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A NEW FORMULA FOR DETERMINATION OF TANGENTIAL AND RESIDUAL STRESSES AROUND A HOLE IN PLATE FOR A VARIETY OF BIAXIAL LOADING

معادله جديده لتحديد الاجهادات المماسه المولده والمتبقيه حول محيط الثقوب فى الالواح المعدنيه تحت تأثير قيم مختلفه للاحمال المزدوجه

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ملخص::

نظرا لاهميه دراسة الاجهادات المولده حول محيط الثقوب تحت تأثير الاحمال الخارجيه فضلا عن الاجهادات المتبقيه بعد ازاله الاحمال الخارجيه وكذلك الاجهادات المتبقيه بعد عمليات تشغيل الثقوب فى الالواح المعدنيه المستخدمه فى الصناعات الهندسيه المختلفه • ونظرا لعدم توافر الصيغ او المعادلات سواء الرياضيه او التجريبيه لتحليل مثل هذه المشكلات فقد اهتم هذا البحث بدراسه وتحليل هذه الاجهادات وامكن ايجاد صيغ مبسطه لمعادلات يمكن استخدامها فى تحليل الاجهادات المماسه (المولده او المتبقيه) حول محيط الثقوب فى الالواح المعدنيه ذات الابعاد المحدوده وكذلك الالواح الغير محدوده الابعاد والواقعه تحت تأثير الاحمال الخارجيه الاحاديه وكذلك المزدوجه التأثير المطبقه فى الاتجاهات المتعامده ذات الطبيعه الواحده ، (شد ـ شد) او المعكوسه الطبيعه (شد ـ ضغط) بنسب مختلفه تحدد حسب قيمه الاحمال الخارجيه الموءثره فى الاتجاه الافقى الى الاحمال الموءثره فى الاتجاه الرأسى• وللتحقق من امكانيه تطبيق هذه المعادلات فى تحليل مثل هذه المشكلات امكن استخدام الطريقه التجريبيه للانعكاس الضوئى وكذلك طريقه قياس الانفعال بالمقاومه الكهربيه لقياس قيمه الاجهادات حول محيط الثقوب تحت تأثير قيم مختلفه للاحمال الخارجيه• واوضحت نتائج التجارب المعمليه توافق النتائج التجريبيه مع النتائج المستنتجه فى المعادلات المقترحه فى هذا البحث • ونظرا لوجود بعض المعادلات الرياضيه المقيده بحالات خاصه والخاصه فقط بقياس الاجهادات المحيطيه حول الثقوب فى الالواح ذات الابعاد الغير محدوده والواقعه تحت تأثير الاحمال الخارجيه المفرده ذات الاتجاه الواحد واخرى لحاله تساوى الاحمال الخارجيه (شد ـ شد) او (شد ـ ضغط) فى الالواح ذات الابعاد المحدوده •
وبمقارنه نتائج هذه المعادلات مع النتائج المستنتجه من المعادلات المقترحه فى هذا البحث وكذلك النتائج التجريبيه والمعمليه اظهرت توافق المعادلات المقترحه لتحليل الاجهادات حول محيط الثقوب وتميزها فى جميع الحالات العامه المدروسه فى هذا البحث مع قصور فى المعادلات المتاحه والمنشورة بالابحاث السابقه وذلك لتقيدها بحالات خاصه جدا • دون مقدرتها على تحليل الاجهادات المتبقيه سواء بعد ازاله الاحمال الخارجيه او بعد اتمام عمليات تشغيل الثقوب•

ABSTRACT

This work deals with the development of a new analytical formula which may satisfy the original problem of both tangential stress distribution as well as residual stress around the boundary of a hole in plate subjected to a variety of biaxial loading.

The developed equation has been employed for computing the tangential stress around a hole for different biaxial loading conditions.

Photoelastic coating technique has been used for measuring the residual stress that remain in plates after performing a hole by piercing , drilling , forming or machining processes. Moreover, based on the strain gauge experimental technique, the residual stress has been computed.

Comparison of the results indicates the applicability of the developed equation in computing both tangential stress distribution as well as residual stress.

Moreover, the use of photoelastic coating technique facilitate the experimental tests and provide full field experimental method for determination of stress distribution at points on the surface and in interior of specimens. Thus, the birefringent coating technique has many advantages for measuring residual stresses as compared to strain gauge method.

