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Mathematical Modeling of Occurrence and Population Density of Predatory Spider and Their Preys on Cotton Plants and Broad Bean in Two Governorates in Egypt

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ABSTRACT

The research presents a mathematical simulation of three kinds of pests and their predator for two plants: Cotton and broad bean. The study was applied in two regions in Egypt representing Upper Egypt and Delta. The study is based on similar field experimental results during the two successive years 2003-2004, 2004-2005 in Qaha station- Qalubia governorate and seds station–Beni sweif governorate.

Four models are introduced according to the plant and the geographical regions. The data were fitted to continuous curves to enable the process of predicting the number of each kind of the involved species. Many functions are suggested to describe the effect of temperature and relative humidity on population. However, for particular plant in particular area, only narrow ranges occur for both temperature and humidity. A process of smoothing the data was necessary to avoid the very extreme points of sudden increase (due to migration or eggs hatching) which can not be taken into account in a mathematical model. After smoothing the data, a least square method was used to fit the points to continuous curves. Then an algorithm has been made aiming to predict the number of the preys and predator at any time by knowing the initial state of each of them.

The modeling for anticipating the expected number of spiders and preys for the different months was achieved using a Microsoft Visual Studio program. Four models are constructed and they have been shown to be easy for users. A calibration for the model was examined using a comparison study between field and model results, and a reasonable matching was observed.

Key Words: Mathematical modeling, spiders, preys, temperature, relative humidity

INTRODUCTION

In Egypt, as in the other countries, plant production for human and live stock consumption is threatened by a wide range of insects and arthropod species. Among the serious pests that cause big loss of plants are the cotton leaf worm, *Spodoptera littoralis* (Boisd) which are known to have more than 270 host plants all over the world, aphids, *Aphis craccivora* (Koch), *Aphis gossypii* (Glover) and the phytophagous mite, *Tetranychus urticae* (Koch).

The traditional unwise use of chemical pesticides for pest control and their consequence of toxic residues, environmental pollution and side effect on beneficial insects (bees, predator and parasites) necessitates search for alternative control agents, mainly of natural origin. Regarding pest natural enemies, the spider is known as a potential factor for reducing insect population. It is characterized by its wide host range and can tolerate hard field conditions (***Agnew et al***., 1985; ***Joon et al***., 1988 and ***El- Erksousy*** 2000).

The present study includes the occurrence and population density of predatory spider and their preys on cotton plant and broad bean, crops, in two governorates in Egypt, one in lower Egypt Qaha (Qualyobia governorate) and in upper Egypt at Seds station (Beni-Suif governorate) in two successive years 2004 and 2005.

In this paper a mathematical simulation has been presented for three kinds of pests and their predator for the two plants: cotton and broad bean. The study was made in two regions of Egypt representing Upper Egypt, and Delta. Thus giving us four different types of models, according to the plant and the geographical region. The importance of the model is that it allows us of predicting the variation of the number of pests and their preys at any time starting from knowing its number at a given time. The simulation is based on data collected half monthly from the fields over two years 2004 – 2005.

The data were fitted to continuous curves to enable the process of predicting the numbers of each kind of the involved species. Discovering and analyzing the classical type of interaction predator-prey, (which would be supposed to be clear during the study), neither was very clear in the relations obtained nor was the main objective of the work. The excess of food for each species, involved in the study, made no such relation clear. Besides, the main goal of the work was to predict the populations during the season. However, the continuous curves obtained here could be used to facilitate a study of the relation between the interacting species under these conditions of excess of food.

This work depends mainly on simulating and fitting of existing data. Other work may depend on the existing models and governing equations to rebuild a modified simulation model **(*El- Messoussi et al., 2007*)**

MATERIALS AND METHODS

Similar field experimental designs were carried out during two successive years 2003-2004 and 2004-2005, in two different localities in Egypt (one at Qaha station, Qalybiya governorate, and the other at Seds station, Beni- Sueif governorate). Five randomly branches ranged in their length between 50 to 100cm from four directions of a tree were shaken five times over the screening cloth. Five trees of each sort were used as replicates after shaking. Samples were taken every 15 days during the period from December to April (on broad bean plant) and from May to October (on cotton plant).

(i)Estimation of spider population

The collection of individuals took place by tree shaking and receiving the falls spiders on silky trap. After that, the individuals were kept in 70% ethyl alcohol for identification.

(ii) Estimation of S.littoralis population

Collection of individual was made by several methods,

a) By net, picking by hand or collecting of plant parts.

b) Sample has been collected carefully from plant parts leaves for determining immature stages and egg patches.

c) The soil under the chosen plants was examined to search for full grown larvae and pupae.

(iii)Estimation of Aphis population

Samples of 100 leaves were randomly collected twice monthly from the three levels of the plant (upper, middle and lower parts).

(iv)Estimation of phytophagous mite population

Phytophagous mite populations were estimated as number of adult and immature stages of mites found on leaf samples periodically picked at random from experimental location. Each sample was composed of 100 leaves of every host. Samples were taken twice monthly.

