Dynamics of Productive Efficiency of Commercial Banks

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ABESTRACT

The performance of the financial institutions is a major concern for both, the regulators and the policy makers, since it has a strong linkage with the performance of the economy.

Banking has the same features like any mature industry. Companies that survive should keep their costs down. The banking sector is considered sensitive and important in any society, as it is the basement and the pillar of success or failure of any economy. Measuring efficiency of resources exploitation of this sector is considered necessary, to rationalize and direct the process of the banking industry for every country

In this article we measure the bank efficiency based on the level of consistency between the head quarter and its branches from the perspective of achieving the main goals and sub goals of the main branch and its dependence respectively. The aim of this paper is to use a bi-level linear programming technique with rough parameters in the constraints to measure the productive efficiency of commercial banks. The proposed linear objective functions will be maximized for different goals that corresponding to the different banks. Based on D&K's goal programming method the described approach was developed to deal with the bi-level decision-making problem. The concept of tolerance membership function together with the branch and bound technique were used to generate the optimal solution for the problem under investigation. Also an auxiliary problem is discussed to illustrate the functionality of the proposed approach.

Key Words - Bi–level programming, rough programming, goal programming, productive efficiency, branch and bound

1- INTRODUCTION

The performance of the financial institutions is a major concern for both, the regulators and the policy makers, since it has a strong linkage with the performance of the economy.

Banking has the same features like any mature industry. Companies that survive should keep their costs down. The banking sector is considered sensitive and important in any society, as it is the basement and the pillar of success or failure of any economy. Hence, measuring efficiency of resources exploitation of this sector is considered necessary, to rationalize and direct the process of the banking industry for every country

In this article we measure the bank efficiency based on the level of consistency between the head quarter and its branches from the perspective of achieving the main goals and sub goals of the main branch and its dependence respectively. [6, 8]

Rough set theory [5] is an effective tool for data mining, and it has been the core problem in academic and application fields. Many articles gave detailed discussions under different background and also obtained many important research results [3].

Commercial banking is a very difficult service industry in which to measure output, technical change, or productivity growth. First, there is disagreement over which services banks produce and over how to measure them.

In recent times, a significant body of literature has evolved which explores the performance of financial institutions in the wake of financial liberalization. These studies are essentially micro-economic in nature and seek to analyze the efficiency and productivity of banking systems. Such analysis is of relevance from the policy standpoint, because as the finance-growth literature suggests, if banks become better-functioning entities, this is expected to be reflected in safety and soundness of the financial system and ultimately, lead to increases in the rate of economic growth. More importantly, such analysis is useful in enabling policymakers to identify the success or failure of policy initiatives or, alternatively, highlight different strategies undertaken by banking firms which contribute to their successes.[11,12]
The productivity is a concerned with real resource use, output from a given set of inputs and measured as the output per unit input (or a set of inputs). This simplistic approach is useful when there is only one technology, one input and one output. However, for a firm, merely getting the maximum output from a given set of inputs is not adequate since different technologies, different inputs and different sets of outputs from the same set of inputs are obtained. Thus, more important is the change in productivity over a period of time, from one period to another. Productivity is hence,[17]

Both, static and dynamic in nature: a measure of, both, the change in technology over time, and optimal use of resources, for the best available technology, at a given time. Moreover, if the objective of the firm is to maximize profits, the productivity measured as ratio of physical units may not be the best criterion. Hence, in addition to conventional measure of productivity, a "monetized value of productivity" may be a better performance measure.

Bi-level programming problem is formulated for a problem in which two decision makers make decisions successively [7]. The majority of researches on bi-level decision making has centered on the linear Programming. A set of approaches and algorithms of linear bi-level programming, such as the well-known Kuhn–Tucker approach, Kth-best approach, Branch-and-bound algorithm and genetic algorithm have been widely used [9].

Execution of strategy

A strategy can be implemented through human resource management, use of technology, and process design. X-efficiency is a measure of how well management aligns technology, human resource management, and other resources to produce a given level of output. It views

banks as a factory that consumes various resources to produce several products and establishes the efficiency with which this transformation takes place. The X-efficiency of banks can be accessed through indicators of financial soundness.

