquad (2)$$

Illuminating the holographic plate H by a parallel laser beam (λ = 632.8 nm) four images are reconstructed as shown in Fig. 2, in which there are two real images H_1', H_2' and two virtual images H_1'', H_2''. To observe interference fringes a lens (f = 50 cm) is used to localize the interference pattern. The experiment is carried out using two separations L = 8 and 15 mm, with small displacements ξ_0 = 0, 30, 50 and 100 um. One of these interferograms as a representative example is shown in Fig. 3. When plotting ξ_0 as a function of R a straight line is obtained as shown in Fig. 4 with a negative slope as expected from eq. (2). The estimated error in case of L = 8 mm and k = 2 is about 1% whilst for L = mm and k = 2 the error becomes about 2%.

2. Analytical analysis of double exposure technique with axial and lateral displacements

a. Axial displacement:

The double-exposure speckle photography has proved to be a simple and convenient method for measuring displacements and strain measurements [7].

In case of double-exposure hologram, an axial translation ξ_0 in the Z-direction of the photographic plate H is made between the two exposures as shown in Fig. 1.

In this case the separation between the diffused object and the plate is L. For each exposure any point (x, y, z) on the photographic plate H receives two waves. The first one is the plane wave or unscattered wave which can be represented by $U_o = A_R e^{iKL}$, and the other one is the scattered wave which can be represented by $U_1 = [A\ (x, y)/L]\ \exp\ ikl\ [1 + (x^2 + y^2)/(2L^2)]$. Taking the central point of the plate H as the origin in which L >> x or y. A_R and A (x, y)/L are the amplitude of the transmitted and scattered waves respectively. The scattered wave U_1 for the first exposure is given by:

$$U_1 = \frac{A\ (x, y)}{L}\ \exp.\ ikl$$

After displacing the object D (Fig. 1) in Z-direction by ξ_o i.e. axial translation, the scattered wave can be represented by:

$$U_2 = \frac{A\ (x, y)}{L}\ \exp\ ik\ (L + \xi_o)$$

The resultant U is given by

$$U = (U_o + U_1) + (U_o + U_2) \qquad (3$$

$$\text{Therefore } U = [2\ A_R + \frac{A\ (x, y)}{L}\ (1 + \exp\ ik\ \xi_o)]\ \exp\ ikl$$

The resultant intensity $I = |U|^2$ and this leads to,

$$I = I_o \cos^2 \frac{k\ \xi_o}{2} \qquad (4)$$

Eq. (4) represents the intensity distribution of two beam interference pattern. The resultant pattern wil be circular fringes. Fig. 3 illustrates some of the interference pattern given by eq. (4) for different axial displacements.

b. Lateral displacement:

For a double-exposure hologram in which the photographic plate is at a distance L (z-direction) from the scattered object D which is lateraly displaced by η_o, say in the x-direction, between the two exposures. Following the same procedure as in the previous section for which the transmitted and scattered waves are represented by $U_o = A_R\ \exp\ ikL$ and

$$U_1 = \frac{A\ (x_1, y)}{L}\ \exp\ ikl\ (1 + \frac{x_1^2 + y^2}{2L^2})\ \text{respectively.}$$

After displacement η_o, the scattered wave at the plate H will be

$$U_2 = \frac{A\,(x_2,\, y)}{L} \;\exp\left(1 + \frac{x_2^2 + y^2}{2L^2}\right),$$

where $\quad x_2 = x_1 + \eta_o$,

The resultant wave at H after the two exposures can be given by eq. (3), from which the resultant intensity $I = U^2$. This leads to

$$I = C_o + C_1 \cos^2 k\left(\frac{x_1 \Delta x}{2L}\right) + C_2 \cos^2 k\left(\frac{x_1 \Delta x}{2L}\right) \cos^2 k\left(\frac{x_1^2 + y^2}{4L}\right) \qquad (5)$$

where $\quad C_o = 4\,A^2$, $C_1 = \left(\frac{2A}{L}\right)^2$ and $C_2 = 16\,\frac{AA_R}{L}$. Eq. (5) represents the resultant intensity distribution pattern of double lateral speckle hologram with a longitudinal separation L. The second term of the right hand side of eq. (5) represent the intensity distribution of straight line fringes perpendicular to the direction of the lateral displacement. The third term represents straight line fringes modulated by circular interference pattern. Figure 5 illustrates different patterns described by eq. (5). In both interferograms L is kept constant whilst η_o takes different values.