(v)Taxonomical studies

Investigated samples were examined in Petri dish filled with 70% ethyl alcohol. Examination was carried out using stereoscopic binocular microscope.

Identification of spider specimens followed the system of ***Petrunkevitch*** (1939) and ***Kaston*** (1978) by aid of **(*El-Hennawy*, 2002).** The mite individuals were identification according to ***Pritchard and Baker*** (1953). The individuals were identified in the Plant Protection Research Institute, Agriculture Research center Cairo, Egypt.

Mathematical modeling of the data

The data were collected every half month for 2 years and for the two successive plants, Cotton: May - October and Broad Bean: December - May, in two different regions representing Delta and Upper Egypt. The number of each species was scored with the average temperature and relative humidity along the time interval.

The data were plotted and examined seeking a trend with either the other species, the weather, or with time. The only clear trend has appeared when the total monthly population was plotted against time. The population of the spider has shown a clear trend with the time. Also the preys, as a whole, have shown a clear trend with time. But, no such trend was clear when examining each kind of the preys alone. Thus it was more convenient to plot the prey against the spider. After that a probabilistic expectation for each kind of the prey is done based on the initial ratio of this kind to the whole preys.

Due to the saturation (huge amounts of food for preys and huge amounts of preys for the spider) the typical expected interaction: predator-prey interaction was not clear as the clearness of the growth with respect to time. A very important factor was clear which is the "carrying capacity" or the maximum number of species that can live together for a long time under the given conditions (by a long time it is meant a sufficient time for interaction). It was found that after any sudden increase of the preys or the predator (due to eggs hatching or migration), a fast decrease happened in order to retain the system to its reasonable state without a considerable change of the other species.

Although many functions are suggested to describe the predation of species by other species **(*Murray,* 2001)**, no such function was used here as discussed in the above paragraph. Instead a time dependent model was built to each individual species.

Also many functions are suggested to describe the effect of the temperature and the relative humidity on populations ***(Mallet et al.,* 1999*; El- Messoussi et al.,* 2007)**. However, in the present work for particular plant, in particular area, only a narrow range occurs for both temperature and humidity in which their small variations are almost not affecting the population. Thus instead of making a function for the temperature and relative humidity, it is stated that the model holds only for the given ranges in each case.

A process of smoothing the data was necessary to avoid the very extreme points of sudden increase (due to migration or eggs hatching) that cannot be taken into account in a mathematical model. However, the model can predict what happens after a sudden change.

Simulation program of population dynamics

After smoothing the data, a least square method was used to fit the points to continuous curves. Then an algorithm has been made aiming at predicting the number of the preys and the predator at any time by knowing the initial state of each of them. The basic idea is to make a one-to-one correspondence between the initial given state and a specific value (the time step) on the fitted curve t_1. The time interval between the initial date T_1 (at which the state is known) and final date T_2 (at which the state is required) is calculated in months to get the number of steps increase, which allows us to calculate t2 on the curve at which the final value of the prey or the predator is calculated.

It is important to know that the time steps are not the *actual dates*. They are related to the population not to the time. For example if the given population is $x(t_1)$ then that value gives us a specified step t_1 on the curve, from which we can predict $x(t_2)$ at the step t_2. The steps are related to the real date in the sense that the population may increase before a certain date and decreases after it. So the initial state (Date, Population) gives us a unique step t_1. Then the final date will give us a unique step t_2 at which we calculate $x(t_2)$.

