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LAMINAR FLOW BETWEEN COAXIAL ROTATING CYLINDERS

السريان الرقائقي بين أسطوانتيـــــن متحدتي المحور تدوران حــول محـــورها

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خلاصة :

يتناول هذا البحث دراسة السريان الرقائقي في الخلوص بين اسطوانتين متحدتي المحور تدوران حول محورها في اتجاه واحد او في اتجاهين مختلفين . هذا السريان يتواجد في العديد من التطبيقات الهندسية مثل نظم التبريد في الماكينات الدوارة كالموتورات ومولدات الكهرباء وغيرها .

يهدف هذا البحث الى حساب المفاقيد الهيدروليكية وكذلك توزيعات السرعة المحورية والسرعة المماسية . اخذ في الاعتبار ان الاسطوانتين لها طول كاف يسمح بحدوث السريان الرقائقي كامل الانتظام وان العوامل المؤثرة على السريان تعتمد على المسافة القطرية فقط ماعدا الضغط فانه يعتمد على الاتجاه المحوري ايضا .

اظهرت النتائج انه في حالة السريان الرقائقي يكون معامل الاحتكاك اكبر في حالة دوران الاسطح . كذلك وجد ان قيم معامل الاحتكاك تزداد بزيادة نسبة السرعة الزاوية (السرعة الزاوية للاسطوانه الاولى / السرعة الزاوية للاسطوانة الثانية) وتزداد ايضا قيم معامل الاحتكاك بزيادة نسبة السرعة (السرعة المماسة / السرعة المحورية) . وجد ايضا ان معامل الاحتكاك يتأثر بتغيير النسبة بين نصفي قطر الاسطوانتين . كذلك امكن الحصول على توزيعات السرعة المحورية والسرعة المماسية لقيم مختلفة من نسب الاقطار عند نسب سرعة مختلفة لنسب سرعات زاوية متعددة .

ABSTRACT

Theoretical investigation concerning the hydraulic resistance in an annulus of concentric rotating cylinders is described. The axial and tangential velocity distributions were introduced. The annulus is assumed to be long enough and a uniform purely developed laminar flow is considered. Flow parameters, except the pressure are assumed as functions of the radial distance only.

Rotation affects the friction coefficient and both axial and tangential velocity distributions. The governing factors are the speed ratio, radii ratio and angular velocity ratio.

INTRODUCTION

The study of fluid flow between two coaxial rotating cylinders is of great importance in many industrial applications, some of these are the cooling systems of rotating machines such as gas turbines, electric generators, motors, etc. Also the journal bearing and the flow between the stationary and rotating parts of axial flow turbo machines and the flow in the rising line of deep well pumps belong to this type of flow [1,5].

Most of the numerous investigations dealing with this problem are concerned with finding the flow pattern at the entrance region [3,7,10]. It was found that the velocity distribution depends to a great extent on the axial distance. If the annulus is sufficiently long, the tangential velocity profile further develops and finally reaches the fully developed state with a Couette velocity profile independent of the axial position. Kaye and Elgar [1] made an important contribution to the understanding of the regimes of flow in a concentric annulus with inner rotating cylinder. They showed that there are four distinct flow regimes; purely laminar, laminar flow with Taylor vortices, turbulent flow with vortices, and purely turbulent flow. The boundaries of the regimes were conveniently represented by a plot of axial Reynolds number versus Taylor number. Astill [3] showed, by means of a smoke visualization technique and hot wire measurements, that transition in a tangential developing flow starts near the rotating inner cylinder as oscillating waves within the tangential boundary layer. He also developed a unique stability criterion, which is a Taylor number based on the tangential boundary layer displacement thickness.

Abdul-Kader and Surceh [6] studied the fully developed turbulent flow in a concentric annulus with the inner cylinder rotating and a steady axial flow taking place under a constant axial pressure gradient. Axial and tangential velocity distributions were obtained. Coney and El-Shaarawi [7] studied numerically the developing laminar flow in the entrance region of concentric annuli with rotating inner cylinder.

Turbulent helical flow in concentric cylinders with fine clearance was studied experimentally by Gelhar and Monkmeyer [4]. It was found that the axial resistance expressed as a discharge coefficient is a function of Reynolds number ,based on axial and tangential velocities ,and also a function of the clearance ratio.

Turbulent flow in axially rotating pipes was studied by Mitsukiyo and Kouji [9]. They studied experimentally the changes in flow pattern, together with the hydraulic loss within the pipe. It was found that increase of pipe rotation continuously reduces the hydraulic loss and gradually changes the flow pattern from a turbulent type to a laminar one. Seiichi Washio et al [11] studied the wave phenomena in coaxial pipes. They found that there exists an equivalent single pipe which can simulate a double pipe for its properties of wave transmission

A finite difference scheme was developed [10] for solving the boundary layer equations governing the laminar free convection flow in open ended vertical annuli with rotating inner walls. It was found that heating the inner surface has always a stabilizing effect, while heating the outer surface has either destabilizing or stabilizing effects.

The present study deals with theoretical purely developed laminar flow in a long concentric annulus for the most general case, when the inner and outer cylinders are rotating in the same or different directions. Steady axial flow is assumed to exist under a constant axial pressure gradient. The objective of the study is to obtain analytical expressions for the pressure drop and velocity distributions across annulus with rotating walls.

FUNDAMENTAL ANALYSIS

For the steady flow of incompressible viscous fluid, in the absence of body forces, the basic equations of motion and continuity are:

z - momentum equation:

$$v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2}\right) \qquad \text{..................(1)}$$

r _ momentum equation:

$$v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \left(\frac{\partial^2 v_r}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{vr}{r}\right) + \frac{\partial^2 v_r}{\partial z^2}\right) \qquad \text{..................(2)}$$

θ - momentum equation:

$$v_r \frac{\partial v_\theta}{\partial r} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} = \nu\left(\frac{\partial^2 v_\theta}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{\partial^2 v_\theta}{\partial z^2}\right) \qquad \text{..................(3)}$$

continuity equation:

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0 \qquad \text{..................(4)}$$

where ; r, θ and z are the radial, tangential and axial coordinates respectively. v_r, v_θ and v_z are the velocity components in radial, tangential and axial directions respectively. p is the pressure, ν is the kinematic viscosity and ρ is the fluid density.

Assumptions are made that the annulus is long enough and having a fully developed uniform laminar flow, the flow parameters, except the pressure, are function of r only. The pressure is function of r and also of axial direction z. Based on these assumptions the radial velocity component $v_r = 0$ for all values of r. Equations (1), (2) and (3) can be reduced to the following simplified form:

$$\nu\left(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r}\right) = \frac{1}{\rho}\frac{\partial p}{\partial z} \qquad \text{.....................(5)}$$

$$\frac{v_\theta^2}{r} = \frac{1}{\rho}\,\frac{\partial p}{\partial r} \qquad \text{.....................(6)}$$

$$\frac{\partial^2 v_\theta}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) = 0 \qquad \text{.....................(7)}$$

With the annulus having inner and outer radii r_1 and r_2 respectively, and the inner cylinder is rotating with angular velocity w_1 while the outer one is also rotating with w_2, equations (5),(6) and (7) are subjected to the following boundary conditions;

$$\left.\begin{array}{ll} \text{at} \quad r = r_1 & \begin{bmatrix} v_z = 0 \\ v_\theta = w_1 r_1 \end{bmatrix} \\ \text{at} \quad r = r_2 & \begin{bmatrix} v_z = 0 \\ v_\theta = w_2 r_2 \end{bmatrix} \end{array}\right| \qquad \text{................(8)}$$