INTRODUCTION

The shearing operations, such as that in piercing or drilling, lead to residual stresses that remain with manufactured part after it has been deformed. Moreover, residual stress may remain around a hole in plate subjected to biaxial loading conditions after all external forces have been removed.

The residual stresses on the surface and in interior of a metal part are considered to be undesirable because they lower the fracture strength of the part. The residual stress can also lead to stress cracking or stress corrosion cracking over a period of time.

The problem of stress distribution and residual stress around a circular hole in plate has been treated analytically and experimentally by several investigators over the past years.

The stress distribution around a circular hole in a composite plate has been studied by the analytical approach discussed by Leknitskii [1]and Savin [2] where a closed form solution is given for a thin, homogeneous, and finite plate with a hole.

Daniel et al. [3] analyzed the deformation and failure of Bron-Epoxy plate with circular hole subjected to uniaxial tension using both the photoelastic coating technique and the finite element method to evaluate the safety of a composite structure.

Soby [4] presented theoretical results for rectangular holes with round corners and he reported that the minimum possible stress concentration factor is 2.85.

Ross [5] has reported results of numerous photoelastic experiments on holes and notches in thin plates under uniaxial tension. His results show a gross overestimate of the stress concentration factor as compared to Sobey's values.

Durelli et al. [6] have shown that, hole shapes can be very effectively optimized by using photoelastic techniques.

Durelli and Murrey [7] studied the optimum shape of a hole in a biaxial field of two loading of the same sign (k = 1) and reported that the stress concentration factor for a circular hole is 2, and for loading of opposite sign no such simple relation has been found.

Durelli and Rajaiah [8], have presented a paper with the object of minimizing stress concentrations. They have developed a quasi shape which introduces a stress concentration in the case of biaxial loading when the ratio of k=-1.

Rajaiah [9] studied the optimized hole shape in circular cylindrical shells under axial load, by using the photoelastic techniques. The process leads to significant decrease in the stress concentration factor.

Durelli and Rajaiah [10] have used photoelastic coating technique with a special reflection polariscope for the optimization processes of the hole shape and strain determination around a hole.

In order to account for residual stresses from cold expansion of the stop-drill hole, fracture mechanics, superposition procedure was employed by Cathey and Grandt [11] and good correlation with test data was shown.

Landy et al. [12], employed an analytical program utilizing finite element approach for determination of residual stress distribution around the stop-drill holes.

Venant, S. [14] summarized that, if a small circular hole is made in plate subjected to a uniform tension, the stress distribution in the neighborhood of the hole will be changed, but the change is negligible at distances which are larger compared with the radius of the hole.

Most of the available literature are concerned with distribution of stress around the boundary of a hole under uniaxial loading, with the exception of few investigated investigated optimization process for a hole shape in a biaxial field of two loading (loading ratio k=1).

Thus, it is necessary to develop an analytical approach to facilitate the determination of both residual stress, as well as tangential stress distribution around the boundary of a hole in a plate for a variety of biaxial loading conditions.

ANALYTICAL APPROACH

The development of a new analytical formula to characterize stress distribution around the boundary of a circular hole in plate subjected to biaxial loading may be useful for experimental estimation of residual stress.

It was essentially evident that the selection of a stress component for this particular problem is difficult since non of the available functions is satisfactory. In order to overcome this difficulty, a method of superposition is commonly used which employs two different stress components.

1- The first component is selected such that the stress associated with it satisfy the condition of stress distribution around the boundary of the hole under the applied stresses conditions. The first stress component , assumed to have the forms

$$\sigma_{\theta i} = \sigma_1 \{ [1+(k/3)]+ 2 [1-(k/2)] \cos 2\theta \} \qquad (1a)$$

$$\sigma_{\theta f} = \sigma_1 \{ [1+(k-2a_{yx})/3]+ 2 [1-((k+a_{yx})/3)] \cos 2\theta]\} \qquad (1b)$$

where k is the biaxial loading ratio ($k= \sigma_2/\sigma_1$)

a_{yx} finite dimension ratio ($a_{yx}= b_y / a_x$)

$\sigma_{\theta i}$ stress distribution for infinite plate.

$\sigma_{\theta f}$ stress distribution for finite plate.