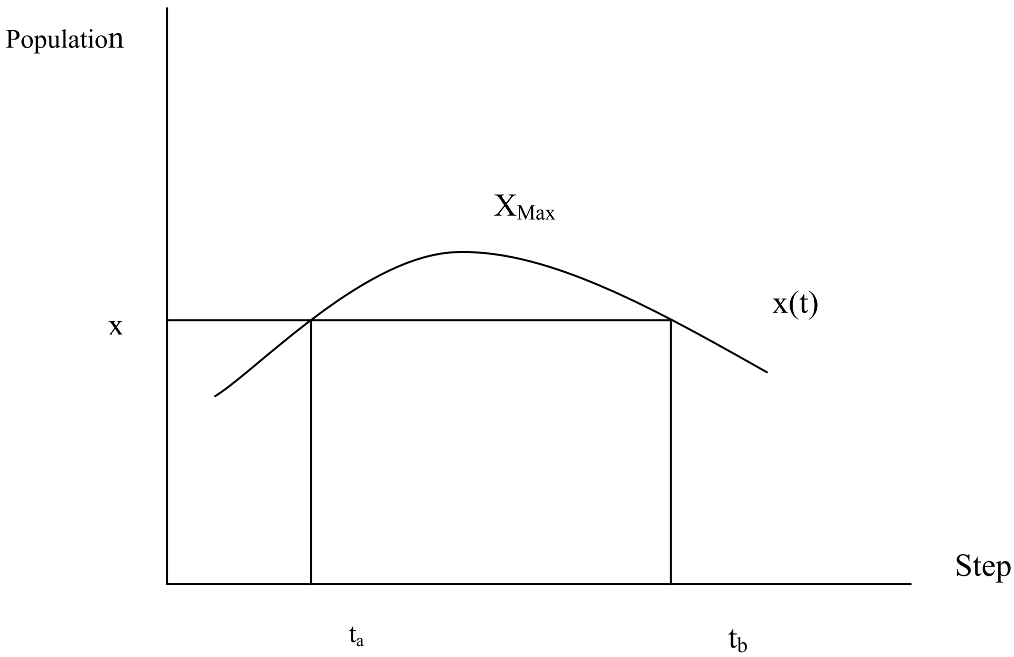

Fig.1: Illustration of the relation of the time steps with the actual dates

For example, in Fig.1, the corresponding value to the population x could be t_a or t_b. In order to get the correct value, we should consider the date which will determine the interval of increase or decrease of the population.

We can summarize the technique of calculation as follows:

1) Enter the initial state: Population x, Date T_1
2) Enter the Date T_2 at which you want to predict the population
3) From the population x, find the possible values of the step t_1
4) From T_1, find the correct step t_1
5) From T_2, find the step t_2
6) Calculate $x(t_2)$

In addition to the above entries, the parameters of temperature and relative humidity are required in order to check the validity of the program, since each group of the data are collected at some condition of both temperature and relative humidity. Also the entered population should be in the range of the real data.

The user is asked to enter the region: Upper Egypt or Delta, and the plant: Cotton or Broad Bean. This will give 4 models

Model 1.1: Cotton in Upper Egypt
Model 1.2: Broad Bean in Upper Egypt
Model 2.1: Cotton in Delta
Model 2.2: Broad Bean in Delta

The ranges found over the 2 years 2004-2005 for temperature and relative humidity are given in the following Tables.

Table 1: The ranges of Temperature in each case (It is to be noted that temperature is measured in the field)

	Cotton	Broad Bean
Upper Egypt	24 – 35 ˚ C	12 – 28 ˚ C
Delta	20 – 30 ˚ C	11 – 26 ˚ C

Table 2: The ranges of Relative Humidity in each case

	Cotton	Broad Bean
Upper Egypt	40 – 55 %	44 – 60 %
Delta	50 -65 %	50 – 70 %

RESULTS

Model 1.1
Cotton in Upper Egypt

For model 1.1-Spider, there was a jump in the 2005 population from 30 to 65 which is not consistent with the smooth population of the distribution of the year 2004. The model is based on the 2004 distribution only, while the data from the sudden increase till the end of the season in 2005 were used to simulate the case after a sudden increase. Thus we have two cases:

Table 3: The governing equations for spiders in model 1.1

Governing equation	Population range	Date condition
$-9.3 + 22.6036\,t - 2.91071\,t^2$	< 35	None
$120. - 67.5\,t + 12.5\,t^2$	≥ 35	None

The unit of the time step t is the month in all the equations (30 days to be precise).

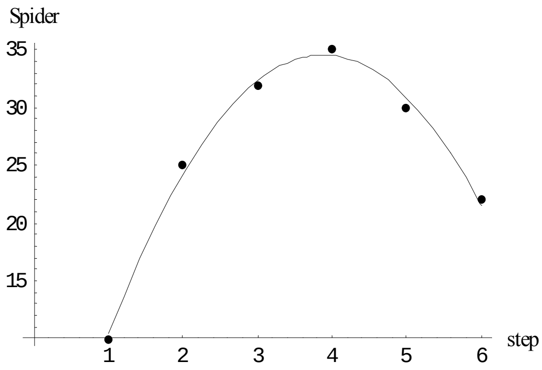

Fig. 2: Spider population in model 1.1 for range < 35

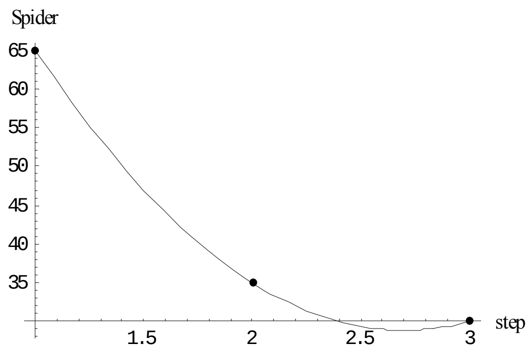

Fig.3: Spider population in model 1.1 for range≥35

For Model 1.1-Prey, it was found that there is no date dependence i.e. in 2004 the population started to decrease, after reaching the value 87 (occurred at July), while in 2005, the population is increasing all the time (because it did not reach its maximum value). We just plotted the 2 years together which gave us a complete picture for the prey population from a small value to its maximum. For best fitting, the curve was defined on 4 cases as follows:

Table 4: The governing equations for preys in model 1.1

Governing equation	Population range	Date condition
$9.6- 1.68571\,t + 1.71429\,t^2$	10- 45	None
$-270. + 93.\,t - 6.\,t^2$	45 – 65	None
$-270. + 93.\,t - 6.\,t^2$	65-87	$15/8 \leq T_1$
$-22.35 + 31.35\,t - 2.25\,t^2$	87 – 65	$T_1 > 15/8$ (15th August)