The articulation of a strategy is a key driver for success in dynamic, competitive environments like that of the financial services industry. The main strategic choices a bank faces concern product mix, client mix, geographical location, distribution channels, and form Of organization. [3, 4, 5]

Diverse forms of rough sets have been proposed and studied with different binary relations. Moreover, some of them are investigated with addition topological or algebraic structures [4,8].

The goal programming (GP) model is useful for decision makers to consider several objectives in finding a set of acceptable solutions simultaneously. Since only partial information can possibly be obtained, precisely determining the goal value of each objective might be difficult for decision makers; the main studies that incorporate uncertainty and imprecision into the GP are [6].

In [1] the author proposed a bi-level integer non-linear programming problem with linear or non-linear constraints, and in which the non-linear objective function at each level are to maximized. Also an interactive approach for solving bi-level integer multi-objective fractional programming problem was suggested in [2].

In [10] Xu and Yao discussed a class of linear multi-objective programming problems with random rough coefficients and gave a crisp equivalent model.

In [7] Saraj and Sadeghi presented a fuzzy goal programming (FGP) technique for solving Quadratic Bi-Level Fractional Multi-Objective Programming (QBL-FMOP) Problem was developed.

In this paper it is assumed that, there is a bank has two branches, one of them (headquarter) in AUE (the Leader player) and the other branch in Egypt (the Follower player), the two branches have their different objectives, the first objective (say N_1) represents the Increment of the deposits amount and the second one (say N_2) represents the Increment of the loans amount, and they Invest in two types of financial bonds X_1, X_2. Of course headquarter will plan the strategy and goals of the follower branch.

2- PROBLEM FORMULATION AND SOLUTION CONCEPT

Let $x_i \in R, (i = 1,2)$ be a vector variables indicating the first and the second decision level's choices respectively. Let $H_i: R^2 \to R^{N_i}, (i = 1,2)$ be the first and the second level's objective functions respectively. Let "FLDM" represents the first level decision maker or the AUE branch and "SLDM" represents the second level decision maker or the Egypt branch and they have N_1 and N_2 objective functions, respectively.[1,2]

Since the AUE branch sets his rough goals firstly then the and the Egypt branch goals, therefore, the bi- level multi-objective linear programming problem with definite goals contains rough parameters in constraints may be stated as follows:

[FLDM]

$$\max_{x_1} H_1(x_1, x_2) = \max_{x_1}\left(h_{11}, \dots, h_{1N_1}\right), \quad (1)$$

Where x_2 solves

[SLDM]

$$\max_{x_2} H_2(x_1, x_2) = \max_{x_2}\left(h_{21}, \dots, h_{2N_2}\right), \quad (2)$$

Subject to

$$G = \{(\xi x_1, \xi x_2) | g_i(\xi x_1, \xi x_2) \le y, i = 1,2, \dots m, (x_1, x_2) \ge 0\}.$$

$$f_{kj} = \sum_{j=1}^{n} \xi_{kj} x_j \quad , i = 1,2, \dots, n_k.$$

Where **G** is the bi-level linear constraint set contains rough the parameters. H_1 and H_2 are linear functions with definite goals.

Now, going back to the bi- level multi-objective linear programming problem contains rough parameters in constraints, An associated goal programming for this problem with (N_1 + N_2) could be rewritten as follows:

[FLDM]

$$\text{Achieve } (h_{11}(x), \dots, h_{1N_1}(x)) = (k_{11}, \dots k_{1N_1}), \quad (4)$$

Where x_2 solves

[SLDM]

$$\text{Achieve}(h_{21}(x), \dots, h_{2N_2}(x)) = (k_{21}, \dots k_{2N_2}), (5)$$

Subject to

$$x \in G.$$

Where k_{1N_1}, k_{2N_2} are scalars and represent the aspiration levels associated with the objectives of the first and the second level decision makers respectively.