2- The second stress component must have associated stresses which satisfy the stresses remaining within the boundary of the hole after the hole has been formed or machined and all external forces have been removed. Thus the photoelasticity can be effectively employed to estimate the second stress component as the following,

$$\sigma_R= N_\theta / 2h [E/(1+\mu) \lambda/C] \qquad (2)$$

where N_θ	fringe order
E	modulus of elasticity
μ	poisson's ratio
λ	wave length of light
c	optical sensitivity and
h	coating thickness

Thus, the required equation for the original problem is obtained by superposition of the first and second stress components. Towards this end one may assume the following:

a- for infinite plate subjected to biaxial loading

$$\sigma_{\theta i} = \sigma_1\{ [1+(k/3)]+ 2 [1-(k/2)] \cos 2\theta \} + N_\theta / 2h [E/(1+\mu) \lambda/C] \qquad (3a)$$

b- for the finite plate subjected to biaxial loading

$$\sigma_{\theta f} = \sigma_1\{ [1+(k-2a_{yx})/3]+ 2 [1-((k+a_{yx})/2)] \cos 2\theta]\}+ N_\theta / 2h [E/(1+\mu) \lambda/C] \qquad (3b)$$

Equations (1) and (2) give the polar tangential stress components as well as residual stress at any point defined by (a, θ) in a body subjected to biaxial loading. The stresses along x-axis can be obtained by setting θ=0 deg. Similarly the stresses along Y-axis can be computed by setting θ = 90 deg., for the circular hole of radius a.

Consequently, the full-field stress distribution around the boundary of a hole for various biaxial loading level can be computed by employing the developed equation (3), which satisfy the original problem in finite as well as infinite plates.

EXPERIMENTAL PROCEDURE

The specimens were 20 Cm. by 20 Cm. aluminum plates of 6 mm thickness with a 8 mm diameter central circular hole. Twelve identical specimens, were instrumented and tested.

Two-element rectangular rosette strain gages were employed to establish the magnitude of stress field. The strain gauges were mounted on the surface of the specimens with its axes coincident with the principal directions, x=0 deg. and y=90 deg.

Ten specimens were prepared for application of birefringent coating technique. A special coating of aluminum paint was applied to one face of specimens and cured to provide a perfectly reflective surface. Subsequently, a thin apoxy sheet of 1 mm. thickness was bonded to the painted surface prior to piercing or drilling.

Two specimens were drilled to create a central hole, then the fringe patterns were photographed with no load. The other eight specimens that were prepared for birefringent and the two instrumented specimens with strain gages were then placed in loading frame. Subsequently, at every loading condition, fringe patterns in the birefringent coating were photographed and strain gauge data were recorded

Finally, all specimens that were prepared for birefringent coating technique were re-photographed to record the fringe patterns that remain after removing the external forces. Fig.1 shows isochromatic fringes that remained (a) after performing a hole by drilling. (b) after removing the external forces. These birefringent are corresponding to the residual stress around the boundary of the hole. The images were then magnified and analyzed using a slide projector.

RESULTS AND DISCUSSION

Stress distribution around the boundary of circular holes for a variety of loading conditions has been computed analytically using the developed equation (3), and different analytical approach. This however will be done subsequently using a different experimental technique.

Resulting residual stress after drilling the hole were obtained experimentally by photoelastic coating method using equation (2).

The tangential stress distribution around the circumference of the holes were determined primarily from photoelastic data. Stresses are related to birefringent by stress optic law, [13].

$$\sigma_1 - \sigma_2 = (N / 2h) [(E / (1+\mu) (\lambda/c)] \qquad (4)$$

Moreover isochromatic fringe data corresponding to non-loading conditions after removing the external forces were then used for the estimation of residual stress around the boundary of the holes

The developed equations (3) were employed for analytical computing of the tangential stress ratio, for every biaxial loading conditions. Strain gauge data was used to determine strains related to stresses on the hole boundary at the horizontal and vertical axes as follows

$$\sigma_1 = E / (1-\mu^2) (\varepsilon_1 + \mu\varepsilon_2) \qquad (5)$$

Experimental results obtained by photoelastic coating technique were compared with results obtained by strain gauge method.

Analytical results were obtained by polar components of stress for infinite plates subjected to uniaxial loading as, [14].

$$\sigma_{\theta i} = \sigma_0 [1 + 2 \cos 2\theta] \qquad (6)$$

and were then compared with the experimental and analytical results previously obtained, in this investigation.