Solving equation (5) gives

$$v_z = \frac{1}{4\mu}\,\frac{\partial p}{\partial z}\,r^2 + C_1 \ln r + C_2 \qquad \text{.................(9)}$$

Using the boundary conditions (8), the constants of integration C_1 and C_2 can be easily determined and equation (9) is expressed in the following form:

$$v_z = \frac{1}{4\mu}\,\frac{\partial p}{\partial z}\,r^2 + \frac{1}{4\mu}\,\frac{\partial p}{\partial z}\,\frac{(r_2^2 - r_1^2)}{\ln(r_1/r_2)}\ln r - \frac{1}{8\mu}\,\frac{\partial p}{\partial z}\,(r_2^2 + r_1^2) -$$

$$- \frac{1}{8\mu}\,\frac{\partial p}{\partial z}\,\frac{(r_2^2 - r_1^2)}{\ln(r_1/r_2)}\ln r_1 r_2 \qquad \text{...............(10)}$$

After some arrangements equation (10) yields to

$$v_z = \frac{1}{4\mu}\,\frac{\partial p}{\partial z}\left[r^2 + \frac{r_2^2 \ln(r/r_1) - r_1^2 \ln(r/r_2)}{\ln(r_1/r_2)}\right] \qquad \text{......(11)}$$

Equation (11) exactly expresses the axial velocity profile of a steady laminar flow in an annulus.

Solving equation (7) gives

$$v_\theta = C_3 r + C_4/r \qquad \text{.......................... (12)}$$

Applying the boundary conditions (8), the constants of integration C_3 and C_4 can be obtained and the tangential component of velocity v_θ is written as follows;

$$v_\theta = \frac{v_2 r_2^2 - v_1 r_1^2}{(r_2^2 - r_1^2)}\, r + \frac{1}{r}\,\frac{r_1^2 r_2^2 (v_1 - v_2)}{(r_2^2 - r_1^2)} \qquad \text{...............(13)}$$

The hydraulic losses can be deduced using the dissipation function, which is defined as the dissipated energy in a unit volume per unit time. The dissipated function is written in cylindrical coordinates in the following form [8]:

$$d\phi = \mu \Big[\, 2\left(\frac{\partial v_r}{\partial r}\right)^2 + 2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2 + 2\left(\frac{\partial v_z}{\partial z}\right)^2 + \left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right)^2 + \left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)^2 + \left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)^2 \Big]\, 2\pi\, r\, dr\, dz \qquad \text{.......(14)}$$

where $d\phi$ is the dissipated energy in an annulus of radius r and width dr having a length of dz.

Based on the above mentioned assumptions, the radial component of velocity $v_r = 0$, equation (14) can be written as follows;

$$d\phi = \mu \left[\left(\frac{\partial v_z}{\partial r}\right)^2 + \left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right)^2\right] 2\pi\, r\, dr\, dz \qquad \text{..........(15)}$$

Substitute the values of v_z and v_θ from equations (11) and (13), into equation (15), then

$$d\phi = \mu \left(\left[\left(\frac{1}{4\mu}\frac{\partial p}{\partial z}\right)\left(2r + \frac{C_1}{r} - \frac{C_2}{r}\right)\right]^2 + \left[2\,\frac{C_4}{r^2}\right]^2 \right) 2\pi\, r\, dr\, dz$$

$$d\phi = \mu \left[\frac{1}{16\mu^2}\left(\frac{\partial p}{\partial z}\right)\left(4r^2 + 4C_5 + \frac{C_5^2}{r^2}\right) + 4\,\frac{C_4^2}{r^4}\right] 2\pi\, r\, dr\, dz \qquad \text{....(16)}$$

where $C_5 = C_1 - C_2$

To get the dissipated energy per unit time in the whole volume of fluid flowing in an annular space between two coaxial rotating cylinders of radii r_1 and r_2 in a length l, the above equation must be integrated as follows;

$$\phi = \int_0^l \int_{r_1}^{r_2} d\phi = \int_0^l \int_{r_1}^{r_2} \left[\frac{2\pi}{16\mu}\left(\frac{\partial p}{\partial z}\right)^2 \left(4r^3 + 4C_5 r + \frac{C_5^2}{r}\right) + 8\pi\mu\,\frac{C_4^2}{r^3}\right] dr\, dz$$

$$\phi = \left[\frac{\pi l}{8\mu} \left(\frac{\partial p}{\partial z}\right)^2 (r^4 + 2C_5 r^2 + C_5^2 \ln r) - 4\pi \mu l \frac{C_4^2}{r^2} \right]_{r_1}^{r_2} \quad ....(17)$$

introducing the values of C_4 and C_5, then

$$\phi = \frac{\pi l}{8\mu} \left(\frac{\partial p}{\partial z}\right)^2 \left[(r_2^4 - r_1^4) + \frac{(r_2^2 - r_1^2)^2}{\ln m} + 4\pi \mu l \frac{r_1^2 r_2^2}{r_1^2 - r_2^2} (w_1 - w_2)^2 \right]$$

where $m = r_1/r_2$ = radii ratio

The dissipated energy in such case is due to energy loss caused by both longitudinal and rotational motion of the fluid. This energy loss can be expressed in the following form;

$$\phi = Q.dp + T.w \quad ....(18)$$

where Q is the volume flow rate through the annulus
dp is the pressure loss caused by friction due to axial motion of the fluid in a system of length l.
T is the torque required for rotating the fluid contained in the system with a relative angular velocity w.

The first part of the right hand side of equation (18) represents the energy loss due to axial motion, while the second part represents that due to rotation and it can be expressed as follows;

$$T\,w = \sum_{n=1,2} \int_0^l \mu \left(\frac{\partial v_\theta}{\partial r}\right) 2\pi\, r_n\, dz\, w_n\, r_n \quad ....(19)$$

where n refers to the cylinder; the inner one is represented by $n = 1$ and the outer $n = 2$

Substituting v_θ from equation (13) into eq. (19) gives:

$$T.w = 2\,\pi\,\mu\,l\,\left[\left(C_3 - \frac{C_4}{r_2^2}\right) w_2 r_2^2 + \left(C_3 - \frac{C_4}{r_1^2}\right) w_1 r_1^2\right]$$

Introducing the values of C_3 and C_4 in the above equation, and rearranging, it becomes

$$T.w = \frac{(r_2^2 + r_1^2)(w_2 r_2^2 - w_1 r_1^2)}{(r_2^2 - r_1^2)} 2\pi\mu l \quad ....(20)$$

Substituting in equation (18) with the value of $T.w$ shown in (20) and putting $Q.dp = V\pi(r_2^2 - r_1^2)\,\rho\, g\, h_l$, where V is the mean axial velocity in the annulus and h_l is the head loss, then

$$\frac{\pi l}{8\mu}\left(\rho g\frac{h_1}{l}\right)^2\left[(r_2^4 - r_1^4) + \frac{(r_2^2 - r_1^2)^2}{\ln m}\right] + 4\pi\mu l\left[\frac{r_1^2 r_2^2}{(r_1^2 - r_2^2)}(w_1 - w_2)^2\right]$$

$$= V.\pi(r_2^2 - r_1^2)\,\rho.g.hl. + \left[\frac{(r_2^2+r_1^2)(w_2^2 r_2^2 - w_1^2 r_1^2)}{r_2^2 - r_1^2}\right] 2\pi\mu l \qquad \ldots\ldots\ldots\ldots(21)$$