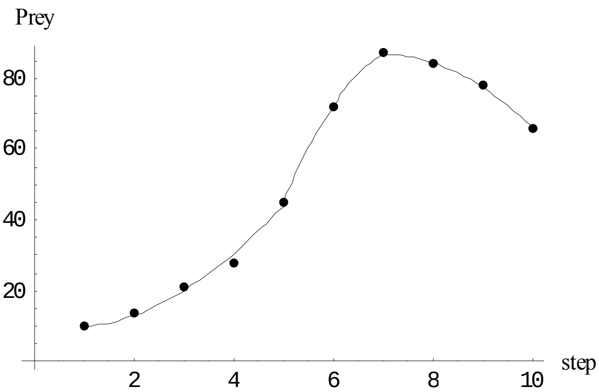

Fig. 4: Prey population in model 1.1

After getting the preys, each type of the three involved preys is found according to each initial ratio to the other preys and to its average ratio over the two seasons over which the model was built. The equations governing these ratios for model 1.1 are

$$Sl_2 = \frac{1}{2}\left(\frac{Sl_1}{P_1} + 0.25\right)P_2, \quad Tu_2 = \frac{1}{2}\left(\frac{Tu_1}{P_1} + 0.65\right)P_2, \quad Ap_2 = P_2 - Sl_2 - Tu_2 \qquad (1)$$

Where, the subscript "1" denotes the initial population of each kind of the preys at time T_1. The symbols used are: "*Sl*" for "*Spodoptera littoralis*", "*Tu*" for "*Tetranychus urticae*", and "*Ap*" for "*Aphis gossypii*". The symbol "P" denotes the total number of the preys, and the subscript "2" indicates the expected value at time T_2. The value 0.25 in the first equation is the mean ratio of the prey *Spodoptera littoralis* to the total preys along the two years of the study. The value 0.65 is the mean ratio of the prey *Tetranychus urticae* to the total preys along the two years of the study. Thus equation (1) is used to expect each kind of the preys by multiplying the average of its mean ratio over the two years 2004 -2005, and its initial ratio, by the total number of the preys P_2. This procedure will be typically applied in models, 1.2, 2.1, and 2.2 using equations (2), (3), and (4) respectively, while in each case the corresponding means will be used instead.

Model 1.2

Broad Bean in Upper Egypt

For Model 1.2-Spider, the behavior in 2004 and 2005 was almost the same, and the populations were very near. Thus we have plotted the average of the two years. The population was increasing all the time which indicates that the maximum population has not been reached. For best fitting we have defined the function of population on two cases.

Table 5: The governing equations for spiders in model 1.2

Governing equation	Population range	Date condition
$-1.+16.5\,t$	< 49	None
$81.-23.\,t+4.\,t^2$	49≥	None

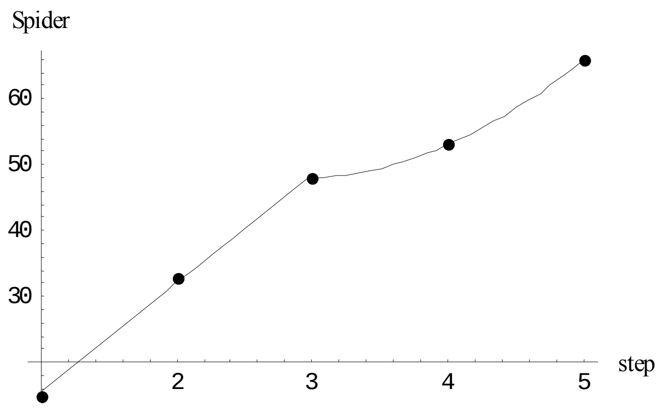

Fig. 5: Spider population in model 1.2

For Model 1.2-Preys, the population was increasing all the time in 2004, which indicates that the maximum value was not reached. While in 2005 the population started to decrease after reaching the value 93. It is noted that the increase or decrease of the preys is affected by the population not the date. We plotted the two years together to get a complete picture of the behavior for population values from 15-93. The population function is defined on 5 cases,

Table 6: The governing equations for preys in model 1.2

Governing equation	Population range	Date condition
$25. - 17.5t + 7.5t^2$	15 – 40	None
$103. - 40.5t + 6.5t^2$	40 – 63	None
$-82. + 39.t - 2.t^2$	63 – 70	None
$-82. + 39.t - 2.t^2$	70 – 93	$1/3 \leq T_1$
$394. - 67.5t + 3.5t^2$	93 – 70	$T_1 > 1/3$