According to the experimental and analytical results, the variation of tangential stress around the boundary of the holes along radial lines located at $\theta = \theta \;\; + \Delta\theta$, for different biaxial loading conditions obtained by employing the developed equation (3) are shown in Fig.2a and Fig.2b.

Changes in tangential stress ratio for biaxial tension-tension to biaxial tension-compression are clearly illustrated in Fig.3a and Fig.3b. Symmetric variation occurs only for biaxial tension-tension of $K = 1$, and tension-compression of $K = -1$. In this case the stress is zero at the point defined by $\theta = 45$ deg., which is commonly referred to as a singular point (isotropic point- the point of zero tangential stress).

In the case of uniaxial where $K = 0$, the isotropic point occurs at $\theta = 54.63$ deg. for finite plates and $\theta = 60$ deg. for infinite plates.

An important and significant result is that the location of isotropic point shift from $\theta = 54.63$ deg. for uniaxial loading to $\theta = 45$ deg. for biaxial loading condition of $K = -1$.

According to Venant's principal, Kirsch, G.,[14] obtained a general solution for the stress function and he found that, the tangential stress is greatest when $\theta = \pi/2$, and the stress concentration factor at this point is 3, and is equal to -1 at $\theta = 0$. Thereafter, he concluded that, the localized character of stress around a hole justifies the application of the equation (6), derived for an infinite large plate, to a plate of finite width, provided that, the width of the plate should not be less than four times of the hole diameter.

By taking a tensile stress and a compressive stress of the same magnitude in two perpendicular directions, he obtained the tangential stresses at the boundary of the hole from the following equation:-

$$\sigma_\theta = \sigma_{app} (1-2 \cos 2\theta - [1-2 \cos (2\theta - \pi)]) \qquad (7)$$

For θ = 90 deg., $\sigma_\theta = 4\ \sigma_{app}$. For θ = 0 deg., $\sigma_\theta = -4\ \sigma_{app}$

Hence, for large plate under pure shear, the maximum tangential stress at the boundary of the hole is four times of the applied pure shear stress.

Examination of the results obtained from equation (7) evidenced that, it is restricted to the equally tension-tension or tension-compression biaxial stress conditions only. Moreover, the results show a gross overestimate in the stress concentration factor, compared to the results that were obtained from the developed equation (3) also, the pattern of change of the tangential stress as a function of angular displacement is in reversed trend as shown in Fig. 2b.

As shown in Fig. 4 the distribution of residual stress around the boundary of the drilled hole is constant. But these holes subjected to biaxial loading of K = 1, 2, 3, -1, -2 and -3, produce a maximum residual stress concentration at θ = 90 deg. and decreases to minimum at θ = 0 deg. that after removing the external forces.

As can be noted, there is a reasonable agreement among the various method of analysis except at the points of highest tensile and compressive stresses obtained by strain gauge method.

Closed agreement between the results obtained analytically by employing the developed equation (3) , and that obtained by equation (6) for infinite plate subjected to uniaxial loading condition where K = 0.

CONCLUSIONS

1- By employing equation (3) developed in this investigation, it has been possible to obtain the stress distribution around the boundary of circular holes in finite as well as infinite plates subjected to a variety of biaxial loading conditions. However, the developed equation can be considered to be reliable.

2- Equation (7) is restricted to the equally tension-tension and tension-compression biaxial stress conditions only. Furthermore, the pattern of change of the tangential stress predicted by this equation is in reversed trend. Thus, this equation is insufficient to satisfy the original problem in finite as well as infinite plates under variable biaxial loading conditions.

3- It has been observed that the residual stress around the boundary of a circular hole performed by drilling in finite plate is fairly constant all around the circumferences. But that hole subjected to biaxial loading produce residual stress that vary from a maximum at θ =90.0 degree to minimum at θ = 0.0 degree that after removing the external forces.

4- The use of photoelastic coating technique facilitates the experimental tests, moreover provides full-field experimental method to determine stress distribution as well as residual stress on surface or in interior of specimen. Thus photoelastic coating technique has many advantages for measuring the residual stress compared to strain gauge method of analysis.

