

RELATION BETWEEN A FUNCTION OF THE EQUATORIAL COORDINATES AND THE ANGULAR DISTANCE TO THE VERTEX FOR HYADES STARS

M. A. Sharaf*& Samiha Abdullah Al-Qari **
*Department of Astronomy, Faculty of Science, King Abdul Aziz University, *Jeddah, Saudi Arabia.
** Dean of girls educational College for Scientific Departments- Makkah Saudi Arabia

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ABSTRACT

In this paper, relation was developed for Hyades stars between a function of the equatorial coordinates and the angular distances from the vertex. The precision criteria of this relation are very satisfactory and a correlation coefficient value of $\cong 1$ was found which proves that the attributes are completely related linearly. The importance of this relation was illustrated through its usages as:

- Acriterion for membership of the cluster,
- Agenerating function for evaluating some parameters of the cluster,
- a generating function for the initial values of the vertex equatorial coordinates which could then be improved iteratively using the procedure of differential corrections.

1.INTRODUCTION

The open star cluster known as the Hyades occupies a unique place in the history and literature of astronomy. It is one of the few star clusters to have been recognized by the ancients, and shares with the Pleiades and the Coma clusters the distinction of being sufficiently close to us for the brighter members to be individually visible to the naked eye. Hyades cluster, with some possible members, a total mass of some 300 – 400 the mass of the sun, and an age of around 600 - 800 Myr, has an extension in the sky of about 20^0. Hyades cluster provides a well known example of a moving cluster, that is a group of stars whose parallel motions in space yield, on the celestial sphere, directions of proper motion that appear to converge to a point called the vertex of the motion (or of the cluster). The determination of the equatorial coordinates of the vertex , that is, its right ascension and declination (A,D) is one of the most important problems in the kinematical and physical studies of moving clusters (Wayman, 1965; Hanson 1975; Eggen 1984; Gunn *et al.*, 1988; Sharaf *et al.*, 2000). In particular, the color -magnitude diagram of the Hyades cluster has been of prime importance in establishing the Zero Age Main Sequence (ZAMS) as well as in calibrating luminosity criteria which permit the determination of the absolute magnitudes of the stars from observable features in their spectra (Perryman *et al.*1998; de Bruije *et al.*, 2001). The kinematic

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distance of the Hyades derived from a combination of proper motions and spectroscopic radial velocities, has been one of the fundamental starting points for the calibration of the photometric distance scale (Hanson, 1975; Gunn *et al.* 1988; Schwan,1991). In fact, the availability of the final results of the Hipparcos astrometry mission, provide a radical improvement in astrometric data on all stars in the Hipparcos observing programme,. The first detailed study of the distance, structure, membership, dynamics and age of the Hyades cluster, using Hipparcos data was by Perrymant *et al.* (1998). Recently (Sharaf, 2003) a relation was established between the apparent magnitude and the parallax for Hyades stars using the best rational approximation technique. The precision criteria of such a relation were very satisfactory and some utilizations of the relation were also given.

In the present paper, a relation was developed between a function of equatorial coordinates, the right ascension and declination (α, δ) and the angular distances from the vertex. We used 195 Hyades stars of Hipparcos data, the precision criteria of the relation are very satisfactory and a correlation coefficient of value $\cong 1$ was found which proves that the attributes are completely related linearly. The importance of this relation was illustrated through its usages as: a criterion for membership of the cluster, generating function for evaluating some parameters of the cluster, and the initial values of the vertex equatorial coordinates which could then be improved iteratively using the procedure of differential corrections (Sharaf *et al.*, 2000).

2.Basic formulations

The material of this section is a summary of the corresponding section of (Sharaf *et al.*, 2000), and is given through the following subsections:

2.1. Determination of the vertex of a moving cluster

If we know the right ascensions α_i, declinations δ_i and the components $\left(\mu_\alpha^{(i)}, \mu_\delta^{(i)}\right)$ of the total proper motion $\mu_\alpha^{(i)}; i = 1,......, N$, where N is the number of the cluster members, then the equatorial coordinates (A, D) of a moving cluster could be determined from

$$A = \tan^{-1}(\eta/\xi), \quad (1)$$

$$D = \tan^{-1}\left[\left(\eta^2 + \xi^2\right)^{-1/2}\right], \quad (2)$$

where

$$\xi = (T_5 T_2 - T_3 T_4)/\Delta, \quad (3)$$

$$\eta = \left(T_3 T_2 - T_5 T_1\right)/\Delta, \tag{4}$$

$$T_1 = \sum_{i=1}^{N} a_i^2, \quad T_2 = \sum_{i=1}^{N} a_i b_i, \quad T_3 = \sum_{i=1}^{N} a_i c_i,$$

$$T_4 = \sum_{i=1}^{N} b_i^2, \quad T_5 = \sum_{i=1}^{N} b_i c_i, \tag{5}$$

$$\Delta = T_2^2 - T_4 T_1,$$

$$a_i = \mu_\alpha^{(i)} \sin\delta_i \cos\alpha_i \cos\delta_i - \mu_\delta^{(i)} \sin\alpha_i,$$

$$b_i = \mu_\alpha^{(i)} \sin\delta_i \sin\alpha_i \cos\delta_i + \mu_\delta^{(i)} \cos\alpha_i, \tag{6}$$

$$c_i = \mu_\alpha^{(i)} \cos^2 \delta_i.$$

2.2. Differential corrections to the vertex coordinates

The differential corrections ΔA and ΔD to the vertex coordinates A and D are given as:

$$\Delta A = \left(G_5 G_2 - G_3 G_4\right)/E, \tag{7}$$

$$\Delta D = \left(G_3 G_2 - G_5 G_1\right)/E, \tag{8}$$

where

$$E = G_2^2 - G_4 G_1, \tag{9}$$

$$G_1 = \sum_{i=1}^{N} \Psi_i^2, \qquad G_2 = \sum_{i=1}^{N} \Psi_i \Phi_i, \qquad G_3 = \sum_{i=1}^{N} \Psi_i \Delta\theta_i,$$

$$G_4 = \sum_{i=1}^{N} \Phi_i^2, \qquad G_5 = \sum_{i=1}^{N} \Phi_i \Delta\theta_i, \tag{10}$$

$$\Psi_i = \sin^2 \theta_{cal}^{(i)} [\cos\delta_i \tan D \cos(A - \alpha_i) - \sin\delta_i] / \sin^2 (A - \alpha_i) \tag{11}$$

$$\Phi_i = -\sin^2 \theta_{cal}^{(i)} [\cos\delta_i \sec^2 D] / \sin(A - \alpha_i) \tag{12}$$

$$\Delta\theta_i = \theta_{obs}^{(i)} - \theta_{cal}^{(i)}, \tag{13}$$

$$\theta_{obs}^{(i)} = \tan^{-1}\left[\frac{\mu_\alpha^{(i)} \cos\delta_i}{\mu_\delta^{(i)}}\right], \tag{14}$$

$$\theta_{cal}^{(1)} = \cos^{-1}\left[\frac{\sin D - \sin\delta_i \cos\lambda_i}{\cos\delta_i \sin\lambda_i}\right], \quad (15)$$

θ_{obs} is the position angle of the total proper motion μ and λ_j is the angular distance of the jth star to the vertex and is given by

$$\lambda_j = \cos^{-1}\left[\sin\delta_j \sin D + \cos\delta_j \cos D \cos\left(A - \alpha_j\right)\right]. \quad (16)$$

Now having obtained the corrections ΔA and ΔD, one can determine the corrected values of the coordinates of the vertex, A^* and D^* from

$$A^* = A + \Delta A, \quad (17)$$

$$D^* = D + \Delta D. \quad (18)$$

This process of corrections could be repeated in an iterative manner until the desired accuracy is achieved such, for instance, $|\Delta A| \le \epsilon_1$ and $|\Delta D| \le \epsilon_2$, where ϵ_1 and ϵ_2 are two given tolerances.