Substitute in the above equation with the value of the hydraulic radius $R_h = 2(r_2 - r_1)$, and Reynolds number $R_N = (\rho\ V\ R_h)/\mu$, then equation (21) gives

$$\frac{R_N}{64}\frac{h_1}{l}\frac{2g}{V^2}R_h\left[\frac{(1 + m^2) + (1 - m^2)/\ln m}{(1 - m)^2}\right] - \frac{8}{R_N}\frac{l}{h_1}\frac{1}{R_h g}\frac{1}{(1 + m)^2}.$$

$$.\left[2(w_1 - w_2)^2 r_1^2 + (r_2^2 + r_1^2)(w_2^2 - m^2 w_1^2)\right] - 1 = 0 \qquad \ldots\ldots\ldots(22)$$

Solving equation (22) for the value h_1, we get

$$h_1 = \frac{16}{R_N}\frac{l}{R_h}\frac{V}{g}\cdot\frac{V + \sqrt{V^2 - \left[\frac{(1+m^2) + (1 - m^2)/\ln m}{(1 - m)^2}\right]\left[\frac{2(w_1-w_2)^2 r_1^2 + (r_1^2+r_2^2)(w_2^2-m^2 w_1^2)}{(1 + m)^2}\right]}}{\frac{(1+m^2) + (1-m^2)/\ln m}{(1-m)^2}}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots(23)$$

Equating the last equation (23) with the known Darcy's equation

$$h_1 = \frac{\lambda l}{d}\frac{V^2}{2g}$$

then the value of the friction factor can be obtained in the following form:

$$\lambda = \frac{32}{R_N}\cdot\frac{1 + \sqrt{1 + \frac{1}{V^2}\left[\frac{(1+m^2)+(1-m^2)/\ln m}{(1-m)^2}\right]\left[\frac{2(w_1-w_2)^2 r_1^2+(r_2^2+r_1^2)(w_2^2-m^2 w_1^2)}{(1+m)^2}\right]}}{\frac{(1+m^2) + (1-m^2)/\ln m}{(1-m)^2}}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots(24)$$

Introducing the values $\alpha = (w_1/w_2)$ = angular velocity ratio and $\beta_2 = (w_2 r_2/V)$ = speed ratio, then equation (24) becomes;

$$\lambda = \frac{32}{R_N} \; \frac{1 + \sqrt{1 + \left[\dfrac{(1+m^2)+(1-m^2)/\ln m}{(1-m)^2}\right]\left[\dfrac{2(\alpha-1)^2 m^2 + (1+m^2)(1-\alpha^2 m^2)}{(1+m)^2}\right]\beta_2^2}}{\dfrac{(1+m^2) + (1-m^2)/\ln m}{(1-m^2)}} \qquad \text{..............(25)}$$

The friction coefficient λ for a fully developed laminar flow in an annulus with both cylinders rotating can be calculated using equation (25). The following special conditions can be deduced from equation (25):

i) <u>laminar flow in a stationary tube</u>

Substituting the value of $m = 0$ and $w_1 = w_2 = 0$, it yields to:

$$\lambda = \frac{64}{R_N} \qquad \text{.........(26)}$$

which is the known equation for the friction coefficient in laminar flow in pipes

ii) <u>laminar flow in a stationary annulus</u>

Substituting $w_1 = w_2 = 0$ gives:

$$\lambda = \frac{64}{R_N} \; \frac{(1-m)^2}{(1+m^2) + (1-m^2)/\ln m} \qquad \text{..............(27)}$$

This equation is the same as that obtained by (2).

iii) <u>laminar flow in an annulus with inner rotating cylinder</u> ($w_2 = 0$)

$$\lambda = \frac{32}{R_N} \; \frac{1 + \sqrt{1 + \left[\dfrac{(1+m^2) + (1-m^2)/\ln m}{(1-m)^2}\right]\left[\dfrac{\beta_1^2 (1-m^2)}{(1+m)^2}\right]}}{\dfrac{(1+m^2) + (1-m^2)/\ln m}{(1-m)^2}} \qquad \text{..(28)}$$

where β_1 is the speed ratio $= \dfrac{w_1 r_1}{V}$

iv) laminar flow in an annulus with outer rotating cylinder ($w_1 = 0$)

$$\lambda = \frac{32}{R_N} \frac{1 + \sqrt{1 + \left[\frac{(1+m^2) + (1-m^2)/\ln m}{(1-m)^2}\right]\left[\frac{\beta_2^2(3m^2+1)}{(1+m)^2}\right]}}{\frac{(1+m^2) + (1-m^2)/\ln m}{(1-m)^2}} \quad .....(29)$$

where β_2 is the speed ratio= $\frac{w_2 r_2}{V}$

v) laminar flow in an annulus with inner and outer rotating cylinders at the same angular velocity in the same direction, ($w_1 = w_2 = w$)

$$\lambda = \frac{32}{R_N} \frac{1 + \sqrt{1 + \left[\frac{(1+m^2) + (1-m^2)/\ln m}{(1-m)^2}\right]\left[\frac{\beta^2(1-m^4)}{(1+m)^2}\right]}}{\frac{(1+m^2) + (1-m^2)/\ln m}{(1-m)^2}} \quad ....(30)$$

where β is the speed ratio = $\frac{w r_2}{V}$

vi) laminar flow in an annulus with inner and outer cylinders rotating at the same angular velocity but in opposite directions; ($w_1 = -w_2 = |w|$)

$$\lambda = \frac{32}{R_N} \frac{1 + \sqrt{1 + \left[\frac{(1+m^2) + (1-m^2)/\ln m}{(1-m)^2}\right]\left[\frac{\beta^2(1+4m^2-m^4)}{(1+m)^2}\right]}}{\frac{(1+m^2) + (1-m^2)/\ln m}{(1-m)^2}} \quad ..(31)$$

Axial velocity distribution

Introducing the value of $\frac{\partial p}{\partial z} = \frac{\rho g h_1}{l} = \frac{\rho g}{R_h} \frac{V^2}{2g}$ into equation

(11) and rearranging, it yields to;

$$V_z = \frac{1}{4\mu}\,\frac{\rho g}{R_h}\,\frac{V^2}{2gI}\left[r^2 + \frac{r_2^2 \ln(r/r_1) - r_1^2 \ln(r/r_2)}{\ln (r_1/r_2)} \right]$$

$$\frac{V_z}{V} = \frac{C}{32\,(1 - m)^2}\left[\left(\frac{r}{r_2}\right)^2 + \frac{\ln (r/r_1) - \ln (r/r_2)}{\ln (m)} \right] \quad .....(32)$$

where $C = \lambda R_N$

RESULTS AND DISCUSSIONS

A computer program was constructed to solve the above obtained expressions for the coefficient of friction and velocity distributions across an annulus of rotating cylinders. Below is a sample of the obtained results.

(1) Coefficient of Friction with Rotating Walls

Figure (1) shows the relation between the friction coefficient ratio (λ/f) and the radii ratio ($m = r_1/r_2$) at different speed ratios ($\beta_2 = w_2 r_2/V$). These curves are obtained for different angular velocity ratios ($\alpha = w_1/w_2$) ranging from - 1.6 to 1.0. From these curves it is clear that for negative values of angular velocity ratio, i.e. for higher angular velocity ratios, the coefficient of friction is higher than that at positive angular velocity ratios. It is also clear that increasing radii ratio increases the friction coefficient ratio for negative values of angular velocity ratios, i.e. when the two cylinders are rotating in opposite directions. For positive angular velocity ratios the friction coefficient decreases with increasing radii ratio.

Values of friction coefficient ratio against the speed ratio are shown in Fig. (2) for radii ratios of 0.5, 0.7 and 0.9. From these curves it is evident that increasing the speed ratio increases the friction coefficient ratio.

