



The Egyptian International Journal of Engineering Sciences and Technology

Vol. 33 (2021)28–36

<https://ejest.journals.ekb.eg/>



Analysis and vibration of rectangular nanoplates - An overview

Atef Eraky^a, Abdallah Salama^b, Karim I. Mohamed^{c*}

^aProfessor of structural engineering, faculty of engineering, Zagazig University, Egypt.

^bLecturer at structural engineering Dept., faculty of engineering, Zagazig University, Egypt.

^cDemonstrator at structural engineering Dept., faculty of engineering, Zagazig University, Egypt.

ARTICLE INFO

Keywords:

Nanoplates
Free vibration
Forced vibration
Nonlocal elasticity theory
Analytical solution

ABSTRACT

This work presents a review on solution methods and analysis of nanoplates structures with different boundary conditions and load cases, under some effects such as magnetic field and the effect of other parameters on the vibration and analysis. Moreover, it represents a review about the theories that are used to study these nanoplates structures such; the nonlocal elasticity theory by Eringen which is introduced to take into consideration the small scale effect of such nano-structure. The equation of motion of the nanoplate is derived then it used to study this type of nano-structures. Nanoplates are used in a lot of branches of life and in different applications because of the high and excellent mechanical, thermal and electrical properties of these nanoplates. Applications of nanoplate structure are also introduced to show the importance of studying and getting a solution of such nanoplates. Also, some parametric studies are discussed to show the effect of the studied parameters on the dynamic behaviour and analysis of these nanoplates.

1. Introduction

Nanotechnology has recently become one of the most attractive areas in research because of its using and development in the different branches of life. As a reason of the widely progress and development at the area of the nanotechnology, nanoplates like another nanostructure has been used in a lot of applications of nano or micro electro mechanical systems and because of this reason it was very important for researchers to study this type of nano structures. The extensive property such as electrical, mechanical and thermal properties [1, 2] of the materials that is used in the manufacture of the nanoplates made this type of nanostructure very important. Nanoplates such as graphene sheet would be one of the eminent new materials for the next

generation of nano devices Fig. (1). This graphene sheets are used in the manufacture of a lot of nano-devices like sensors and memory devices [3] (Fig. (2)) and there are a lot of another applications like nano-sheet resonator [4], mass sensors [5] and gas sensors [6]. Studying of such type of small scale structures, understanding the performance and experiments on the nano scale level is a difficult and expensive duty. This is the reason why many researchers made their studies and researches on the development of theoretical models. The using of nanoplates in a lot of work fields made the understanding of dynamic behavior, structure and vibration of these nanoplates an important issue. Therefore, this point of study has become a subject of interest in recent studies because it is very important for manufacture and design of such that devices.

* Corresponding author. Tel.: +2-01003502134
E-mail address: Alaomdakarim@yahoo.com

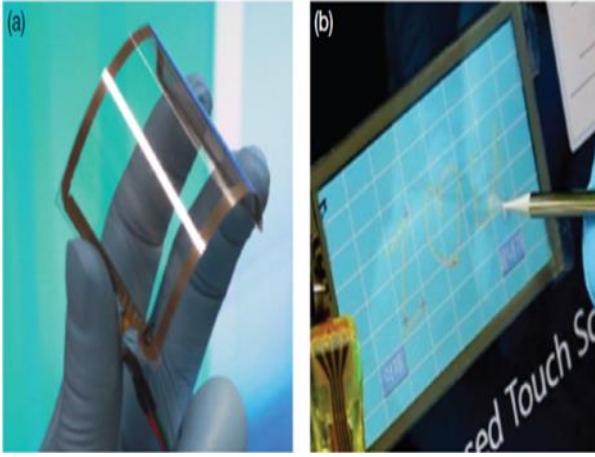


Fig. 1. Nano devices. (a) A graphene touch panel showing flexibility [1]. (b) A graphene touch screen panel connected to a computer with control software [1].

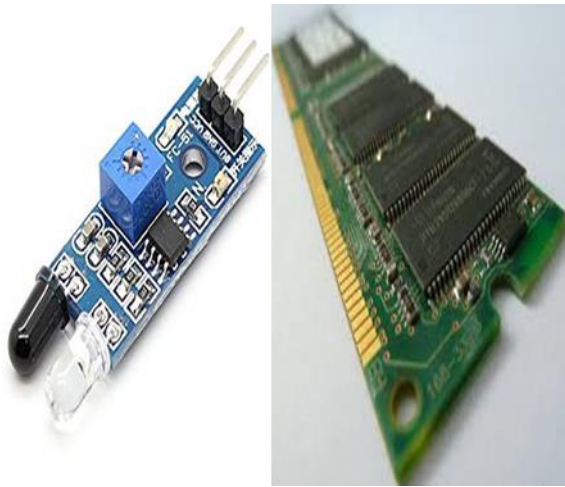


Fig. 2. Sensor and Memory device.