Discussion

It can be shown from the above results that speckle patterns of a displaced diffused object could be reconstructed holographically. The real and virtual reconstructed images interfere with each other to give interference fringes. Hence the axial displacement of the object is correlated to the radii of the fringes as two separate identical diffusers (Fig. 4). The experimental results are in good agreement with eq. (2) for two separate identical diffusers. The maximum experimental error of the measurements is within 2%.

The new analytical formula for the axial and lateral displacements, i.e. equations (4) and (5), are confirmed experimentally by measuring the displacement of the diffused object. The lateral shift of an axial placed object is represented by eq. (5). The intensity distribution curve of figure 5 is illustrated in Fig. 6. The straight line fringe corresponding to the first order in figure 6.a is modulated by two bright circular fringes, meanwhile the second order is modulated by eight circular bright fringes. The intensity distribution of Fig. 5.b is illustrated by Fig. 6.b. The drop in intensity between two successive straight line bright and dark fringes is 57%. This is represented by the second term of Eq. 5. In Fig. 6.b, the second order of the straight line frignes is moduated by two superimposed circular fringes as shown in the third term of Eq. 5. The drop in intensity between each two successive bright and dark circular fringes inside this bright straight line fringe equals 25%.

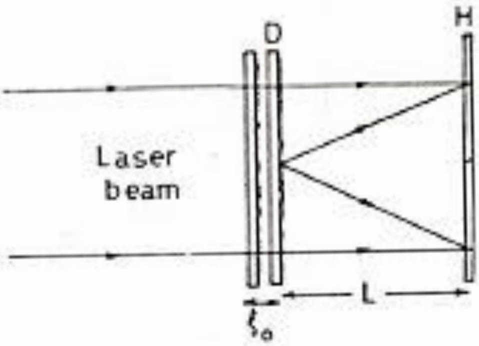

Fig.1: Recording of double-exposure hologram due to interference of scattered and transmitted laser beam through the diffused object D with an axial displacement.

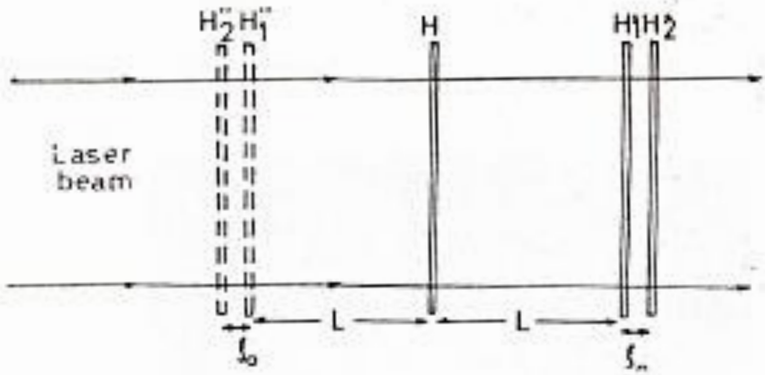

Fig.2: Reconstruction of real and virtual impages from the hologram H.