2.3. Velocity components V_α, V_δ and ρ

Let V_α, V_δ and ρ be the components of the velocity V (as a basic assumption, all members of a moving cluster have the same $|V|$) along coordinates system whose center is a star and consisting of three mutually perpendicular unit vectors $(\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\delta}}, \hat{\mathbf{r}})$ defined as follows:

- The unit vector $\hat{\boldsymbol{\alpha}}$ tangent to the circle of constant declination and pointing in the direction of increasing right ascension.
- The unit vector $\hat{\boldsymbol{\delta}}$ tangent to the circle of constant right ascension and pointing towards the north celestial pole.
- The unit vector $\hat{\mathbf{r}}$ lying on the radius vector, which joins the sun to the star. These components are given as,

$$V_\alpha = 4.74\mu_\alpha \cos\delta / p, \quad (19)$$

$$V_\delta = 4.74\mu_\delta / p, \quad (20)$$

$$V_t^2 = V_\alpha^{\,2} + V_\delta^{\,2}, \quad (21)$$

where p is the parallax. Also we have

$$V_t = V \sin\lambda, \quad (22)$$

$\rho = V\cos\lambda.$ (23)

V is the velocity of the cluster.

3. Numerical applications

3.1.Data

For the present applications we used $195(=N)$ stars of Hipparcos data, by these data and the algorithm of Section 2.1 we get for A and D the values

A=97.902663° (24)

D= 6.7732° (25)

3.2 Relation between B and λ

Using Hipparcos data, listed in Table I of Appendix A and the formulations of Section 2 together with the numerical values of Equations (24) and (25) we get very simple and very significant relation between the equatorial coordinates (α,δ) and the angular distance to the vertex λ, this relation is:

$$\lambda = c_1 + c_2 B \qquad (26)$$

$$B = \sin\alpha\cos\delta - \cos\alpha \qquad (27)$$

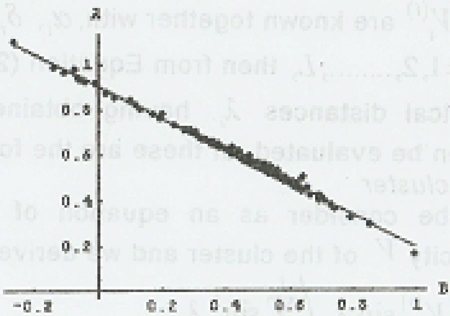

Figure 1: Graphical representation between the raw and the fitted data

The analysis of this relation is given as:

(1) The used formula is: λ= 0.901915 -0.699866 *B*

(2) The probable errors for the coefficients of the formula is: (± 0.00126563 ,± 0.00259902)

(3) The estimated variance of the fit is 0.0000893146

(4) The confidence intervals for the coefficients are respectively: 0.877306, 0.890400) and (-0.710652 , -0.683983)

(5) The average squared distance between the exact solution and the least squares solution (Kopal and Sharaf,1980) is = 0.0000183685

(6) The linear correlation coefficient between (λ , *B*) is: -0.997191

(7)The graphical representation between the raw and the fitted data is given in Fig1.

4.Utilizations of the relation

Assuming Equation (26) as a given relation for Hyades cluster, then it could be utilized in generating some important knowledge about the cluster, of these are the following:

4.1 Equatorial coordinates criterion

Equation (26) may provide an additional membership criterion of Hyades cluster as follows.

Assume the equatorial coordinates (α_0, δ_0) of a star, suspected of being a member of the cluster was known. Now from these values of (α_0, δ_0) the spherical distance λ_0 of the star from the vertex could be obtained from Equation (26). With the spherical distance λ_c calculated by means of Equation (16) and, if $|\lambda_0 - \lambda_c|$ is reasonably small, we can conclude that possible the star is a member of the cluster. This is what we may call it *equatorial coordinates criterion*.

4.2. Generating function

Equation (26) may also be used as generating function for evaluating some important parameters of the cluster. Assuming that the tangential velocities $V_t^{(i)}$ are known together with, α_i, δ_i, $\mu_\alpha^{(i)}$ and $\mu_\delta^{(i)}$ for L (say) stars; $i = 1, 2, ..., L$, then from Equation (26) we get the corresponding spherical distances λ_i. having obtained λ's, some parameters could then be evaluated, of these are the following.