2. Small size challenge

As a reason of the very small size effect of such type of structure, the atomic simulation and laboratory experiments methods are very difficult, time and money consuming to study this type of nanostructures with these methods. So that, at 1972 Eringen [7] introduced the first form of nonlocal model taking into consideration the very small scale size effect and intermolecular cohesive forces to study such nanostructures.

2.1 Nonlocal elasticity theory (integral)

As it mentioned before the big challenge is to take into account the small scale affect so Eringen [7] succeeded to overcome this point. At this study the stress at any reference point of the domain depends on the strain field at all points of the domain. Experimental investigations on phonon dispersion and lattice dynamics have defended this statement. This theory introduced the relationship of stress and strain for a homogeneous elastic solid according to:

$$\sigma_x = D * \epsilon_x \quad (1)$$

$$\sigma_{ij} = \int \Psi(|x - \bar{x}|) * C_{ijlh} * \epsilon_{lh}(\bar{x}) * dV(\bar{x}) \quad (2)$$

where $\Psi(|x - \bar{x}|)$ is the kernel function or nonlocal modulus whose argument is the Euclidean distance ($|x - \bar{x}|$), C_{ijlh} is the elastic modulus tensor, D is the elastic modulus, σ_{ij} is the nonlocal stress tensor and ϵ_{lh} is the nonlocal strain tensor.

2.2 Nonlocal continuum theory (differential)

After the previous form of integral model, an equation in differential equivalent form of Eringen's nonlocal elasticity theory was presented to simplify the previous integral form of the nonlocal model and that was according to [8]. At this part of study the kernel function or nonlocal modulus at the previous integral model equation is satisfied and the differential equation can be obtained with the differential operator that is given by $(1 - (e_0 a)^2 \nabla^2)$. Therefore, the integral nonlocal form equation can be simplified to:

$$(1 - (e_0 a)^2) * \sigma = E \epsilon \quad (3)$$

where (a) is an internal characteristic length (lattice parameter, granular size, or molecular diameters) and (e_0) is a material constant for adjusting the model in matching some reliable results by experiments or other models.

3. Displacements and strains

According to the Kirchhoff plate theory [9], the displacements of any point of the nanoplate can be expressed in terms of the middle surface

displacement components. The displacement field is given by:

$$u(x, y, z, t) = u_0(x, y, t) - z \frac{\partial w_0}{\partial x} \tag{4}$$

$$v(x, y, z, t) = v_0(x, y, t) - z \frac{\partial w_0}{\partial y} \tag{5}$$

$$w(x, y, z, t) = w_0(x, y, t) \tag{6}$$

where u_0 , v_0 and w_0 are the middle surface displacement component in the x , y , z directions respectively.

The strains which are related to the previous three displacement equations can be computed using either the nonlinear strain-displacement relations or the linear strain-displacement relations. Then, the strain components of any arbitrary point in the nanoplate are expressed as the following:

$$\epsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \tag{7}$$

$$\epsilon_{yy} = \frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \tag{8}$$

$$\epsilon_{xy} = \frac{1}{2} \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right] \tag{9}$$

After substituting by the form of displacement that is given by Kirchhoff plate theory, the previous equations of strains will be in the form of:

$$\begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \epsilon_{xx}^0 \\ \epsilon_{yy}^0 \\ \gamma_{xy}^0 \end{bmatrix} + Z \begin{bmatrix} \epsilon_{xx}^1 \\ \epsilon_{yy}^1 \\ \gamma_{xy}^1 \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} \epsilon_{xx}^0 \\ \epsilon_{yy}^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2} \left(\frac{\partial w_0}{\partial x} \right)^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2} \left(\frac{\partial w_0}{\partial y} \right)^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \frac{\partial w_0}{\partial x} \frac{\partial w_0}{\partial y} \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} \epsilon_{xx}^1 \\ \epsilon_{yy}^1 \\ \gamma_{xy}^1 \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2 \frac{\partial^2 w_0}{\partial x \partial y} \end{bmatrix} \tag{12}$$

where $(\epsilon_{xx}^0, \epsilon_{yy}^0, \gamma_{xy}^0)$ are the membrane strains and $(\epsilon_{xx}^1, \epsilon_{yy}^1, \gamma_{xy}^1)$ are the bending strains which are known as the curvatures.

4. Equation of motion

The study of the analysis of any structure and understanding its behavior cannot be done without reaching to the equation of motion that describes the construction of this structure. The equation of motion that describes this type of structure is derived in detail in [9]; the equation of motion is derived using the principle of virtual work displacements. The dynamic formulation of the principle of virtual work is:

$$\int_0^T (\delta U + \delta V - \delta K) dt = 0 \tag{13}$$

where (δU) , (δV) and (δK) represent the virtual strain energy, the virtual work done by the applied forces and the virtual kinetic energy respectively.