3- THE TRANSFORMATION OF RANDOM ROUGH COEFFICIENT [10]

To convert the bi-level multi-objective linear programming problem with random rough coefficient in the constraints into the respective crisp equivalent forms for solving this trust probability constrains, this process is usually hard for most of the cases, in our case the transformation process is introduced in the following theorems.[10]

Theorem 1

Suppose that random variables $\tilde{\bar{e}}_{rj}(\lambda)$ and $\tilde{\bar{b}}_r(\lambda)$ are characterized by

$\tilde{\bar{e}}_{rj}(\lambda) \sim \mathcal{N}\left(e_{rj}(\lambda), V_r^e\right), \tilde{\bar{b}}_r(\lambda) \sim \mathcal{N}\left(b_r(\lambda), \left(\sigma_r^b\right)^2\right)$,where $e_{rj}(\lambda), b_r(\lambda)$ are rough variables, and $V_r^e, \left(\sigma_r^b\right)^2$ are positive definite covariance's. Then

$e_r(\lambda)^T x - b_r(\lambda) = [(a,b),(c,d)](c \le a \le b \le d)$is also a rough variable. We assume that it is characterized by the following trust measure function:

$$\operatorname{Tr}\{e_r(\lambda)^T x - b_r(\lambda) \le t\} = \begin{cases} 0 & \text{if } t \le c, \\ \dfrac{t-c}{2(d-c)} & \text{if } c \le t \le a, \\ \dfrac{1}{2}\left(\dfrac{t-c}{d-c} + \dfrac{t-a}{b-a}\right) & \text{if } a \le t \le b, \\ \dfrac{1}{2}\left(\dfrac{t-c}{d-c} + 1\right) & \text{if } b \le t \le d, \\ 1 & \text{if } d \le t. \end{cases}$$

where

$\left(e_{rj}(\lambda)\right)_{nx1} = \left(e_{r1}(\lambda), e_{r2}(\lambda), \dots, e_{rn}(\lambda)\right)^T$. then, we have

$$\operatorname{Tr}\left\{\lambda \middle| \Pr\left\{\tilde{\bar{e}}_r(\lambda)^T x \le \tilde{\bar{b}}_r(\lambda)\right\} \ge \theta_r\right\} \ge \eta_r$$

if and only if

$$\begin{cases} a \geq M \geq c + 2(d - c)\eta_r & \text{if } c \leq M \leq a, \\ b \geq M \geq \dfrac{2\eta_r(d - c)(b - a) + C(b - a) + a(d - c)}{d - c + b - a} & \text{if } a \leq M \leq b, \\ d \geq M \geq (2\eta_r - 1)(d - c) + c & \text{if } b \leq M \leq d, \\ M \geq d & \text{if } M \geq d. \end{cases}$$

The proof of theorems1is given in [10]

4- THE EQUIVALENT CRISP PROBLEM OF BI-LEVEL ROUGH LINEAR PROBLEM

The equivalent bi-level multi-objective linear programming problem equivalent to the bi- level multi-objective linear programming problem contains rough parameters in constraints with definite goals may be stated as follows:

[FLDM]

$$\max_{x_1} H_1(x_1, x_2) = \max_{x_1}\left(h_{11}, \ldots, h_{1N_1}\right), (6)$$

Where x_2 solves

[SLDM]

$$\max_{x_2} H_2(x_1, x_2) = \max_{x_2}\left(h_{21}, \ldots, h_{2N_2}\right), \quad (7)$$

Subject to

$$G = \{(\xi x_1, \xi x_2) | g_i(\xi x_1, \xi x_2) \leq y, i = 1,2, \ldots m, (x_1, x_2) \geq 0\}. \ (8)$$

Where h_1, h_2 are the objective functions of the first and the second level decision makers “FLDM”, “SLDM” respectively.