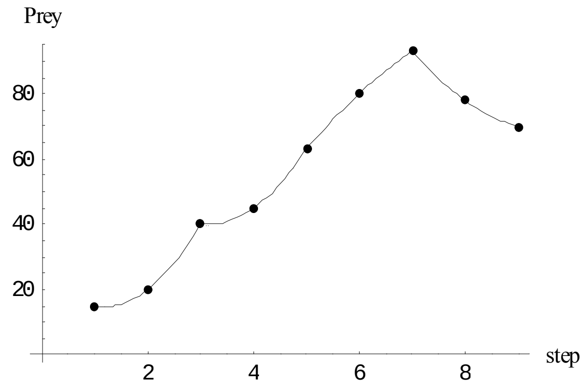

Fig. 6: Prey population in model 1.2

Finally, each kind of the preys is found in a similar manner to that in model 1.1. The equations of ratios here are

$$Sl_2 = \frac{1}{2}(\frac{Sl_1}{P_1} + 0.8)P_2, \text{ if } P_1 \leq 60, \quad Sl_2 = \frac{1}{2}(\frac{Sl_1}{P_1} + 0.2)P_2, P_1 > 60$$

$$Tu_2 = \frac{1}{2}(\frac{Tu_1}{P_1} + 0.1)P_2, \quad Ap_2 = P_2 - Sl_2 - Tu_2 \tag{2}$$

Model 2.1
Cotton in Delta

For Model 2.1-Spider, the population for values less than 90 was governed by the function.

Table 7: The governing equations for spiders < 90 in model 2.1

Governing equation	Population range	Date condition
$3.8 + 24.4857t - 2.t^2$	< 90	Non

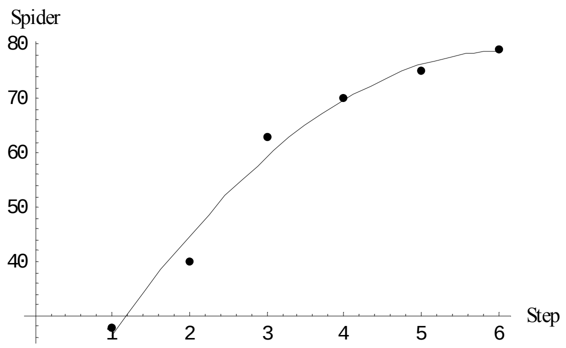

Fig. 7: Spider population < 90 in model 2.1

In 2004 an unexpected sudden increase occurred from 75 to 180. The trend after that was simulated in another case which illustrates that a sudden increase is followed by a sharp decrease.

Table 8: The governing equations for spiders < 90 in model 2.1

Governing equation	Population range	Date condition
$328. - 176.5\,t + 28.5\,t^2$	90≥	Non

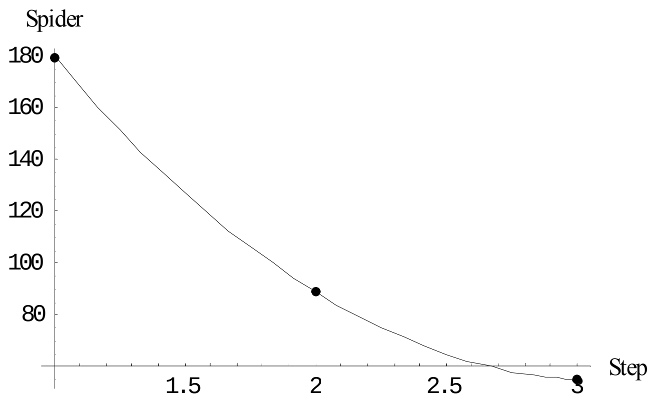

Fig. 8: Spider population ≥ 90 in model 2.1

The preys are found to increase before 1/ 7 (1st of June), and decrease after it. The governing equations are as follows:

Table 9: The governing equations for preys in model 2.1

Governing equation	Population range	Date condition
$-15.7 + 38.5036\,t - 4.125\,t^2$	< 75	None
216. -63. t	75≥	None

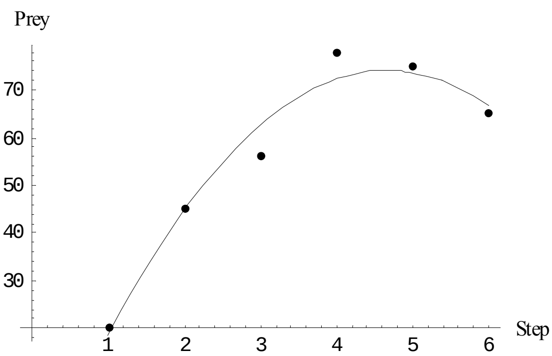

Fig. 9: Prey population < 75 in model 2.1

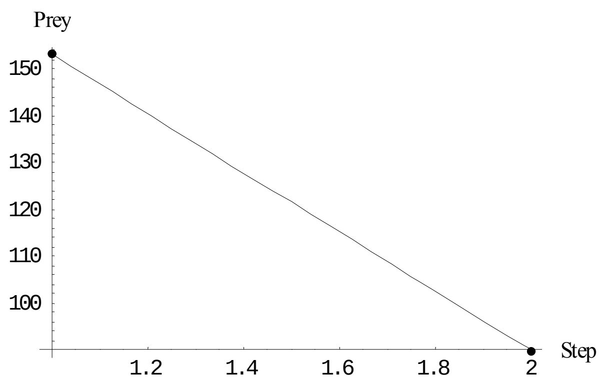

Fig. 10: Prey population ≥ 75 in model 2.1

The equations of each kind of preys are

$$Sl_2 = \frac{1}{2}(\frac{Sl_1}{P_1} + 0.8)P_2, \text{ if } P_1 \le 60, \quad Sl_2 = \frac{1}{2}(\frac{Sl_1}{P_1} + 0.2)P_2, P_1 > 60$$

$$Tu_2 = \frac{1}{2}(\frac{Tu_1}{P_1} + 0.1)P_2, \quad Ap_2 = P_2 - Sl_2 - Tu_2 \qquad (3)$$