Substituting by (δU) virtual strain energy, (δV) virtual work done by applied forces and (δK) virtual kinetic energy into the dynamic principle virtual work equation and making simplification we will get a simplified form to the equation of motion by equaling the coefficient of virtual strain energy (δU) , virtual work done by applied forces (δV) , and the virtual kinetic energy (δK) to zero; we get the following form:

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \frac{\partial^2 u_0}{\partial t^2} - I_1 \frac{\partial^2}{\partial t^2} \left(\frac{\partial w_0}{\partial x} \right) \tag{14}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = I_0 \frac{\partial^2 v_0}{\partial t^2} - I_1 \frac{\partial^2}{\partial t^2} \left(\frac{\partial w_0}{\partial y} \right) \tag{15}$$

$$\frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial^2 M_{yy}}{\partial y^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + q = I_0 \frac{\partial^2 w_0}{\partial t^2} \tag{16}$$

where quantities (N_{xx}, N_{yy}, N_{xy}) are the in-plane force resultants, and (M_{xx}, M_{yy}, M_{xy}) are the moment resultants, and (I_0, I_1) are the mass moments of inertia.

5. Solution methods and analysis

The solution methods and analysis [9] that is used to get the solutions of the nanoplates with different boundary conditions of the edges are Navier method, Levy method, Ritz method and the analytical method. The Navier solution method [10, 11] can be used for any rectangular nanoplates with a simply supported ends for all edges. The Levy solution [12] can be used for any rectangular nanoplates with a simply supported any two opposite edges and the other two edges having one of any possible combination of boundary conditions: simple support, free or fixed support. The Ritz method [13] can be used to get approximate solutions for any different boundary conditions, as long as this method can find a suitable approximation functions for the problem. The analytical solution is one of the most popular methods to get an exact solution for such that nanostructure [14, 15].

6. Medium which nanoplates embedded in

The modeling of the medium which nanoplates embedded in it or the foundation which nanoplates are resisting on it; is very important issue. So that, it is very important to show the different types of this medium.

6.1 Elastic medium

The elastic medium can be modeled as Winkler foundation [16]. The Winkler model consists of a group of infinite set of springs connected with the nanoplate in parallel and shear layer stiffness Fig. (3). the spring stiffness is depicted (K1) and shear layer stiffness (K2).

6.2 Viscoelastic medium

The viscoelastic medium can be modeled as Kelvin–Voigt foundation [10]. The Kelvin–Voigt model consists of a group of infinite springs which represent the stiffness and dashpots which represent the damping coefficient connected with the nanoplate in parallel Fig. (4). The damping coefficient and the spring stiffness are depicted as (C)

and (K) respectively.

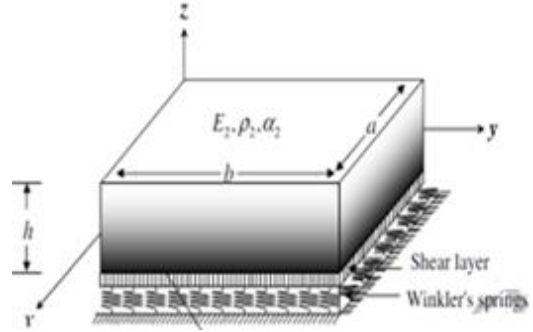


Fig. 3. Nanoplate resting on elastic foundation [16].

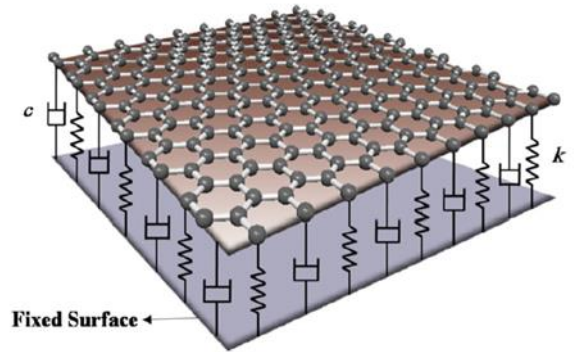


Fig. 4. Nanoplate resting on viscoelastic foundation [10].

7. Vibration and dynamic analysis of nanoplates

The using of nanoplates in a lot of work fields made the understanding of dynamic behavior, structure and vibration of these nanoplates a normal issue. Therefore, this point of study has become a subject of research and interest in recent studies because it is very important for manufacture of such devices and for optimization and effective design.

7.1 Free vibration

Malekzadeh et al. [17] investigated the free vibration of an orthotropic straight-sided quadrilateral nanoplate. This study was based on two main theories, the nonlocal elasticity theory to take the small scale size into consideration and the first order shear deformation theory (FSDT). The results of this study showed that, the quadrilateral nanoplates were influenced by the small scale size effect more than the rectangular nanoplates with the same thickness

for the two types. Also, the results showed that the high order frequencies were influenced by the nonlocal parameter greater than the lower ones and this occurs by increasing the thickness ratio. Also, by increasing the nonlocal parameter the frequency decreases for all type of boundary conditions.

