SIMULATION OF I-V STEPS SHARPENING FOR JOSEPHSON JUNCTIONS

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Abstract:

The effect of rf frequency on the induced I-V steps for Josephson junction is discussed. The resistively shunted junction is applied for the case of an applied rf rectified sinsoidal current in which the amplitude is modulated by a saw toothed function. It is shown that the step rise sharpens on increasing the rf frequency. We also show that the first fundamental step is nearly absent for low rf frequencies, while on increasing the rf frequency, the step evolves and assume its well known configuration.

Introduction

There has been considerable interest to investigate the phenomena associated with Josephson junctions due to their potential use in superconducting quantum interference devices (SQUID'S), high speed ligic and memory devices, mixer modulators, and ac-amplifiers. One such area of interest, is the induced steps in the I-V characteristics of a Josephson junction driven by either an ac- or dc-curent sources or both. The effect of critical current amplitude modulation(1), radio and micro wave power, and thermal noise (thermal current)(2-5) on the induced steps has been widely investigated through the use of several mathematical models. In figure (1), the effect of varying the rf frequency on the I-V characteristics is shown(6). This figure shows several interesting features that occurs on increasing the rf frequency. Such features are, the evolution of the first step, higher steps are better evolved than the first step, but still their step rise sharpens, and finally the over all curve rise increases (i.e. approaching the ohmic line faster). It is to the best of our knowledge that such phenomena has not been investigated and presented elsewhere.

Hence, it is the purpose of this paper to investigate the effect of rf frequency on the features discussed above, from the theoretical point of view. In our work we applied the resistively shunted junction model(7) (RSJ), for a Josephson junction biased by an rf rectified sinsoidal current phase relation, in which its amplitude is modulated by a saw toothed function. The model is worked for three rf frequencies

(10,50,100 MHz) at zero temperature and for an rf frequency (10 MHz) at a normalized temperature (t = 0.75). It turns out the calculated I-V curves agree very well (on a qualitative agreement basis) with the I-V curves of figure (1). In sec. 2 the model is discussed for the zero and non zero temperature, together with the simulation technique used. In sec. 3, the theoretical resusts are presented, discussed and compared to the experimental results. Finally in sec. 4 we present some closing conclusions.

Theory:

The response of a Josephson junction to an ac current source $I_{ac} \sin \omega t$, ω is the applied external frequency), while biased by a dc current source is given by:

$$C\ dV/dt + \sigma V + I_c \sin\phi = I_{dc} + I_{ac} \sin\omega t + I_N , \qquad (1\text{-a})$$

$$2eV = \hbar\ d\phi/\ dt , \qquad (1\text{-b})$$

Where C is the Josephson junction capacitance, V is the voltage across the junction, junction conductance, I_c junction critical current, ϕ is the phase difference across the junction, and I_N is the thermal current generated in the resistor $R = 1/\sigma$ at a given temperature. In eq. (1-b), $\hbar$ is Planck's constant divided by 2π, and e is the electron charge. It is worth noted that eq. (1-a), (for $I_N = 0$), is applicable to other physical systems such as the driven damped pendulum, a charged density wave in the presence of a time varying field pinned to an underlaying lattice, and a charged particle moving in a viscous medium driven by a periodic potential. In our work we will only consider the case for C = 0 and I_{dc} = 0. The rf current source will be replaced by:

$$I_o \ \frac{t}{T} \left|\sin \omega t\right| , \qquad (2)$$

where (T) is the modulation period of the sawtoothed function $(\frac{T_o}{T} t)$, and I_o is the current amplitude at t = T. Next, by setting u = t, equation (1-b) can be expressed as

$$V = \frac{\hbar\omega}{2e} \frac{d\phi}{du} \qquad (3)$$

From (1-a) and (3), we then have:

$$\frac{d\phi}{du} = \frac{2eRI_c}{\hbar} \left[\frac{i_o}{T} u \sin u + i_N - \sin \phi\right] \qquad (4)$$

where i_o and I_N are the rf and thermal currents normalized with respect to the critical current I_c. For the thermal current, the average distribution is given by:

$$\langle i_N \rangle_{rms} = \left(\frac{4k_B T f}{I_c^2 R} \right)^{1/2} \tag{5-a}$$

$$\Delta f = \frac{2e R I_c}{\hbar} (2 \Delta u)^{-1} \tag{5-b}$$

Where k_B is Boltzmann constant, and u is the normalized time step involved in the numerical integration of equation (4).

