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A constant force function for the lattice Boltzmann equation in isotropic turbulence

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ABSTRACT

A constant force function for the lattice Boltzmann method is introduced and parts of the programming code are presented. Adding force to the lattice Boltzmann equation is a challenging issue because the nature of the forcing techniques and the nature of the lattice Boltzmann equation (LBE) itself. The LBE is a difference equation and it is coded in the physical space, however, the forcing is usually done in the Fourier space at low wave-numbers. This paper discusses the force function and its coding to be readable by the LBE without losing of any physical meaning of the engineering problem.

1. Introduction

Simulation of forced isotropic turbulent flow is an important engineering problem because it is a standard problem to study statistical properties of turbulence and it is the best kind to investigate turbulent theories, visualizing vortices and test the Kolmogorov theories of turbulence. The forcing of the lattice Boltzmann equation is done by adding the force to the collision term or by shifting the velocity field or by adding the force to the collision process and shifting the velocity field at the same time. This process is done randomly each time step and the force is also added to the space at random points in the box each time step. In this study, the force function will be coded and all parameters will be discussed. The coding will be considered using the FORTRAN language and the code can be modeled easily with any other programming languages. Many studies have been presented to study the forcing of the lattice Boltzmann method such as Luo [1], Shan and Chen [2], Guo et al. [3], Succi [4], Cosgrove et al. [5], Siggia and Patterson [6], Mohseni et al. [7]. Generating the velocity field of isotropic turbulence using the forcing function was introduced by Abdel Kareem et al. [8] where extracting of multiscale vortical structures was investigated with resolution of 128^3 and the function is also used to resolution of 256^3 by comparing the lattice D3Q15 and D3Q19 models [9]. It was shown that the D3Q19 model is more stable than the D3Q15 model. Some recent studies of isotropic turbulent flows are presented using the same forcing function by Albernaz et al.[10,11].

Elghobashi[12] applied the same force at low wave-numbers at every time step to generate a statistically stationary velocity field for Reynolds number between 73 and 133. Gkoudesnes and Deiterding[13] use the same forcing function to their LBM solver for homogeneous isotropic turbulence(HIT) in a periodic box and they started with zero initial velocity and unit density. A comparison between the direct numerical simulations (DNS) and large eddy simulations (LES)[14] is also considered using the same forcing function where they use the forcing scheme in their HIT for different resolutions starting from 32^3 to 512^3. Also, Abdel kareem et al.[15] applied the force function in their study of filtering isotropic turbulent data in addition to synthetic data that are generated by solutions of Navier-Stokes equations. This paper is organized as follows: section 1 is the current introduction and section 2 presents the forcing function and its coding in FORTRAN language. In section 3, the results and discussion which are mainly concentrated in depicting the energy spectrum and the vortical structures resulted from the forcing function and finally, sec.4 is the conclusion of the study.

2. The forcing function and FORTRAN code

The lattice Boltzmann equation (LBE) can be written as

$$f_\alpha(\boldsymbol{x} + \boldsymbol{e}_\alpha \delta t, t + \delta t) - f_\alpha(\boldsymbol{x}, t) = \frac{-1}{\tau}(f_\alpha(\boldsymbol{x}, t) - f_\alpha^{eq}(\boldsymbol{x}, t)) + 3\rho w_\alpha(\boldsymbol{e}_\alpha \cdot \boldsymbol{F}).$$

Where the last term represents the force term which is added to the collision term in the lattice Boltzmann method (LBM). The basic motivation of this study is to present a

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function ***F*** that appeared in the last term in the equation. All the parameters and definitions of the functions introduced in the above LBE can be found in references [8,9], respectively. Also, the force code will be discussed and some parts will be introduced. The most important point is the function itself which should be clear in containing the coordinate variables and the wavenumbers. The function used can be written as:

$$\boldsymbol{F}_1 = \begin{cases} F_x = A \sum_{k_1=-k_{max}}^{k_{max}} \sum_{k_2=-k_{max}}^{k_{max}} \sum_{k_3=-k_{max}}^{k_{max}} [(\frac{k_2 k_3}{k^2}) \sin(\frac{2\pi}{N}(k_1 x + k_2 y + k_3 z) + \phi)] \\ F_y = -2A \sum_{k_1=-k_{max}}^{k_{max}} \sum_{k_2=-k_{max}}^{k_{max}} \sum_{-k_{max}}^{k_{max}} [(\frac{k_1 k_3}{k^2}) \sin(\frac{2\pi}{N}(k_1 x + k_2 y + k_3 z) + \phi)], \\ F_z = A \sum_{k_1=-k_{max}}^{k_{max}} \sum_{k_2=-k_{max}}^{k_{max}} \sum_{k_3=-k_{max}}^{k_{max}} [(\frac{k_1 k_2}{k^2}) \sin(\frac{2\pi}{N}(k_1 x + k_2 y + k_3 z) + \phi)] \end{cases}$$