NOMENCLATURE

a_{yx}	finite dimension ratio
K_o	uniaxial stress ratio
$K_{1,2..n}$	biaxial stress ratio
θ	angular displacement from the horizontal axis
σ_o	uniaxial applied stress
σ_1	applied stress in y-direction
σ_2	applied stress in x-direction
$\sigma_{\theta f}$	stress distribution for finite plate
$\sigma_{\theta i}$	stress distribution for infinite plate
σ_θ / σ_1	normalized tangential stress

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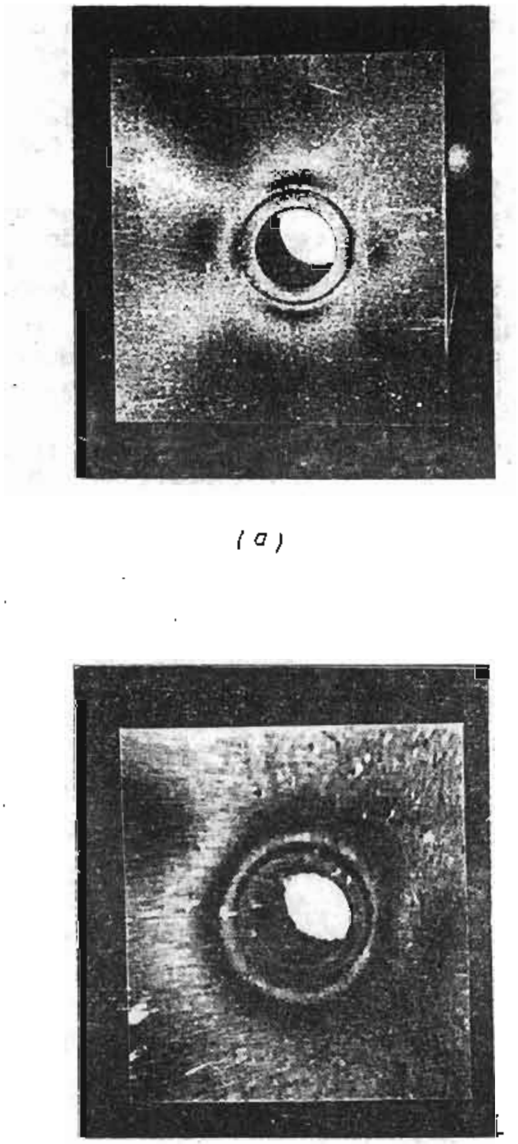

(a)

(b)

Fig.1. Isochromatic fring patterns in photoelastc coating around a hole in plate (a) after drilling the hole, and (b) after removing the external forces.