Equation (4) was solved numerically by applying the Runge Kutta method. Since our aim was to understand the qualitative nature of the solution, and not to get higher precisions, which would necessitate much longer complutation, i_0 was taken up be 10^3, and $T = 10^{-3}$ sec for all three frequencies (10, 50, 100 MHz), and u = /100. The normalized voltage is given by

$$\langle v \rangle = \left\langle \frac{V}{RI_c} \right\rangle = \frac{\hbar \omega}{2eRI_c} \left\langle \frac{d\phi}{du} \right\rangle \tag{6}$$

Where the averaging is taken over half the rf cycle, which is also the case for the current. Finally its worth noting that for the rf current, keeping T constant and changing the frequency, has the same effect of keeping w constant and varying T.

Discussion:

Figure (2) shows the numerical I-V curves at zero temperature and three rf frequencies, 10, 50 and 100 MHz. There are two regions of interest in all curves. The first is that part of the curve that lies below a normalized current equal to two, which is the main interest of this paper. The second region is that for normalized currents values above two. In the first region it is clearly shown how the first step evolves with increasing rf frequency. In figure (3), the inset shows a generals step shape induced by an applied rf current. On the path OA several rf cycles elapse before v reaches v_c. On increasing the current the system will move from A to C, where the duration between transitions decreases. At point C the phase transition occurs every cycle. Beyond point C, the transition occurs every few cycles on both the positive and negative rf swings. At point B the transition rate isnow twice every cycle, and the system moves upward to point D at which a second step appears(5). Bearing this in mind, we nowe pay our attentions to figure (3), in which we placed the 10⁹ MHz curve, (curve I), along side that of the 100 MHz rf frequency, (curve II), and try to reach a physical interpretation of how the first step is evolved. On increasing the current from 1 to $i = i_1$, the voltage on curve I is v_1 (point A) which is less than v_2 (point B) for curve II.

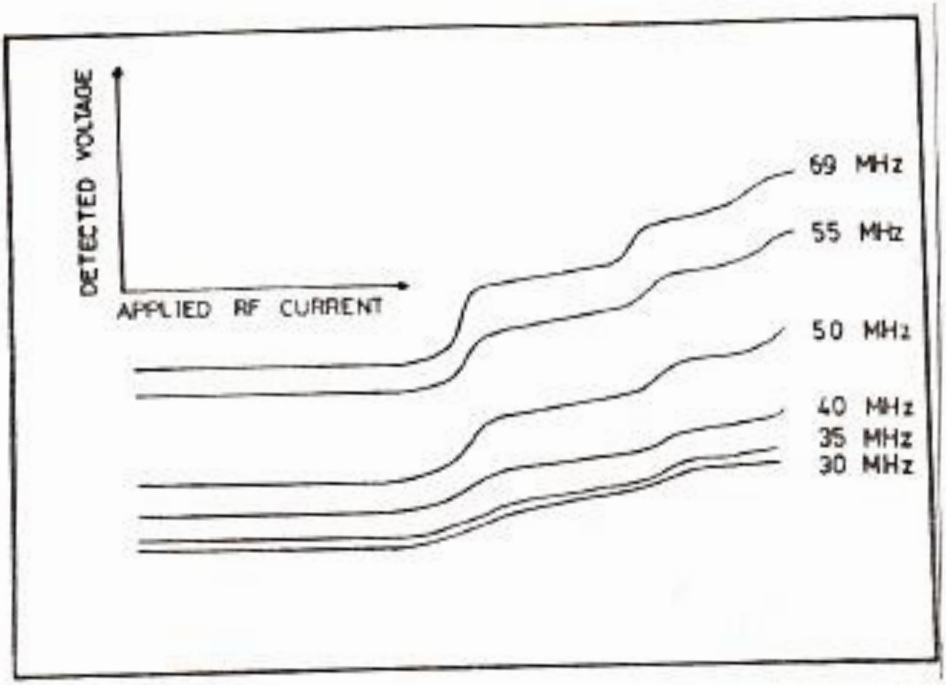

Figure (1): Experimental[1] I-V curves for different rf frequencies.