where, the random phase is represented by ϕ and the wavenumbers are k_1, k_2 and k_3 in the 3D cartesian directions and the disturbance is caused by the sinusoidal function. A is the force amplitude and can be chosen equal to 10^{-4} . This force function has been used in several studies in the last ten years such as[10,11,12,13,14]. The first part of the code is generating the random phase function ϕ by a system random number generating function rand( ):

```
do k1=-kmax, kmax
do k2=-kmax, kmax
do k3=-kmax, kmax
   ϕ(k1,k2,k3)=2π*rand( )
     endddo
          enddo
              enddo,
```

k_{max} is the maximum cutoff wavenumber. The second part of the code is the main body of the code where the sine function is calculated at the low wavenumbers - $k_{max} \leq |k| \leq k_{max}$:

```
      do i=1,nx
       dx=2π*REAL(i)/REAL(nx)
    do j=1,ny
       dy=2π*REAL(j)/REAL(ny)
     do k=1,nz
       dz=2π *REAL(k)/REAL(nz)
        ux(i,j,k)=0.0
        uy(i,j,k)=0.0
              uz(i,j,k)=0.0
       do k1=-kmax, kmax
       do k2=-kmax, kmax
       do k3=-kmax, kmax
index=REAL(k1*k1+k2*k2+k3*k3)
if(√(index).gt.0..and.√(index).le.kmax)then
F=sin(k1*dx+k2*dy+k3*dz+ϕ(k1,k2,k3))
ux(i,j,k)=ux(i,j,k)+1.0/index*k2*k3*F
uy(i,j,k)=uy(i,j,k)-2.0/index*k1*k3*F
uz(i,j,k)=uz(i,j,k)+1.0/index*k1*k2*F
      endif
                 enddo
                   enddo
                    enddo
       ux(i,j,k)=A*ux(i,j,k)
       uy(i,j,k)=A*uy(i,j,k)
       uz(i,j,k)=A*uz(i,j,k)
  Write(file-number) ux(i,j,k),uy(i,j,k),uz(i,j,k)
        enddo
              enddo
                enddo
```

Here n_x, n_y and n_z are the box lengths in x, y and z-directions. The output velocities are the low-wavenumber velocity modes that are calculated in the sphere with a radius of kmax. In all cases, one can set kmax equals to 2 or 3 or 4 and the forcing amplitude as $A=10^{-4}$. The force is divergence free, where it is clear that **div F=0**.

3. Results and discussion

Figure 1 shows the energy spectrum for this force, where the cutoff wavenumber effect can be noticed in the figure. Figure 2 shows the vortical structures that are depicted using the method introduced by Abdel Kareem[16] whose definition is :

$$Q_S^S = [(Q_W^3 + Q_S^3) + (\Sigma^2 - R_s^2)]^{\frac{1}{3}}, .$$

Where Q_W represents the rotation tensor strength and Q_S represents the deformation tensor strength, Ξ and R_s are the enstrophy production term and the strain rate production, respectively. The mathematical definitions of the tensors are:

$$Q_W = \frac{1}{2}\Omega_{ij}\Omega_{ij}, Q_s = -\frac{1}{2}S_{ij}S_{ij}, \Sigma = \omega_i S_{ij} \omega_j, R_s = -\frac{1}{3}S_{ij}S_{jk}S_{ki}.$$

Where Ω_{ij} is the rotation tensor and S_{ij} is the strain tensor.

After few time steps, this force can develop the simulations of the lattice Boltzmann equation and the spectrum can transformed from low wavenumbers to higher wavenumbers as depicted in Figure 3. Also, the large vortices that are visualized in Figure 1 will be developed to thinner and longer vortices with time advancement and it is shown in Figure 4. The developed figures 3& 4 indicate that the forcing function is a good candidate for isotropic turbulence and can generate steady and stationary isotropic turbulent flow data. This forcing is succeeded to generate 3D isotropic turbulent data with statistical results that are close to the results obtained by Fourier spectral numerical simulations of Navier-Stokes equations. In recent studies, the same function is compared with other different forces to generate 3D isotropic turbulent data [17] and also used in comparison of different forcing techniques of the LBM [18].

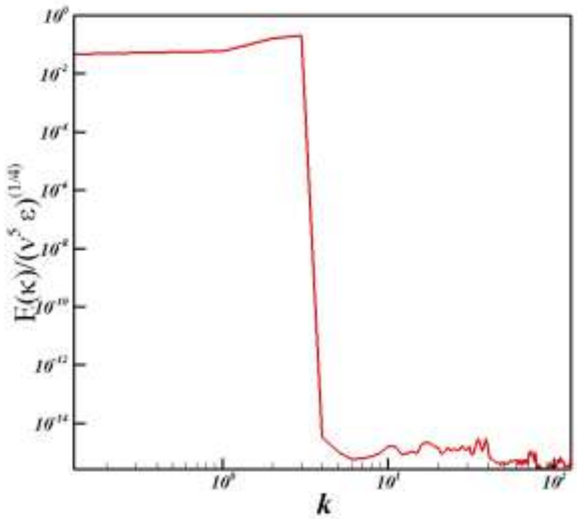

Figure 1: Energy spectrum of the forcing function at $n_x=n_y=n_z=256$.