The effect of rotation of inner cylinder only , i.e. when $w_2 = 0$ is shown in Fig. (3). It can be noticed that the friction coefficient ratio is always greater than unity, which agrees with the results of reference [7] for inner cylinder rotation. Also increasing the speed ratio increases the friction factor, and increasing radii ratio decreases the friction coefficient ratio. This can be explained as ncreasing radii ratio decreases the tangential velocity gradient which in turn decreases losses. Figure (4) illustrates the effect of outer cylinder rotation only, i.e. when $w_1 = 0$. In this case also the friction coefficient ratio increases with increasing the speed ratio, but increasing radii ratio has an opposite effect on the friction coefficient ratio. Increasing the radii ratio increases the rotating area which tends to increase losses, therefore in this case ($w_1 = 0$) the friction coefficient ratio increases with increasing radii ratio. The effect of speed ratio on the friction coefficient ratio for different radii ratios when only the inner or the outer cylinder is rotating is shown in Figs.(5) and (6). It is clear that for both cases the effect of increasing the speed ratio is always to increase the friction coefficient ratio.

Axial and Tangential Velocity Distributions

Figures (7) to (10) show the axial and tangential velocity distributions when either the inner cylinder or the outer cylinder is rotating and when both cylinders are rotating in the same or in opposite directions. The results in these Figures are provided for a wide range of the radii ratio ($m = r_1/r_2$) and for different values of speed ratio ($\beta = wr/V$) and at different values of angular velocity ratio ($\alpha = w_1/w_2$).

The axial velocity distributions indicate a trend in the shift of the point of the maximum velocity towards the axis of the annulus as the radii ratio becomes larger as shown in Figs.(7) and (8). Also it can be noticed that increasing radii ratio has a small effect on the axial velocity distributions. The same results can be noticed in Figs.(9) and (10) when only the inner or the outer cylinders is rotating. However for inner cylinder rotation a noticeable change in the axial velocity distribution near the rotating wall can be observed, while no significant change can be noticed near the stationary side as shown in Fig.(9). When the outer cylinder is rotating only a noticeable change near both walls is indicated as shown in Fig.(10).

The tangential velocity distributions for all the above cases are shown in Figs. (7) to (10). It can be noticed in all cases that there is a zone in the middle third of the gap, in which the tangential velocity variation is almost linear and outside which the variation is more rapid. These results considerably agree enough with that obtained in reference [6] for turbulent flow in an annulus with inner wall rotation.

CONCLUSIONS

Laminar flow in the annulus of concentric rotating cylinders was investigated theoretically. The friction coefficient, axial and tangential velocity distributions were introduced. The results are summarized as follows:

(1) In general for laminar flow in an annulus of rotating walls, the coefficient of friction is higher than that for stationary walls.

(2) Increasing both speed ratio (wr/V) and relative angular velocity ratio (w_1/w_2) increase the friction coefficient ratio.

(3) The friction coefficient decreases with increasing radii ratio when the inner cylinder is rotating only, while for outer cylinder rotation only increasing radii ratio has an opposite effect.

(4) Axial and tangential velocity distributions were introduced for a wide range of radii ratio and different speed ratios at different angular velocity ratios.

(5) The maximum value of the axial velocity tends to shift towards the axis of the annulus with increasing radii ratio.

REFERENCES

(1) Kaye,J., and Elgar,E.C.," Modes of Adiabatic and Diabatic Flow Between Concentric Rotating Cylinders," Transactions ASME, Vol. 80, 1958, PP. 753-765.

(2) Knudsen,G.L., and Katz,D.L.,"Fluid Dynamics and Heat Transfer,"

McGraw Hill Book Comp. New York, 1958.

(3) Astili,K.N.," Studies of the Developing Flow Between Concentric Cylinders with the Inner Cylinder Rotating," ASME Journal of Heat Transfer Vol. 86, 1964, PP. 383-391.

(4) Gelhar,L.N., and Monkmeyer,P.L.," Turbulent Helical Flow Through Annulus," ASCE Journal of the Engineering Mechanics Division, Vol .94, No. EM1, Feb.,1968, PP. 295-316.

(5) Miyazaki,H.," Combined Free and Forced Convective Heat Transfer and Fluid Flow in a Rotating Curved Circular Tube," Int. Journal of Heat and Mass Transfer, Vol. 14, 1971, PP 1295-1309.

(6) Abdul-Khader,M.H., and Suresh Roa,H.," Turbulent Helical Flow Through Annulus," ASCE Journal of the Engineering Mechanics Division, Vol. 98, No. EM2, April 1972, PP. 447-456.

(7) Coney,J.E.R. and El-Shaarawi,M.A.I.," A Contribution to the Numerical Solution Of Developing Laminar Flow in the Entrance Region of Concentric Annuli with Rotating Inner Walls," ASME Journal of Fluid Engineering, Vol. 96, No. 4, Dec. 1974, PP. 333-340.

(8) Loietceiancky,L.G.,"Mechanics of Fluid and Gases" Moscow, Naoka, 1978. (In Russian)

(9) Mitsukiyo Murakami and Kouji Kikuyama," Turbulent Flow in Axial Rotating Pipes"ASME Journal of Fluids Engineering, Mar.1980, Vol. 102, PP. 97-102.

(10) El-Shaarawi,M.A.I., and Sarhan,A.S.," Developing Laminar free Convection in an Open Ended Vertical Annulus with a Rotating Inner Cylinder" ASME Journal of Fluids Engineering, Vol. 103, Aug. 1981, PP. 552-558.

(11) Seiichi Washio and Tadataka Konishi," Research on Wave Phenomena in Hydraulic Lines" Bulletin of JSME, Vol. 28, No. 241, July 1985.

NOMENCLATURE

$C_1, C_2,$ C_3, C_4	} Constants of integrations.
C_5	Constant = $C_1 - C_2$
f	Coefficient of friction for laminar flow in stationary conduit.
h_l	Head loss.
l	Pipe length.
m	Radii Ratio = r_1/r_2
P	Pressure.
Q	Discharge.
r	Radial distance.
r_1, r_2	Inner and outer cylinders radii respectively.
R_h	Hydraulic radius.
R_N	Reynolds Number.
T	Torque.
V	Mean axial velocity.

V	Mean axial velocity.
V_r	Radial velocity.
V_z	Axial velocity.
V_θ	Tangential velocity.
z	Axial distance.
θ	Tangential coordinate.
ν	Kinematic viscosity.
μ	Dynamic viscosity.
ρ	Density.
Φ	Dissipation function.
α	Angular seed ratio (w_1/w_2).
w	Angular speed.
w_1, w_2	Angular speed of inner and outer cylinders respectively.
β	Speed ratio (wr/V).
β_1	Speed ratio based on the inner cylinder = (w_1r_1/V).
β_2	Speed ratio based on the outer cylinder = (w_2r_2/V).
λ	Coefficient of friction for laminar flow in an annulus of rotating walls.
λ/f	Coefficient of friction ratio.

