MAGNETIC PINNING FOR A TYPE II SUPERCONDUCTING WEDGE

L.N. Shehata & A.G. Saif
Atomic Energy Establishment, Cairo, Egypt.

Abstract:

The total work done by a moving flux line from the sample surface to infinity is evaluated. The magnetic pinning force is then calculated and certain limited values are considered. Special cases are deduced from our general formula which shows an agreement with the available data.

Introduction

The pinning forces can be calculated from the change in magnetic energy of a type II superconductor when the material approachs London limits(1). The flux lines (FL,s) may move freely within a completely homogeneous type II superconductor when the external magnetic field strength is above H_{c_1} - the first critical field. In this case only, mutual interactions between the FL,s appear. On the other hand, if the superconductor is not homogeneous, i.e., if it includes strains, dislocations, grain boundaries, inclusions or variations in composition, then the motion of the FL,s is not free(1,2). The structural defects act as pinning sites i.e., as barriers to the flux motion.

The sample inhomogenities change the distribution of the magnetic fields and supercurrents, which in turn affect the spatial variations of the flux line energy. The presence of macroscopic inclusions or voids(1-3) in the superconducting matrix give rise to magnetic pinning interaction. At low fields the FL,s are well separated and it is possible in this case to obtain an expression for the pinning force(3). Simple examples for such type of interaction is a smooth plan boundary, which is parallel to the FL,s and perpendicular to the driving force, between an insulator and a superconductor.

The interaction between straight or curved FL,s and cylindrical(4,5), spherical(3) and ellipsoidal(6) inclusions were considered. The interaction of a curved or straight FL with the surfaces of an edged type II superconductor as well as the distribution of the magnetic field and the current density were calculated in(7). In the last work the magnetic pinning of a FL in a superconductor occupying a "quarter space" was given. Therefore, it is interested to study the magnetic pinning in a superconductor of a "wedged" shape. This problem will be considered here by different approach. Throughout this work an expression for the total work done by moving a FL from the sample surface to infinity is derived and investigated.

Magnetic Pinning

We assume our superconductor in the form of a "wedge" with two inclined surfaces by an angle and defined by $0 \leq \varphi \leq \alpha$, $0 \leq \rho \leq \alpha$ and $-\infty < z < +\infty$ where (ρ, φ, Z) are the cylinderical coordinates. As it was shown in(7) an energy barrier exists near the boundaries of the "edged" superconductor. This barrier arises from the character of the interaction near the boundaries between the flux lines (repulsive) and their images (attractive). The repulsive force is caused by the surface current I_s, the flux density difference of which is,

$$\vec{M}_{rev} = (\mu_o \vec{H}_o - \vec{B}), \tag{1}$$

where $\vec{H}_o$ and $\vec{B}$ are, respectively, the external longitudinal magnetic field and the flux density ($\vec{B} = n\vec{\phi}_o$; n is the equilibrium flux density and ϕ_o in the flux quantum) and μ_o is the permeability. On the other hand, the attractive force is due to the distortion of the flux line structure in the vicinity of the surface. Thus the potential energy U of a flux line may be expressed as the sum of the two terms U_1, the potential energy due to the interaction of the FL,s and the external field, and U_2, which is due to the attractive interaction of the FL,s and their images through the surface boundaries.

At the flux position (ρ_o, φ_o) the penetrating and proper fields $\vec{H}_L(\rho_o, \varphi_o)$ and H_v (ρ_o, φ_o), respectively, can be calculated from(10), these fields are

$$\vec{H}_L(\rho_o, \varphi_o) = \frac{2\vec{H}_o}{\pi} \int_o^{\infty} \frac{\cosh \frac{\pi \tau}{2}}{\sin h} [\sin h\, \varphi_o \tau + \sin h\, (\alpha - \varphi_o)\tau] \times$$

$$K_{i\tau}(\rho_o)\, d\tau \equiv \frac{2H_o}{\pi} \vec{f}, \tag{2}$$

$$\vec{H}_v(\rho_o, \varphi_o) \simeq \frac{\vec{e}_z}{\delta\ell} (2h_{c_1} - e^{-2\rho_o \sin \varphi_o/2}) \equiv \frac{g}{\delta\ell}, \tag{3}$$