Definition 1:

For any

$x_1(x_1 \in G_1 = \{x_1|(x_1, x_2) \in G\})$ Achieves the first level decision maker goals with under-attainment or over-attainment, if the decision-making variable

$x_2(x_2 \in G_2 = \{x_2|(x_1, x_2) \in G_1\})$ Achieves the second level decision maker goals with under-attainment or over-attainment. Hence (x_1, x_2) is a feasible solution of the rough goal bi-Level multi-objective linear programming problem.[12]

Definition 2:

If (x_1^*, x_2^*) is a feasible solution of the rough goal bi-Level multi-objective linear programming problem, such that the first level decision maker achieves all goals; so (x_1^*, x_2^*) is the Pareto optimal solution of the rough goal bi-level multi-objective linear programming problem.

A GOAL APPROACH FOR THE BI- LEVEL MULTI-OBJECTIVE LINEAR PROGRAMMING PROBLEM

To solve the bi- level multi-objective linear programming problem with definite goals, one first get the optimal solution of the "FLDM" with definite goals, and the "SLDM" should get his optimal solution with definite goals, as follows:

4-1 THE FIRST LEVEL DECISION MAKER

First, the "FLDM" solves the following problem:

$$\text{Achieve } (h_{11}(x), \dots, h_{1N_1}(x)) = (k_{11}, \dots k_{1N_1}),$$

(9) Subject to

$$x \in G.$$

Where $k_{11}, \dots, k_{1N_1}$ are scalars, and represent the aspiration levels associated with the objectives, $h_{11}, \dots, h_{1N_1}$, respectively.

We consider the following bi-level multi-objective linear programming problem associated to the first goal as:

$$P_{11}: \text{ Minimize } D_{11} = d_{11}^{-} + d_{11}^{+}, \qquad (10)$$

Subject to

$$h_{11}(x) + d_{11}^{-} - d_{11}^{+} = k_{11},$$

$$x \in G,$$

$$d_{11}^{-}, d_{11}^{+} \geq 0.$$

Where d_{11}^{-} and d_{11}^{+} are the under-attainment and over-attainment, respectively, of the first goal and

$$d_{11}^{-} X\, d_{11}^{+} = 0.$$

Then the attainment problem associated with the second goal is equivalent to the optimization problem P_{12}, where:

$$P_{12}: \text{ Minimize} D_{12} = d_{12}^{-} + d_{12}^{+}, \qquad (11)$$

Subject to

$$h_{12}(x) + d_{12}^{-} - d_{12}^{+} = k_{12},$$

$$h_{11}(x) + d_{11}^{-} - d_{11}^{+} = k_{11},$$

$$d_{11}^{-} + d_{11}^{+} = D_{11}^{*},$$

$$x \in G,$$

$$d_{1t}^{-} + d_{1t}^{+} \geq 0.$$

The optimal solution of the linear goal programming model is given by

$$x^{*} = (x_1^F, x_2^F).$$

4-2 THE SECOND LEVEL DECISION MAKER

Second, in the same way, the "SLDM" independently solves:

Achieve

$$(h_{21}(x), \dots, h_{2N_2}(x)) = (k_{21}, \dots k_{2N_2}), \quad (12)$$

Subject to

$$x \in G.$$

Where $k_{21}, \dots, k_{2N_2}$ are scalars, and represent the aspiration levels associated with the objectives, $h_{21}, \dots, h_{2N_2}$, respectively.

The "SLDM" will do the same action as the "FLDM" till he obtains his optimal solution

$$x^{*} = (x_1^S, x_2^S).$$

5-FUZZY APPROACH OF BI-LEVEL LINEAR PROGRAMMING WITH ROUGH PARAMETERS PROBLEM

Now the solution of the AUE branch and Egypt branch are disclosed. But, the two solutions are usually different and conflicting because of the nature between two levels goals. The AUE branch knows that using the optimal decisions x_1^F as a control factors for the EGYPT branch, is not a practical solution, in the real case it will be more reasonable to permit some tolerance for the EGYPT branch to extend its feasible region for searching an optimal solution, and at the same time to reduce the searching time and/or the interactions between the two Decision Makers.