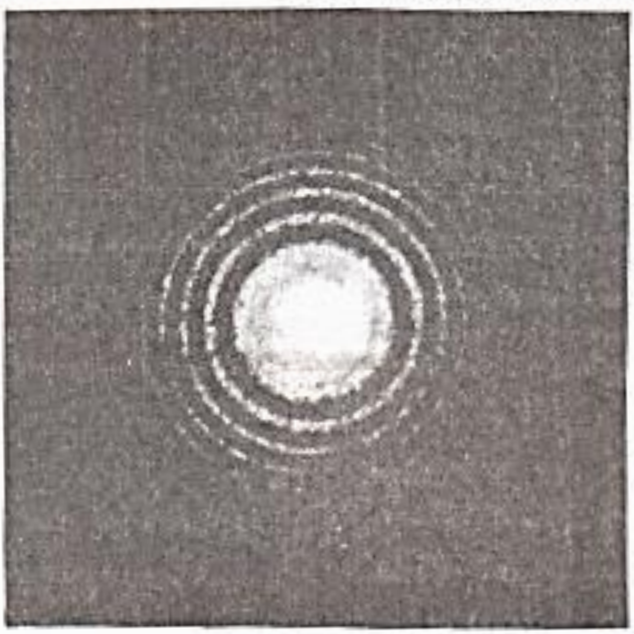

Fig.3: Interferogram of axially shifted double exposure hologram at λ = 63.8 nm, L = 8 mm and ξ_o = 100 um.

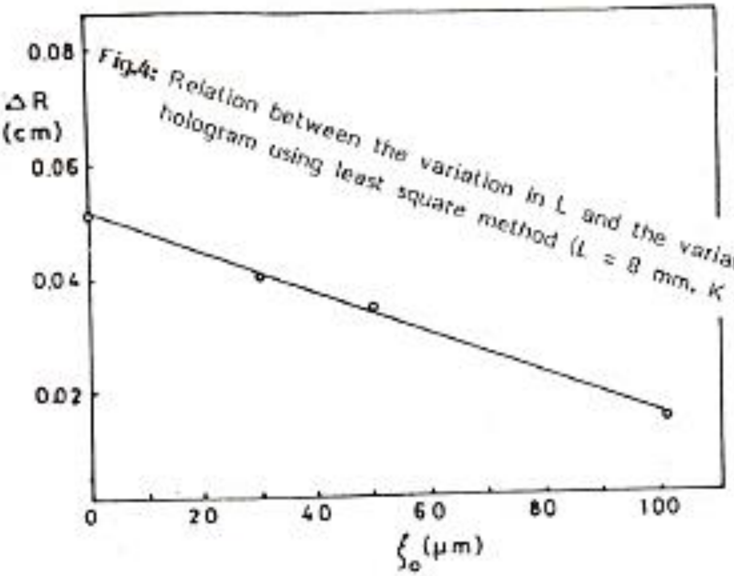

Fig.4: Relation between the variation in L and the variation in R for a double exposure hologram using least square method (L = 8 mm, K = 2).