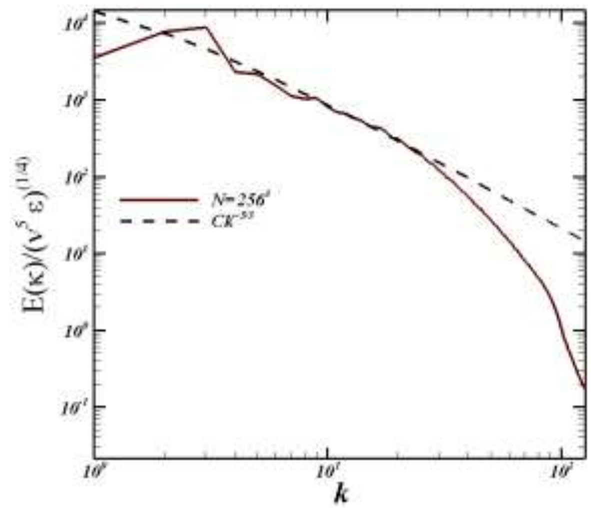

Figure 3: Energy spectrum of a developed time step generated using the forcing function at $n_x=n_y=n_z=256$ [9].

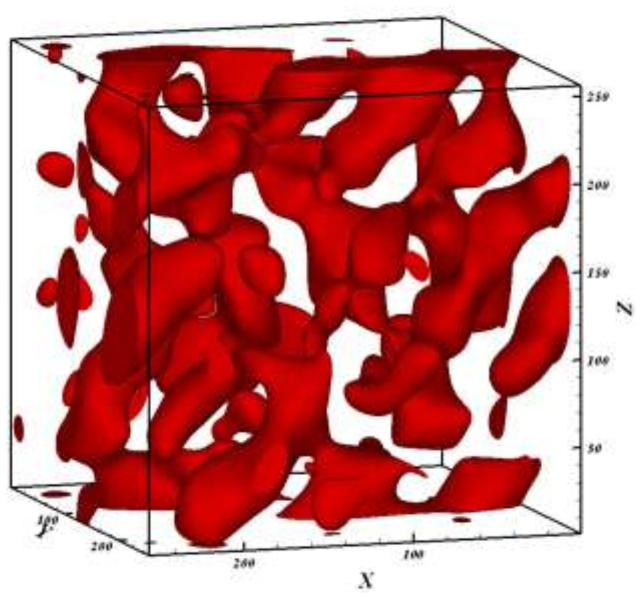

Figure 2: Isosurfaces of the vortices generated from the forcing function initial velocities at the first calculations with $n_x=n_y=n_z=256$.

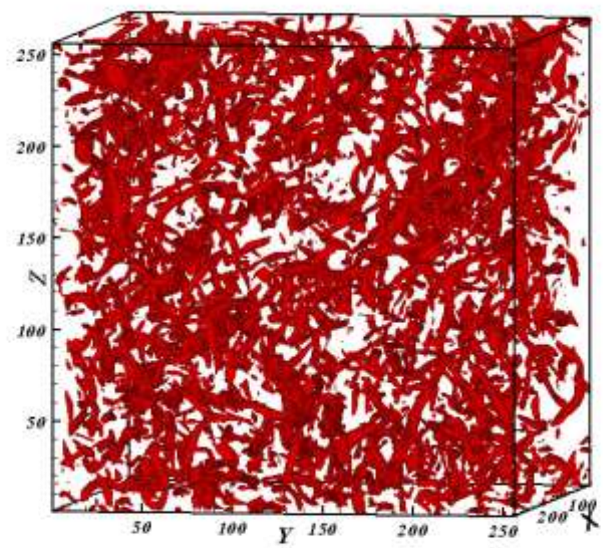

Figure 4: Isosurfaces of the vortices for a developed time step that is generated from the forcing function velocities at later time step with $n_x=n_y=n_z=256$ [9].

4. Conclusion

A force function is introduced to discuss the linear forcing of the lattice Boltzmann equation and the method can be also used with other numerical methods for the same purpose. The forcing function is divergence free and can compensate the turbulent velocity with adding energy and preventing the decaying of the flow field. The coding of the function using FORTRAN is also presented in a clear and a simple way to help the readers to adjust the programming of the function using any programming language. The constant force can be saved and then used at every time step of the simulations.

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