So, the range of the decision variable x_1 should be around x_1^F with a maximum tolerance t_1 and the following membership function will specifyx_1:

$$\mu(x_1) = \begin{cases} \frac{x_1-(x_1^F-t_1)}{t_1} x_1^F - t_1 \le x_1 \le x_1^F, \\ \frac{-x_1+(x_1^F+t_1)}{t_1} x_1^F \le x_1 \le x_1^F - t_1, \end{cases} \quad (13)$$

where X_1^F is the most preferred solution the $(X_1^F - t_1)$ and $(X_1^F + t_1)$ are the worst acceptable decision; and that satisfaction is linearly increasing with the interval of $\lfloor X_1^F - t_1, X_1 \rfloor$ and linearly decreasing with$\lfloor X_1, X_1^F - t_1 \rfloor$, and the other decisions are not acceptable.

First, the "FLDM" goals may reasonably consider that it is absolutely acceptable if $h_1 \ge h_1^F$ and absolutely unacceptable if $h_1 < \grave{h}_1$, and consider that the preference with $[\grave{h}_1, h_1^F]$ is linearly increasing. This due to the fact that the "SLDM" obtained the optimum at (x_1^S, x_2^S), which in turn provides the "FLDM" the objective function values $\grave{h}_1$, make any $h_1 < \grave{h}_1 = h_1(x_1^S, x_2^S)$ unattractive in practice.

The following membership functions of the "FLDM" can be stated as:

$$\grave{\mu}[h_1(x)] = \begin{cases} 1 & \text{if } h_1(x) > h_1^F, \\ \frac{h_1(x)-\grave{h}_1}{h_1^F-\grave{h}_1} & \text{if } \grave{h}_1 \le h_1(x) \le h_1^F, \quad (14) \\ 0 & \text{if } \grave{h}_1 \ge h_1(x). \end{cases}$$

Second, the "SLDM" goals may reasonably consider that it is absolutely acceptable if $h_2 \geq h_2^S$ and absolutely unacceptable if $h_2 < \grave{h}_2$, and that the preference with $[\grave{h}_2, h_2^S]$ is linearly increasing. So, the "SLDM" has the following membership functions for its goal:

$$\grave{\mu}[h_2(x)] = \begin{cases} 1 & \text{if } h_2(x) > h_2^S, \\ \frac{h_2(x)-\grave{h}_2}{h_2^S-\grave{h}_2} & \text{if } \grave{h}_2 \le h_2(x) \le h_2^S, (15) \\ 0 & \text{if } \grave{h}_2 \ge h_2(x). \end{cases}$$

Finally, in order to generate the satisfactory solution, which is also a Pareto optimal solution with overall satisfaction for all decision-makers, we can solve the following problem.

$max\, \delta$, (16)Subject to

$$\frac{x_1 - (x_1^F - t_1)}{t_1} \geq \delta,$$

$$\frac{-x_1 + (x_1^F + t_1)}{t_1} \geq \delta,$$

$$\grave{\mu}[h_1(x)] \geq \delta,$$

$$\grave{\mu}[h_2(x)] \geq \delta,$$

$$x \in G,$$

$$t_i > 0 \,, \delta \in [0,1].$$

where δ represents the overall satisfaction for the two decision makers.

If the "FLDM" is satisfied with the obtained solution, then the satisfactory solution is reached. Otherwise, he should provide a new membership function for the control variable and objectives to the "SLDM", the process will repeated until a satisfactory solution is reached.

6-Numerical Example

Banks have three principal activities: taking deposits, making loans, and investing in securities. To do this they use labor (skilled and unskilled), physical capital, and financial capital. Two main questions are addressed in this study: (i) How efficient are banks in SSA and what determines their degree of efficiency? (ii)What other factors may explain the low level of financial development in SSA? To answer these questions, the paper uses stochastic frontier analysis to assess banking efficiency and its determinants. This method makes it

Possible to determine the cost frontier while taking into account factors related to both the technological process of banks and the environment in which they operate. To investigate the low level of financial development, the paper employs the generalized method of moments [9]

(GMM) system of Arellano and Bover (1995)