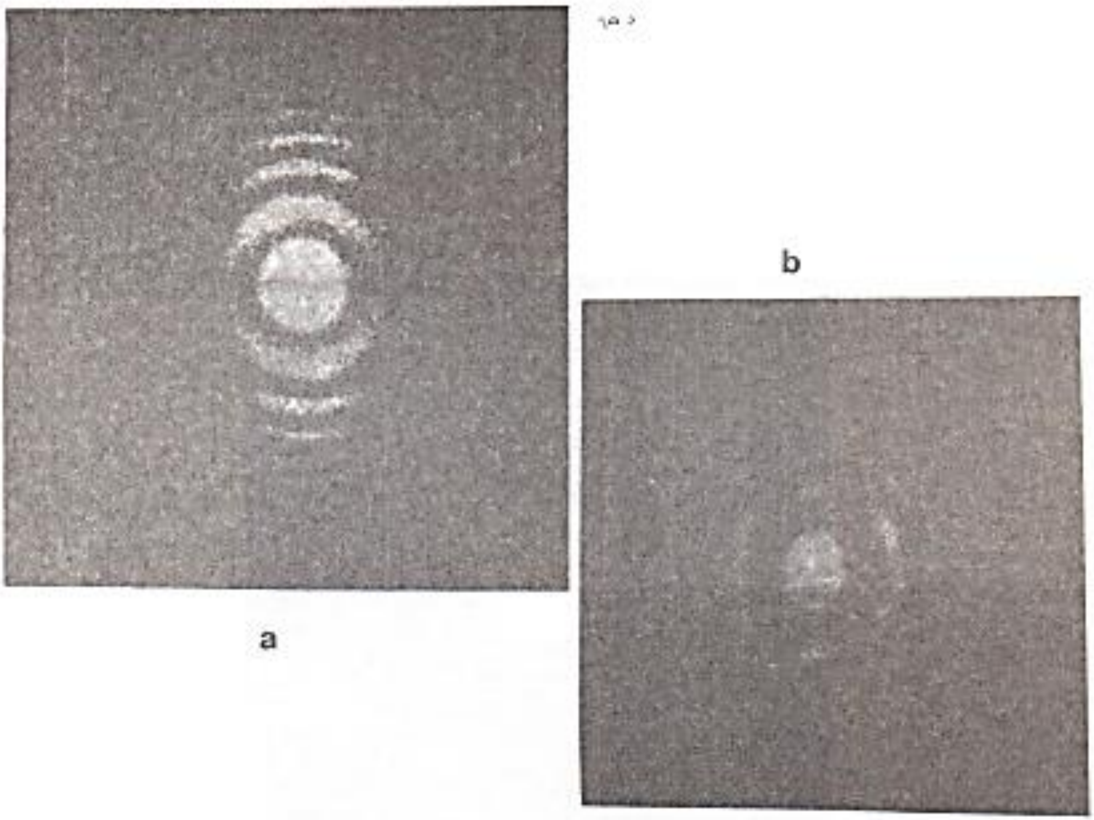

Fig.5: Interferogram of double exposure speckle hologram with lateral translation η_o, separation 3 mm and wavelength 632.8 nm. (a) η_o = 30 um; (b) η_o = 50 um

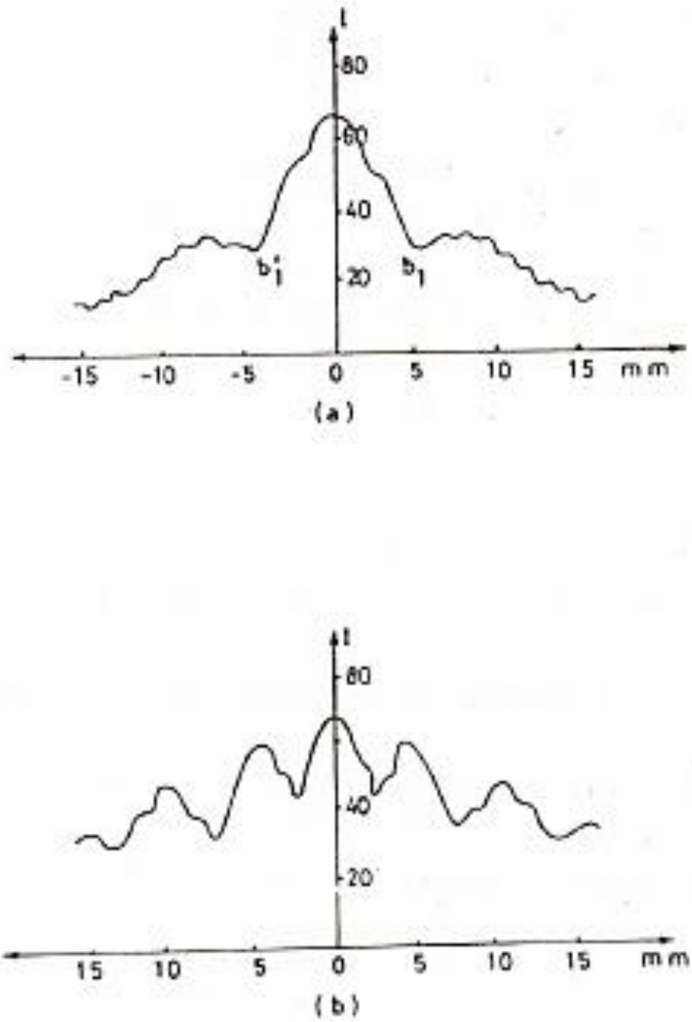

Fig.6: The intensity distribution curves of the corresponding interferograms of Fig. 5.

Conclusions:

1. The speckle holograms of a semitransparent object can be investigated either as two separate identical diffusers or as a diffused object in two orthogonal positions. In both cases an agreement between the theoretical and experimental data is obtained within an acceptable error.
2. The equation representing the lateral displacement can be used to determined simultaneously both lateral and axial positions of the diffused object.
3. The interference pattern consists of straight line fringes overimposed by the same straight line fringes modulated by circular fringes.

References:

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