Ansari et al. [18] investigated the free vibration of single layer graphene sheets (SLGSs). This study was based on the generalized differential quadrature method and the nonlocal elasticity theory was used to take the small scale effect into account. The frequencies solutions were introduced for two types of supports ends of the single layer graphene sheets; the simple and clamped support and this solutions based on the generalized differential quadrature method. The molecular dynamic simulation was made for this type of nanoplates and the results were with good agreement with the results introduced by the previous method with different boundary conditions. The results showed that the nonlocal parameter values were not dependent on any variable in the geometry of the nanoplate.

Pouresmaeeli et al. [10] investigated the free vibration analysis of an orthotropic viscoelastic nanoplates resting on viscoelastic foundation with simply supported ends. This study was based on the Classical Laminated plate Theory (CLPT) and the nonlocal elasticity theory. The Navier solution method was used to get the frequency of the simply supported nanoplates. The results showed that the frequency was influenced by the damping coefficient of the surrounding medium, damping coefficient of the nanoplate, stiffness of the surrounding medium and aspect ratio. The frequency is increasing by decreasing of damping coefficient of surrounding medium and structural damping but the frequency increases by increasing the stiffness of the foundation and the aspect ratio.

Karlicic et al. [19] studied the free vibration of an orthotropic viscoelastic nanoplates consisting of multi nanoplate system resting on viscoelastic surrounding medium. This study was based on the Kirchhoff plate Theory and the nonlocal elasticity theory. The exact natural frequencies were derived and introduced for the simply supported nanoplates with a different number of these nanoplates. The results showed that by increasing the nonlocal parameter the frequency decreases as a result of the decreasing of stiffness and rigidity. Also, by increasing the number of the nanoplates in system, the frequency decreases.

Li et al. [20] introduced solutions of the free vibration of rectangular thin plates with all edges

free. This study is based on the Hamiltonian system and this based on the symplectic super-position method. This work can be used as a benchmark for the future work. The exact solution of the deflection of such plates was given and the results were with a good agreement with the results from the finite element method.

7.2 Forced vibration

Abbas Assadi [21] studied the forced vibration of rectangular nanoplates using an analytical method with taking into consideration the surface properties and effects such as surface stresses, surface mass density and surface elasticity. This study was based on the Kirchhoff plate theory and the results were in a good agreement with the results obtained from experiment. The results for the forced vibration study showed that there was an ellipse region on the nanoplate's surfaces at which if any point load is applied on any point of this region, there will no effects of the surface properties on the response of the nanoplates. Also, the previous ellipse region has a major and a minor axis which the length of these axis equal to half of the nanoplates edges length in the direction which is parallel to these axis. The effect of surface properties decreases by increasing the excitation frequency and will have no effect at the resonance excitation.

Hashemi et al [22] investigated the forced vibration of a rectangular isotropic viscoelastic single layer graphene sheets resting on a viscoelastic foundation by using an analytical method. The Navier solution method was used in this study to get the response and the dynamic deflection of the simply supported nanoplates with a distributed load. The viscoelastic foundation or surrounding medium of the nanoplate was modelled as Kelvin-Voigt model. This study was based on the classical laminated plate theory and the nonlocal elasticity theory. The results showed that by increasing the nonlocal parameter, the dynamic deflection and the natural frequency of the system increase and this occur because the stiffness and rigidity of the nanoplate decreases. Also, by increasing the nonlocal parameter and decreasing applied load area, the higher resonant frequency moves to left. The dynamic deflection changes with the location and area of the applied distributed load. The results showed that the maximum deflection increases by increasing the area of the dynamic load and when the applied load moves away from the ends of the nanoplates. By increasing the surrounding medium

stiffness, the dynamic deflection decreases and the natural frequency increases.

Rahbar and Rostami [23] studied the forced vibration of rectangular orthotropic plates with different arbitrary types of boundary conditions and the plates were subjected to a non-uniform distributed dynamic load by using a semi analytical solution. The dynamic displacement and bending moments were determined and introduced at this paper. This study was based on the Extended Kantorovich Method. At this method the partial differential equations were reduced to ordinary differential equations which will be easy to use these equations. The results obtained from this method were in a good agreement with the results in the previous studies.