* *The velocity of the cluster*

Equation (22) can be consider as an equation of condition for determining the velocity V of the cluster and we derive

$$V = \sum_{i=1}^{L} V_t^{(i)} \sin \lambda_i \Big/ \sum_{i=1}^{L} \sin^2 \lambda_i . \qquad (28)$$

* *The radial velocities of the cluster stars*

From Equations (22) and (23) the radial velocities $\rho_i, i = 1, 2, ..., L$ follow from:

$$\rho_i = V_t^{(i)} \cot \lambda_i . \qquad (29)$$

* *The parallaxes of the cluster stars*

The total proper motions μ_i; $i = 1, 2, ..., L$ are computed from:

$$\mu_i = \sqrt{\left(\mu_\alpha^{(i)} \cos \delta_i\right)^2 + \left(\mu_\delta^{(i)}\right)^2} ,$$

then the parallaxes p_i; $i = 1, 2, \ldots\ldots, L$ are computed from

$$p_i = 4.738\,\mu_i / V_r^{(i)}. \tag{30}$$

• *The absolute magnitudes of the cluster stars*

If the apparent magnitudes m_i, $i = 1, 2, \ldots\ldots., L$ of the cluster stars are also known, then their absolute magnitudes can be found from

$$M_i = m_i + 5 + 5\log p_i, \tag{31}$$

where P_i are given from equation (30) .

• *The center of the cluster*

The center of the cluster (x_c, y_c, z_c) can be derived by simple method of finding the equatorial coordinates of the center of mass for a number of discrete objects, so

$$x_c = \left[\sum_{i=1}^{L} \cos\delta_i \cos\alpha_i \Big/ p_i\right] \Big/ L, \tag{32}$$

$$y_c = \left[\sum_{i=1}^{L} \cos\delta_i \sin\alpha_i \Big/ p_i\right] \Big/ L, \tag{33}$$

$$z_c = \left[\sum_{i=1}^{L} \sin\alpha_i \Big/ p_i\right] \Big/ L, \tag{34}$$

where P_i are given from equation (30).

• *The distance of the cluster*

The distance of the cluster is given by

$$d = L \Big/ \left[\sum_{i=1}^{L} p_i\right],$$

where P_i are given from equation (30).

4.3. Initial values of the vertex coordinates

Select a few N_0 (say) stars which are adopted as Hyades members. In what follows we shall illustrate the usage of Equation (26) in generating initial values of the vertex equatorial coordinates A_0 and D_0. These values could then be improved iteratively using the procedure of differential corrections as mentioned in Section 2.

1. Compute $\theta_{obs}^{(i)}$; $i = 1, 2, \ldots\ldots., N_0$ from equation (14).
2. Compute λ_i; $i = 1, 2, \ldots\ldots., N_0$ from equation (26).
3. Compute $D_0^{(i)}$ and A_0; $i = 1, 2, \ldots\ldots, N_0$ from

$$D_0^{(i)} = \sin^{-1}\left(\cos\theta_{obs}^{(i)} \cos\delta_i \sin\lambda_i + \sin\delta_i \cos\lambda_i\right),$$

$$A_0^{(i)} = \alpha_i + \cos^{-1}\left[\left(\cos\lambda_i - \sin\delta_i \sin D_0^{(i)}\right)\Big/\cos\delta_i \cos D_0^{(i)}\right].$$

4. Compute A_0 and D_0 from

$$A_0 = \sum_{i=1}^{N_0} A_0^{(i)} \Big/ N_0 ,$$

$$D_0 = \sum_{i=1}^{N_0} D_0^{(i)} \Big/ N_0 .$$

CONCLUSION

In concludingThe present paper represents the second phase [the first phase is given in reference of Sharaf *el al.*, 2004] of group researches aiming at establishing relations between coordinates and kinematical parameters for Hyades stars. In the first phase relation was developed for Hyades stars between a function of the right ascensions and the angular distances from the vertex

In the present paper, a relation was developed for Hyades stars between equatorial coordinates and the angular distances from the vertex. Precision criteria of this relation which are:

1- the probable errors for the coefficients of the formula,

2-the estimated variance of the fit ,

3- the confidence intervals for the coefficients and

4-the Q value are all very satisfactory.

Also a correlation coefficient of value $\cong 1$ was found, which proves that the attributes are completely related linearly. The importance of this relation was illustrated through its usages as:

- A criterion for membership of the cluster which we may call it *equatorial coordinates criterion*
- A generating function for evaluating some parameters of the cluster for examples, the velocity and the center of the cluster, also the parallaxes, radial velocities and the absolute magnitudes of the cluster stars.
- Finally, the relation could be utilized to generate initial values of the vertex equatorial coordinates which could then be improved iteratively using the procedure of differential correction.

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