Model 2.2

Broad Bean in Delta

For Model 2.2, Spiders have shown a trend that is very similar to that of Upper Egypt in the broad bean. (See figures 5 and 11). The population governing equations are as follows:

Table 10: The governing equations for spiders in model 2.2

Governing equation	Population range	Date condition
10.6667 +21. t	11-74	Non
$143.- 44.t+7.t^2$	> 74	Non

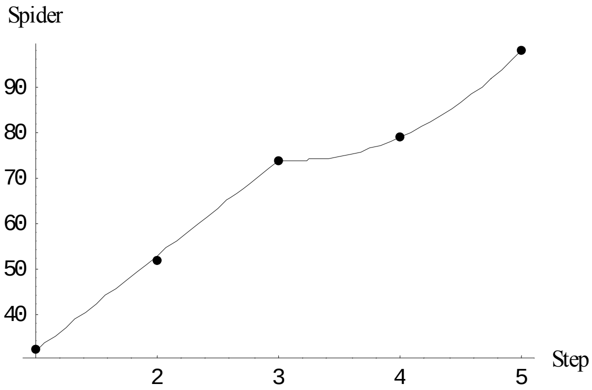

Fig 11, Spider population in model 2.2

For Model 2.2-Prey, we have the following equations

Table 11: The governing equations for preys in model 2.2

Governing equation	Population range	Date condition
$-13.+35.0714t-4.92857t^2$	5 - 49	Non
$211.5-71.8t+8.t^2$	> 49	Non

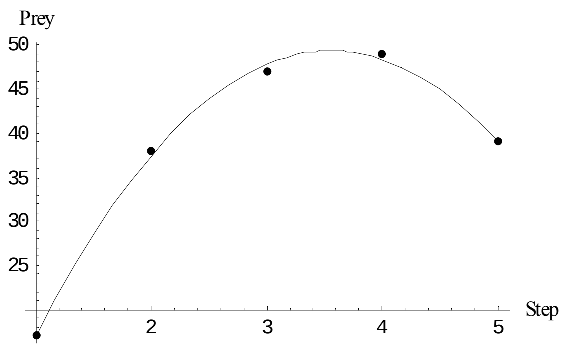

Fig. 12: Prey population <50 in model 2.2

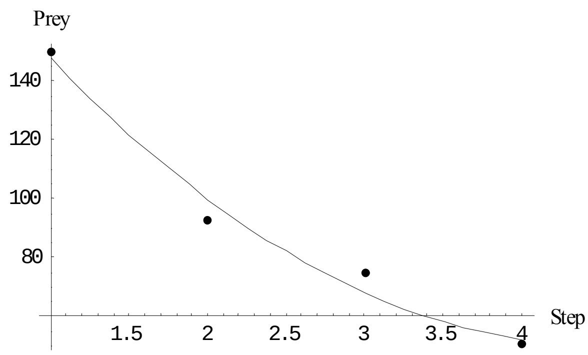

Fig. 13: Prey population ≥ 50 in model 2.2

The equations for each kind of the preys are

$$Sl_2 = \frac{1}{2}(\frac{Sl_1}{P_1} + 0.2)P_2, \quad Tu_2 = \frac{1}{2}(\frac{Tu_1}{P_1} + 0.1)P_2, \quad Ap_2 = P_2 - Sl_2 - Tu_2 \qquad (4)$$

The program interface

In what follows, we present two snap shots of the working interference program, to illustrate the ideas above:

First, the user is asked to choose the model among the four given models.