This example aims at determining the extent to which commercial banks held banking efficiency. The banking sector is one of the main players in any society, hence, measuring the efficiency of performance of the sector members sector is necessary, to rationalize and direct the process of the banking industry for every country. The standard analytical method has been used in this study by using the solution method for bi-level multi-objective linear programming problem under random rough coefficient in constrain [[13]

The results show that on average, SSA banks are cost-efficient in producing their main outputs—deposits and short-term loans. However, efficiency could be improved by enhancing the credit environment through better functioning judicial and legal processes and

The accessibility of information on borrowers. This should allow banks more effectively to play their financial intermediary role of transforming deposits into loans for investment. The estimations show that financial development has been hindered by inflation and somewhat by concentration in the banking sector. Better macroeconomic stabilization policies that keep

Inflation under control and a more competitive banking system could help financial development.[14]

In our case study the central main branch will be considered as the first level decision maker (call it FLDM) for all the other branches and it is located at Dubai. The second level decision maker (call it SLDM) is one of the dependent branches and it is located in Cairo, Egypt.

The FLDM determines specific objectives which are increasing both Loans, and deposits.

The targets for the FLDM is fifty millions dollars for the first objective, and eighty millions dollars for the second one, while the targets for the SLDM is twenty millions dollars for the first objectives and thirty eight millions dollars for the second objectives.

Based on the difficulty of determining the market fluctuation, the objectives are determined in a rough environment, this situation makes the obtaining of the problem solution impossible under the traditional techniques .[15]

So the bi-level multi-objective linear programming problem under random rough coefficient in constraint is the suitable choice to solve this kind of problems.

All these objectives are determined in a rough environment (based on the fluctuation of the market status). In this environment it is difficult to determine the market fluctuation which makes finding an accurate solution of the problem is almost impossible using the traditional techniques. So the bi-level multi-objective linear programming problem (BLMOLPP) under random rough coefficient in constraint will be the suitable choice to solve these particular types of problems.

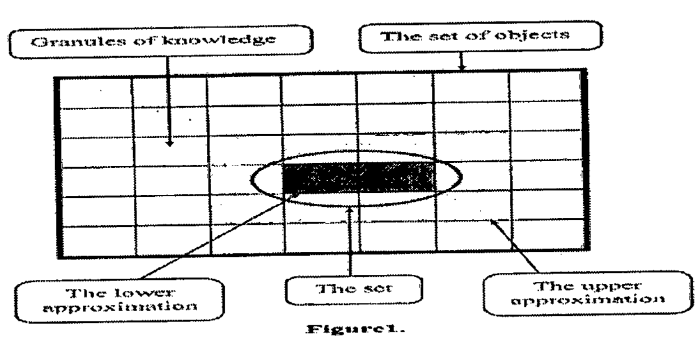

Figure1.

Let's assume that the FLDM has the following goals $h_1 = (50, 80)$ and the SLDM has the following goals $h_2 = (20, 38)$ to satisfy the overall satisfaction for both decisions makers. Now the mathematical form of the above description is:

FLDM

$$\max_{x_1} h_{11}(x) = 5x_1 + x_2,$$

$$\max_{x_1} h_{12}(x) = 2x_1 + x_2,$$

Where x_2 solves

SLDM

$$\max_{x_2} h_{21}(x) = x_1 + 2x_2,$$

$$\max_{x_2} h_{22}(x) = 2x_1 + 2x_2,$$

Subject to

$$\xi_1 x_1 + \xi_2 x_2 <= 45,$$

$$\xi_3 x_1 + \xi_4 \ x_2 <= 30,$$

$$x_1, x_2 \geq 0.$$

Assume that the rough parameters are defined as follows:

$\xi_1 \sim \mathcal{N}(\rho_1, 1)$, with $\rho_1 = ([2,3],[1,4])$, $\xi_2 \sim \mathcal{N}(\rho_2, 4)$, with $\rho_2 = ([1,2],[1,3])$,

$\xi_3 \sim \mathcal{N}(\rho_3, 1)$, with $\rho_3 = ([1,2],[0,3])$, $\xi_4 \sim \mathcal{N}(\rho_4, 2)$, with $\rho_4 = ([3,4],[2,5])$,

Let also$\eta_r = 0.9$.