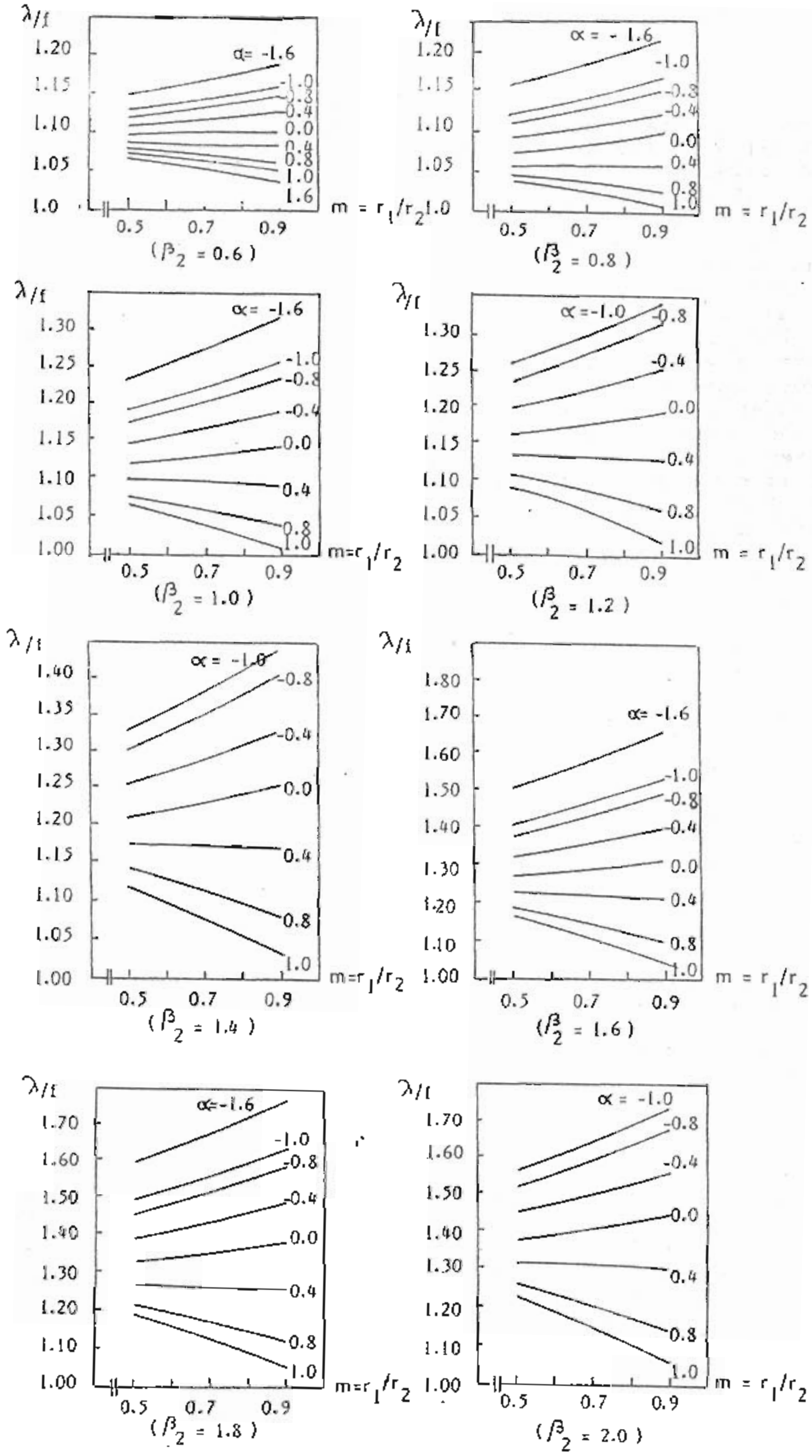
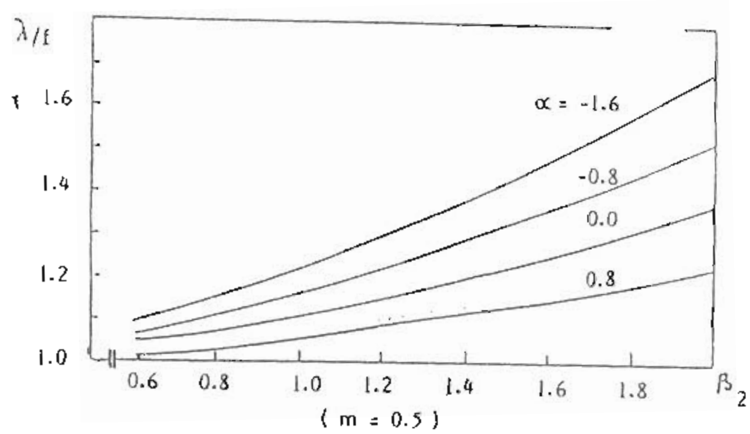
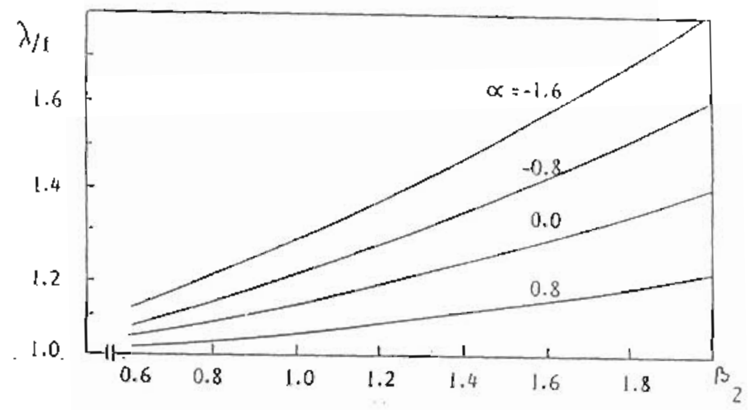


Fig.(1) Relation Between The Friction Coefficient Ratio (λ/f) and Radii Ratio ($m = r_1/r_2$) at Different Angular Velocity ratios ($v1/v2$).
Speed Ratio $\beta_2 = v_2 r_2/V$ = 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0.



(m = 0.5)



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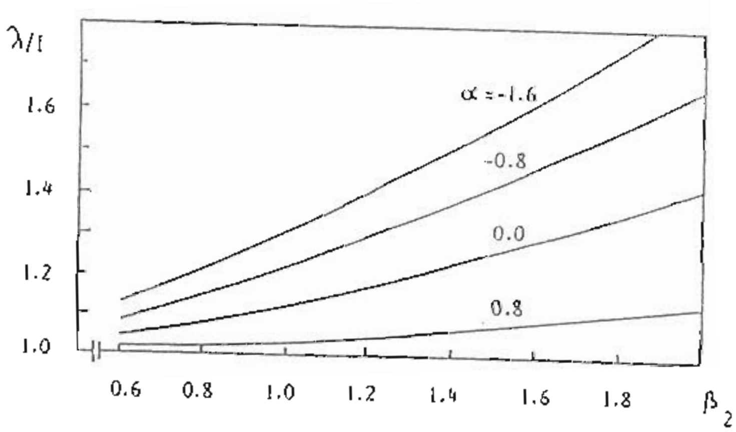


(m = 0.9)

Fig.(2) Relation Between The Friction Coefficient Ratio (λ/f) and Speed Ratio ($\beta_2 = w_2r_2/V$) at Different Angular Velocity Ratio ($\alpha = w_1/w_2$ = 0.8, 0.0, -0.8, -1.6) for Radii Ratio (m = 0.5, 0.7, 0.9)