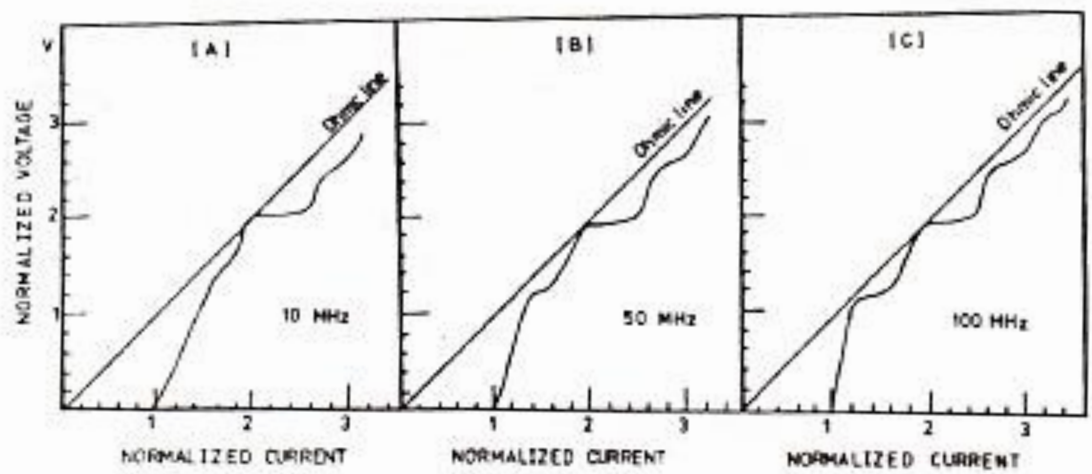

Figure (2): Calculated I-V curves for frequencies (a) 10 MHz, (b) 50 MHz and (C) 100 MHz at zero temperature.

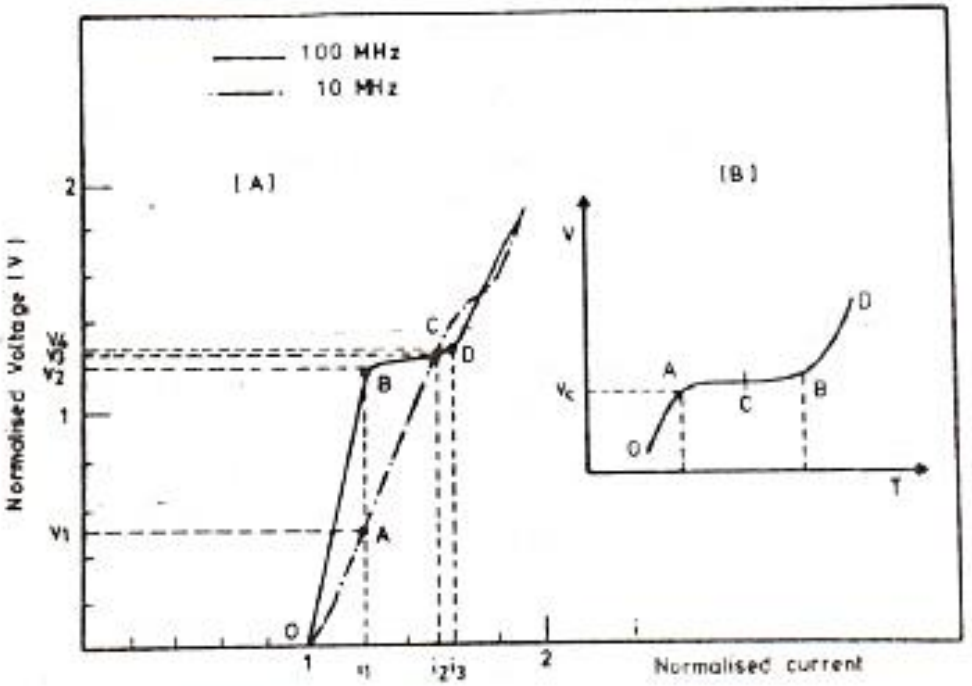

Figure (3): Comparison between I-V curves for rf frequencies 10 and 100 MHz. The effect of the rf frequency is clearly shown on the sharpening of the first step.