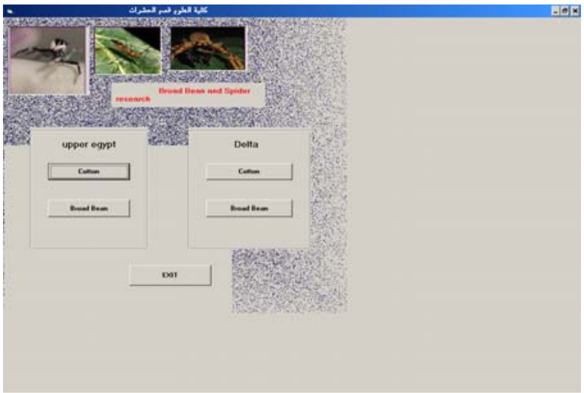

Fig. 14

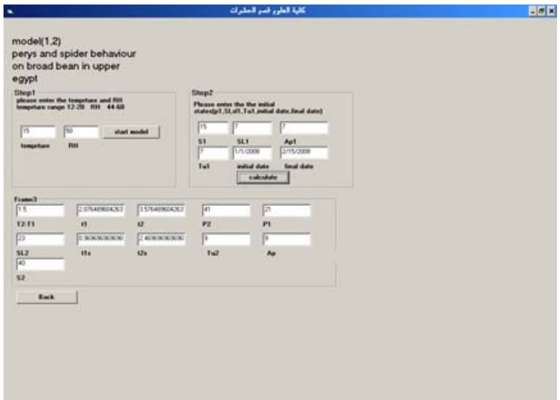

Fig. 15

Then after checking that the temperature and relative humidity are in the range of the chosen model, the user is asked to input the initial state: (Date, Population), and the final date. The output gives the state at the required final time.

NUMERICAL RESULTS

The modeling for anticipating the expected number of spiders and number of preys for the different months of the year considering different temperatures and relative humilities has been achieved using Microsoft Visual Studio program.

The model is based on modeling the governing equations generated in the previous section. Four models are generated to model four cases,

Case 1: (model 1. 1) cotton plant in Upper Egypt

Case 2: (model 1. 2) Broad Bean plant in Upper Egypt

Case 3: (model 2. 1) cotton plant in Delta

Case 4: (model 2. 2) Broad Bean plant in Delta

The input and output data sheets are shown in Figs. (14, 15) and they designed as easy as possible for the users. The model is calibrated by comparing the field results with model results for case (1.2) and case (2.1). The comparison is shown in Tables (12, 13) and Figs. (16 , 17).

Model (1, 2)

Table 12: Comparison between field results and model results for model (1.2)

Temperature (C°)	**Relative Humidity (RH %)**	**Field Results**		**Model Results**	
		No. of spiders	Total No. of Preys	No. of spiders	Total No. of Preys
18.9 December	57 December	16	15	16	15
16.2 January	53 January	41	20	33	26
14.4 February	55.5 February	55	40	48	40
20.5 March	49.5 March	60	50	54	51
26. April	51 April	72	63	68	71

Model (2, 1)

Table 13: Comparison between field results and model results for model (2,1)

Temperature (C°)	Relative Humidity (RH %)	Field Results		Model Results	
		No. of spiders	Total No. of Preys	No. of spiders	Total No. of Preys
23.7 May	59 May	25	45	25	45
25.8 June	59.5 June	52	56	44	63
26.9 July	57.5 July	75	78	59	73
27.2 August	58 August	80	75	70	73
28.3 September	57 September	89	65	76	66
29.2 October	54 October	55	65	79	50

Model (1, 2)

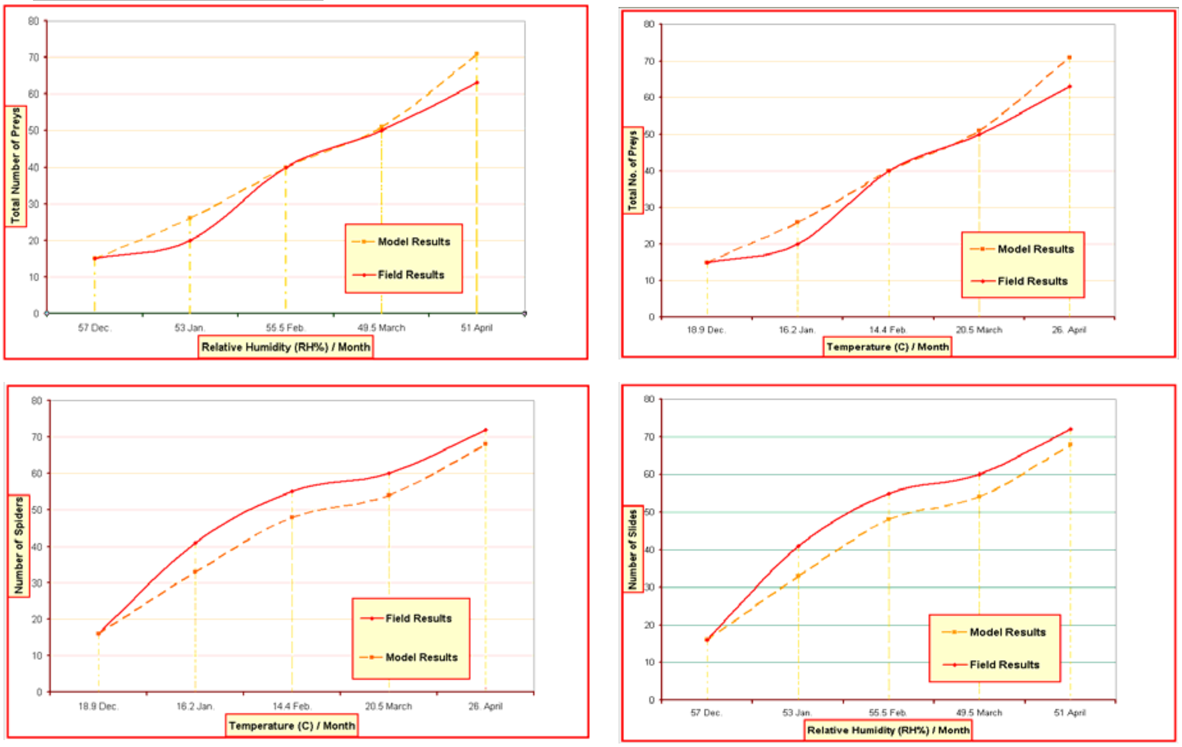

Fig. 16: Calibration of model 1.2

Model (2, 1)