Now by using theorem 1, the equivalent crisp problem which equivalent to a BLMOLPP under rough parameters in constraints with definite goals, as follows:

FLDM

Achieve $h_{11} = (5x_1 + x_2) = k_{11},$

Achieve $h_{12} = (2x_1 + x_2) = k_{12},$

Where x_2 solves

SLDM

Achieve $h_{21} = (x_1 + 2x_2) = k_{21},$

Achieve$h_{22} = (2x_1 + 2x_2) = k_{22}.$

Subject to

$x \in G = \{3.4x_1 + 2.6x_2 <= 45,$

$2.4x_1 + 4.4x_2 <= 30,$

$x_1 \geq 0, x_2 \geq 0 \geq 0\}.$

By calculating the trust measure for every rough coefficient by using trust measure function in theorem 1 the following will obtained.

$\mathrm{Tr}\{\xi_1\} = 0.9, \mathrm{Tr}\{\xi_2\} = 0.9, \mathrm{Tr}\{\xi_3\} = 0.9, \mathrm{Tr}\{\xi_4\} = 0.9, \mathrm{Tr}\{\xi_5\} = 0.6, \mathrm{Tr}\{\xi_6\} = 0.9.$

So, according to the above results, the trust more than or equal $\propto$ ($\propto = 0.6$) is the equivalent crisp problem, which equivalent to bi-Level multi-objective linear programming problem under rough parameters in constraints.

Now, the associated goal programming for the above problem with $(N_1 + N_2)$ goals can be rewritten an as follows:

FLDM

$$\max_{x_1} h_1(x_1, x_2) = \max_{x_1}\lfloor 5x_1 + x_2, 2x_1 + x_2 \rfloor,$$

Where $x_2$solves

SLDM

$$\max_{x_2} h_2(x_1, x_2) = \max\lfloor x_1 + 2x_2, 2x_1 + 2x_2 \rfloor,$$

Subject to

$$x \in G = \{3.4x_1 + 2.6x_2 <= 45,$$

$$2.4x_1 + 4.4x_2 <= 30,$$

$$x_1 \geq 0, x_2 \geq 0 \geq 0\}.$$

First, the "FLDM" solves his problem as the following:

Achieve $5x_1 + x_2 = k_{11}$,

Achieve $2x_1 + x_2 = k_{12,}$

Subject to

$$x \in G.$$

And the aspiration levels of the goals are assumed to be $k_{11} = 50$, $k_{12,} = 20$, respectively.

Then, the optimization problem associated with the first goal is formulated as follows:

$$P_{11}: \quad Minimize\ D_{11} = d_{11}^- + d_{11}^+,$$

Subject to

$$5x_1 + x_2 + d_{11}^- - d_{11}^+ = k_{11},$$

$$x \in G,$$

$$d_{11}^-, d_{11}^+ \geq 0.$$

Where the maximum degree of attainment of problem P_{11} is $D_{11}^* = 0.0002$ with an optimal solution

$$x_1 = (9.9998, 0.0011) \ \& \ d_{11}^- = 0, d_{11}^+ = 0.0002.$$

Now the attainment problem for the second goal of the FLDM is equivalent to problemP_{12}:

$$P_{12}: \quad Minimize\ D_{12} = d_{12}^- + d_{12}^+$$

Subject to

$$2x_1 + x_2 + d_{12}^- - d_{12}^+ = k_{12},$$

$$5x_1 + x_2 + d_{11}^- - d_{11}^+ = 50,$$

$$d_{11}^- + d_{11}^+ = 0.0002,$$

$$x \in G,$$

$$d_{12}^- + d_{12}^+ \geq 0.$$

Where the maximum degree of attainment of the problem P_{12} is $D_{12}^* = 0.0006$

Therefore, the optimal solution of the problem P_{12} is given by

$$x_2 = (9.9999,0.0008),$$

$$d_{11}^{-} = 0, d_{11}^{+} = 0.0002,$$

$$d_{12}^{-} = 0 \;, d_{12}^{+} = 0.0006 \;,$$

So the optimal solution of the BLMOLPP model is given by x^* which is the optimal solution of the FLDM$x^* = (x_1, x_2) = (9.9999,0.0008)$.