T. Aksencer and M. Aydogdu [12] investigated the forced vibration and buckling of an isotropic rectangular single layer grapheme sheets. This study was based on the Classical Laminated plate Theory (CLPT) and the nonlocal elasticity theory. The Navier solution method was used in this study to solve the nanoplates with simply supported ends and the Levy solution method was used to solve the nanoplates with two opposite ends simply supported and the other two ends with any type of support. The results of this study showed that the nonlocal parameter should be taken into consideration for any nanoplate with length or width less than 30 nm. As a reason of the small scale effect, the nonlocal dynamic deflection was larger than the classical dynamic deflection at which the nonlocal parameter equal to zero. Also, the results showed that the clamped supported ends of nanoplates were more sensitive to nonlocal parameter effect than the simply supported ends. The frequency decreases by increasing the nanoplate's aspect ratio. By increasing the half wave numbers, the effect of the nonlocality on vibration increases. The buckling of the nanoplate increases by decreasing the nonlocal parameter. At the larger length of nanoplates, the buckling load ratio becomes constant and the effect of nonlocality is lost. The amplitude coefficient of the vibration is introduced also at this study.

Junhai et al [14] studied the forced vibration of a rectangular viscoelastic orthotropic nanoplate which was embedded in viscoelastic surrounding medium with a completely free ends. The viscoelastic foundation was modelled using Kelvin-Voigt foundation. This study was based on the nonlocal elasticity theory by Eringen to take into consideration the small scale effect and the D'Alembert's principle was used also. The exact solution for this type of

nanoplate was introduced by using an analytical method and this method was the Hamiltonian-based method and by using a superposition of some boundary conditions. At this analytical method the vibration of this nanoplate was introduced by using a new total unknown vector which was reduced after that to an eigen-problem in symplectic space. The molecular dynamic simulation was used and the results were in a good agreement with the results from this method. The results showed that, by increasing the nonlocal parameter, the resonant frequency increases except the first resonant frequency is not effect by the nonlocal parameter. Also, by increasing the modulus ratio, the resonant frequency increases. Besides, the resonant frequency increases by increasing the stiffness of the viscoelastic foundation and with the decreasing of the damping coefficient of the viscoelastic foundation.

Atanasov et al [24] studied the forced vibration of simply supported rectangular elastic orthotropic double layered nanoplates with the effect of the in-plane Lorentz magnetic force and the exact solution of this vibration was introduced by using Bernoulli-Fourier method. The elastic foundation was modeled using the Winkler foundation. This study was based on the Kirchhoff plate theory and the nonlocal elasticity theory by Eringen. The results showed that, with neglecting the nonlocal parameter coefficient and the magnetic force, the amplitude of the response becomes lower for the orthotropic nanoplates than the isotropic nanoplates. It was noticed from this study that the dynamic response of the nanoplates increases by decreasing the magnetic force and the nonlocal parameter. Also, the dynamic response of the nanoplate increases by increasing the external excitation and this phenomenon occurs with the lower value of the nonlocal parameter coefficient but with a larger value of the nonlocal parameter coefficient, the dynamic response decreases by the increasing of the external excitation.

Majid et al [25] studied the nonlinear vibration of a rectangular single layer grapheme sheets resting on viscoelastic foundation which was modelled as Kelvin-Voigt medium and taken into account for this study the thermo magnetic force and the effect of the multi frequency excitation. The exact solution of this forced vibration behaviour was obtained by using the multiple time scales method and the Galerkin principle which transformed the partial differential equations of motion to the ordinary equations. The results were in a good agreement with the results of the other literatures. The results showed that, the frequency response of any nanoplate especially single

layer graphene sheets decreases by increasing the coefficient of Visco-paster and Winkler and the resonance behaviour was affected by the combination of the force amplitude. Also, with the long periods of time of the frequency excitation made the nonlinear dynamic behaviour was complex and no one can predict it. It is noticed that, by increasing the force amplitude, the frequency response increases. By increasing the damping coefficient, the frequency response decreases and this because the damping coefficient makes the system to be hard. The nonlocal parameter coefficient effect on the dynamic response at the nonlinear vibration counter to the effect on the dynamic response at linear vibration.

Liu and Chen [26] studied the vibration response of the finite periodic single layer graphene sheets with different boundary conditions by using the wave method. This study was based on the nonlocal Mindlin plate theory. The results obtained from the molecular dynamic simulation (MD) were agreed well with the results obtained from the current wave method. The results obtained from this study showed that, at the region of the high periodic number, the displacement response was smaller than those at the low periodic number and the shear strain response was much more at the region of the low periodic number than the region of the high periodic number. For the nanoplates with two edges were simply supported and the other two edges were free ends and with different width, the increasing of nanoplates width leads to a decrease in the first band gap bandwidth and also at this case, the shear strain response and displacement response increasing. It was noticed that, the boundary supports were affected on the first order gap bandwidths of the periodic nanoplates and the boundary constrains were affected on the high order gap bandwidths of the periodic nanoplates. The band gap bandwidths of the shear strains response and the displacements response of the periodic nanoplates were affected by the thickness of the nanoplates and all bandwidths of the periodic nanoplates increased by decreasing the nanoplate's thickness.