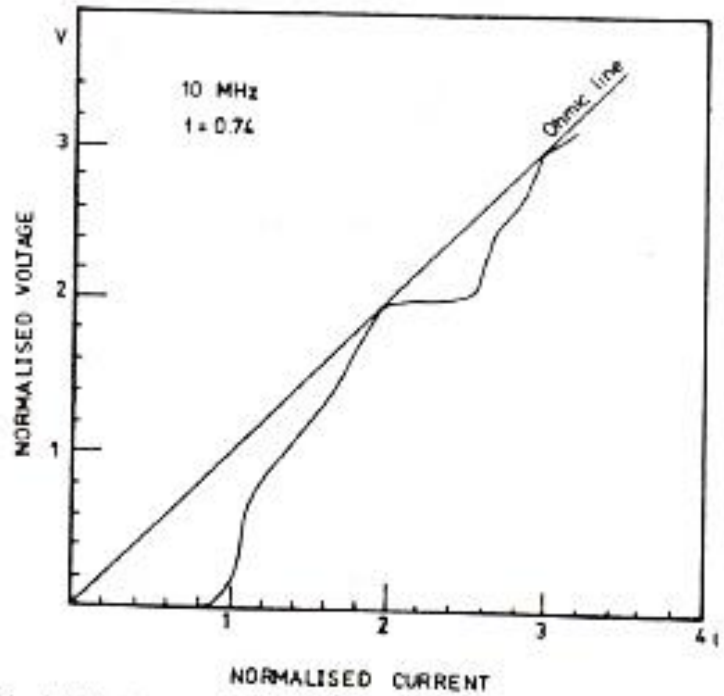

Figure (4): Calculated I-V curve for an rf frequency 10 MHz and normalized temperature t = .74.

Now since curve II is at a higher frequency, then there are more rf cycles involved than that for curve I during the same range of current. More rf cycles means that the winding process is much greater, hence a higher phase transition rate is expected for curve II, which leads to the fact that $v_2 > v_1$. Thus we see that as the rf frequency is increased, the step rise (OB) sharpens. On the other hand for curve I, we see that on further increasing the current, it will rise to point C, beyond which a step is observed at $v \sim 1.5$, $i \sim 1.5$ which is a subharmonic step (n = 3/2). Now back to curve II, on increasing the current beyond point B, a step width is observed. From B to C, the transition rate increases untill at point C we have a single phase transition every complete rf cycle. If our rf current was not rectified, we would then have the step extended to $i \sim 2$ where two transitions occurs every rf cycle. But since we have a rectified current the step width will terminate at point D and a new riser starts again leading to the second step. Thus from the figure and our discussion our work suggest that at low rf frequency there is a step (second subharmonic), and as the rf frequency is increased the step is shifted downwards untill it completly assumes the position of the first fundamental step with a width equal to half the critical current.

In the second region, the model shows steps existing at $V \sim 2$, 2.5 and ~ 3, in other words we have fundamental and subharmonic steps. Such behavior is due to the nature of the rf current considered in this work. The general features of these steps are the same as discussed elsewhere, hence there is no need to repeat them again. It is worth noting that earlier, we mentioned that the effect of varying the rf frequency and keeping the modulation frequency, is the same if the opposite is done. Thus our work predicts that by changing the modusation frequency and keeping the junction at a constant rf frequency we would be able to observe such phenomena . In order to more test our model, and get an extra-support for our work we have also solved the equations for a normalized temperature t = .74 (i.e $i_N > 0$) and an rf frequency 10 MHz, with no change in the modulation frequency. The results are shown in figure (4) where we notice the following. The second subharmonic step is completely washed out, while that for the third it is less pronounced than that for T = 0. The fundamental steps show a slight rounding on their knees. Hence the effect of thermal current is only very strong on subharmonic steps, this is in agreement with ref (3).

Conclusions

There are several important conclusion as a result of our work discussed above. First we have shown the effect of rf frequency on the evolution of the first step. Second, our work proves that such effect can also be observed by keeping the rf frequency constant, and varying the modulation frequency. Last, the rectified current considered in our work leads to the appearance of subharmonic steps. This last result suggest a new mean of experimentally detecting subharmonic steps.

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