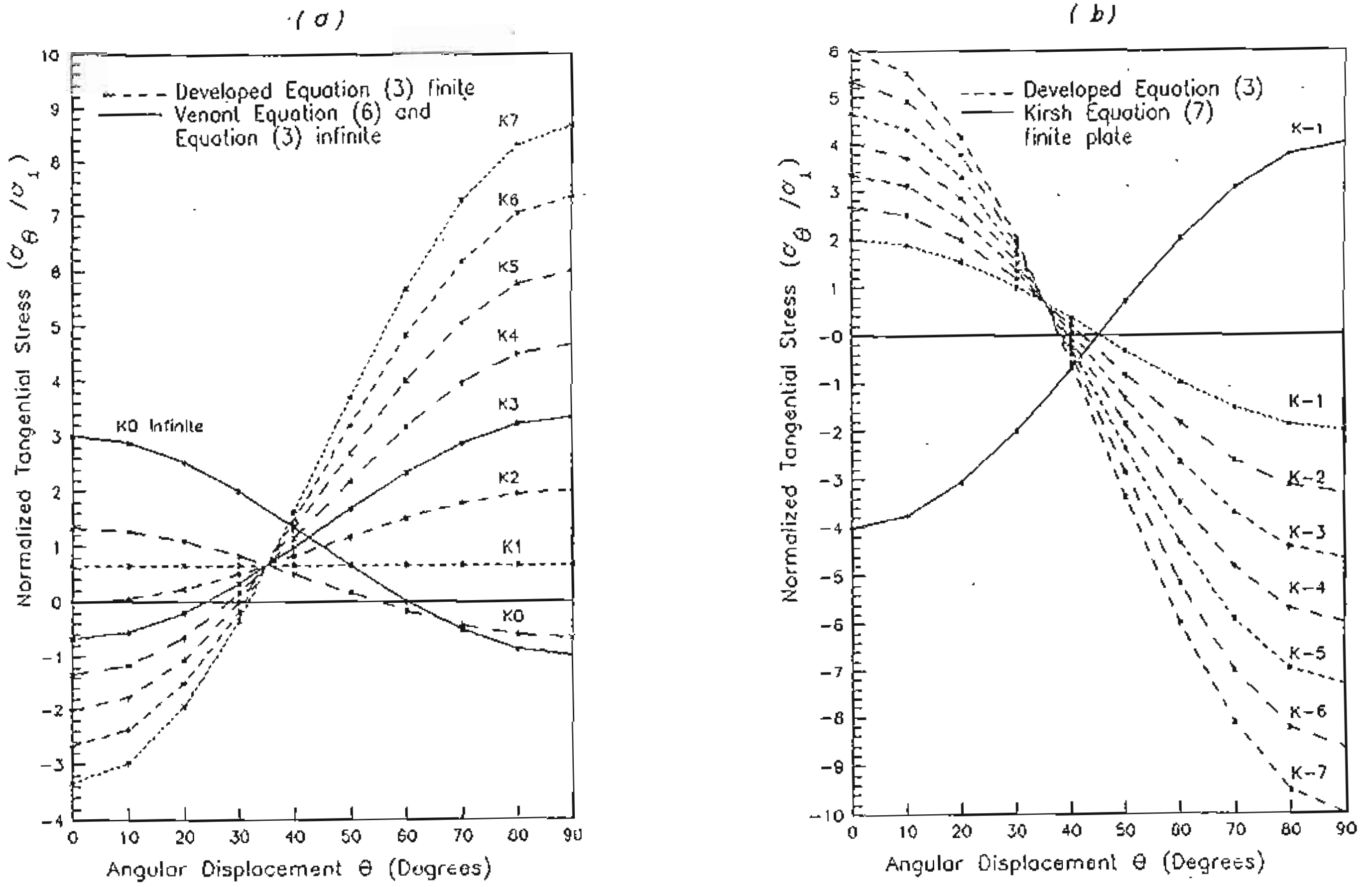

Fig.2. Variation of tangential stress around the boundary of circular holes in plate subjected to a varity of biaxial loading (a) for tension-tension, and (b) for tension-compression biaxial loading, using the developed equation (3).