where $K_{i\tau}(z)$ is the cylinderical function of the complex argument with imaginary index (for $\tau = o$, $K_o(z)$ is Hankel's function, and $(\vec{e}_\rho, \vec{e}_\varphi, \vec{e}_z)$ are the unit vectors in cylinderical coordinates. The surface current density $\vec{J}_s = \frac{c}{4\pi}$ curl $\vec{H}_L$ may be calculated may be calculated as,

$$\vec{J}_s \sim \frac{M_{rev}}{\mu_o} \left[\frac{\delta f}{\delta \rho_o} \vec{e}_\rho + \frac{1}{\rho_o} \frac{\delta f}{\delta \varphi_o} \vec{e}_\varphi \right] = \frac{M_{erv}}{\mu_o} \text{curl } \vec{f}, \tag{4}$$

where $M_{rev} = \left| M_{rev} \right| \equiv \left| \mu_o H_o \right|$ for $H_o \sim H_{c_1}$

The lorentze force (or driving force) per unit length of the FL has the form

$$\frac{\emptyset_o}{\mu_o} \vec{M}_{rev} \times \text{curl } \vec{f}.$$

In addition on a FL exerts and attractive force A x curl H_v, where A is a constant vector to be determined, i.e.

$$\vec{A} \times \text{Curl } \vec{H}_v \quad A \times \text{Curl } \vec{g}.$$

In equilibrium, the total work done by moving the FL from the edge of the sample to infinity must be zero(1,2,8). Using this, o e can determine the constant vector A. Long calculations give

$$\int_0^{\infty} \left(\frac{\emptyset_o M_{rev}}{\mu_o} \frac{\delta f}{\delta \rho_o} - A \frac{\delta g}{\delta \rho_o}\right) d \rho_o + \int_0^{\infty} \frac{1}{\rho_o} \left(\frac{\emptyset_o M_{rev}}{\mu_o} \frac{\delta f}{\delta \varphi_o} - \right.$$
$$\left. - A \frac{\delta g}{\delta \varphi_o}\right) \rho_o \; d \varphi_o = 0. \tag{5}$$

i.e.

$$A = - \frac{\emptyset_o M_{rev}}{\mu_o} \left[2 - e^{-2 \rho_o \sin \alpha/2}\right]^{-1} \tag{6}$$

therefore,

The force due to magnetic pinning may be expressed as (compare(1,3,6)):

$$\vec{F}_p = \frac{\emptyset_o \vec{M}_{rev}}{\mu_o} \times \text{curl } \vec{f} - A \times \text{curl } \vec{g} \tag{7}$$

Where the expressions for Lorentz and attreactive forces and (6) are to be used in (7).

From (7) it is easy to show that the magnetic pinning force $\vec{F}_p$ attains its minimum value when $\varphi_o = \frac{\alpha}{2}$ (assuming that $H_o \sim H_{c_1}$, i.e., $M_{rev} \sim H_{c_1}$), in this case

$$(\vec{F}_p)_{min} = (\emptyset_o H_{c_1} \times \text{curl } \vec{f})_{\varphi_o = \frac{\alpha}{2}} \tag{8}$$

The maximum value of $\vec{F}_p$ is reached at $\rho_o \to \frac{\xi}{\lambda}$, where is the coherence length and is the penetration depth, i.e.,

$$(\vec{F}_p)_{max} = (\vec{\emptyset}_o H_{c_1} \times \text{curl } \vec{f})_{\rho_o = \frac{\xi}{\lambda}} \tag{9}$$

For planer surface $\alpha = o$ (or $= 2\pi$) and the constant vector value is (compare(1,3))

$$A = -\frac{\emptyset_o M_{rev}}{\mu_o}; \tag{10}$$

in this case the corresponding magnetic pinning force (7) leads to the same formula given in refs.(1,3).

Summary and Conclusion:

1. The magnetic pinning due to a planer surface can be deduced from our general formula (7). (Compare with refs.(1,3)).
2. The pinning force may be calculated from the change in the magnetic energy as the sample approaches London Limit. This force is the sum of the pinning force due to FLs entering and exiting the sample.
3. The magnetic pinning attains its minimum value when $\varphi_o = \frac{\alpha}{2}$. While it reaches its maximum value at $\rho \sim \gamma\xi^{-1}$. These results agree with our previous calculations given in ref.(7).

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