Adhikari et al [27] investigated the transverse vibration of rectangular nanoplates such single layer graphene sheets with simply supported boundary conditions for all edges of the nanoplate and studied the nonlocal normal modes of the nanoplates by introducing the mass matrix and stiffness matrix for these nanoplates. Also, the nonlocal functions of mode shapes, natural frequency and frequency response were derived and introduced at this study. The results of the natural frequency were in

agreement with results of the finite element simulation. This study was based on the dynamic behaviour of three nonlocal systems and these systems are beam, rod and plate. The results of the three studied systems for the functions of the nonlocal mode shape, natural frequency and frequency response were with a high accuracy of all systems except the nonlocal rod model.

Khaniki and Hosseini [28] studied the dynamic response of a simply supported viscoelastic orthotropic nanoplates which consist of double layers of the nanoplates and this study was investigated under the effect of a moving nanoparticles with different biaxial loads on each layer. The main theory which this study was used is the Kelvin-Voigt theory to take into account the coupling between the double layers. This study was based on D'Alembert principle, nonlocal elasticity theory and the Kirchhoff plate theory. The exact solution was introduced by using the Laplace and Galerkin transform methods. The results showed that, increasing the stiffness coefficient of the surrounding medium to the nanoplates leads to an increase in the coupling between the two layers of the nanoplates and this phenomenon leads to make the deflection or deformation in the first layer increases but decreases in the second layer. Increasing the small scale size effect which represented by the nonlocal parameter leads to an increase in the deformation of the two layers of nanoplates. Increasing the damping coefficient of the surrounding medium leads to a decrease in the deformation of the two layers of the nanoplates. Also, with the effect of the biaxial compression load, the deformation of the two layers increases but with the effect of the biaxial tension load, the deformation of the two layers decreases. The results also showed that, with all angles of the moving of nanoparticles on surface of nanoplates, the nonlocal parameter increases and this leads to an increase in the dynamic behavior deformation of the two layers of nanoplates. Finally, increasing the radius in circular moving nanoparticles path leads to a decrease in the dynamic behavior deformation of the two layers of nanoplates.

Jomehzadeh et al [29] investigated the dynamic behavior of the nonlinear response of an isotropic rectangular simply and clamped supported graphene sheet matrix subjected to sub-harmonic and harmonic resonance. This study was based on the Von-Karman principle and the nonlocal elasticity theory by Eringen. The exact solution was introduced by using the averaging method. The results from this work were in a good agreement with results from the

molecular dynamic simulation using the REBO potential simulation. The results showed that, the frequency response of the graphene sheets was one third of the excitation frequency. The simply supported graphene sheets have a nonlinear behavior compared to the clamped supported ones and this because simply supported ones have low stiffness than the clamped ones. At the nonlinear behavior of graphene sheets, the frequency and amplitude increase with each other and this mean that, the graphene sheets nanoplates are hardening system. The increase in the elasticity of the graphene sheet matrix leads to a decrease in the nonlinear dynamic behavior and this phenomenon has an effect on the simply supported graphene sheets than clamped supported ones.

Nami and Janghorban [30] investigated the dynamic resonance of rectangular simply supported functionally graded nanoplates. This study was based on the strain gradient elasticity theory, nonlocal elasticity theory and the Kirchhoff plate theory. The results showed that, the increasing of the nonlocal parameter and the power law indexes leads to a move of the resonance position to the lower frequencies of the load but the power law indexes have more effect than the nonlocal parameter. Also, increasing the gradient parameter leads to move the position of the resonance to the higher load frequencies and by increasing the aspect ratio, the resonance frequency decreases.

Ghorbanpour [31] studied the forced vibration and dynamic analysis of rectangular poly-vinylidene fluoride nanoplates resting on elastic foundation with the effect of moving nanoparticles. The elastic foundation was modeled as Pasternak foundation model. This study was based on the classical laminated plate theory, Hamilton's principle theory and the nonlocal elasticity theory. The exact solution of the dynamic deflection and vibration was introduced by using the Galerkin method. The results showed that, by increasing the mode numbers, the frequency ratio decreases and this is clear on curves. For all modes numbers, the increasing of the nonlocal parameter leads to a decrease in the frequency ratio and this because of decreasing the interaction force between atoms. Also, increasing the nanoparticle mass leads to a decrease in the frequency ratio and the effect of the nanoparticle mass become more at higher nonlocal parameter coefficient. The increase in the stiffness of the foundation leads to an increase in the natural frequency and the increasing of the initial stress leads to a decrease in the values of the frequency ratio. Besides, frequency ratio of the poly-

vinylidene fluoride nanoplates increases by increasing the length of the square nanoplate and with applying of the positive electric potential.