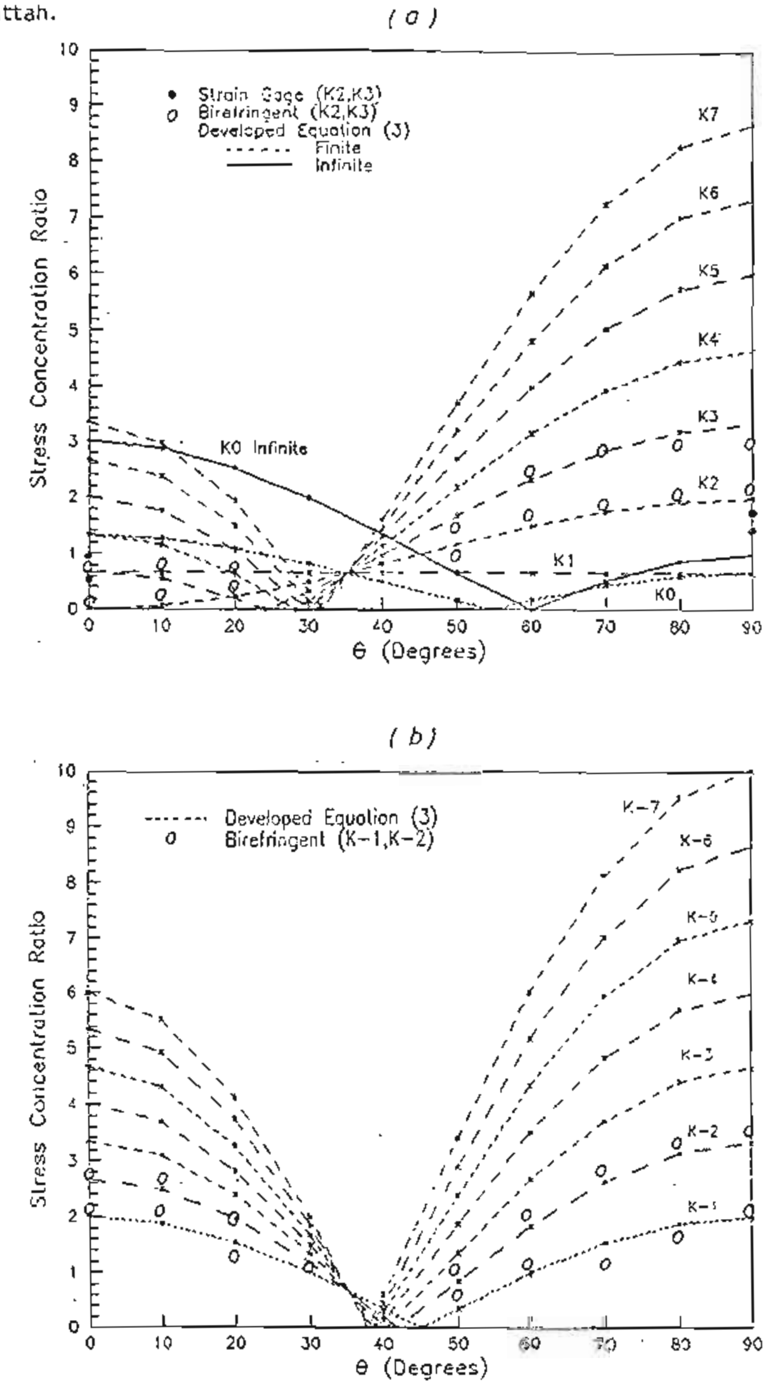

Fig.3. Comparison of the results analytically obtaind using the developed equation (3) with the experimental results, (a) for tension-tension (b) for tension-compression biaxial loading conditions.

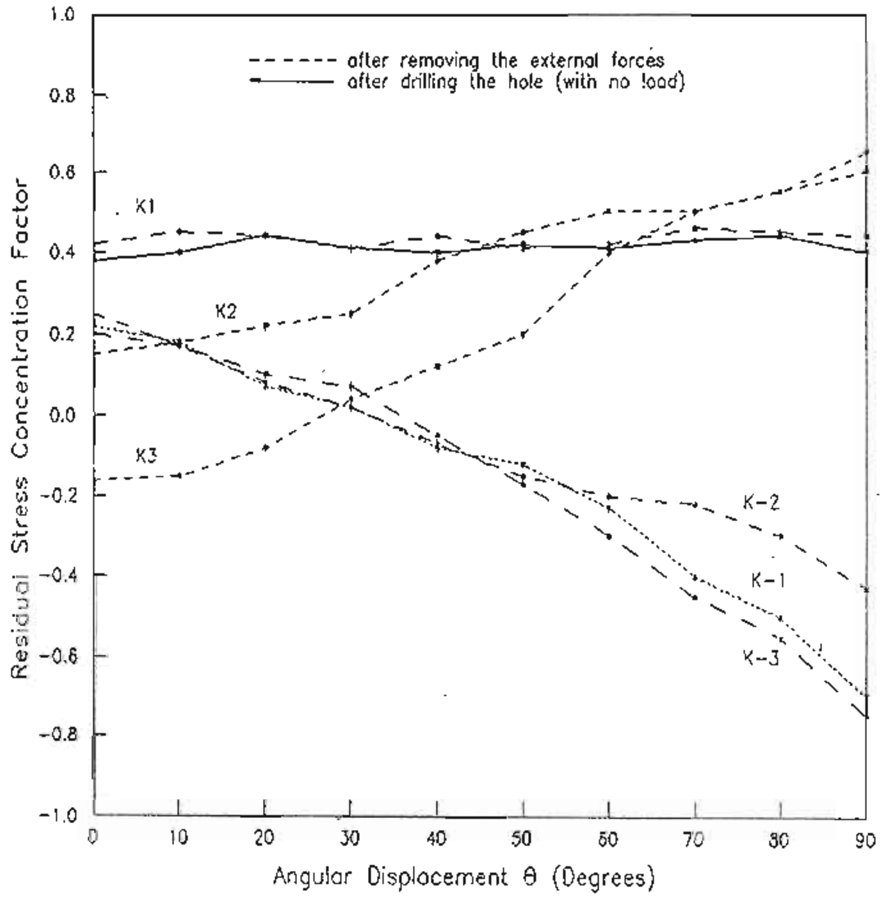

Fig.4. Distribution of the residual stress around the circmference of the holes remaining after removing the external forces for a varitry of biaxial loading conditions.