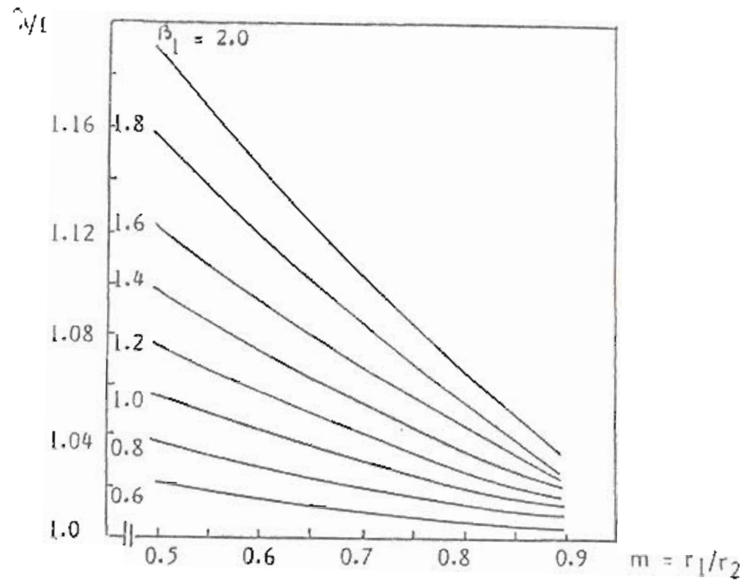


Fig.(3) Relation Between The Friction Coefficient Ratio (λ/f) and Radii Ratio ($m = r_1/r_2$) with Inner Cylinder Rotating Only ($w_2 = 0$)

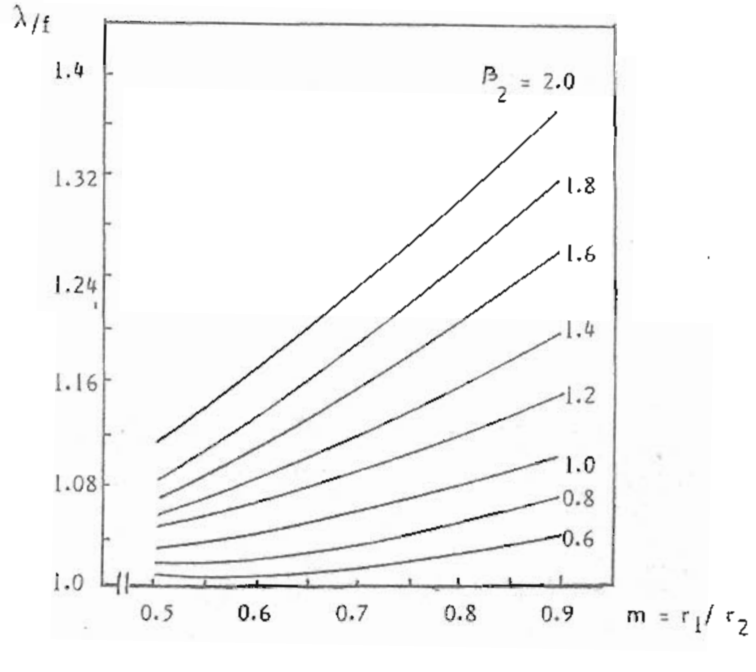


Fig.(4) Relation Between The Friction Coefficient Ratio (λ/f) and Radii Ratio ($m = r_1/r_2$) with Outer Cylinder Rotating Only ($w_1 = 0$)

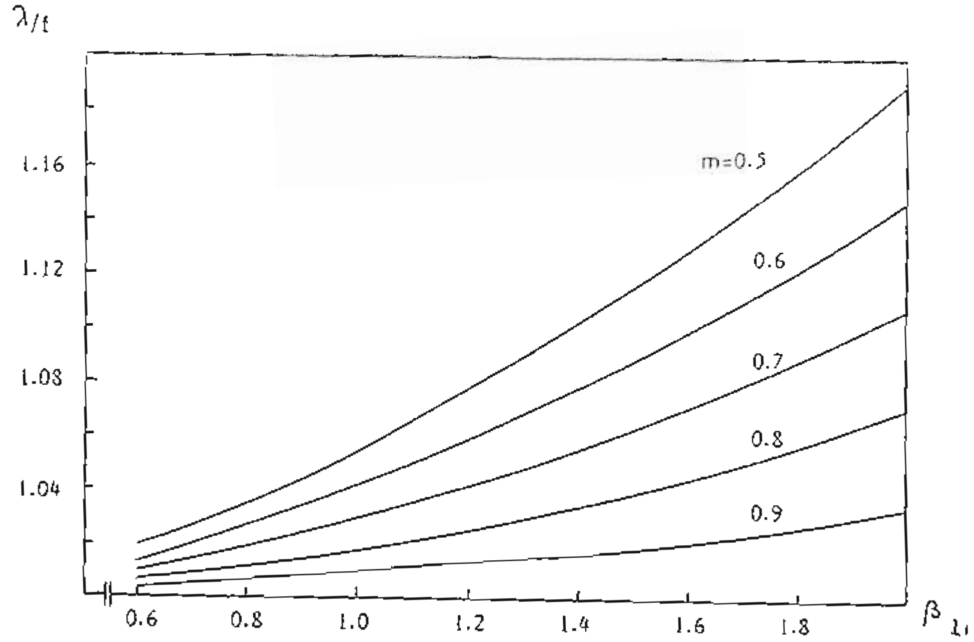


Fig.(5) Relation Between The Friction Coefficient Ratio (λ/f) and Speed Ratio ($\beta_1 = w_1 r_1/V$) at Different Radii Ratios ($m = r_1/r_2$) with Inner Cylinder Rotating Only ($w_2 = 0$)

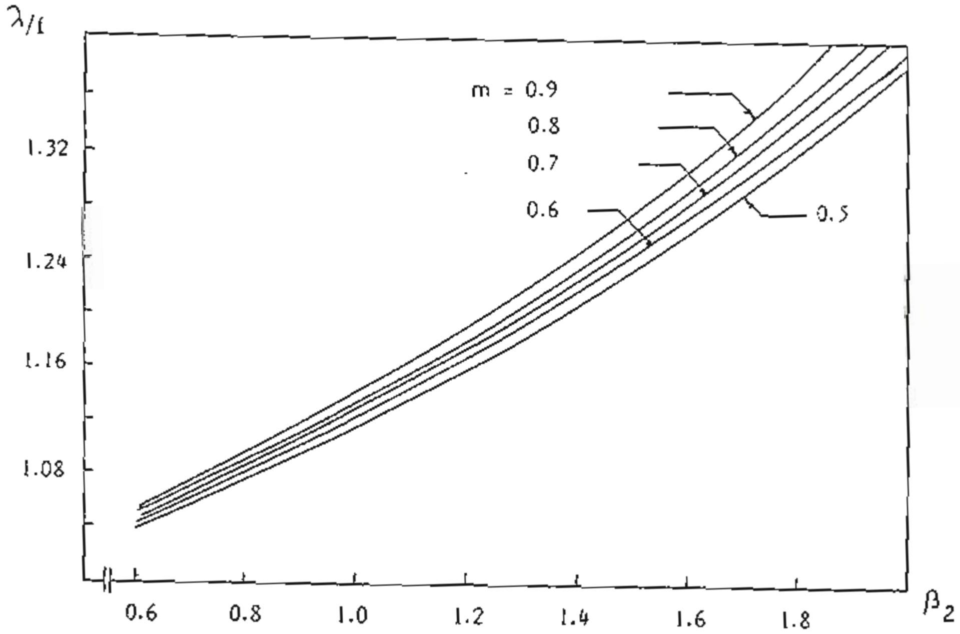
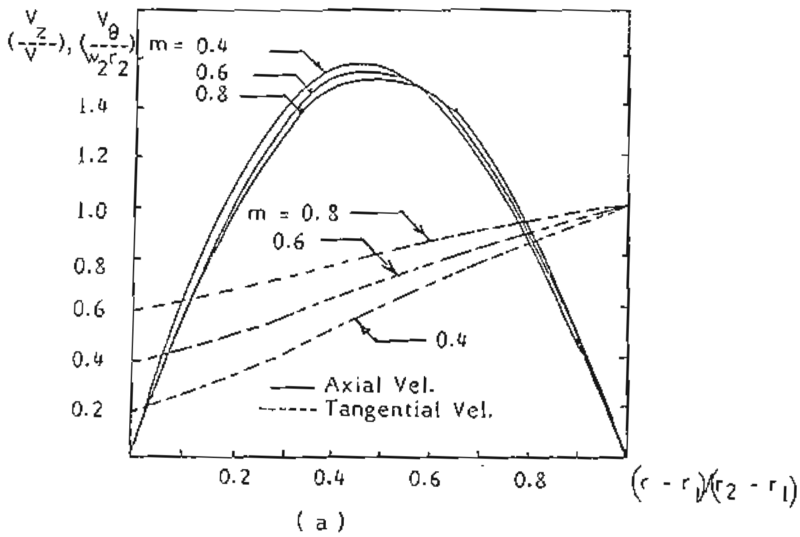