8. Conclusion

With the widely development of nano technology, nanoplates have been used in a lot of applications and because of this reason it was very important for researchers to study this type of nano structures. This work presents an overview on solution methods and analysis of nanoplates structures. Moreover, it considers a review about the theories that are used to study these nanoplates. Also, parametric studies are discussed to show the effect of the studied parameters on the dynamic behaviour and analysis of nanoplates. A lot of results are introduced at this overview to facilitate access to these results. Some of these results showed that, vibration is influenced by nanoplate dimension, damping coefficient of the foundation and of the structure, stiffness of the foundation and aspect ratio. Nonlocal parameter should be taken into account for any nanoplate with length or width less than 30nm. Clamped supported ends of the nanoplates are more sensitive to nonlocal parameter than the simply supported ends. By increasing the magnetic force, dynamic response is decreasing. Finally, theories and results which are introduced at this study are very important for the next researches at this part of study.

References

- [1] G. Jo, M. Choe, S. Lee, W. Park, Y. H. Kahng, and T. Lee, "The application of graphene as electrodes in electrical and optical devices," *Nanotechnology*, vol. 23, no. 11, 2012, doi: 10.1088/0957-4484/23/11/112001.
- [2] M. Arefi and A. M. Zenkour, "Nonlocal electro-thermo-mechanical analysis of a sandwich nanoplate containing a Kelvin-Voigt viscoelastic nanoplate and two piezoelectric layers," *Acta Mech.*, vol. 228, no. 2, pp. 475–493, 2017, doi: 10.1007/s00707-016-1716-0.
- [3] Y. Ji et al., "Organic nonvolatile memory devices with charge trapping multilayer graphene film," *Nanotechnology*, vol. 23, no. 10, 2012, doi: 10.1088/0957-4484/23/10/105202.
- [4] J. S. Bunch et al., "Electromechanical resonators from graphene sheets," *Science* (80-.), vol. 315, no. 5811, pp. 490–493, 2007, doi: 10.1126/science.1136836.
- [5] A. Sakhaee-Pour, M. T. Ahmadian, and A. Vafai, "Applications of single-layered graphene sheets as mass sensors and atomistic dust detectors," *Solid State Commun.*, vol. 145, no. 4, pp. 168–172, 2008, doi: 10.1016/j.ssc.2007.10.032.
- [6] G. Lu, L. E. Ocola, and J. Chen, "Reduced graphene oxide for room-temperature gas sensors,"