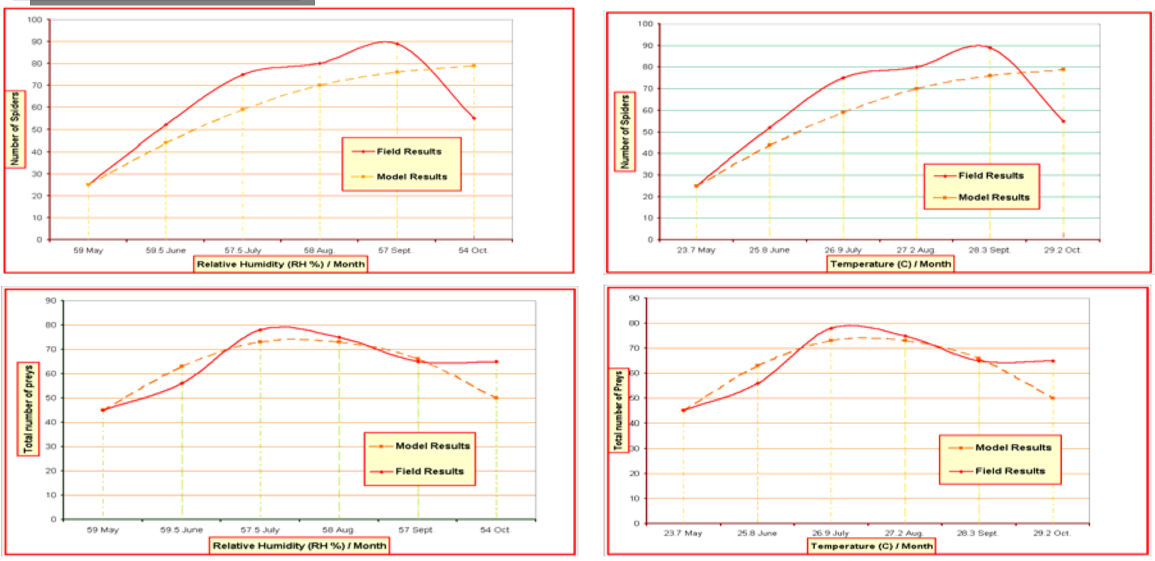

Fig. 17: Calibration of model 2.1

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ARABIC SUMMERY

موديل رياضى لتواجد العناكب وعوائلها على نبات القطن والفول وكثافتهما فى محافظتين بجمهورية مصر العربية

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البحث يقدم تمثيل رياضي لثلاثة أنواع من الافات والعناكب المفترسة وتأثيرها على نبات القطن و نبات الفول. تم تطبيق الدراسة في منطقتين في مصر(الدلتا و الصعيد). تعتمد الدراسة على نتائج سابقة تم رصدها عمليا في الحقل في الاعوام 2003-2004 و 2004-2005 في محطة قها بمحافظة القليوبية و محطة سدس بمحافظة بني سويف.

تم استحداث وتقديم أربعة نماذج تم تصنيفها طبقا لنوع النبات و الموقع الجغرافي. تم استخدام المنحنيات التي تنطبق وتمثل النتائج المطلوبة لتوقع الاعداد لكل نوع من الافات التي تضمنتها الدراسة. تم استخدام عدد من المعادلات لوصف تاثير درجات الحرارة و الرطوبة على النتائج لكل نبات و لكل منطقة وتم رصد تأثير محدود لكل من درجة الحرارة و الرطوبة. ظهر ضرورة ضبط النتائج لتجنب القيم الشاذه الصغرى والعظمى للنتائج نتيجة هجرة أو فقس البيض والتي لا يمكن ادراجها في التمثيل الرياضي. ثم يلي ذلك وضع النموذج الرياضي و الذي يهدف الى توقع اعداد الافات والعناكب المفترسة عند أي زمن بعد معرفة الأعداد الابتدائية لكل منهم.

تم انشاء أربعة نماذج رياضية للأعداد المتوقعة للافات و العناكب المفترسة لأشهر السنة المختلفة باستخدام برنامج Microsoft Visual Studio و تم بناؤها بأسلوب ميسر وسهل الاستخدام. نم اختبار النماذج الرياضية باجراء دراسة مقارنة بين نتائج النماذج والنتائج المعملية الحقلية ووجدت متقاربة بشكل كاف.