Fig.(6) Relation Between The Friction Coefficient Ratio (λ/f) and Speed Ratio ($\beta_2 = w_2 r_2/V$) at Different Radii Ratios ($m = r_1/r_2$) with Outer Cylinder Rotating Only ($w_1 = 0$)



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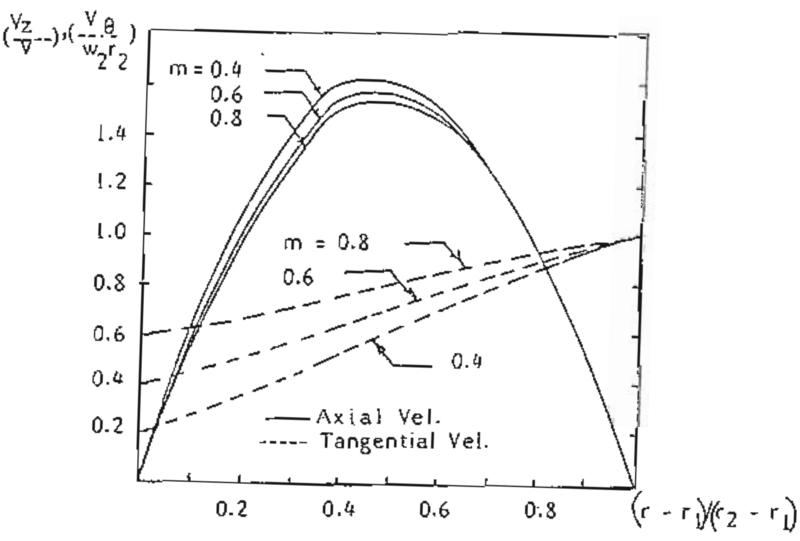


(b)

Fig.(7) Axial and Tangential Velocity Distributions Across an Annulus. Angular Velocity Ratio $\alpha = w_1/w_2 = 1.0$
(a) Speed Ratio $\beta_2 = w_2 r_2/V = 0.5$
(b) Speed Ratio $\beta_2 = w_2 r_2/V = 1.0$

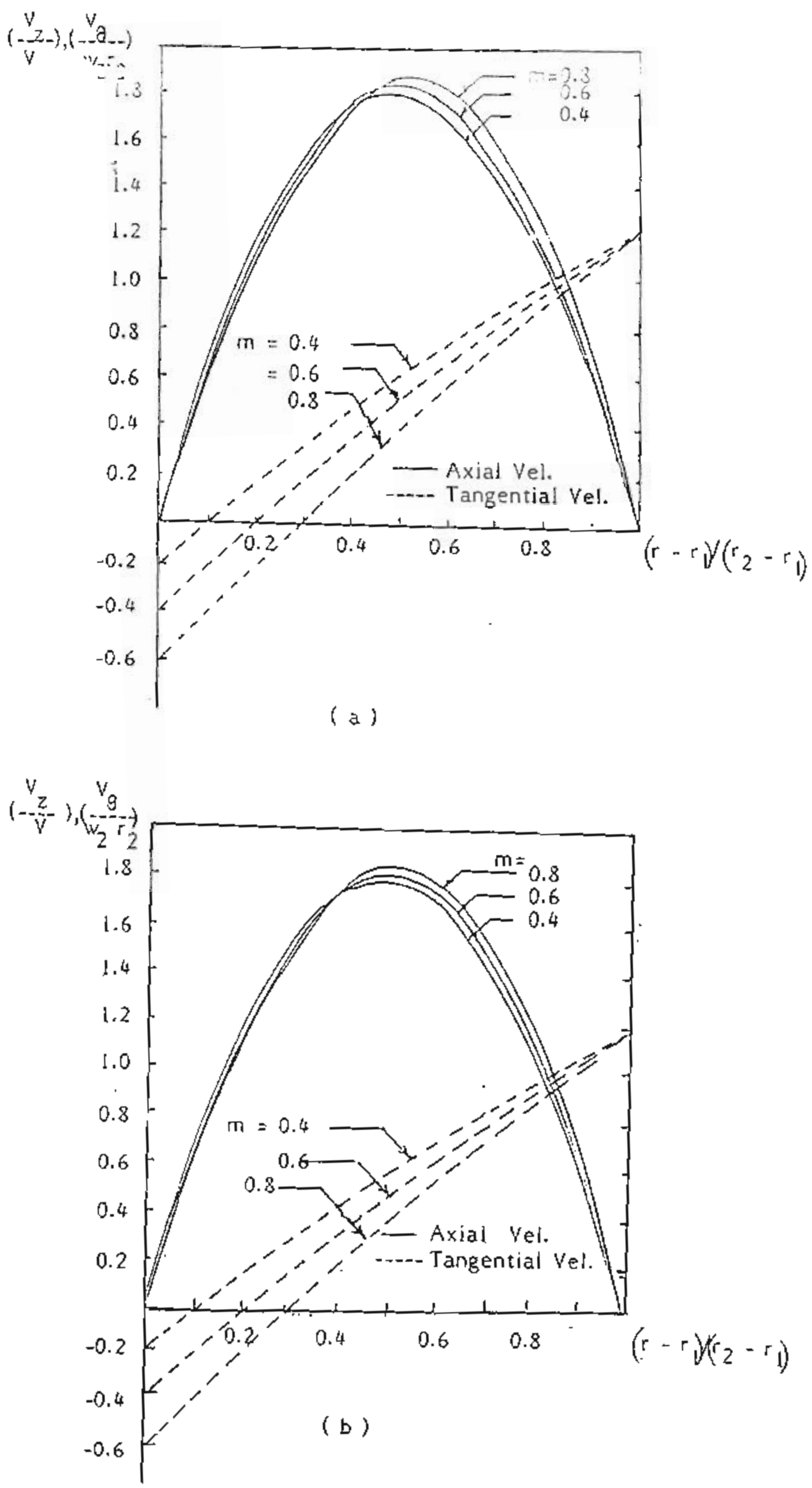


Fig.(8) Axial and Tangential Velocity Distributions Across an Annulus. Angular Velocity Ratio $\alpha = w_1/w_2 = -1.0$
(a) Speed Ratio $\beta_2 = w_2 r_2/V = 0.5$
(b) Speed Ratio $\beta_2 = w_2 r_2/V = 1.0$