- Nanotechnology, vol. 20, no. 44, p. 445502, 2009, doi: 10.1088/0957-4484/20/44/445502.
- [7] A. C. Eringen, “Linear theory of nonlocal elasticity and dispersion of plane waves,” *Int. J. Eng. Sci.*, vol. 10, no. 5, pp. 425–435, 1972, doi: 10.1016/0020-7225(72)90050-X.
- [8] A. C. Eringen, “On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves,” *J. Appl. Phys.*, vol. 54, no. 9, pp. 4703–4710, 1983, doi: 10.1063/1.332803.
- [9] J.N. Reddy, “Mechanics of laminated composite plates and shells”: theory and analysis. CRC Press; 2004.
- [10] S. Poursmaeeli, E. Ghavanloo, and S. A. Fazelzadeh, “Vibration analysis of viscoelastic orthotropic nanoplates resting on viscoelastic medium,” *Compos. Struct.*, vol. 96, pp. 405–410, 2013, doi: 10.1016/j.compstruct.2012.08.051.S.C.
- [11] S. C. Pradhan and J. K. Phadikar, “Nonlocal elasticity theory for vibration of nanoplates,” *J. Sound Vib.*, vol. 325, no. 1–2, pp. 206–223, 2009, doi: 10.1016/j.jsv.2009.03.007.
- [12] T. Aksencer and M. Aydogdu, “Forced transverse vibration of nanoplates using nonlocal elasticity,” *Phys. E Low-Dimensional Syst. Nanostructures*, vol. 44, no. 7–8, pp. 1752–1759, 2012, doi: 10.1016/j.physe.2011.12.004.
- [13] S. Chakraverty and L. Behera, “Free vibration of rectangular nanoplates using Rayleigh-Ritz method,” *Phys. E Low-Dimensional Syst. Nanostructures*, vol. 56, pp. 357–363, 2014, doi: 10.1016/j.physe.2013.08.014.
- [14] J. Fan, D. Rong, Z. Zhou, C. Xu, and X. Xu, “Exact solutions for forced vibration of completely free orthotropic rectangular nanoplates resting on viscoelastic foundation,” *Eur. J. Mech. A/Solids*, vol. 73, no. May 2018, pp. 22–33, 2019, doi: 10.1016/j.euromechsol.2018.06.007.
- [15] B. Babu and B. P. Patel, “Analytical solution for strain gradient elastic Kirchhoff rectangular plates under transverse static loading,” *Eur. J. Mech. A/Solids*, vol. 73, no. March 2018, pp. 101–111, 2019, doi: 10.1016/j.euromechsol.2018.07.007.
- [16] M. Sobhy, “A comprehensive study on FGM nanoplates embedded in an elastic medium,” *Compos. Struct.*, vol. 134, pp. 966–980, 2015, doi: 10.1016/j.compstruct.2015.08.102.
- [17] P. Malekzadeh, A. R. Setoodeh, and A. A. Beni, “Small scale effect on the free vibration of orthotropic arbitrary straight-sided quadrilateral nanoplates,” *Compos. Struct.*, vol. 93, no. 7, pp. 1631–1639, 2011, doi: 10.1016/j.compstruct.2011.01.008.
- [18] R. Ansari, S. Sahmani, and B. Arash, “Nonlocal plate model for free vibrations of single-layered graphene sheets,” *Phys. Lett. Sect. A Gen. At. Solid State Phys.*, vol. 375, no. 1, pp. 53–62, 2010, doi: 10.1016/j.physleta.2010.10.028.
- [19] D. Karličić, P. Kozić, and R. Pavlović, “Free transverse vibration of nonlocal viscoelastic orthotropic multi-nanoplate system (MNPS) embedded in a viscoelastic medium,” *Compos. Struct.*, vol. 115, no. 1, pp. 89–99, 2014, doi: 10.1016/j.compstruct.2014.04.002.
- [20] R. Li, B. Wang, G. Li, and B. Tian, “Hamiltonian system-based analytic modeling of the free rectangular thin plates’ free vibration,” *Appl. Math. Model.*, vol. 40, no. 2, pp. 984–992, 2016, doi: 10.1016/j.apm.2015.06.019.
- [21] A. Assadi, “Size dependent forced vibration of nanoplates with consideration of surface effects,” *Appl. Math. Model.*, vol. 37, no. 5, pp. 3575–3588, 2013, doi: 10.1016/j.apm.2012.07.049.
- [22] S. Hosseini-Hashemi, H. Mehrabani, and A. Ahmadi-Savadkoobi, “Forced vibration of nanoplate on viscoelastic substrate with consideration of structural damping: An analytical solution,” *Compos. Struct.*, vol. 133, pp. 8–15, 2015, doi: 10.1016/j.compstruct.2015.07.068.
- [23] A. R. Ranji and H. R. Hoseynabadi, “A semi-analytical solution for forced vibrations response of rectangular orthotropic plates with various boundary conditions,” *J. Mech. Sci. Technol.*, vol. 24, no. 1, pp. 357–364, 2010, doi: 10.1007/s12206-009-1010-3.
- [24] M. S. Atanasov, D. Karličić, and P. Kozić, “Forced transverse vibrations of an elastically connected nonlocal orthotropic double-nanoplate system subjected to an in-plane magnetic field,” *Acta Mech.*, vol. 228, no. 6, pp. 2165–2185, 2017, doi: 10.1007/s00707-017-1815-6.
- [25] M. Ghadiri, S. Hamed, and S. Hosseini, “Nonlinear dual frequency excited vibration of viscoelastic graphene sheets exposed to thermo-magnetic field,” vol. 83. Elsevier B.V., 2020.
- [26] C. C. Liu and Z. B. Chen, “Dynamic analysis of finite periodic nanoplate structures with various boundaries,” *Phys. E Low-Dimensional Syst. Nanostructures*, vol. 60, pp. 139–146, 2014, doi: 10.1016/j.physe.2014.02.016.
- [27] S. Adhikari, D. Gilchrist, T. Murmu, and M. A. McCarthy, “Nonlocal normal modes in nanoscale dynamical systems,” *Mech. Syst. Signal Process.*, vol. 60, pp. 583–603, 2015, doi: 10.1016/j.ymsp.2014.12.004.
- [28] H. B. Khaniki and S. Hosseini-Hashemi, “Dynamic response of biaxially loaded double-layer viscoelastic orthotropic nanoplate system under a moving nanoparticle,” *Int. J. Eng. Sci.*, vol. 115, pp. 51–72, 2017, doi: 10.1016/j.ijengsci.2017.02.005.
- [29] E. Jomehzadeh et al., “Nonlinear subharmonic oscillation of orthotropic graphene-matrix composite,” *Comput. Mater. Sci.*, vol. 99, pp. 164–172, 2015, doi: 10.1016/j.commatsci.2014.12.019.
- [30] M. R. Nami and M. Janghorban, “Resonance behavior of FG rectangular micro/nano plate based on nonlocal elasticity theory and strain gradient theory with one gradient constant,” *Compos. Struct.*, vol. 111, no. 1, pp. 349–353, 2014, doi: 10.1016/j.compstruct.2014.01.012.
- [31] A. G. Arani, R. Kolahchi, and H. Gh. Afshar, “Dynamic analysis of embedded PVDF nanoplate subjected to a moving nanoparticle on an arbitrary elliptical path,” *J. Brazilian Soc. Mech. Sci. Eng.*, vol. 37, no. 3, pp. 973–986, 2015, doi: 10.1007/s40430-014-0215-2.