Second the "SLDM" solves his problem as the following:

Achieve$x_1 + 2x_2 = k_{21}$,

Achieve $2x_1 + 2x_2 = k_{22}$,

Subject to

$$x \in G.$$

And the aspiration levels of the goals are assumed to be $k_{21} = 10, k_{22} = 18$ respectively. Then, the optimization problem associated with the first goal is formulated as follows:

$$P_{21}: \; Minimize \; D_{21} = d_{21}^{-} + d_{21}^{+},$$

Subject to

$$x_1 + 2x_2 + d_{21}^{-} - d_{21}^{+} = 10,$$

$$x \in G,$$

$$d_{21}^{-} + d_{21}^{+} \geq 0.$$

Where the maximum degree of attainment of problem $P_{21}$is$D_{21}^{*} = 0$ with an optimal solution

$$x = (9.9987,0.0006) \;\&$$

$d_{21}^{-} = 0, d_{21}^{+} = 0.$

Now the attainment problem for the second goal of the SLDM is equivalent to problem P_{22}

$$P_{22}: \ Minimize\ D_{22} = d_{22}^{-} + d_{22}^{+},$$

Subject to

$$2x_1 + 2x_2 + d_{22}^{-} - d_{22}^{+} = 18,$$

$$x_1 + 2x_2 + d_{21}^{-} - d_{21}^{+} = 10,$$

$$x \in G.$$

Finally assume that the FLDM control decision is $x_1^F = 9.9999$ with an available tolerance equal to "one", then the "SLDM" solves a Tchebycheff problem with an optimal solution

$$(d_{21}^F, d_{22}^F) = (10.0001, 0.9869),$$

$$x_1^F = 0.9999\ \&\ h_1 = (50.9874, 20.9871),$$

$$h_2 = (11.9739, 21.974)$$

With an overall satisfaction for both decisions makers

- The final results of the problem are:
- The AUE branch achieves the following goals h_1= (77.98,48.98), and
- Also the EGYPT branch achieves the following goals h_2= (18.97,36.97),
- With an overall satisfaction for both decisions makers.

5- CONCLUSION

Efficiency and productivity change have long been the focus of banking research. Findings in relation to these areas of research have important implications to policymakers who are concerned about bank safety and soundness, as well as to bank managers who seek to improve operating performance.

The suggested linear objective functions are to be maximized with different goals to measure the productivity efficiency for commercial banks with different level of decision making players. The suggested approach in this paper was mainly based on the goal programming method of Dauer and Krueger to develop the optimal solution of the bi- level decision- makers. The concepts of tolerance membership function together with the branch and bound technique were applied on a goal programming models to obtain the optimal solution for the bi-level problem. An overall satisfaction solution is obtained to hold the efficiency of the performance of the dependent branch with respect to the main decision maker.

Now after solving the problem, the solution of the AUE branch and Egypt branch are disclosed. But usually the two solutions are different because of the two levels goals structure's nature.

From the AUE branch side it is known that, using their optimal decisions as control factors for the EGYPT branch, is not a practical approach.

So, it is more reasonable to have some tolerance that gives the EGYPT branch an extent feasible region (flexibility) to search for their optimal solution, and also to reduce the searching time and/or the actual interactions.

The declining efficiency and productivity in the later years indicates potential for increase in outputs -Loans and investments in the banking sector. Banks have been successful in mobilizing resources deposits through improved technology/innovations and product offering. Now this should translate into increased output of loans and investments, through reduced prices (interest rate spread) and improve on their productivity. Policies to enhance Loan advancement at good prices be encouraged or policies to enhance competitive pricing to ensure advancement of loans may be pursued. This will improve intermediation in the banking sector.

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