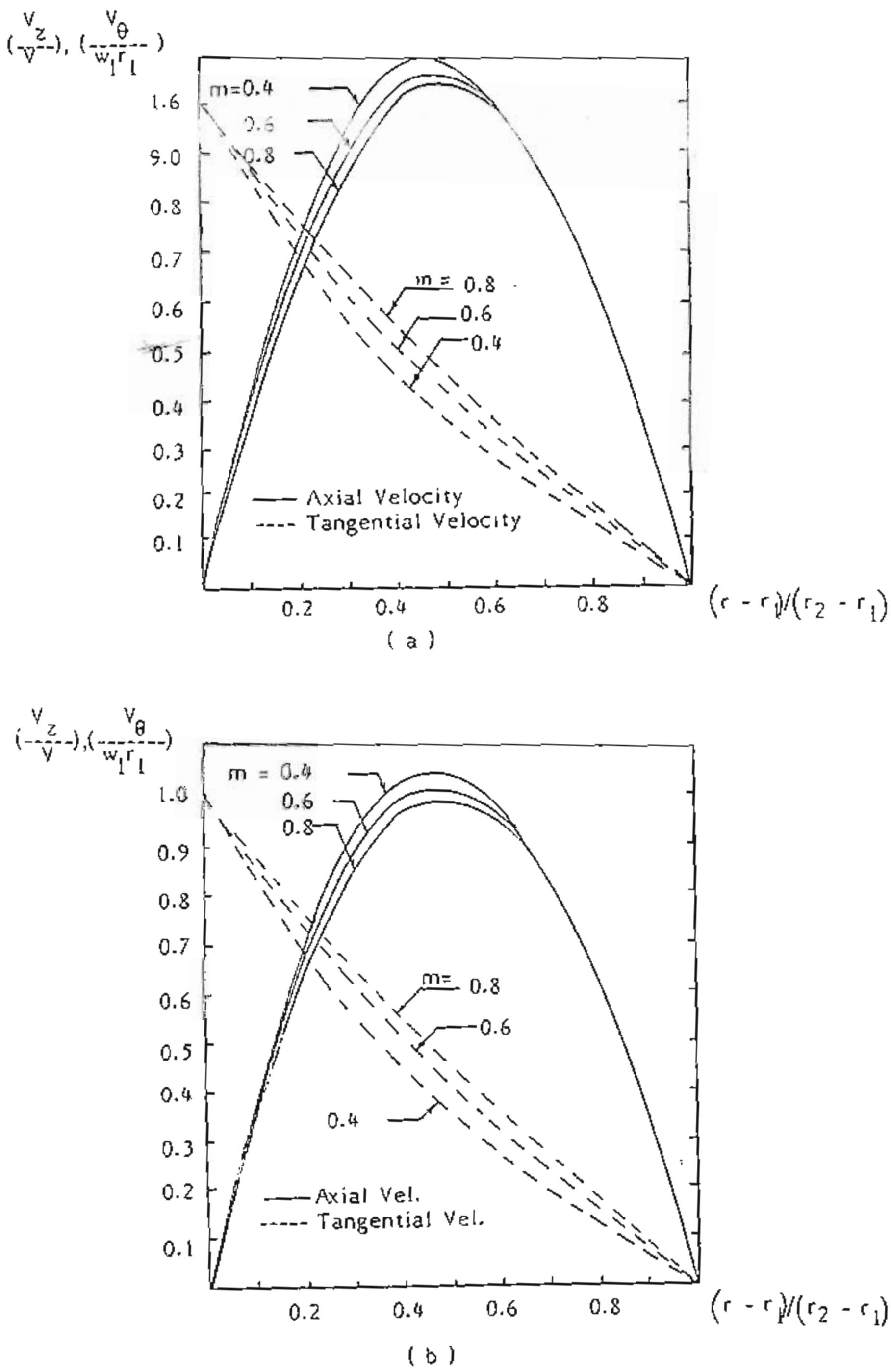
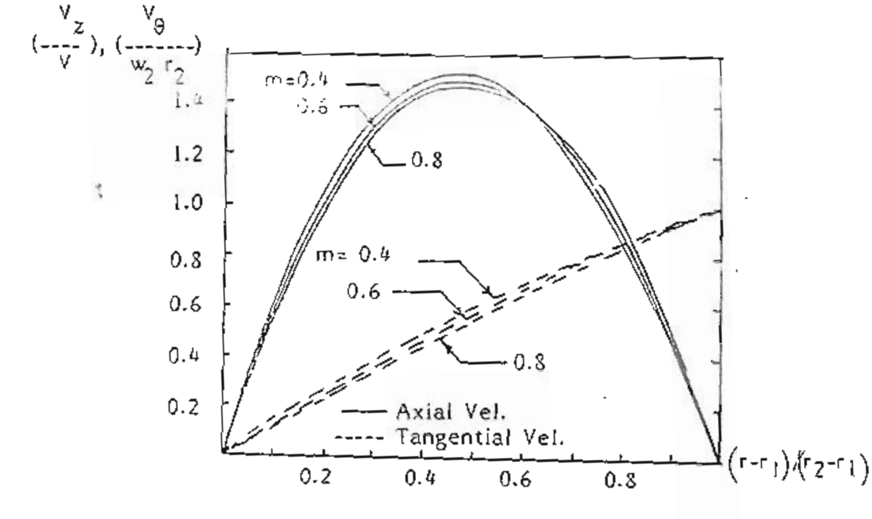
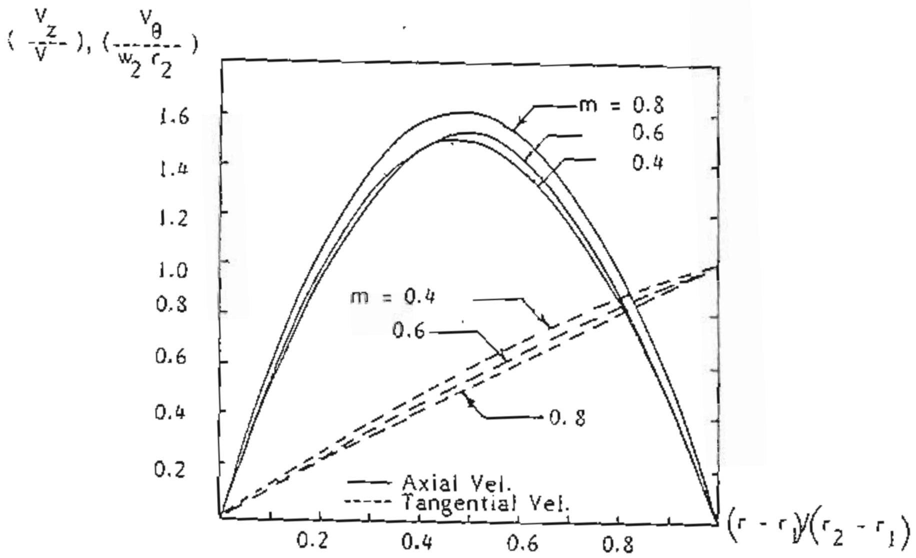


Fig.(9) Axial and Tangential Velocity Distributions Across an Annulus with Inner Cylinder Rotating Only ($w_2 = 0$) for Different Radii Ratios.
(a) Speed Ratio $\beta_1 = w_1 r_1/V = 0.5$
(b) Speed Ratio $\beta_1 = w_1 r_1/V = 1.0$



(a)



(b)

Fig.(10) Axial and Tangential Velocity Distributions Across an Annulus with Outer Cylinder Rotating Only ($w_1 = 0$) for Different Radii Ratios.
(a) Speed Ratio $\beta_2 = w_2 r_2/V = 0.5$
(b) Speed Ratio $\beta_2 = w_2 r_2/V = 1.0$